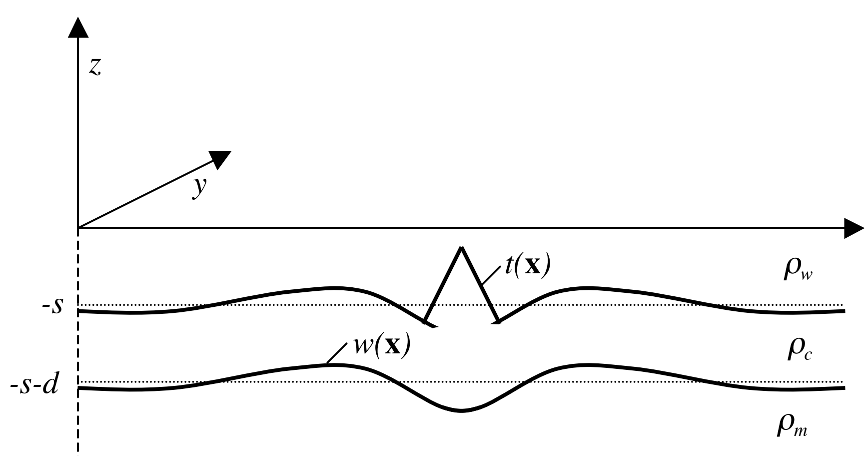
Gravity/Topography Transfer Function and Isostatic Geoid Anomalies

This lecture combines thin-elastic plate flexure theory with the solution to Poisson's equation to develop a linear relationship between gravity and topography. This relationship can be used in a variety of ways.

(1) If both the topography and gravity are measured over an area that is several times greater then the flexural wavelength, then the gravity/topography relationship (in the wavenumber domain) can be used to estimate the elastic thickness of the lithosphere and/or the crustal thickness. There are many good references on this topic including *Dorman and Lewis* [1972], *McKenzie and Bowin*, [1976]; *Banks et al.*, [1977]; *Watts*, [1978]; *McNutt*, [1979].
(2) At wavelengths greater than the flexural wavelength where features are isostatically-compensated, the geoid/topography ratio can be used to estimate the depth of compensation of crustal plateaus and the depth of compensation of hot-spot swells [*Haxby and Turcotte*, 1978].
(3) If the gravity field is known over a large area but there is rather sparse ship-track coverage, the topography/gravity transfer function can be used to interpolate the seafloor depth among the sparse ship soundings [*Smith and Sandwell,* 1994].

Flexure theory

In a previous lecture we developed an analytic solution for the response of a thin-elastic plate floating on a fluid mantle that is subjected to a line load. Here we follow the same approach but solve the flexure equation for an arbitrary vertical load representing, for example, the loading of the lithosphere due to the weight of a volcano as shown in the following diagram



where s is the mean ocean depth (~4 km) and d is the thickness of the crust (~6 km). The topography of the Moho is equal to deflection of the elastic plate $w(\mathbf{x})$. The topography of the seafloor, $t(\mathbf{x})$, has two components; the topographic load, $t_o(\mathbf{x})$, and the deflection of the elastic plate $w(\mathbf{x})$.

$$t(\mathbf{x}) = t_o(\mathbf{x}) + w(\mathbf{x}) \tag{1}$$

For this calculation, we make the following assumptions: the thickness of the elastic plate is less than the flexural wavelength; the deflection of the elastic plate is much less than the flexural wavelength; the flexural rigidity, D, is constant; and there is no end-load on the plate so $F = 0$. The vertical force balance for flexure of a thin elastic plate floating on the mantle is described by the following differential equation

$$D\left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\right) w(\mathbf{x}) + (\rho_m - \rho_w) g w(\mathbf{x}) = -(\rho_c - \rho_w) g t_o(\mathbf{x}) \tag{2}$$

where the parameters are defined in the following table.

Parameter	Definition	Value/Unit
$w(\mathbf{x})$	deflection of plate (positive up)	m
$D = \dfrac{Eh^3}{12(1-\nu)}$	flexural rigidity	N m
h	elastic plate thickness	m
ρ_w	seawater density	1025 kg m^{-3}
ρ_c	seawater density	2800 kg m^{-3}
ρ_m	mantle density	3330 kg m^{-3}
g	acceleration of gravity	9.82 m s^{-2}
E	Young's modulus	6.5 x 10^{10} Pa
ν	Poisson's ratio	0.25

Take the 2-D fourier transform of (2) to reduce the differential equation to an algebraic equation,

$$D(2\pi)^4\left(k_x^4 + 2k_x^2 k_y^2 + k_y^4\right) W(\mathbf{k}) + (\rho_m - \rho_w) g W(\mathbf{k}) = -(\rho_c - \rho_w) g [T(\mathbf{k}) - W(\mathbf{k})] \tag{3}$$

where we have used equation (1) to replace $T_o(\mathbf{k})$. With a little algebra and noting that $|\mathbf{k}|^4 = \left(k_x^2 + k_y^2\right)^2$ this can be re-written as

$$D(2\pi|\mathbf{k}|)^4 W(\mathbf{k}) + (\rho_m - \rho_c) g W(\mathbf{k}) = -(\rho_c - \rho_w) g T(\mathbf{k}) \tag{4}$$

Now one can solve for the deflection of the elastic plate in terms of the observed topography

$$W(\mathbf{k}) = \frac{-(\rho_c - \rho_w)}{(\rho_m - \rho_c)}\left[1 + \frac{D(2\pi|\mathbf{k}|)^4}{g(\rho_m - \rho_c)}\right]^{-1} T(\mathbf{k}) \tag{5}$$

This equation is called the isostatic response function because it describes the topography of the Moho in terms of the topography of the seafloor. Define the flexural wavelength

$$\lambda_f = 2\pi\left[\frac{D}{g(\rho_m - \rho_c)}\right]^{1/4} = \sqrt{2}\pi\alpha \tag{6}$$

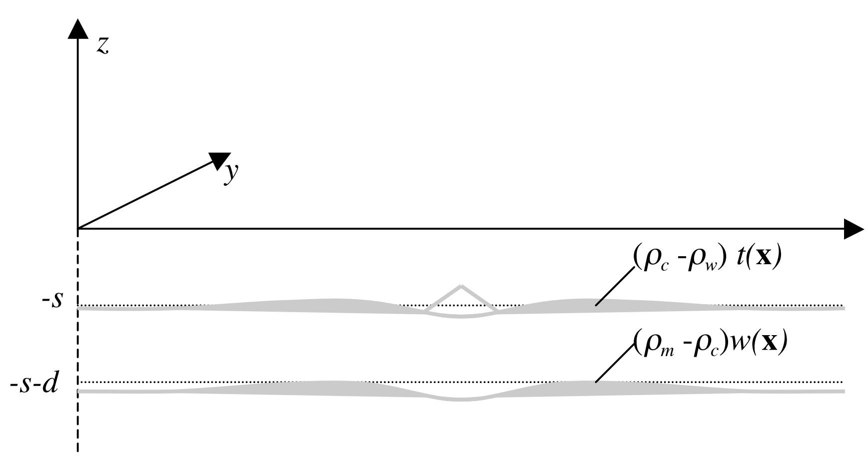
(Note α is the flexural parameter from a previous lecture.) When the wavelength of the topography is much greater than the flexural wavelength, then the topography of the Moho follows the Airy-compenstion model; this is *compensated* topography.

$$W(\mathbf{k}) = \frac{-(\rho_c - \rho_w)}{(\rho_m - \rho_c)} T(\mathbf{k}) \tag{7}$$

In contrast, when the wavelength of the topography is much less than the flexural wavelength, the topography of the Moho is zero; this is *uncompensated* topography. The gravity field of the earth is very sensitive to the degree of isostatic compensation so it is useful to develop the gravity field for this model.

Gravity/topography transfer function

The gravity anomaly for this model is approximated by compressing the topography into a sheet mass where the surface density is $(\rho_c - \rho_w)t(\mathbf{x})$. Similarly the Moho topography is compressed into a sheet mass with surface density $(\rho_m - \rho_c)w(\mathbf{x})$. Finally the gravity anomaly in each layer is upward-continued to the ocean surface.



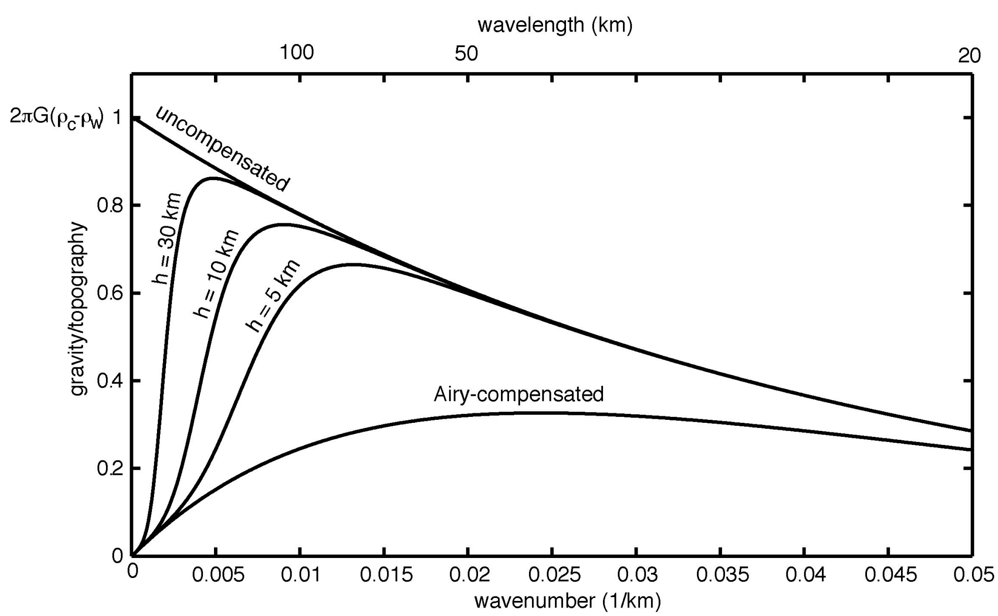
The solution to Poisson's equation (equation 13 in the previous lecture) provides an approximate method of constructing a gravity model for the combined model.

$$\Delta g(\mathbf{k}) = 2\pi G(\rho_c - \rho_w)e^{-2\pi|\mathbf{k}|s}T(\mathbf{k}) + 2\pi G(\rho_m - \rho_c)e^{-2\pi|\mathbf{k}|(s+d)}W(\mathbf{k}) \tag{8}$$

Using equation 5, this can be re-written in terms of the observed topography

$$\Delta g(\mathbf{k}) = 2\pi G(\rho_c - \rho_w)e^{-2\pi|\mathbf{k}|s}\left\{1 - \left[1 + \frac{D(2\pi|\mathbf{k}|)^4}{g(\rho_m - \rho_c)}\right]^{-1} e^{-2\pi|\mathbf{k}|d}\right\}T(\mathbf{k}). \tag{9}$$

This formulation provides a direct approach to constructing gravity anomaly models from seafloor topography: i) take the 2-D fourier transform of the topography; ii) multiply by the gravity-to-topography transfer function; and iii) take the inverse fourier transform of the result. The most important parameter is the elastic plate thickness that is used to estimate the flexural rigidity. The figure below shows the gravity/topography transfer function for a range of elastic thicknesses.



Since the asthenosphere relieves stresses on geological timescales, there is no truly-uncompensated topography. Thus the gravity anomaly for very large-scale structures such as continents and hot-spot swells is small or zero far from the edges of these features. It is only the sharp topographic features such as large seamounts that will have prominent gravity expressions.