Isostacy: Compensation of Topography and Isostatic Geoid Anomalies

November 3, 2016

1 Isostacy

One of the major goals of interpreting planetary gravity fields is to understand how topography – positive (*e.g.*, mountain) or negative (e.g., impact basin) is supported. This is a somewhat different goal from in exploration geophysics where usually the goal is to understand why gravity anomalies occur where there is no local topographic expression. In the latter case gravity anomalies may be the key to high or low densities associated with e.g., ore deposits (high density) or e.g., oil reserves (nearby salt domes).

The Bouguer gravity formula removes the effect of topography as if the topography can just sit there forever. In other words it is as though the lithosphere has infinite strength, and so does not deform in response to a load. We refer to this as *uncompensated* topography (figure on left below). Short wavelength features can be supported by the lithosphere and so the Bouguer correction effectively accounts for the gravitational attraction of the anomalous mass. Hence uncompensated topography has a free air anomaly, but zero Bouguer anomaly.

Long wavelength topography (*e.g.*, a mountain range) causes the lithosphere to "sag" into the mantle. Since the Moho (crust /mantle boundary is generally embedded in the lithosphere) such *compensated* features have low density roots associated with them (figure on right below).

At these long wavelengths, the lithosphere has zero strength. If the mantle behaves like a fluid, the pressure will be constant along level surfaces at depth. If the pressure were not constant the mantle material would flow until the lateral pressure variation was removed. In this case we know that, at some depth, the total mass in vertical columns of material must be equal. This

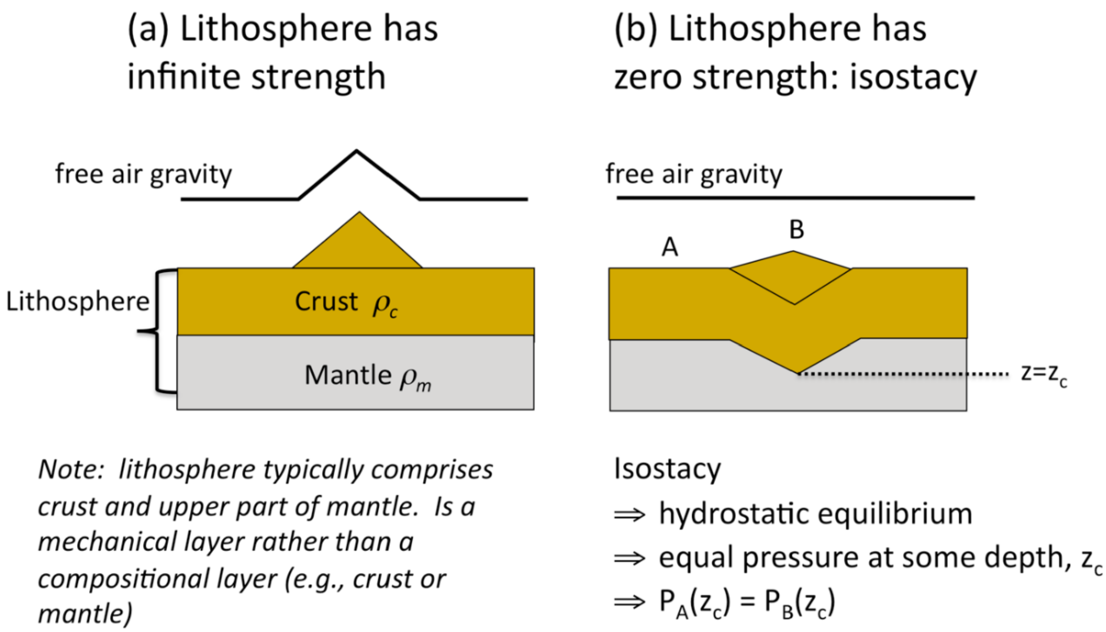

Figure 1:

depth is called the *depth of compensation* (z_c in right hand figure above). This is the concept of *isostacy*.

In our picture the total mass in a column of material under A is the same as the total mass in a column under B. This means that the free-air anomaly must be zero *i.e.*, the only variation in g is due to the fact that at position B we are further from the center of the Earth than at position A. For positive topography that is compensated, the Bouguer anomaly will be negative.

The condition for isostatic compensation can be written as

$$p_B - p_A = g \int_{-\infty}^{h} \rho_B(z) dz - g \int_{-\infty}^{h} \rho_A(z) dz = 0$$

$$\Longrightarrow \int_{-\infty}^{h} \Delta\rho(z) dz = 0 \tag{1}$$

where $\Delta\rho(z)$ is the difference in density between two columns as a function of depth and h is the depth of compensation. This equation tells us something about lateral variation in density as a function of depth. It turns out that the *geoid* anomalies associated with isostatic compensation can give us more information about $\Delta\rho(z)$.

2 Isostatic Geoid Anomalies

To compute the geoid anomaly over topography we proceed as we did in computing the Bouguer gravity anomaly. The increment in U caused by a ring of material is

$$\delta U = -\frac{2\pi G r \Delta\rho(z) dr dz}{[r^2 + (z+b)^2]^{1/2}} \qquad \left[\text{from} \quad \delta U = -\frac{G\rho(\vec{r}')dV'}{|\vec{r} - \vec{r}'|}\right]$$

The anomaly in the potential caused by a disc of material is found by integrating this expression over z ($-\infty \to h$, and this integral actually becomes $0 \to h$ in practice) and over r ($0 \to R$). After some intermediate steps and assuming again (as in the Bouger correction calculation) that the disk is broad compared with the observation height we get

$$\begin{aligned} \Delta U &= -2\pi G \left\{ R \int_{-\infty}^{h} \Delta\rho(z) dz - \int_{-\infty}^{h} \Delta\rho(z)(z+b) dz \right\} \\ &= -2\pi G \left\{ R \int_{-\infty}^{h} \Delta\rho(z) dz - \int_{-\infty}^{h} \Delta\rho(z) z dz - b \int_{-\infty}^{h} \Delta\rho(z) dz \right\} \end{aligned}$$

For perfect isostatic compensation, we have

$$\int_{-\infty}^{h} \Delta\rho(z) dz = 0$$

so

$$\Delta U = 2\pi G \int_{-\infty}^{h} z \Delta\rho(z) dz \tag{2}$$

The gravitational potential anomaly due to a shallow long wavelength isostatic density distribution is proportional to the dipole moment of the density distribution below the measurement point.

Geoid anomalies (ΔN) are measured in meters and are related to anomalies in potential by using the equation developed earlier:

$$\Delta N = -\frac{\Delta U}{g_R} = -\frac{2\pi G}{g_R} \int_{-\infty}^{z_c} z \Delta\rho(z) dz \tag{3}$$

where g_R is gravity on the reference geoid, and z_c is the compensation depth. We can use this equation to tell us about $\Delta\rho(z)$ – in fact it can be used to distinguish between different models of compensation.

Isostatic compensation can be achieved in several ways. We discuss three simplified models but in reality compensation is probably achieved by a complicated mixture of these models (and of other effects).

3 Airy Compensation

In Airy isostacy, compensation is achieved by varying the thickness of a constant density crust:

The thickness of continental crust with zero elevation (*i.e.*, the surface at sea level) is H. Topography of height h is associated with a low density root of thickness b. Using isostasy (principle of hydrostatic equilibrium) we have

$$\rho_c H + \rho_m b = \rho_c(h + H + b)$$

which we can rearrange to give

$$b = \frac{\rho_c h}{\rho_m - \rho_c} \tag{4}$$

The geoid anomaly associated with compensated positive topography can also be found. If we choose sea level (or whatever our reference surface is) as our origin and note that $\Delta\rho(z)$ in equation (3) above is $\rho(z)$ in column B minus $\rho(z)$ in column A we have

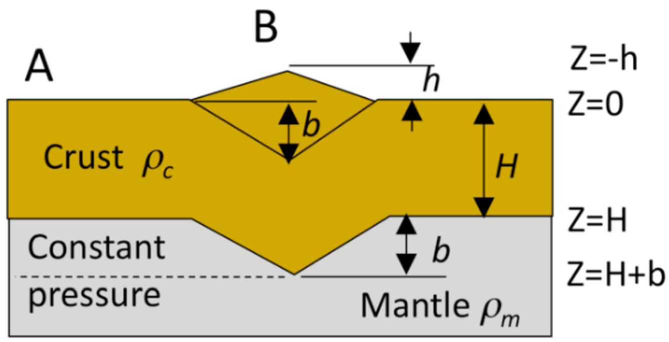

Figure 2:

$$
\begin{aligned}
\Delta N &= -\frac{2\pi G}{g_R}\left[\int_{-h}^{0}\rho_c z dz + \int_{0}^{H}(\rho_c-\rho_c)zdz + \int_{H}^{H+b}(\rho_c-\rho_m)zdz\right] \\
&= -\frac{2\pi G}{g_R}\left[-\frac{\rho_c h^2}{2} + \left(\frac{\rho_c-\rho_m}{2}\right)\left[(H+b)^2 - H^2\right]\right] \\
&= +\frac{\pi G}{g_R}\left[\rho_c h^2 + (\rho_m-\rho_c)(2Hb+b^2)\right]
\end{aligned}
$$

we can now eliminate b using equation 4 giving

$$\Delta N = \frac{\pi G}{g_R}\rho_c\left[2Hh + \frac{\rho_m h^2}{\rho_m-\rho_c}\right] \tag{5}$$

If $\frac{h}{H} << 1$, then the second term can be neglected and we can estimate the geoid-to-topography ratio (GTR)

$$GTR = \frac{\Delta N}{h} = \frac{2\pi G}{g_R}\rho_c H \tag{6}$$

We see there is a simple linear relationship between GTR and the depth of compensation, H.

This kind of model (crustal thickening) can explain geoid anomalies for several features of the terrestrial planets:

1. Earth: passive continental margins - see Turcotte & Schubert, figure 5-21
2. Earth: several major mountain belts, *e.g.*, the Himalayas
3. Mars: the elevation of the southern highlands
4. Venus: flat plateau-like regions, *e.g.*, Ovda Regio
5. the Moon: the lunar highlands

4 Pratt compensation

An alternative way to achieve isostatic compensation is to have horizontal variations in density over some prescribed depth range, W. (*i.e.*,, the densities of adjacent blocks are different, but the block bases are all at the depth of compensation). In this case we want to solve for the lateral variations in density.

Suppose the reference density corresponding to no topography is ρ_0 and suppose the p^{th} column has positive topography of height h and density ρ_p. Then

$$\rho_0 W = \rho_p (W + h)$$

which we can rearrange to give ρ_p

$$\rho_p = \frac{\rho_0 W}{W + h}$$

The geoid anomaly is

$$\Delta N = -\frac{2\pi G}{g_R} \int_{-h}^{W} z \Delta\rho(z) dz$$

Here $\Delta\rho$ is the density difference betweeen the p^{th} column and the reference density. Thus

$$\begin{aligned}\Delta N &= -\frac{2\pi G}{g_R}\left\{\int_{-h}^{0}\rho_p z dz + \int_{0}^{W}(\rho_p - \rho_0) z dz\right\} \\ &= -\frac{2\pi G}{g_R}\left[\frac{-\rho_p h^2}{2} + (\rho_p - \rho_0)\frac{W^2}{2}\right]\end{aligned}$$

Eliminating ρ_p gives

$$\Delta N = \frac{\pi G}{g_R}\rho_0 W h \tag{7}$$

In this case ΔN is proportional to h.

5 Thermal Isostacy

Long-wavelength topography may be supported by lateral density variations within the mantle rather than or as well as by lateral density variations in the crust. To support high topography buoyant forces in the mantle may arise from either compositional or temperature variations (so high topography is supported by either mantle that is regionally less dense by virtue of either being compositionally distinct or hotter).

Earlier we saw

$$\Delta N = -\frac{2\pi G}{g_R}\int_{-\infty}^{z_c} z\Delta\rho(z) dz$$

In the case of thermal isostacy $\Delta\rho(z) = -\rho_0\alpha\Delta T(z)$, where α is the coefficient of thermal expansion (assumed to be constant over the depth range of the mantle considered), ρ_0 is the ambient density, and ΔT is the temperature contrast with the surrounding mantle (so $\Delta T(z) = T(z) - T_m$). The negative sign arises since an increase in temperature results in a decease in density. We assume that the density anomaly persists from the base of the crust H, downwards to some depth $H+m$. Thus

$$\begin{aligned}\Delta N &= -\frac{2\pi G}{g_R}\left\{\int\limits_{-h}^{0} \rho_c z \; dz + \int\limits_{H}^{H+m} -\rho_m \alpha (T(z) - T_m) z \; dz\right\} \\ &= -\frac{2\pi G}{g_R}\left\{\frac{\rho_c h^2}{2} - \rho_m \alpha \int\limits_{H}^{H+m} (T(z) - T_m)\, z \; dz\right\}\end{aligned} \tag{8}$$

Thus if we know $T(z)$ we can compute the integral above.