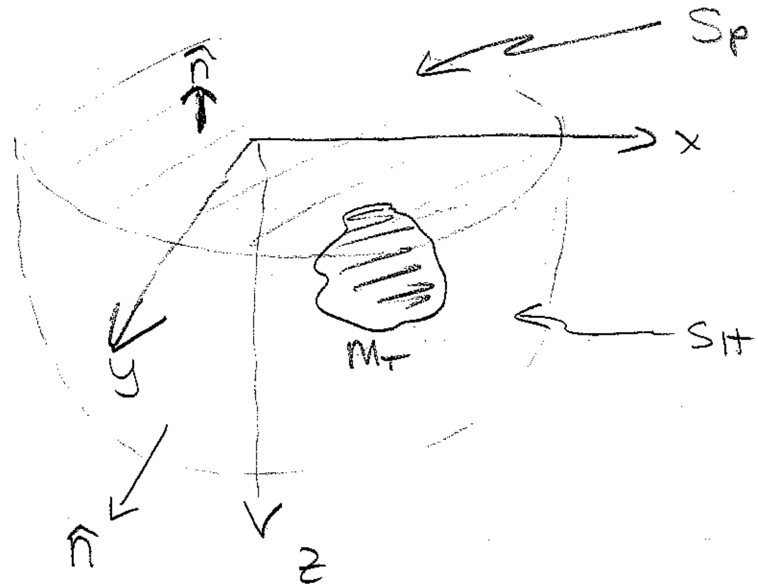
Application of Gauss's Law



--> want mass, M_T of some subsurface structure.
--> Measure normal comp gravity on plane S_P
--> Mass is enclosed by a surface $S = S_P + S_H$

From Gauss's Law

$$-4\pi G M_T = \int_S \hat{g}\cdot\hat{n}\,ds = \underbrace{-}_{\text{b/c of } \hat{n} \text{ dir}^n}\int_{S_P} g_z\,ds + \int_0^{2\pi}\int_{\pi/2}^{\pi} \underbrace{\boxed{\frac{\partial\phi}{\partial r}}}_{\text{normal comp of } \vec{g}} \underbrace{r^2\sin\theta\,d\theta\,d\phi}_{\text{elemental surf area}}$$

Potential of a volume density distribution viewed @ distance doesn't depend on details of the distbⁿ. (B)

$$\text{So} \quad \phi(P) = G\int_R \frac{\rho}{r}\,dv$$

↳ region enclosing M.

$$= \frac{G}{r}\int_R \rho\,dv = \frac{G M_T}{r}$$

ie M_T looks like a pt. mass @ large distances.

For large r $\quad \dfrac{\partial\phi}{\partial r} = -\dfrac{G M_T}{r^2}$

So $\boxed{B} = \int_0^{2\pi}\int_{\pi/2}^{\pi} \frac{\partial \Phi}{\partial r} r^2 \sin\theta \, d\theta \, d\phi$

$= -2\pi G M_T \int_{\pi/2}^{\pi} \sin\theta \, d\theta$

$= -2\pi G M_T \left[-\cos\theta\right]_{\pi/2}^{\pi} = -2\pi G M_T (1-0)$

$= -2\pi G M_T.$

$$\Rightarrow -\int_{S_p} g_z \, dS - 2\pi G M_T = -4\pi G M_T$$

$\Uparrow$

Gauss's Law

$$\Rightarrow \boxed{\int_{S_p} g_z \, dS = 2\pi G M_T}$$

Thus the vertical comp gravity integrated over an ∞ plane is proportional to total anomalous mass, below plane.

in practise $\Rightarrow$ large in spatial extent of M_T.