HEAT

CONTENTS

3 Energy cannot be created or destroyed, according to classical physics. However, it can change form. Heat is one form.

Energy enters the atmosphere predominantly as short-wave radiation from the sun. Once here, some of the energy becomes sensible heat, and some evaporates water. The energy can change form many times while driving the weather. It finally leaves as long-wave terrestrial radiation.

An energy-conservation relationship known as the **First Law of Thermodynamics** forms the basis of a heat budget. We can apply this budget within two frameworks: Lagrangian and Eulerian.

Lagrangian means that we follow an air parcel as it moves in the atmosphere. This is useful for determining the temperature of a rising air parcel, and anticipating whether clouds form.

Eulerian means that we examine a volume fixed in space, such as over a farm or town. This is useful for forecasting temperature at any location, such as at your house. Eulerian methods must consider the transport (flux) of energy to and from the fixed volume. Transport can be via radiation, advection, turbulence, and conduction.

SENSIBLE AND LATENT HEATS

Sensible

Sensible heat ΔQ_H (in units of J) can be sensed by humans; namely, it is that portion of total heat content of air associated with temperature change ΔT at constant pressure. The sensible heat per unit mass of air m_{air} is given by

$$\frac{\Delta Q_H}{m_{air}} = C_p \cdot \Delta T \qquad \bullet(3.1)$$

Sensible heat is also known as **enthalpy** (see focus box in the subsection on the First Law of Thermo.).

The **specific heat at constant pressure**, C_p, depends on the material being heated. For dry air, C_{pd} = 1004 J·kg^{-1}·K^{-1}. For moist air (namely, for a gas consisting of a mixture of dry air molecules plus water vapor molecules), the specific heat is

Solved Example

How much sensible heat is needed to warm 2 kg of dry air by 5°C?

Solution

Given: m_{air} = 2 kg , ΔT = 5°C
C_{pd} = 1004.67 J·kg^{-1}·K^{-1}.
Find: ΔQ_H = ? J

Rearrange eq. (3.1)

$\Delta Q_H = m_{air} \cdot C_{pd} \cdot \Delta T$
$= (2 \text{ kg}) \cdot (1004.67 \text{ J·kg}^{-1} \cdot \text{K}^{-1}) \cdot (5°\text{C}) =$ **10.046 kJ**

Check: Units OK. Physics reasonable.
Discussion: This amount of air takes up a space of about 2.45 m^3 — the size of a small closet.

Science Graffito

"He who can no longer pause to wonder and stand rapt in awe is as good as dead; his eyes are closed."
- Albert Einstein.

Science Graffito

"Virtual particles are composed entirely of math and exist solely to fill otherwise embarrassing gaps in physics."
- Bruno Maddox.

FOCUS • Internal Energy

In thermodynamics, **internal energy** consists of the sum of microscopic kinetic and potential energy associated with matter. Microscopic kinetic energy is associated with random motions of masses, while microscopic potential energy is associated with forces that bind masses together.

Microscopic kinetic energies include random movement (translation) of molecules, molecular vibration and rotation, electron motion and spin, and nuclear spin. The sum of these kinetic energies is called **sensible energy**, which we humans can sense and measure as **temperature**.

For binding energies, it takes energy to pull two masses apart and break their bonds. This is analogous to increasing the microscopic potential energy of the system. When the two masses snap back together, their microscopic potential energy is released back into other energy forms. Three forms of binding energy are:

- **latent** — bonds between molecules
- **chemical** — bonds between atoms
- **nuclear** — sub-atomic bonds

We will ignore chemical reactions and nuclear explosions here.

Latent energy is associated with the phase change (solid, liquid, gas). In solids, the molecules are bound closely together in a somewhat rigid lattice. In liquids, molecules can more easily move relative to each other, but are still held close together. In gases, the molecules are further apart and have much weaker bonds.

For example, when you add energy to ice (a process shown by the big arrow in Fig. 3F.1), initially the ice warms (sensible energy increases). Then, energy is consumed to break the bonds of the solid to melt the ice (latent energy increases), after which the liquid can warm (sensible). Next, energy is used to vaporize the liquid (latent), after which the gas can warm (sensible).

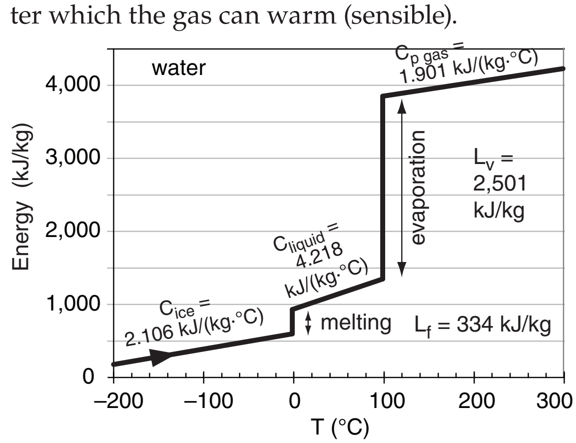


Fig. 3F.1. *Sensible and latent energy for water.*

FOCUS • Cp vs. Cv

C_v — Specific Heat at Constant Volume

Consider a sealed box of fixed volume V filled with air, as sketched in Fig. 3F.2a below. The number of air molecules (idealized by the little spheres) can't change, so the air density ρ is constant. Suppose that initially, the air temperature T_o is cool, as represented by the short arrows denoting the movement of each molecule in box 3F.2a. When each molecule bounces off a wall of the container, it imparts a small force. The sum of forces from all molecules that bounce off a wall of area A results in an air pressure P_o.

If you add Δq thermal energy to air in the box, the temperature rises to T_2 (represented by longer arrows in Fig. 3F.2b). Also, when each molecule bounces off a wall, it imparts a greater force because it is moving faster. Thus, the pressure P_2 will be larger. This is expected from the ideal gas law under constant density, for which

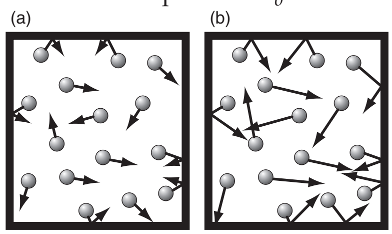


Fig. 3F.2. *Molecules in a fixed volume.*

$$P_o/T_o = P_2/T_2 = \text{constant} = \rho \cdot \Re \qquad \text{(3F.1)}$$

Different materials warm by different amounts when you add heat. If you know how much thermal energy Δq you added per kilogram of material, and you measure the resulting increase in temperature $T_2 - T_o$, then you can empirically determine the **specific heat at constant volume:**

$$C_v = \Delta q/(T_2 - T_o) \qquad \text{(3F.2)}$$

which is about $C_v = 717 \text{ J·kg}^{-1}\text{·K}^{-1}$ for dry air.

C_p — Specific Heat at Constant Pressure

For a different scenario, consider a box (Fig. 3F.3c) with a frictionless moveable piston at the top. The weight of the stationary piston is balanced by the pressure of the gas trapped below it. If you add Δq thermal energy to the air, the molecules will move faster (Fig. 3F.3d), and exert greater pressure against the piston and against the other walls of the chamber. But the weight of the piston hasn't changed, so the increased pressure of the gas causes the piston to rise.

But when any molecule bounces off the piston and helps move it, the molecule loses some of its microscopic kinetic energy. (An analogy is when a billiard ball bounces off an empty cardboard box sitting in the middle of the billiard table. The box moves a bit when hit by the ball, and the ball returns more slowly.) The result is that the gas temperature T_1 in Fig. 3F.3e is not as warm as in Figs. 3F.2b or 3F.3d, but is warmer than the initial temperature; namely, $T_o < T_1 < T_2$.

(continues in next column)

(continuation of Focus on Cp vs. Cv)

The molecules spread out within the larger volume in Fig. 3F.3e. Thus, air density ρ decreases, causing fewer molecules near the piston to push against it. The combined effects of decreasing density and temperature cause the air pressure to decrease as the piston rises. Eventually the piston stops rising at the point where the air pressure balances the piston weight, as shown in Fig. 3F.3e.

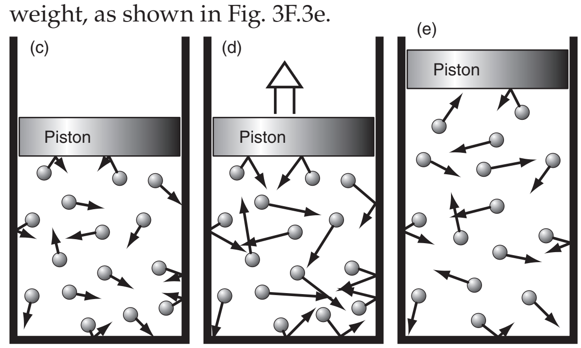


Fig. 3F.3. *Molecules in a constant-pressure chamber.*

Hence, this is an isobaric process (determined by the weight of the piston in this contrived example). The ideal gas law for constant pressure says:

$$\rho_o \cdot T_o = \rho_1 \cdot T_1 = \text{constant} = P / \Re \qquad \text{(3F.3)}$$

If you know how much thermal energy Δq you added per kilogram of material, and you measure the resulting increase in temperature $T_1 - T_o$, then you can empirically determine the **specific heat at constant pressure:**

$$C_p = \Delta q/(T_1 - T_o) \qquad \text{(3F.4)}$$

which is about $C_p = 1004 \text{ J·kg}^{-1}\text{·K}^{-1}$ for dry air.

C_p vs. C_v

Thus, in a constant pressure situation, the addition of Δq thermal energy results in less warming [$(T_1 - T_o) < (T_2 - T_o)$] than at constant volume. The reason is that, for constant pressure, some of the random microscopic kinetic energy of the molecules is converted into the macroscale **work** of expanding the air and moving the piston up against the pull of gravity. Such conservation of energy is partly described by the First Law of Thermodynamics.

In the atmosphere, the pressure at any altitude is determined by the weight of all the air molecules above that altitude (namely, the "piston" is all the air molecules above). If you add a small amount of thermal energy to air molecules at that one altitude, then you haven't significantly affected the number of molecules above, hence the pressure is constant. Thus, for most atmospheric situations it is appropriate to use C_p, not C_v, when forecasting temperature changes associated with the transfer of thermal energy into or from an air parcel.

Solved Example

Find the specific heat at constant pressure for humid air holding 10 g of water vapor per kg of dry air.

Solution

Given: $r = 10 g_{vapor}/1000 g_{dry\ air} = 0.01$ g/g
Find: $C_p = ?$ $J \cdot kg^{-1} \cdot K^{-1}$

Use eq. (3.2):

$C_p = (1004\ J \cdot kg^{-1} \cdot K^{-1}) \cdot [\ 1 + (1.84 \cdot (0.01\ g/g))\ ]$
$= \underline{\mathbf{1022.5\ J \cdot kg}}^{-1} \cdot K^{-1}$

Check: Units OK. Magnitude reasonable.
Discussion: Even a modest amount of water vapor can cause a significant increase in specific heat. See Chapter 4 for typical ranges of r in the atmosphere.

FOCUS • Specific Heat Cp for Air

The specific heat at constant pressure C_p for air is the average of the specific heats for its constituents, weighted by their relative abundance:

$$m_T \cdot C_p = m_d \cdot C_{pd} + m_v \cdot C_{pv} \qquad \text{(3F.5)}$$

where $m_T = m_d + m_v$ is the total mass of air (as a sum of mass of dry air m_d and water vapor m_v), and C_{pd} and C_{pv} are the specific heats for dry air and water vapor, respectively.

Define a mixing ratio r of water vapor as $r = m_v / m_d$. Then eq. (3F.5) becomes:

$$C_p = (1 - r) \cdot C_{pd} \cdot [1 + r \cdot C_{pv} / C_{pd}] \qquad \text{(3F.6)}$$

Given: $C_{pd} = 1004\ J \cdot kg^{-1} \cdot K^{-1}$ at 0°C for dry air, and $C_{pv} = 1850\ J \cdot kg^{-1} \cdot K^{-1}$ for water vapor, eq. (3F.6) becomes

$$C_p \approx C_{pd} \cdot [1 + 1.84 \cdot r] \qquad \text{(3.2)}$$

Even for dry air, the specific heat varies slightly with temperature, as shown in figure 3F.4, below:

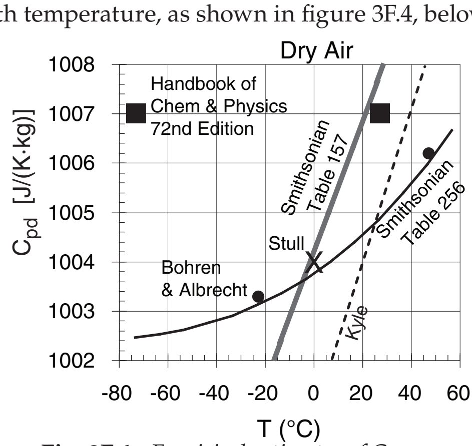


Fig. 3F.4. *Empirical estimates of C_{pd}.*

In this book, we will use $C_{pd} = 1004\ J \cdot kg^{-1} \cdot K^{-1}$, and will approximate it as being constant.

$$C_p \approx C_{pd} \cdot [1 + 1.84 \cdot r] \qquad \text{(3.2)}$$

where r is the mixing ratio of water vapor, in units of $g_{vapor}/g_{dry\ air}$. The specific heat of liquid water is much greater: $C_{liq} \approx 4200\ J \cdot kg^{-1} \cdot K^{-1}$.

Latent

Latent heat is a hidden heat related to water **phase changes**. Evaporation of liquid-water droplets cools the air by removing sensible heat and storing it as latent heat. The following list summarizes those water phase changes that cool the air:

vaporization: liquid to vapor
melting: solid (ice) to liquid
sublimation: solid to vapor

Phase changes of water in the opposite direction cause sensible warming of the air by releasing latent heat:

condensation: vapor to liquid
fusion: liquid to solid (ice)
deposition: vapor to ice

When air parcels or fixed volumes contain moist air, there is the possibility that water phase changes can alter the temperature, even without heat transport across the volume boundaries. The amount of heat ΔQ_E per mass of phase-changed water Δm_{water} is defined to be

$$\frac{\Delta Q_E}{\Delta m_{water}} = L \qquad \bullet\text{(3.3)}$$

where L is known as the **latent-heat** factor.

Values of the latent-heat factor for water are:

$L_v = \pm\ 2.5 \times 10^6\ J \cdot kg^{-1} \quad = L_{\ condensation\ \ or\ vaporization}$

$L_f = \pm\ 3.34 \times 10^5\ J \cdot kg^{-1} \quad = L_{\ fusion\ or\ \ melting}$

$L_d = \pm\ 2.83 \times 10^6\ J \cdot kg^{-1} \quad = L_{\ deposition\ or\ sublimation}$

where the sign depends on the direction of phase change, as described above. See Appendix B for tables of geophysical constants.

LAGRANGIAN HEAT BUDGET – PART 1: UNSATURATED

Air Parcels

Consider a hypothetical blob of air that we follow and examine as it moves through the atmosphere. We can think of it as a very large air-filled balloon about the diameter of two city blocks, but without any latex skin. This lack of a latex skin leaves the air blob unprotected.

When these air blobs move through the atmosphere, myriad eddies (swirls of turbulent motion) tend to mix some of the outside air with the air just inside the blob (such as the mixing you see in smoke rising from a campfire). Thus, warmer or colder air could be added to (**entrained** in through the sides of) the blob, and some air from inside could be lost (**detrained**) to the surrounding atmosphere. Also, in the real atmosphere, atmospheric radiation can heat or cool the air blob. These processes combine to make thermodynamic study of real air blobs extremely difficult.

But to gain some insight into the thermodynamics of air, we can imagine a simplified situation where radiative effects are relatively small, and where the turbulent entrainment/detrainment happens only in the outer portions of the air blob, leaving an inner core somewhat protected. This is indeed observed in the real atmosphere. So consider the protected inner core (about the diameter of a city block) as an **air parcel**.

As the parcel moves we can compute its thermodynamic and dynamic state, based on work done on it, changes within it, fluxes across its surface, and mixing with its surrounding environment. This concept is often employed in **Lagrangian** studies.

In Part 1 of this Lagrangian description, we will study only those situations where there is NO condensation or evaporation of water; namely, **unsaturated** conditions. This does not mean that the air is dry — only that any water vapor within the air is not undergoing phase changes.

Historically, these processes with no phase changes are called "**dry**" processes, even though there may be water vapor in the air. In the Moisture chapter, after moisture variables are introduced, the analysis will continue in a section called " Lagrangian Heat Budget – Part 2: Saturated", where "**moist**" (saturated) processes are included.

Solved Example

How much latent heat is released when 2 kg of water vapor condenses into liquid?

Solution

Given: m_{vapor} = 2 kg , L_v = 2.5×10^6 J·kg^{-1} .
Find: ΔQ_E = ? J

Rearrange eq. (3.3)
ΔQ_E = $(2.5\times10^6$ J·kg$^{-1})$ · (2 kg) = **5,000 kJ**

Check: Units OK. Physics reasonable.
Discussion: This amount of liquid water would fill a typical bathtub to only a depth of 2 mm. Thus, a small amount of water can hold a large amount of latent heat. Water vapor is a very important source of energy to drive storms.

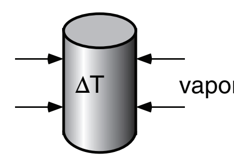
Solved Example

How much dew must condense on the sides of a can of soda for it to warm the soda from 1°C to 16°C?

Hints: Neglect the heat capacity of the metal can. The density of liquid water is 1000 kg·m^{-3}. Assume the density of soda equals that of pure water. Assume the volume of a can is 354 ml (milliliters), where 1 l = 10^{-3} m^3.

Solution

Given: ρ_{water} = 1000 kg·m^{-3}.
C_{liq} = 4200 J·kg^{-1}·K^{-1}
Volume (*Vol*) in Can = 354 ml
L_{cond} = + 2.5×10^6 J·kg^{-1}
ΔT = 15 K
Find: Volume of Condensate
Sketch:



Equate the latent heat release by condensing water vapor (eq. 3.3) with the sensible heat gained by fluid in the can (eq. 3.1)

$$\Delta Q_E = \Delta Q_H$$

$$\rho_{condensate} \cdot (\Delta Vol\ of\ Condensate) \cdot L_{cond} = \rho_{soda} \cdot (Vol\ of\ Can) \cdot C_{liq} \cdot \Delta T$$

Assume the density of condensate and soda are equal, so they cancel. The equation can then be solved for *ΔVolume of Condensate*.

ΔVolume of Condensate = (*Vol of Can*)·C_{liq}·$\Delta T/L_{cond}$
= (354 ml)·(4200 J·kg^{-1}·K^{-1})·(15 K)/(2.5×10^6 J·kg^{-1})
= **8.92 ml**

Check: Units OK. Sketch OK. Physics OK.
Discussion: Latent heats are so large that an amount of water equivalent to only 2.5% of the can volume needs to condense on the outside to warm the can by 15°C. Thus, to keep your can cool, insulate the outside to prevent dew from condensing.

FOCUS • Enthalpy vs. Heat

Let Δq (J/kg) be the amount of thermal energy you add to a stationary mass m of air. Some of this energy warms the air. But as air warms, its volume expands by amount ΔV and pushes against the surrounding atmosphere (which to good approximation is pushing back with constant pressure P). The **work** W done on the atmosphere is $W = P \cdot \Delta V$. (To see this, recall that pressure is force F per unit area A. Thus work $W = P \cdot \Delta V = F \cdot \Delta V / A = F \cdot \Delta d$ = force times distance, where volume ΔV divided by area A is distance Δd.)

The **First Law of Thermodynamics** says that energy is conserved, thus the thermal energy input must balance the sum of warming and work done per unit mass:

$$\Delta q = C_v \cdot \Delta T + P \cdot (\Delta V / m) \tag{3.4a}$$

where C_v ($J \cdot kg^{-1} \cdot K^{-1}$) is the **specific heat of air** at constant volume.

But $P \cdot (\Delta V / m) = \Delta(P \cdot V / m) - V \cdot \Delta P / m$ (from the chain rule of calculus). Also, $P \cdot V / m = P / \rho = \Re \cdot T$ from the ideal gas law, where $\rho = m / V$ is air density and $\Re$ is the gas *constant*. Using this in eq. (3.4a) gives:

$$\Delta q = C_v \cdot \Delta T + \Re \cdot \Delta T - \Delta P / \rho \tag{3.4b}$$

By definition for an ideal gas: $C_v + \Re = C_p$. Thus, you can combine the first two terms on the right to give a form of the **First Law of Thermodynamics** that you can use for the atmosphere:

$$\underbrace{\Delta q}_{heat} = \underbrace{C_p \cdot \Delta T}_{enthalpy} - \Delta P / \rho \tag{3.4}$$

In engineering thermodynamics, the word **heat** always refers to the amount the thermal energy transferred to or from a system; namely, heat per unit mass is Δq. However, the amount of thermal energy possessed by a mass of air at constant pressure is given the name **enthalpy**, h. Thus, $h = C_p \cdot T$, and $\Delta h = C_p \cdot \Delta T$ is the change in enthalpy associated with a temperature change. Engineers carefully discriminate between the words **heat** (heat transferred to/from an object) and **enthalpy** (heat possessed by an object).

This same careful definition should also be used in meteorology. However, by tradition, meteorologists often use the word **sensible heat** in place of the word enthalpy, to describe the heat possessed by an object. Hence, we must be very careful when we talk about "heat". With this in mind, the First Law of Thermo can be annotated as follows:

$$\underbrace{\Delta q}_{\text{heat transferred}} = \underbrace{C_p \cdot \Delta T}_{\text{sensible heat}} - \Delta P / \rho \tag{3.4}$$

This form is useful in meteorology, when rising air parcels experience a decrease in surrounding atmospheric pressure, hence, the last term is non-zero.

First Law of Thermodynamics

The temperature of an air parcel of mass m_{air} changes by amount ΔT when heat (Δq) is transferred to or from the parcel. The temperature also changes when work is done on or by the parcel. This relationship, called the **First Law of Thermodynamics**, can be written (see the Focus box at left) as:

$$C_p \cdot \Delta T = \Delta q + \frac{\Delta P}{\rho} \tag{•3.4}$$

Heat transferred Δq has units of J/kg. ΔP is the pressure change of air in the parcel, and ρ is air density. The pressure of the parcel usually equals that of its surrounding environment, which decreases exponentially with height. The last term will be non-zero for a rising or sinking air parcel as its pressure changes to match the pressure of its environment.

The first law of thermodynamics can be reformulated using the hydrostatic equation to yield the temperature change of a rising or sinking air parcel

$$\Delta T = -\left(\frac{|g|}{C_p}\right) \cdot \Delta z + \frac{\Delta q}{C_p} \tag{•3.5}$$

Solved Example

What is the temperature change of a 10 kg air parcel when heated at rate H = 100 W for 10 minutes? Water-vapor mixing ratio is $r = 0.01\ g_{vapor}/g_{air}$, and the air parcel is stationary.

Solution

Given: H = 100 W, m_{air} = 10 kg, $\Delta z = 0$,
$r = 0.01\ g_{vapor}/g_{air}$, Δt = 10 min

Find: ΔT = ? K

First, calculate the specific heat from eq. (3.2):
$C_p = (1004.67\ J \cdot kg^{-1} \cdot K^{-1}) \cdot [1 + 0.84 \cdot 0.01]$
$= 1013.11\ J \cdot kg^{-1} \cdot K^{-1}$.

Next, calculate the total heat added:
$\Delta Q_H = H \cdot \Delta t = (100\ W) \cdot (10\ min)$
$= (100\ J/s) \cdot (600\ s) = 6x10^4\ J$.

Finally, use eq. (3.5):
$\Delta T = (6x10^4\ J) / [(10\ kg) \cdot (1013.11\ J \cdot kg^{-1} \cdot K^{-1})]$
= **5.92 K**.

Check: Units OK. Physics reasonable.

Discussion: Air density is about 1.225 kg/m^3 at sea level. Thus, 10 kg occupies 8.16 m^3, which is about the size of a large closet. According to the first law of thermodynamics, the air in this closet would be warmed by almost 6 °C if we turned on a 100 W electric light bulb and left it on for 10 minutes, assuming no heat is lost to the walls of the closet.

The heat-transfered-per-mass term (Δq) on the right can be caused by radiative heating, latent heating during condensation, dissipation of turbulence energy into heat, heat from chemical reactions, and convective or turbulent interactions between air inside and outside of the parcel. Advection and convection do not transport heat to/from the parcel, but they do move the parcel about (Fig. 3.1).

The first law of thermodynamics is also called the **conservation of heat**. It says that the temperature of an air parcel will not change, unless heat is added or removed, or unless the pressure of the parcel changes. Thus, even without adding heat to a parcel, its temperature can change during vertical parcel motion. Vertical motion is very important in atmospheric circulations and cloud development.

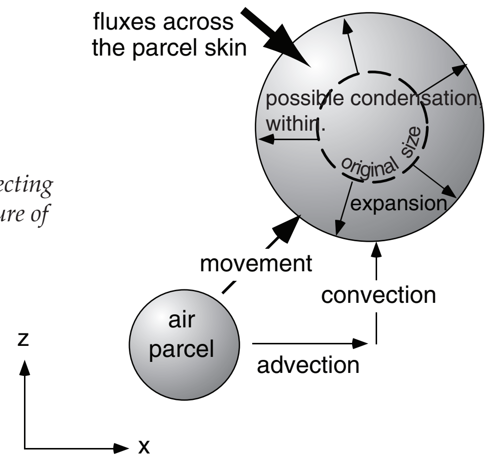


Figure 3.1
Processes affecting the temperature of an air parcel.

Lapse Rate

The decrease of temperature with height is known as the **lapse rate**, Γ. Namely,

$$\Gamma = -\frac{T_2 - T_1}{z_2 - z_1} = -\frac{\Delta T}{\Delta z} \tag{3.6}$$

Two types of lapse rate are **environmental** lapse rates and **process** lapse rates (Fig. 3.2).

When the existing temperature-state of the environment is measured with aircraft, weather balloons, or remote sensors, the resulting graph of temperature vs. height is called an **environmental temperature profile**, or an **environmental sounding**. The temperature decrease within this sounding gives the **environmental lapse rate** as a function of height. It is a static measure of the state of the environment.

Fig. 1.10 is an example of an environmental sounding based on standard-atmosphere conditions. The actual environmental sounding on any day might differ. A wide range of soundings is possible.

A **process lapse rate** gives the temperature decrease associated with some action or process. Examples are the temperature changes with height inside rising air parcels, inside rising cloud parcels, and due to external processes such as radiative heating or cooling. The change of temperature with height for a process is usually defined by a physical relationship, such as the first law of thermodynamics. One common process is the adiabatic process, described next.

Solved Example

Find the lapse rate in the troposphere for a standard atmosphere.

Solution

Given: Std. Atmos. Table 1-5 in Chapter 1, , where
T= –56.5°C at z = 11 km, and T = +15°C at z = 0 km.
Find: Γ = ? °C/km
Use eq. (3.6): Γ = – (–56.5 – 15) /(11–0) = **+6.5 °C/km**

Check: Positive Γ, because T decreases with z.
Discussion: This is the environmental lapse rate of the troposphere.

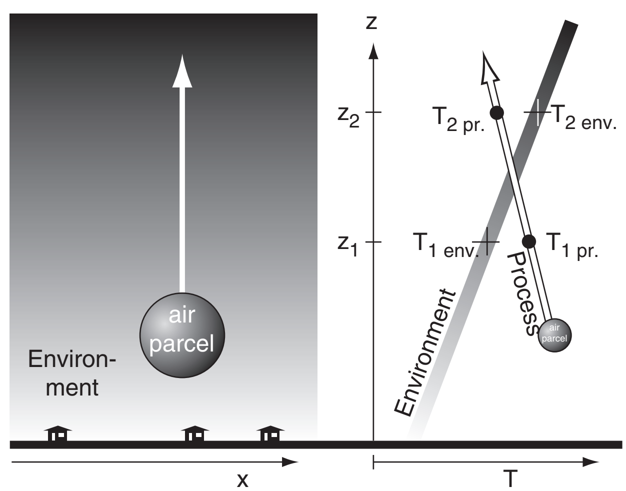


Figure 3.2
Left: Sketch of a physical situation, showing an air parcel moving through an environment. In the environment, darker colors indicate warmer air. Right: Temperature profiles for the environment and the air parcel. The environmental air is not moving. In it, air at height z_1 has temperature $T_{1\ env}$, and air at z_2 has $T_{2\ env}$. The air parcel has an initially warm temperature, but its temperature changes as it rises: becoming $T_{1\ pr}$ at height z_1, and later becoming $T_{2\ pr}$ at height z_2. In the environment of this example, temperature increases as height increases, implying a negative environmental lapse rate. However, the air parcel's temperature decreases with height, implying a positive parcel lapse rate.

Solved Example

A 15°C air parcel at $z = 0$ rises to 2 km. Find final T.

Solution

Given: $T_{initial} = 15°C$, $\Delta z = 2$ km
Find: $T_{final} = ?$ °C

Assume: dry adiabatic process. Use eq. (3.7):
$\Delta T/\Delta z = (T_{final}-T_{init.}) / (z_{final}-z_{init.}) = -9.8$ °C/km
$T_{final} = T_{initial} - (9.8$ °C/km$) \cdot \Delta z$
$= 15$ °C $- (9.8$ °C/km$)\cdot(2$ km$) =$ **–4.6 °C**.

Check: Units OK. Physics reasonable.
Discussion: Sufficient cooling to freeze water.

BEYOND ALGEBRA • Adiabatic Lapse Rate

Calculus can be used to get eq. (3.10) from (3.4). First, write the First Law of Thermo. in its more-accurate differential form, for an adiabatic process ($\Delta q = 0$):

$$dP = \rho \cdot C_p \cdot dT_v$$

Then substitute for density using the ideal gas law $\rho = P / (\Re_d \cdot T_v)$, to give:

$$dP = \frac{P \cdot C_p \cdot dT_v}{\Re_d \cdot T_v}$$

Move pressures to the LHS, and temperatures to the RHS:

$$\frac{dP}{P} = \frac{C_p}{\Re_d} \cdot \frac{dT_v}{T_v}$$

Integrate between pressure P_1 where temperature is T_1, to pressure P_2 where temperature is T_2:

$$\int_{P_1}^{P_2} \frac{dP}{P} = \frac{C_p}{\Re_d} \cdot \int_{T_{v1}}^{T_{v2}} \frac{dT_v}{T_v}$$

where $C_p / \Re_d$ is relatively constant, and was moved out of the integral on the RHS. The result is:

$$\ln(P)\Big|_{P_1}^{P_2} = (C_p / \Re_d) \cdot \ln(T_v)\Big|_{T_{v1}}^{T_{v2}}$$

Plug in the limits, and use $\ln(a) - \ln(b) = \ln(a/b)$:

$$\ln\left(\frac{P_2}{P_1}\right) = (C_p / \Re_d) \cdot \ln\left(\frac{T_{v2}}{T_{v1}}\right)$$

Move $C_p / \Re_d$ to the LHS:

$$(\Re_d / C_p) \cdot \ln\left(\frac{P_2}{P_1}\right) = \ln\left(\frac{T_{v2}}{T_{v1}}\right)$$

Recall that $a \cdot \ln(b) = \ln(b^a)$, thus:

$$\ln\left[\left(\frac{P_2}{P_1}\right)^{\Re_d / C_p}\right] = \ln\left(\frac{T_{v2}}{T_{v1}}\right)$$

Finally, take the anti-log of both sides ($e^{LHS} = e^{RHS}$):

$$\left(\frac{P_2}{P_1}\right)^{\Re_d / C_p} = \frac{T_{v2}}{T_{v1}} \qquad (3.10)$$

Adiabatic Lapse Rate

An **adiabatic** process is one where there is no heat transfer to or from the air parcel ($\Delta q = 0$). This is reasonable if we remember that the air parcel is the protected inner core of a larger blob of air. An air parcel that rises "dry adiabatically" (with no cloud formation) experiences a temperature change given by eq. (3.5), which can be rearranged as:

$$\frac{\Delta T}{\Delta z} = -\left(\frac{|g|}{C_p}\right) = -9.8 \text{ K/km} \qquad \bullet(3.7)$$

Thus, the **"dry" adiabatic lapse rate** Γ_d for a rising air parcel is

$$\Gamma_d = 9.8 \text{ K/km} = 9.8 \text{ °C/km} \qquad (3.8)$$

and it is a process lapse rate. (Degrees K and °C are interchangeable in this equation, because it represents a temperature change with height.) As mentioned earlier, "dry" adiabatic processes also apply to humid air, but only if there is no condensation, clouds, or precipitation in it (i.e., only if the air is unsaturated).

The adiabatic lapse rate can also be found in terms of pressure instead of height. Substituting the ideal gas law into the first law of thermodynamics for an adiabatic process yields:

$$\frac{\Delta T}{T} = \frac{\Re_d}{C_p} \cdot \left(\frac{\Delta P}{P}\right) \qquad (3.9)$$

or

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\Re_d / C_p} \qquad \bullet(3.10)$$

where $\Re_d/C_p = 0.28571$ (dimensionless) for dry air, and where temperatures are in Kelvin.

A different adiabatic lapse rate ("moist" lapse rate) must be used if there is liquid or solid water in the air parcel. This is discussed later in the Moisture chapter.

Solved Example

An air parcel with initial (P, T) = (100 kPa, 20°C) rises adiabatically to $P = 80$ kPa. Find its new T.

Solution

Given: $P_1 = 100$ kPa, $P_2 = 80$ kPa, $T_1 = 20°C = 293K$
Find: $T_2 = ?$ °C

Use eq. (3.10): $T_2 = (293K) \cdot [(80kPa)/(100kPa)]^{0.28571}$
$T_2 = 293K \cdot 0.9382 = 274.9$ K $=$ **1.9°C**

Check: Units OK. Value good, since 80 kPa is about 2 km above 100 kPa, we expect about 20°C cooling.

When calculating temperature differences, units of °C and K are interchangeable, because a change of ΔT = 1°C equals a change of ΔT = 1 K. However, when extracting a temperature from such a difference, always convert it to K before using it in a radiative or ideal gas law.

Potential Temperature

When an air parcel rises/sinks "dry" adiabatically into regions of lower/higher pressure, its temperature changes due to work done by/on the air, even though no thermal energy has been removed/added. Define a new temperature variable called the **potential temperature** θ that is proportional to the sensible heat contained in the parcel, but which is unaffected by work done by/on the parcel.

Namely, the potential temperature is constant (i.e., is a **conserved variable**) for adiabatic processes (i.e., Δq = 0) such as air-parcel ascent, but can increase/decrease when sensible heat is added/removed. Such **diabatic** (non-adiabatic) heat transfer processes include turbulent mixing, condensation, and radiative heating (i.e., $\Delta q \neq 0$).

If height is used as the vertical coordinate, then you can calculate the value of potential temperature θ from:

$$\theta(z) = T(z) + \Gamma_d \cdot z \qquad \bullet(3.11)$$

The units (°C or K) of θ(z) are the same as the units of T(z).

For pressure coordinates, find the value of θ from:

$$\theta = T \cdot \left(\frac{P_o}{P}\right)^{\Re_d / C_p} \qquad \bullet(3.12)$$

where P_o is a reference pressure, $\Re_d/C_p$ = 0.28571 (dimensionless) and where absolute temperatures (K) must be used. The word "potential" arises because θ corresponds to the temperature T that a parcel would potentially have if it were moved adiabatically to the ground or to a reference pressure.

It is common practice to use a reference level of P_o = 100 kPa in eq. (3.12), but this is not a strict definition. For example, boundary-layer meteorologists often use eq. (3.11) where z is height above local ground level, not above the 100 kPa altitude.

To include the buoyant effects of water vapor and liquid water in air, we can define a **virtual potential temperature** for air with no liquid water or ice (3.13), and for air containing liquid water and/or ice (3.14) such as cloud or rain drops, or snow:

$$\theta_v = \theta \cdot [1 + (a \cdot r)] \qquad (3.13)$$

$$\theta_v = \theta \cdot [1 + (a \cdot r) - r_L - r_I] \qquad \bullet(3.14)$$

Solved Example

Find θ for air at z = 500 m with T = 10°C?

Solution

Given: z = 500 m, T = 10°C
Find: θ = ? °C

Assume no liquid water, and use eq. (3.11)
θ = 10°C + (9.8 °C/km)· (0.5 km) = **14.9 °C**

Check: Units OK. Physics reasonable.
Discussion: This is the T that air would have when lowered dry adiabatically to the surface. θs are always greater than the actual T, for z above the ref. level.

Solved Example

Find θ for air at P = 70 kPa with T = 10°C?

Solution

Given: P = 70 kPa , T = 10°C = 283 K, P_o = 100 kPa
Find: θ = ? °C

Use eq. (3.12): θ = (283 K) · [(100 kPa) / (70 kPa)]$^{0.28571}$
θ = 313.6 K = **40.4 °C**

Check: Physics OK. θ is always greater than the actual T, for P smaller than the reference pressure.

Solved Example

What is the virtual potential temperature of air at having potential temperature 15°C, mixing ratio 0.008 $g_{water\ vapor}/g_{air}$, and liquid water mixing ratio of: a) 0 ; b) 0.006 $g_{liq.water}/g_{air}$?

Solution

Given: θ =15°C = 288 K, r = 0.008 $g_{water\ vapor}/g_{air}$,
a) r_L = 0 ; b) r_L =0.006 $g_{liq.water}/g_{air}$.
Find: θ_v = ? °C
Abbreviate "water vapor" with "wv" here.

a) Use eq (3.13): θ_v = (288K)·
[1 + (0.61 g_{air}/g_{wv}) · (0.008 g_{wv} /g_{air})] .
Thus, θ_v = 289.4 K. Or subtract 273 to get Celsius: θ_v = **16.4 °C**.

a) Use eq (3.14): θ_v = (288K)·
[1 + (0.61 g_{air}/g_{wv}) · (0.008 g_{wv}/g_{air}) – 0.006 $g_{liq.}/g_{air}$] = 287.7 K.
Subtract 273 to get Celsius: θ_v = **14.7°C**.

Check: Units OK. Physics reasonable.
Discussion: When no liquid water is present, virtual pot. temperatures are always warmer than potential temperatures, because water vapor is lighter than air.

However, liquid water is heavier than air, and has the opposite effect. This is called **liquid-water loading**, and makes the air act as if it were colder.

Solved Example

Given cloudy air at P = 70 kPa with T = –1°C, r_s = 5 $g_{water\ vapor}/kg_{air}$, r_L = 2 $g_{liq\ water}/kg_{air}$. Find θ_v.

Solution

Given: P = 70 kPa , T = –1°C = 272K,
r_s = 5 $g_{water\ vapor}/kg_{air}$ = 0.005 g_{wv}/g_{air}
r_L = 2 $g_{liq\ water}/kg_{air}$. = 0.002 g_{liq}/g_{air}
Find: θ_v = ? °C

First, use eq. (3.12) to find the potential temperature
θ = (272K) · [(100 kPa)/(70 kPa)]$^{0.28571}$ = 301 K

Next, use eq. (3.15):
θ_v = (301 K) · [1 + (0.61 g_{air}/g_{wv})· (0.005 g_{wv}/g_{air}) – (0.002)] = 301K ·(1.001) = **301.3 K**
θ_v = 301.3 K – 273 K = **28.3°C**

Check: Units OK. Physics reasonable.
Discussion: For this example, there was little effect of the vapor and liquid water. However, for situations with greater water vapor or liquid water, the virtual potential temperature can differ by a few degrees, which can be important for estimating thunderstorm intensity.

where r is mixing ratio of water vapor, r_L is the mixing ratio of liquid water (cloud and rain drops), r_I = ice mixing ratio, a = 0.61 $g_{air}/g_{water\ vapor}$, and both θ values must be in units of Kelvin. Mixing ratio is described in the Moisture chapter; it is the ratio of grams of water per gram of air. θ_v is used to determine how high air parcels can rise, or how quickly they sink. θ_v is constant only when there is no phase changes and no heat transfer; namely, no latent or sensible heat is absorbed or released.

For air that is rising within clouds, with water vapor condensing, it is usually the case that the air is saturated (= 100% relative humidity; see the Moisture chapter for details). As a result, the water-vapor mixing ratio r can be replaced with r_s , the saturation mixing ratio.

$$\theta_v = \theta \cdot [1 + (a \cdot r_s) - r_L] \qquad \bullet(3.15)$$

where a = 0.61 $g_{air}/g_{water\ vapor}$, as before.

However, there are other situations where the air is NOT saturated, but contains liquid water. An example is the non-cloudy air under a cloud base, through which rain is falling at its terminal velocity. For this case, eq (3.14) should be used with an unsaturated value of water-vapor mixing ratio. This situation occurs often, and can be responsible for damaging downbursts of air (see the Thunderstorm chapters).

Why use potential temperature? Because it makes it easier to compare the temperatures of air parcels at two different heights — important for determining if air will buoyantly rise to create thunderstorms. For example, suppose air parcel A has temperature T_A = 20°C at z = 0, while air parcel B has T_B = 15°C at z = 1 km. Parcel A is warmer than parcel B.

Does that mean that parcel A is buoyant (warmer and wants to rise) relative to parcel B? The answer is <u>no</u>, because when parcel B is moved dry adiabatically to the altitude of parcel A, then parcel B is 5°C warmer than parcel A due to adiabatic warming. In fact, you can move parcels A and B to any common altitude, and after considering their adiabatic warming or cooling, parcel B will always be 5°C warmer than parcel A.

The easiest way to summarize this effect is with potential temperature. Using eq. (3.11), we find that θ_A = 20°C and θ_B = 25°C approximately. θ_A and θ_B keep their values (because θ is a conserved variable) no matter to what common altitude you move them, thus θ_B is always 5 °C warmer than θ_A in this illustration.

Another application for potential temperature is to label lines on a thermodynamic diagram, such as described next.

Thermodynamic Diagrams –Part 1: Dry Adiabatic Processes

It is often necessary to compare environmental lapse rates with process or parcel lapse rates, in order to determine parcel buoyancy, cloudiness and storm growth. Sometimes it is easier to utilize a diagram that already indicates the proper thermodynamic relationships, rather than recalculating the thermodynamic equations for every situation. Such a diagram is called a **thermodynamic diagram**, or **thermo diagram** for short.

These diagrams usually include relationships between so many variables that they become quite cluttered with lines and numbers. We will develop our own thermodynamic diagrams little by little as the book progresses. In this first part, we will include only "dry" adiabatic processes.

Most thermodynamic diagrams have temperature along the horizontal axis, and pressure or height along the vertical axis. We will use pressure with a reversed logarithmic scale along the vertical, as an approximation to height. Fig. 3.3 is an example of such a diagram.

To this background chart, we will add diagonal lines showing the temperature decrease with height associated with air parcels rising adiabatically from the surface. These lines are called **"dry" adiabats**. They are also sometimes called **isentropes**.

We can draw different lines for parcels starting at the bottom of the graph (bottom of the atmosphere) with different temperatures. Fig. 3.3 shows the result, as calculated using a computer spreadsheet (see the Focus box).

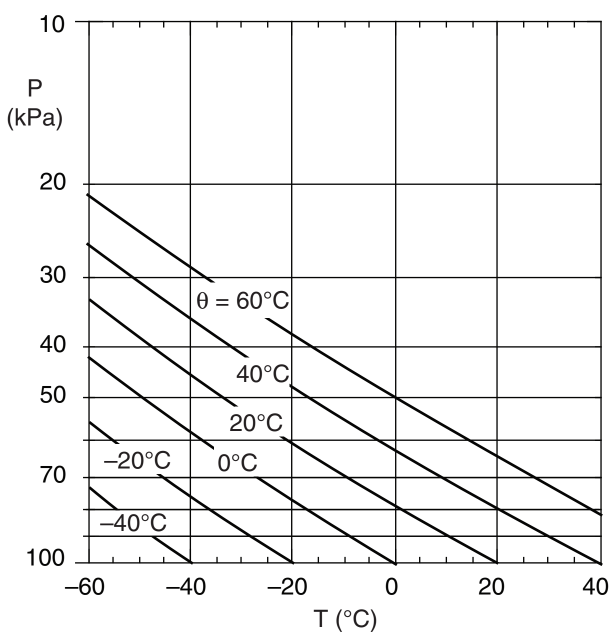


Figure 3.3 (at left)
Thermodynamic diagram showing the temperature change of air parcels of several different initial surface (at P = 100 kPa) temperatures, as they rise dry adiabatically. Recall from Chapter 1 that a logarithmic pressure decrease corresponds to roughly a linear height increase. Thus, height increases upward in this diagram. The thick diagonal lines are called "dry" adiabats, and are labeled with potential temperature.

FOCUS • Spreadsheet Thermodynamics

Thermodynamic diagrams can be produced with spreadsheet programs on personal computers. To calculate the dry adiabats, first type in the following row and column headers. Also shown in the spreadsheet below is the first set of temperature values representing the dry adiabats for air parcels starting at four of the different temperatures. These temperatures were just typed in as numbers.

	A	B	C	D	E
1	Dry Adiabat Example				
2					
3	P(kPa)	T(°C)	T(°C)	T(°C)	T(°C)
4	100.0	60.0	40.0	20.0	0.0

Next, in cell A5, increment to the next pressure by typing equation: =A4 – 10 .

In cell B5, enter eq. (3.10) for the temperature along a dry adiabat: =((B$4+273)*($A5/A4)^0.28571) – 273

The dollar sign implies an absolute reference; namely one that is not changed as the equation is auto-filled down or across.

Next select cell B5, and fill it across C5 and D5 to E5:

	A	B	C	D	E
1	Dry Adiabat Example				
2					
3	P(kPa)	T(°C)	T(°C)	T(°C)	T(°C)
4	100.0	60.0	40.0	20.0	0.0
5	90.0	50.1	30.7	11.3	-8.1

Then, select cells A5 through E5. Fill down to row 12

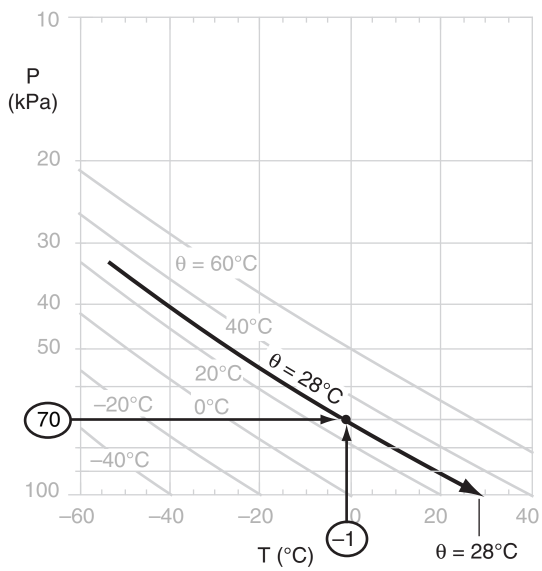
	A	B	C	D	E
1	Dry Adiabat Example				
2					
3	P(kPa)	T(°C)	T(°C)	T(°C)	T(°C)
4	100.0	60.0	40.0	20.0	0.0
5	90.0	50.1	30.7	11.3	-8.1
6	80.0	39.4	20.7	1.9	-16.9
7	70.0	27.7	9.7	-8.4	-26.4
8	60.0	14.8	-2.5	-19.8	-37.1
9	50.0	0.2	-16.2	-32.6	-49.0
10	40.0	-16.7	-32.1	-47.5	-62.9
11	30.0	-36.9	-51.1	-65.3	-79.5
12	20.0	-62.7	-75.4	-88.0	-100.6

Finally, select all the cells holding numbers, and make a graph. Switch the axes such that pressure is on the vertical axis, reverse the order so pressure decreases upward, and finally make the vertical axis logarithmic. The result should look like Fig. 3.3.

Solved Example
Given air at P = 70 kPa with T = –1°C. Find θ using the thermo diagram of Fig. 3.3.

Solution
Given: P = 70 kPa , T = –1°C
Find: θ = ? °C

First, use the thermo diagram to find where the 70 kPa isobar and the –1°C isotherm intersect. (Since the –1°C isotherm wasn't drawn on this diagram, we must mentally interpolate between the lines that are drawn.) The adiabat that passes through this intersection point indicates the potential temperature (again, we must interpolate between the adiabats that are drawn). By extending this adiabat down to the reference pressure of 100 kPa, we can read off the temperature 28°C, which corresponds to the potential temperature θ = **28°C** .



Check: Units OK. Physics reasonable.
Discussion: This exercise is the same as the first part of the previous exercise, for which we calculated θ = 301 K = 28°C. Yes, the answers agree.

The advantage of using an existing printed thermo diagram is that we can draw a few lines and quickly find the answer without doing any calculations. So it can make our lives easier, once we learn how to use it.

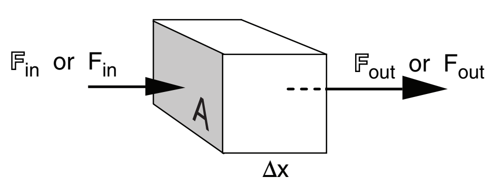


Figure 3.4
Change of flux F across distance Δx can cause changes within the volume. A is area of the face of the cube through which the flux flows. F is a kinematic flux, and 𝔽 is a dynamic flux.

Potential temperature is constant along these diagonal lines, so the lines are usually labeled with θ. Sometimes the labeling is implicit; namely, by the temperature near the bottom of the graph where the adiabat intercepts P = 100 kPa. For example, the diagonal line third from the left in Fig. 3.3 is the θ = 0°C adiabat.

To use this diagram, plot the initial temperature and pressure of the air parcel as a point on the graph. If that air parcel rises or descends to a new pressure, draw a line parallel to the diagonal lines, starting at the initial point, and ending at the final pressure. This line shows how the parcel temperature changes, and what its final temperature is. This approach is easier than solving equations, but not as accurate.

EULERIAN HEAT BUDGET

First Law of Thermo. – Revisited

If the heat flux into one side of a fixed volume is less than the flux out of the opposite side (Fig. 3.4), then some amount of heat Δq is obviously removed from the volume. According to the first law of thermodynamics eq. (3.5), a heat loss causes a temperature decrease (assuming no other work is done).

Thus, it is not the flux itself, but the **gradient of flux** (i.e., the change of kinematic flux ΔF_x or dynamic flux $\Delta \mathbb{F}_x$ across distance Δx) that causes temperature change within the volume. The gradient of flux is also known as a **flux divergence**. A positive value of $\Delta F_x/\Delta x$ gives a positive divergence (more flux leaving the volume than entering). Negative flux divergence is possible, which is sometimes called a **flux convergence**.

Considering the fluxes in all three dimensions, the first law of thermodynamics can be rewritten in Eulerian form as a **heat budget**, **heat balance** , or **heat conservation equation** for a stationary volume (e.g., $Vol = A \cdot \Delta x$ in Fig. 3.4):

$$\frac{\Delta T}{\Delta t} = -\frac{1}{\rho \cdot C_p}\left[\frac{\Delta \mathbb{F}_x}{\Delta x} + \frac{\Delta \mathbb{F}_y}{\Delta y} + \frac{\Delta \mathbb{F}_z}{\Delta z}\right] + \frac{\Delta So}{C_p \cdot \Delta t} \tag{3.16}$$

where ΔS_o is the internal source of heat per unit mass (e.g., J·kg^{-1}), such as caused by **latent heating**, radioactive decay, and exothermal chemical reactions. ΔS_o is negative for net cooling.

The kinematic-flux (F) form of the budget is:

$$\frac{\Delta T}{\Delta t} = -\left[\frac{\Delta F_x}{\Delta x} + \frac{\Delta F_y}{\Delta y} + \frac{\Delta F_z}{\Delta z}\right] + \frac{\Delta So}{C_p \cdot \Delta t} \qquad \bullet(3.17)$$

In these equations, Δx, Δy, and Δz represent the length of sides of the volume, not movement of the volume. Finally, the heat budget can be rewritten using potential temperature:

$$\frac{\Delta\theta}{\Delta t} = -\left[\frac{\Delta F_x}{\Delta x} + \frac{\Delta F_y}{\Delta y} + \frac{\Delta F_z}{\Delta z}\right] + \frac{\Delta So}{C_p \cdot \Delta t} \qquad \bullet(3.18)$$

As discussed in Appendix A, you must be careful when using gradients to ensure that the numerator and denominator are in the same direction. Thus, the flux gradient in the x-direction is:

$$\frac{\Delta F_x}{\Delta x} = \frac{F_{x\ right} - F_{x\ left}}{x_{right} - x_{left}} \qquad (3.19)$$

Flux gradients are similar in the other directions. Gradients are like slopes, when plotted.

Equations (3.16 - 3.18) are important because they are used to make temperature forecasts. To solve these equations, we must determine the flux gradients. The flux gradients that appear in eqs. (3.16 - 3.18) can be caused by **advection** (*adv*), **conduction** (*cond*), **turbulence** (*turb*), and **radiation** (*rad*):

$$\frac{\Delta F_x}{\Delta x} = \left.\frac{\Delta F_x}{\Delta x}\right|_{adv} + \left.\frac{\Delta F_x}{\Delta x}\right|_{cond} + \left.\frac{\Delta F_x}{\Delta x}\right|_{turb} + \left.\frac{\Delta F_x}{\Delta x}\right|_{rad} \qquad (3.20)$$

$$\frac{\Delta F_y}{\Delta y} = \left.\frac{\Delta F_y}{\Delta y}\right|_{adv} + \left.\frac{\Delta F_y}{\Delta y}\right|_{cond} + \left.\frac{\Delta F_y}{\Delta y}\right|_{turb} + \left.\frac{\Delta F_y}{\Delta y}\right|_{rad} \qquad (3.21)$$

$$\frac{\Delta F_z}{\Delta z} = \left.\frac{\Delta F_z}{\Delta z}\right|_{adv} + \left.\frac{\Delta F_z}{\Delta z}\right|_{cond} + \left.\frac{\Delta F_z}{\Delta z}\right|_{turb} + \left.\frac{\Delta F_z}{\Delta z}\right|_{rad} \qquad (3.22)$$

We will describe each of these processes next. Finally, the source term ΔS_o will be estimated, and all the results will be put back into the Eulerian heat budget equation.

Advection

The word advect means "to be transported by the mean wind". **Temperature advection** relates to heat being blown to or from a region by the wind. The amount of **advective flux** of heat increases linearly with mean temperature, and with mean wind speed:

$$F_{x\ adv} = U \cdot T \qquad (3.23)$$

$$F_{y\ adv} = V \cdot T \qquad (3.24)$$

$$F_{z\ adv} = W \cdot T \qquad (3.25)$$

Solved Example

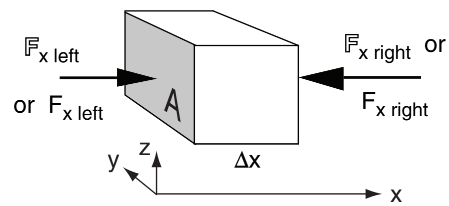
Given a cube of dry air at sea level, with side 20 m. An eastward moving heat flux of 3 W·m^{-2} flows in through the left, while a westward moving heat flux of 4 W·m^{-2} flows in through the right. There are no other fluxes or internal sources of heat. What is the kinematic heat flux through each side, and at what rate will temperature change inside the cube?

Solution

Given: $\mathbb{F}_{x\ right} = -4$ W·m^{-2} (negative because it is moving in the negative x direction)
$\mathbb{F}_{x\ left} = 3$ W·m^{-2}, $\Delta x = 20$ m

Find: $F_{x\ right}$ = ? K·m/s, $F_{x\ left}$ = ? K·m/s
$\Delta T/\Delta t$ = ? K/s

Sketch:



Use eq. (2.11): $F = \mathbb{F} / (\rho \cdot C_p)$

$F_{x\ left}$ = (3 W·m^{-2}) / [1231 (W·m^{-2}) / (K·m·s^{-1})]
= **<u>2.437x10^{-3}</u> <u>K·m·s^{-1}</u>** .

$F_{x\ right}$ = (–4 W·m^{-2}) / [1231 (W·m^{-2})/(K·m·s^{-1})]
= **<u>– 3.249x10^{-3}</u> <u>K·m·s^{-1}</u>** .

The flux gradient (eq. 3.19) is thus:

$$\frac{\Delta F_x}{\Delta x} = \frac{\left[(-3.249\times10^{-3}) - (2.437\times10^{-3})\right](\text{K}\cdot\text{m}\cdot\text{s}^{-1})}{[20-0]\ (\text{m})}$$

= – 2.843x10^{-4} K·s^{-1}.

Plugging this flux gradient into (3.17) gives
$\Delta T/\Delta t$ = **<u>+ 2.843x10^{-4}</u> <u>K·s^{-1}</u>**.

Check: The sign is positive implying warming with time, because heat is flowing into both sides of the cube.

Discussion: The heating rate is equivalent to 1 K/h.

Solved Example

Given a cube of air with winds as sketched in Fig 3.4. Let the mean wind speed be 10 m/s from west to east, but the temperature entering from the left is 20°C while that leaving on the right is 18°C. If the cube has side 20 km, then what is the advective flux gradient? (That is, what is the temperature-tendency contribution from advection.)

Solution

Given: U = 10 m/s, ΔT = 18 – 20°C = –2°C,
Δx = 20 km

Find: $\Delta F_{x\ adv}$ / Δx = ? °C/s

Use (3.26): $\Delta F_{x\ adv} / \Delta x = U \cdot (\Delta T/\Delta x)$
= (10 m/s) · [–2°C / 20000 m] = **– 0.001 °C/s**.
= **–3.6 °C/h**

Check: Units OK. Sketch OK. Physics OK.

Discussion: If this flux gradient were used in the heat-budget equation (3.17), the warming rate would be +3.6 °C/h due to advection. This is called **warm-air advection**, because warmer air is entering the region than is leaving.

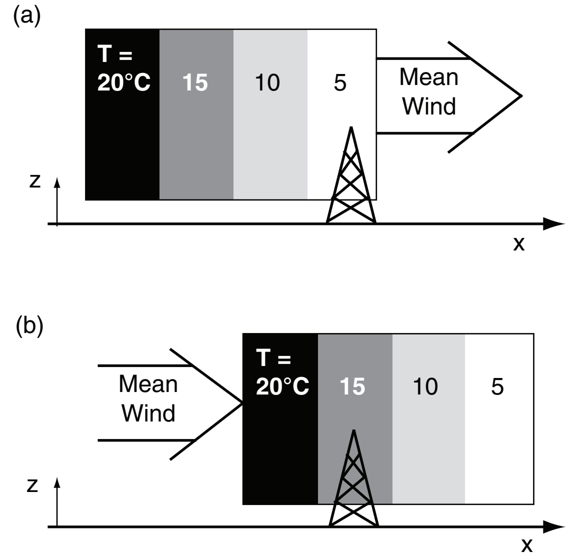


Figure 3.5
Advection illustration. (a) Initial state of the air with an east-west temperature gradient. (b) Later, after the air moves east.

Solved Example

Suppose the average air temperature gets colder with height, such that T = 15°C at z = 200 m and T = 10°C at z = 1000 m, with a linear change in between. If the mean vertical velocity is blowing the cold air downward from aloft, then what is the advective cooling rate at z = 600 m, given a vertical velocity of W = –0.1 m/s? There are no other heat-transfer processes.

Solution

Given: W = –0.1 m/s, z = 600 m
$\Delta T/\Delta z$ = (10–15°C)/(1000–200 m)
= –0.00625 °C/m

Find: $\Delta T/\Delta t$ = ? °C/s

This is a trick question. Although the air is colder aloft, we anticipate that it will warm as it descends due to the adiabatic compression discussed in the previous sections. When the air gets to the height of interest (600 m), it might be so hot as to cause warming rather than cooling. So we must be careful in solving this.

Plugging (3.28) into (3.17), and neglecting all other heat-transfer processes, we find

$\Delta T/\Delta t = - W\,(\Delta T/\Delta z + \Gamma_d)$
= – (–0.1 m/s) · (–0.00625+ 0.0098°C/m)
= **+ 3.55x10^{-4} °C/s**

Check: Units OK. Physics reasonable.

Discussion: The positive sign indicates a warming rate, not cooling. This advective warming rate is the same at all heights between 200 and 1000 m, not just at 600 m. The warming rate is equivalent to 1.28 °C/hour, which could cause a significant temperature increase if it continues for many hours.

Vertical motion is also called **advection** if by the mean wind, but is called **convection** if caused by buoyancy.

The heat budget (eqs. 3.16 - 3.18), however, uses the flux gradient, not the raw flux. As illustrated in Fig. 3.5a, the temperature at a fixed point such as at the top of a tall tower might initially be 5 °C. However, if the air temperature is greater in the west than the east, and if the wind is blowing from the west, then after a while the warmer air will be over the tower (Fig. 3.5b). Thus, the temperature increases with time at the fixed point for a situation with positive mean wind (blowing from west to east) and a negative horizontal temperature gradient (temperature decreasing from west to east).

Although Fig. 3.5 illustrates only horizontal advection in one direction, we need to consider advective effects in all directions, including vertical. If the mean wind speed varies little across the region of interest, then

$$\frac{\Delta F_{x\ adv}}{\Delta x} = \frac{U \cdot \left(T_{right} - T_{left}\right)}{x_{right} - x_{left}} = U \cdot \frac{\Delta T}{\Delta x} \qquad (3.26)$$

Similarly:

$$\frac{\Delta F_{y\ adv}}{\Delta y} = V \cdot \frac{\Delta T}{\Delta y} \qquad (3.27)$$

and

$$\frac{\Delta F_{z\ adv}}{\Delta z} = W \cdot \left[\frac{\Delta T}{\Delta z} + \Gamma_d\right] \qquad (3.28)$$

The dry adiabatic lapse rate Γ_d = 9.8 °C/km is added to the last term to compensate for the adiabatic temperature change that normally accompanies vertical motion as parcels move into regions of different pressure. This correction term must always be added to temperature whenever vertical motions (vertical advections) are considered.

Substituting eq. (3.11) into eqs. (3.26 - 3.28) allows potential temperature to be used for advection:

$$\frac{\Delta F_{x\ adv}}{\Delta x} = U \cdot \frac{\Delta\theta}{\Delta x} \qquad \bullet(3.29)$$

$$\frac{\Delta F_{y\ adv}}{\Delta y} = V \cdot \frac{\Delta\theta}{\Delta y} \qquad \bullet(3.30)$$

$$\frac{\Delta F_{z\ adv}}{\Delta z} = W \cdot \frac{\Delta\theta}{\Delta z} \qquad \bullet(3.31)$$

Solved Example

The potential temperature of the air increases 5°C per 100 km distance north. If a north wind of 20 m/s is blowing, find the advective flux gradient, and the temperature change associated with this advection.

Solution

Given: $\Delta\theta/\Delta y$ = 5°C/100 km = 5x10^{-5} °C/m
V = – 20 m/s (a north wind comes from the north)
Find: $\Delta F/\Delta y$ = ? °C/s, and $\Delta T/\Delta t$ = ? °C/s

Use eq. (3.30): $\Delta F/\Delta y$ = (– 20 m/s)·(5x10^{-5} °C/m)
= **–0.001 °C/s**
Use eq. (3.17) neglecting all other terms:
$\Delta T/\Delta t$ = – $\Delta F/\Delta y$ = – (–0.001°C/s) = **+0.001 °C/s**

Check: Physics reasonable. Sign appropriate, because we expect warming as the warm air is blown toward us from the north in this example.
Discussion: $\Delta T/\Delta t$ = 3.6°C/h, a rapid warming rate.

Molecular Conduction & Surface Fluxes

Heat can be transported by individual molecules bouncing into each other. This conduction process works in solids, liquids and gases, and works with or without any mean or turbulent wind. It is the primary way heat is transferred from the Earth's surface to the atmosphere, and also the primary way that heat is transported deeper into the ground.

The amount of conductive heat flux in the vertical is

$$\mathbb{F}_{z\ cond} = -k \cdot \frac{\Delta T}{\Delta z} \qquad (3.32)$$

where the molecular conductivity k is a property of the material through which heat flows. For air at standard conditions near sea level, k = 2.53x10^{-2} W·m^{-1}·K^{-1}.

Temperature gradients $\Delta T/\Delta z$ in most interior parts of the troposphere are relatively small, and k is very small. Thus, molecular conduction is negligible in those regions. Thus, for simplicity we will use

$$\frac{\Delta F_{x\ cond}}{\Delta x} \approx \frac{\Delta F_{y\ cond}}{\Delta y} \approx \frac{\Delta F_{z\ cond}}{\Delta z} \approx 0 \qquad (3.33)$$

in the heat budget equation (3.17) everywhere except at the ground.

Within the bottom few millimeters of air near the surface, vertical temperature gradients can be quite large (Fig. 3.6). For example, on a sunny summer day, the sun might heat the surface of an asphalt road to temperatures greater than 50°C, while the adjacent air temperatures are about 20 or 30°C. Such a large gradient is sufficient to drive a substantial heat flux.

Solved Example

What vertical temperature difference is necessary across the bottom 1 mm of atmosphere to conduct 300 W·m^{-2} of heat flux?

Solution

Given: $\mathbb{F}_{z\ cond}$ = 300 W·m^{-2} , Δz = 0.001 m
k = 2.53x10^{-2} W·m^{-1}·K^{-1}
Find: ΔT = ? °C
Sketch: (see Fig. 3.6)

Rearrange eq. (3.32) to solve for ΔT:
ΔT = – $\Delta z \cdot \mathbb{F}_{z\ cond}$ / k
ΔT = –(10^{-3} m)·(300 W·m^{-2}) /
(2.53x10^{-2} W·m^{-1}·K^{-1})
ΔT = **– 11.9°K**

Check: Units OK. Physics OK.
Discussion: The air 1 mm above ground must be almost 12°C cooler than the surface to conduct the required heat flux. Such large temperature differences are often observed in the real atmosphere very near the ground, especially on sunny days when the ground becomes very hot.

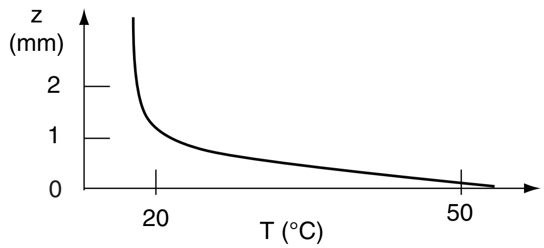


Figure 3.6
Strong temperature gradients within a few millimeters of the ground on a sunny day can drive strong conductive heat fluxes there, but higher in the atmosphere conduction is negligible.

Solved Example

Find the effective surface kinematic heat flux over a forest when the wind speed is 5 m·s^{-1} at a height of 10 m, the surface temperature is 25°C, and the air temperature at 10 m is 20°C.

Solution

Given: M = 5 m/s, z = 10 m
$T_{sfc} - T_{air}$ = 25 – 20°C = 5 °C
Find: F_H = ? K·m·s^{-1}

Use eq. (3.35):
F_H = (2x10^{-2})·(5 m/s)·(5°C) = **0.5 °C·m·s^{-1}**

Check: Units OK. Physics OK.
Discussion: Using conversion (2.11), the dynamic heat flux is 615.5 W·m^{-2}, which is almost half of the solar constant of roughly 1366 W·m^{-2}. This suggests that 50% of the incoming solar radiation heats the air, while the remainder causes evaporation, is conducted into the ground, or is reflected, for this example.

When conduction and turbulence are considered together, we see that turbulence is zero at the ground where conduction can be important, while turbulence is important in the rest of the lower atmosphere where conduction is negligible. For this reason, turbulence and conduction are often combined into an **effective surface turbulent heat flux**, F_H.

In practice, it is virtually impossible to use eq. (3.32) directly, because the temperature gradient across the bottom few millimeters of air is rarely measured or predicted. However, on **windy** days the combination of conduction and turbulence causes the effective surface heat flux to be:

$$F_H = C_H \cdot M \cdot (\theta_{sfc} - \theta_{air}) \qquad \bullet(3.34)$$

or

$$F_H \cong C_H \cdot M \cdot (T_{sfc} - T_{air}) \qquad \bullet(3.35)$$

M is the mean wind speed at 10 m, T_{air} is the air temperature at a height of 10 m, T_{sfc} is the surface skin temperature, and θ_{air} and θ_{sfc} are the corresponding potential temperatures.

C_H is the **bulk heat transfer coefficient**. This coefficient is unfortunately not constant, but varies with surface conditions and turbulence intensity. It has a dimensionless value in the range of 2x10^{-3} over smooth surfaces to 2x10^{-2} over rough or forested surfaces. Eqs. (3.34 - 3.35) are useful during strong advection.

The bottom 1 to 2 km of the troposphere is called the **atmospheric boundary layer** (**ABL**, see the ABL chapter). During **calm** days with strong solar heating and convection, warm rising **thermals** (buoyant air parcels) often occur within the ABL. This type of ABL is called a **mixed layer** (**ML**). The rising thermals cause a substantial effective surface heat flux. Instead of using eqs. (3.34 - 3.35) on these calm sunny days, the surface flux can be found from:

$$F_H = b_H \cdot w_B \cdot (\theta_{sfc} - \theta_{ML}) \qquad \bullet(3.36)$$

or

$$F_H = a_H \cdot w_* \cdot (\theta_{sfc} - \theta_{ML}) \qquad \bullet(3.37)$$

where θ_{ML} is the potential temperature of the air at a height of about 500 m, in the middle of the mixed layer. The empirical **mixed-layer transport coefficient** is a_H = 0.0063, and the **convective transport coefficient** is b_H = 5x10^{-4}. Both are independent of surface roughness.

A **buoyancy velocity scale** w_B gives the effectiveness of thermals in producing vertical heat transport:

$$w_B = \left[\frac{|g| \cdot z_i}{T_{v\ ML}} \cdot (\theta_{v\ sfc} - \theta_{v\ ML}) \right]^{1/2} \qquad \bullet(3.38)$$

where z_i is the depth of the convective ABL (i.e., the mixed layer), and $|g| = 9.8$ m/s^2 is gravitational acceleration. Virtual temperatures and virtual potential temperatures (both in absolute units of K) are used in this last expression, but not in eqs. (3.36 - 3.37). Typical vertical velocities are about $w \approx 0.02 \cdot w_B$. To good approximation, the denominator in eq. (3.38) can be approximated by $\theta_{v\,ML}$ (also in units of K).

The **Deardorff velocity** w_* is another convective velocity scale, and is defined as

$$w_* = \left[\frac{|g| \cdot z_i}{T_v} \cdot F_{Hsfc} \right]^{1/3} \qquad \bullet(3.39)$$

where T_v is an average absolute virtual temperature in the ABL, and $F_{Hsfc} = F_H$ is surface kinematic heat flux. Typical values are $w_* = 1$ to 2 m·s^{-1}, and $w_* \approx 0.08 \cdot w_B$.

Another way to find F_H is with a surface heat budget or a Bowen ratio, as described later in this chapter. You could also write an expression similar to eqs. (3.36 - 3.37), except for surface moisture flux in terms of the difference in water-vapor mixing ratio between the surface and the mid-mixed layer.

Turbulence

Turbulence is the quasi-random movement of air by small (order of 2 mm to 2 km) swirls of motion called **eddies**. Although the amount of air leaving one region is always replaced by the same amount returning from somewhere else (conservation of air mass), the departing air might have a different temperature than the arriving air. By moving temperature around, the net result is a heat flux.

Because there are so many eddies of all sizes moving in such a complex fashion, meteorologists do not even try to describe the heat flux contribution from each individual eddy. Instead, meteorologists use statistics to describe the average heat flux caused by all the eddies within a region (see the Atmos. Boundary Layer chapter). The expressions below represent such statistical averages.

The net effect of turbulence is to mix together air from different initial locations. Thus, turbulence tends to homogenize the air. Potential temperature, wind, and humidity gradually become mixed toward more uniform states by the action of turbulence. The amount of mixing varies with time and location, as turbulence intensity changes.

Fair Weather (no thunderstorms)

During fair-weather, turbulence is strong during daytime over non-snowy ground, and can cause significant heat transport within the atmospheric

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Solved Example

On a calm sunny day, a 2 km thick mixed layer is dry with θ = 295 K. If T_{sfc} = 325 K, find the effective surface kinematic heat flux.

Solution

Given: θ_{ML} = 295 K, θ_{sfc} = 325 K
z_i = 2000 m, $|g|$ = 9.8 m·s^{-2}
Find: $F_{z\,eff.sfc.}$ = ? K·m·s^{-1}

For dry air: $\theta = \theta_v$ (see eq. 3.13).
Use eq. (3.38) first, then eq. (3.36):

$$w_B = \left[\frac{\left|9.8\text{m/s}^2\right| \cdot 2000\text{m}}{295\text{K}} \cdot (325\text{K} - 295\text{K}) \right]^{1/2}$$

= (1993 m^2/s^2)$^{1/2}$ = 44.6 m·s^{-1}

F_H = (5x10^{-4})·(44.6 m·s^{-1})·(325 K – 295 K)
=**<u>0.67 K·m·s</u>**$^{-1}$

Check: Units OK. Physics reasonable.
Discussion: Greater surface-atmosphere temperature differences drive more vigorous thermals, and each thermal then transports more heat.

Solved Example

Given an effective surface kinematic heat flux of 0.67 K·m·s^{-1}, find the Deardorff velocity for a dry, 1 km thick boundary layer of temperature 25°C

Solution

Given: F_H = 0.67 K·m·s^{-1}, z_i = 1 km = 1000 m ,
$T_v = T$ (because dry) = 25°C = 298 K.
Find: w_* = ? m/s

Use eq. (3.39):
w_* = [(9.8 m·s^{-2})·(1000m)·(0.67 K·m·s^{-1})/(298K)]$^{1/3}$
= **<u>2.8 m/s</u>**

Check: Units OK. Physics reasonable.
Discussion: Over land on hot sunny days, warm buoyant thermals often rise with a speed of the same order of magnitude as the Deardorff velocity.

Solved Example

On a sunny fair-weather day with effective surface kinematic heat flux of 0.1 K·m·s^{-1}, what is the vertical flux divergence across an atmospheric boundary layer of depth 1 km?

Solution

Given: F_H = 0.1 K·m·s^{-1}, z_i = 1000 m
Find: $\Delta F_{z\ turb} / \Delta z$ = ? (K/s)

Use eq. (3.41):

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{-1.2 \cdot F_H}{z_i}$$

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{-1.2 \cdot (0.1\ \mathrm{K \cdot m/s})}{1000\mathrm{m}}$$

$$= \underline{\mathbf{-0.00012\ K{\cdot}s^{-1}}}$$

Check: Units OK. Physics OK.
Discussion: The negative sign on the gradient implies heating with time according to eq. (3.17), which makes sense during daytime.

The answer is equivalent to 0.43°C·h^{-1}, or about 5°C/12 h. This seems a bit small, compared to typical amounts of warming during the daylight hours. Thus, one would guess that daytime effective surface heat fluxes can be larger than 0.1 K·m·s^{-1}, and/or ABLs can be shallower than 1 km.

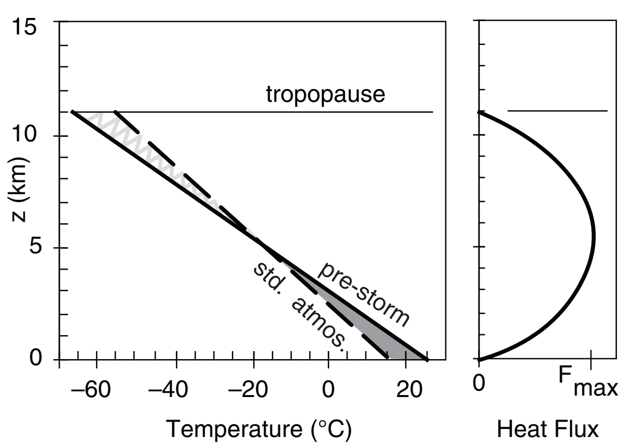

Figure 3.7
Pre-storm air at the bottom of the troposphere that is warmer (shaded dark grey) than the standard atmosphere tends to rise and exchange places with cooler air aloft (hatched light grey) during a thunderstorm. The result is a vertical heat flux.

boundary layer (ABL). [Reasons for different turbulence intensities are explained in the Atmospheric Boundary Layer chapter.] Due to the turbulent homogenization, turbulent fluxes vary roughly linearly with height, and can be estimated from the fluxes at the top and bottom of the ABL.

In fair weather (**wx**) during daytime, the heat flux at the top of the ABL is often roughly 20% of the magnitude of the surface heat flux, but has opposite sign. The flux at the bottom of the ABL is associated with conduction, and was given in the previous subsection as an effective turbulent surface flux F_H. This gives a turbulent flux divergence of:

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{F_{z\ top} - F_{z\ bottom}}{z_i} \qquad (3.40)$$

$0 < z < z_i$, fair wx

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{-1.2 \cdot F_H}{z_i} \qquad (3.41)$$

where z_i is the depth of the ABL (typically of order 200 m to 3 km).

Above the ABL during fair weather, turbulence is often negligible:

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx 0 \qquad \text{for } z > z_i \text{, fair wx} \quad (3.42)$$

At night, turbulence is often much weaker. Thus, vertical flux divergence is often quite small during fair-weather nights, and can be neglected everywhere except in the bottom 100 m or so.

Stormy Weather

During stormy weather, deep cumulus clouds and thunderstorms can cause vigorous turbulent mixing throughout the depth of the troposphere. Such convective clouds form in response to warm, buoyant air underlying cooler air (Fig. 3.7). This type of pre-storm environment is said to be unstable, as will be discussed in more detail in the chapters on Stability and Thunderstorms. The atmosphere tries to re-stabilize itself by letting the warm air rise and the cold air sink. This mixing process is called **moist convective adjustment**. Such mixing causes a vertical heat flux.

The mixing continues until some equilibrium profile of temperature is reached. As a first approximation, the **standard atmospheric lapse rate** (Γ_{sa} = $-\Delta T/\Delta z$ = +6.5 K/km) is used here for the equilibrium profile. If the pre-storm lapse rate is Γ_{ps} = $-\Delta T/\Delta z$, then the vertical turbulent flux gradient is approximately:

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{z_T}{\Delta t} \cdot \left[\Gamma_{ps} - \Gamma_{sa}\right] \cdot \left(\frac{1}{2} - \frac{z}{z_T}\right) \qquad \text{stormy wx} \quad (3.43)$$

where Δt is the lifetime of the thunderstorms (order of 1 h) , and z_T is depth of the troposphere (order of 11 km). The term in square brackets is positive for an unstable pre-storm environment.

As shown in Fig. 3.7, the heat flux profile is shaped like a parabola. The turbulent vertical heat flux is zero at the top and bottom of the troposphere, and reaches a maximum of

$$F_{\max} = z_T^{\ 2} \cdot \left[\Gamma_{ps} - \Gamma_{sa} \right] / (8 \cdot \Delta t) \qquad \text{stormy wx} \quad (3.44)$$

in the middle of the troposphere.

The convective adjustment effect of cooling at low altitudes and heating aloft must be added to the precipitation effect (see the source term) of latent heating at all heights, to find the net affect of the thunderstorm. An even more accurate accounting would include IR longwave radiation divergence from the top of the thunderstorm anvil cloud.

In those regions where turbulence exists, turbulent transport in any horizontal direction often cancels transport in the opposite direction, yielding negligible net flux gradients. Also, in non-turbulent regions there is obviously no turbulent flux gradient. Thus, as an idealized approximation, we can assume the following everywhere:

$$\frac{\Delta F_{x\ turb}}{\Delta x} \approx \frac{\Delta F_{y\ turb}}{\Delta y} \approx 0 \qquad (3.45)$$

Radiation

Air in the troposphere is relatively opaque to most wavelengths of infrared (IR) radiation, causing radiation emitted at one altitude to be reabsorbed into the air at a neighboring altitude, some of which is re-radiated back to the first level. The net result of this very complex process is a relatively small vertical IR flux gradient within the air, and a nearly zero horizontal flux gradient:

$$\frac{\Delta F_{x\ rad}}{\Delta x} \approx \frac{\Delta F_{y\ rad}}{\Delta y} \approx 0 \qquad (3.46)$$

$$\frac{\Delta F_{z\ rad}}{\Delta z} \approx 0.1 \text{ to } 0.2 \text{ (K/h)} \qquad (3.47)$$

In the absence of clouds, haze, or smoke, direct radiative warming of the air due to solar radiation is negligible. However, sunlight warms the air indirectly by heating the surface, which then transfers and distributes its heat to the air by conduction and turbulent convection. There can also be substantial solar warming of clouds during daytime.

Solved Example

What is the vertical turbulence flux gradient in stormy weather at 2 km altitude, if the pre-storm vertical temperature gradient was –10 °C/km? Also, what is the maximum value of vertical heat flux near the middle of the troposphere?

Solution

Given: $\Gamma_{ps} = -\Delta T/\Delta z =$ +10 K/km, $\Gamma_{sa} = 6.5$ K/km
Find: $\Delta F_{z\ turb}/\Delta z = ?$ (K/s) , $F_{max} = ?$ K·m/s
Assume: $z_T = 11$ km, $\Delta t = 1$ h = 3600 s.

Use eq. (3.43):

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{z_T}{\Delta t} \cdot \left[\Gamma_{ps} - \Gamma_{sa} \right] \cdot \left(\frac{1}{2} - \frac{z}{z_T} \right)$$

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{11\text{km}}{3600\text{s}} \cdot \left[(10 - 6.5) \frac{\text{K}}{\text{km}} \right] \cdot \left(\frac{1}{2} - \frac{2\text{km}}{11\text{km}} \right)$$

$\Delta F_{z\ turb}/\Delta z$ = **0.0034 K/s**

Use eq. (3.44):

F_{max} =(11,000m)·(11km)·[(10–6.5)K/km)]/[8·(3600s)]
= **14.7 K·m/s**

Check: Units OK. Physics OK.

Discussion: The positive flux gradient implies cooling over time, because of the negative sign on the flux gradient terms in eq. (3.17). This makes sense, because $z = 2$ km is in the bottom half of the troposphere, where the initially-warm air is becoming cooler with time due to mixing with the colder air from above.

The magnitude of the max heat flux due to thunderstorms is much greater than the heat flux due to thermals in fair weather. Thunderstorms move large amounts of heat upward in the troposphere.

ON DOING SCIENCE • Expert vs. Novice

Expert scientists and engineers often solve problems, organize knowledge, and perceive structure differently than students and other novices.

Problem Solving	Novice	Expert
... is ...	a recall task	a process
... begins with ...	hunt for "the equation"	qualitative analysis
... uses classification based on ...	surface features	deep structure
...tools include ...	"the equation"	graphs, limits, diagrams, conservation laws, etc.

Organizing Knowledge	Novice	Expert
Memory recall is ...	piecemeal	effortless retrieval of relevant collected facts
Reasoning by ...	jumping to hasty, unfounded conclusions	fast mental scan through a chain of possibilities
Conflicting data, ideas & conclusions are...	not recognized	recognized, pointing to need for more info
Related ideas are...	memorized as separate facts	integrated into a coherent big picture

Structure Perception	Novice	Expert
Cues about the structure are ...	missed	recognized and trigger new lines of thought
Disparate instances are...	separately classified based on surface features	recognized as having the same underlying structure
Tasks are performed...	before thinking about the organization	after data is organized to find structure
Theories that don't agree with data ...	are used without revision	identify ideas ripe for revision

(Paraphrased from Wendy Adams, Carl Wieman, Dan Schwartz, and Kathleen Harper.)

Internal Source: Latent Heat

The amount of latent heat given to the air during condensation of $\Delta m_{condensing}$ grams of water vapor that are already contained in the Eulerian volume is $L_v \cdot \Delta m_{condensing}$. When distributed throughout the mass of air m_{air} within the volume, the net latent heating rate for the source term is:

$$\frac{\Delta So}{C_p \cdot \Delta t} = \frac{L_v}{C_p} \cdot \frac{\Delta m_{condensing}}{m_{air} \cdot \Delta t} \tag{3.48}$$

For evaporation of existing liquid water, the sign is negative (to cool the air).

Consider a column of air equal to the troposphere depth of roughly z_{Trop} = 11 km, which is appropriate for thunderstorms. The average condensational heating rate of the whole column is proportional to the amount of precipitation that falls out of the bottom. If RR denotes **rainfall rate** measured as the change of depth of water in a rain gauge with time, then the latent heating source term is:

$$\frac{\Delta So}{C_p \cdot \Delta t} = \frac{L_v}{C_p} \cdot \frac{\rho_{liq}}{\rho_{air}} \cdot \frac{RR}{z_{Trop}} \tag{3.49}$$

where ρ_{liq} = 1000 kg·m^{-3} is the approximate density of liquid water, ρ_{air} is air density averaged over the whole column, and $L_v \ / \ C_p$ = 2500 K·kg$_{air}$·kg$_{liq}^{-1}$ is the ratio of latent heat of condensation to specific heat at constant pressure.

Utilizing the standard atmosphere with a tropopause depth of 11 km gives an **average tropospheric density** of ρ_{air} = 0.689 kg·m^{-3}. Thus, eq. (3.49) can be rewritten as:

$$\frac{\Delta So}{C_p \cdot \Delta t} = a \cdot RR \tag{3.50}$$

where a = 0.33 K/(mm of rain), and for RR in (mm of rain)/s. Divide by 3600 for RR in mm/h.

Net Heat Budget

The net Eulerian heat budget is found by combining the first law of thermodynamics eqs. (3.17 or 3.18) with the definition of all the flux gradients eqs. (3.20 - 3.22). The long messy equation that results can be simplified for many atmospheric situations as follows.

Assume negligible vertical mean temperature advection. Assume negligible horizontal turbulent transport. Assume negligible conduction. Neglect direct solar heating of the troposphere, but approximate IR cooling as a small constant.

The resulting simplified Eulerian net heat budget equation is:

$$\left.\frac{\Delta T}{\Delta t}\right|_{x,y,z} = \underbrace{-\left[U\cdot\frac{\Delta T}{\Delta x} + V\cdot\frac{\Delta T}{\Delta y}\right]}_{\text{advection}} \underbrace{-\,0.1\frac{\text{K}}{\text{h}}}_{\text{radiation}}$$

$$\underbrace{-\frac{\Delta F_{z\,turb}(\theta)}{\Delta z}}_{\text{turbulence}} \underbrace{+\frac{L_v}{C_p}\cdot\frac{\Delta m_{condensing}}{m_{air}\cdot\Delta t}}_{\text{latent heat}} \qquad \bullet(3.51)$$

where the θ in the turbulence term is to remind us that this turbulence flux divergence is for heat. In later chapters we will see similar turbulence flux divergences, but for moisture or momentum. If there is no condensation or evaporation (i.e., for non-cloudy air), the source-sink (latent-heat) term can be set to zero.

The net heat budget is important because you can use it to forecast air temperature at any altitude. Or, if you already know how the air temperature changes with time, you can use the net heat budget to see which processes are most important in causing this change.

The net heat budget applies to a blob of air having a finite volume and mass. For the special case of the Earth's surface (infinitesimally thin; having no mass), you can write a simplified heat budget, as described next.

SURFACE HEAT BUDGET

While the Lagrangian and Eulerian heat budgets give the temperature change and fluxes for a volume (moving or stationary, respectively), we can also examine the fluxes across the Earth's surface. Because the surface has zero thickness, all fluxes must sum to zero. This holds for both heat and moisture.

Heat Budget

During daytime, the net radiative heat-flux to the surface is $-\mathbb{F}^*$, which is balanced by three output fluxes from the surface. They are: turbulent sensible-heat transport into the air (**sensible heat flux** $\mathbb{F}_H$, sometimes called the heat flux), turbulent latent-heat transport into the air (**latent heat flux** $\mathbb{F}_E$ associated with evaporation from the surface), and **molecular conduction** flux from the ground $\mathbb{F}_G$. At night, the directions (signs) of the fluxes are reversed. Regardless of the surface scenario (Fig. 3.8), the fluxes toward the surface must balance the fluxes away from the surface.

Solved Example

Find the tropospheric average latent heating for a rainfall rate of $5\ \text{mm·h}^{-1}$ from a thunderstorm.

Solution

Given: $RR = 5\ \text{mm·h}^{-1}$.
Find: $\Delta S_o/(C_p\cdot\Delta t) = ?\ \text{K·h}^{-1}$

Use eq. (3.50): $\Delta S_o/(C_p\cdot\Delta t) = 0.33\ \text{(K/mm)}\cdot(5\ \text{mm/h}) = \underline{\mathbf{1.65\ K\cdot h}}^{-1}$

Check: Units OK. Physics OK.
Discussion: This is the same order of magnitude as other terms in the Eulerian heat budget.

Solved Example

Find the temperature change during one hour at a fixed point, given $U = 5$ m/s, $V = 0$, $\Delta T/\Delta x = 1°\text{C}/10$ km, $\Delta m_{cond}/m_{air} = 2\ \text{g}_{water}/\text{kg}_{air}$, for a daytime 800 m thick ABL with effective surface kinematic flux of 0.2 K·m/s. Use $L_v/C_p = 2.5\ \text{K}/(\text{g}_{water}/\text{kg}_{air})$.

Solution

Given: (see above)
Find: $\Delta T = ?$ °C

Use eq. (3.51). Do each term separately, times Δt:

$$\text{Adv}\cdot\Delta t = -\left[(5\text{m/s})\cdot\left(\frac{1°\text{C}}{10^4\,\text{m}}\right)\right]\cdot(3600\text{s}) = -1.8°\text{C}$$

$$\text{Lat.Heat\_Source}\cdot\Delta t = \left(2.5\frac{\text{K}\cdot\text{kg}_{air}}{\text{g}_{water}}\right)\cdot\left(2\frac{\text{g}_{water}}{\text{kg}_{air}}\right) = +5.0°\text{C}$$

$$\text{Rad}\cdot\Delta t = \left(-0.1\frac{\text{K}}{\text{h}}\right)\cdot(1\text{h}) = -0.1°\text{C}$$

$$\text{Turb}\cdot\Delta t = -\frac{-1.2\cdot(0.2\text{K}\cdot\text{m/s})}{800\text{m}}\cdot(3600\text{s}) = +1.1°\text{C}$$

where (3.41) was used for the turbulence term.

Thus: $\Delta T = (\text{Adv} + \text{Latent} + \text{Rad} + \text{Turb})\cdot\Delta t$
$= (-1.8 + 5.0 - 0.1 + 1.1)°\text{C} = \underline{\mathbf{4.2\ °C}}$

Check: Units OK. Physics OK.
Discussion: Many processes act simultaneously, some adding and some subtracting from the total. This is one of the complexities of weather forecasting.

During daytime, direct radiative heating of the air is small, and can be neglected. However, strong heating of the ground by solar radiation allows the warm ground to heat the air via the surface heat flux. Thus, radiative heating from the sun is getting to the air indirectly via conduction of sensible heat from the ground, and via surface evaporation of water that transports latent heat. Latent heating was quite important here.

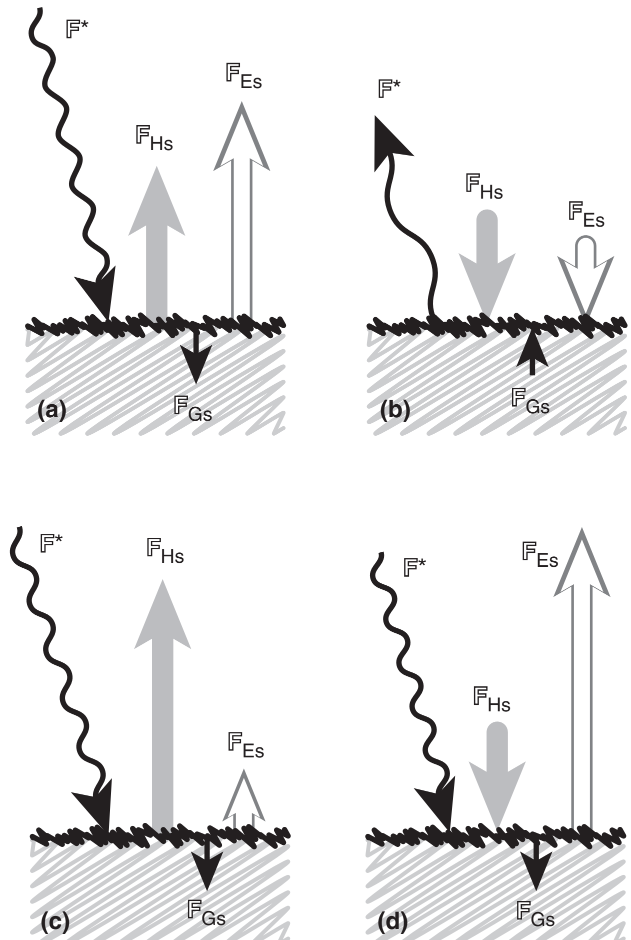

Figure 3.8
Fluxes affecting the surface heat budget. (a) Daytime over vegetated surface. (b) Nighttime over vegetated surface. (c) Daytime over a desert. (d) Oasis effect of warm, dry air advection over a cool, moist surface. $\mathbb{F}^*$ *= net radiative flux,* $\mathbb{F}_{Hs}$ *= sensible heat flux,* $\mathbb{F}_{Es}$ *= latent heat flux,* $\mathbb{F}_{Gs}$ *= conductive heat flux into the ground, and subscript s refers to values at Earth's surface.*

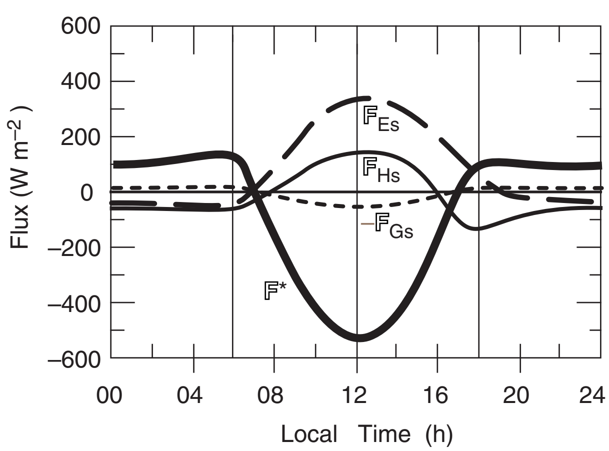

Figure 3.9
Typical diurnal variation of the surface heat budget. The $\mathbb{F}^*$ *net radiation curve comes from Chapter 2. Fluxes are positive upward, as defined in the previous caption.*

For an infinitesimally thin surface, this balance is

$$-\mathbb{F}^* = \mathbb{F}_H + \mathbb{F}_E - \mathbb{F}_G \qquad \bullet(3.52)$$

or in kinematic form (after dividing by $\rho_{air}\cdot C_p$):

$$-F^* = F_H + F_E - F_G \qquad \bullet(3.53)$$

where <u>all fluxes are defined positive upward</u>. All terms have units of W·m^{-2} in the first equation, and K·m·s^{-1} in the second. Also, Fig. 3.9 shows an example of the daily variation of these fluxes for a moist vegetated surface.

One can think of the net radiation as a forcing on the system, and the other fluxes in eqs. (3.52 - 3.53) as the responses. Namely, all of the radiation absorbed at the surface must go somewhere.

To good approximation, the flux into the ground is proportional to the net radiative flux, which in dynamic and kinematic forms are:

$$\mathbb{F}_G \approx X \cdot \mathbb{F}^* \qquad (3.54)$$

$$F_G \approx X \cdot F^* \qquad (3.55)$$

where X = 0.1 during the day, and X = 0.5 at night.

Expressions for the effective sensible heat flux at the surface were presented in eqs. (3.34 - 3.37). Similar expressions can be used in the Moisture chapter for the latent heat flux. A Bowen-ratio method for measuring the sensible and latent flux is described in the next subsection.

Note the difference between the heat budget across the surface (eq. 3.53) where the fluxes must balance, and the heat budget within a volume of air (eq. 3.17) where any imbalance of the fluxes causes a net heating or cooling within the volume.

Bowen Ratio

The ratio of the sensible to latent heat fluxes is called the **Bowen ratio**, B:

$$B = \frac{\mathbb{F}_H}{\mathbb{F}_E} = \frac{F_H}{F_E} \qquad (3.56)$$

The Bowen ratio can be 0.1 over the sea, 0.2 over irrigated crops, 0.5 over grassland, 5 over semi-arid regions, and 10 over deserts.

Many observations have confirmed that the sensible heat flux is proportional to the vertical gradient of potential temperature $\Delta\theta$ in the bottom 20 m of the atmosphere. Namely, $F_H = -K_H \cdot \Delta\theta/\Delta z$, where K_H is an eddy diffusivity (see the Atmos. Boundary Layer chapter), and the negative sign says that the heat flux flows down the local gradient (from hot toward cold air). Similarly, the moisture flux is proportional to the vertical gradient of mixing ratio Δr (grams of water vapor in each kilogram of dry air, as will be described in detail in the Moisture chapter). Namely, $F_E = -K_E \cdot \Delta r / \Delta z$.

By forming a ratio (the Bowen ratio) of these two fluxes and assuming $K_E = K_H$, the result is:

$$B = \gamma \cdot \frac{\Delta\theta}{\Delta r} \tag{3.57}$$

where $\gamma = C_p/L_v = 0.4$ $(\mathrm{g_{water\ vapor}/kg_{air}}) \cdot \mathrm{K^{-1}}$ is called the **psychrometric constant**.

Thus, the Bowen ratio can easily be measured using **thermometers** and **hygrometers**. The potential temperature $\Delta\theta$ and mixing ratio Δr differences must be measured between the same pair of heights z to use this approach (see Fig. 3.10). This gives, $\Delta\theta = T_2 - T_1 + (0.0098\ \mathrm{K/m}) \cdot (z_2 - z_1)$, and $\Delta r = r_2 - r_1$.

When eq. (3.56) is combined with the surface energy balance eq. (3.52) and the approximation for ground flux eq. (3.54), the result for sensible heat flux is:

$$\mathbb{F}_H = \frac{-0.9 \cdot \mathbb{F}^*}{\dfrac{\Delta r}{\gamma \cdot \Delta\theta} + 1} \tag{3.58}$$

or kinematically:

$$F_H = \frac{-0.9 \cdot F^*}{\dfrac{\Delta r}{\gamma \cdot \Delta\theta} + 1} \tag{3.59}$$

Similarly, for latent heat flux associated with the evaporation or condensation of water at the surface:

$$\mathbb{F}_E = \frac{-0.9 \cdot \mathbb{F}^*}{\dfrac{\gamma \cdot \Delta\theta}{\Delta r} + 1} \tag{3.60}$$

or

$$F_E = \frac{-0.9 \cdot F^*}{\dfrac{\gamma \cdot \Delta\theta}{\Delta r} + 1} \tag{3.61}$$

Instead of calculating both equations, one can save time by calculating the first, and then using the surface energy balance to find the other:

$$\mathbb{F}_E = -0.9 \cdot \mathbb{F}^* - \mathbb{F}_H \tag{3.62}$$

or

$$F_E = -0.9 \cdot F^* - F_H \tag{3.63}$$

Solved Example

Given a net radiative surface flux of $-600\ \mathrm{W \cdot m^{-2}}$ over grassland, find the other terms of the surface heat budget.

Solution

Given: $\mathbb{F}^* = -600\ \mathrm{W \cdot m^{-2}}$ (probably daytime)
$B = 0.5$ for grassland
Find: $\mathbb{F}_G$, $\mathbb{F}_H$, $\mathbb{F}_E$ = ? $\mathrm{W \cdot m^{-2}}$

Use eq. (3.54) with $X = 0.1$ for day:
$\mathbb{F}_G = 0.1 \cdot \mathbb{F}^* = 0.1 \cdot (-600\ \mathrm{W \cdot m^{-2}}) = \underline{\mathbf{-60\ W \cdot m^{-2}}}$

Combine eqs. (3.52) & (3.56):
$\mathbb{F}_H = B \cdot (\mathbb{F}_G - \mathbb{F}^*) / (1 + B)$
$\mathbb{F}_E = (\mathbb{F}_G - \mathbb{F}^*) / (1 + B)$
Thus,
$\mathbb{F}_H = 0.5 \cdot (-60 + 600\ \mathrm{W \cdot m^{-2}}) / (1 + 0.5)$
$= \underline{\mathbf{180\ W \cdot m^{-2}}}$

$\mathbb{F}_E = (-60 + 600\ \mathrm{W \cdot m^{-2}}) / (1 + 0.5)$
$= \underline{\mathbf{360\ W \cdot m^{-2}}}$

Check: Units OK. Physics OK.
Discussion: Does the energy budget really balance? To answer this, use eq. (3.52):

$-\mathbb{F}^* = \mathbb{F}_H + \mathbb{F}_E - \mathbb{F}_G$
$600 = 180 + 360 + 60\ \mathrm{W \cdot m^{-2}}$ Yes, it balances.

The Bowen ratio is quite variable, even over a single type of surface such as grassland. For example, some grasses might be more moist or might be transpiring more than other grasses, depending on the maturity of the grass, insolation, air temperature, and soil moisture. For this reason, it is usually impossible to use the technique in this example to find the sensible and latent heat fluxes. A better alternative is to measure the Bowen ratio first, such as shown by Fig. 3.10 and eq. (3.57).

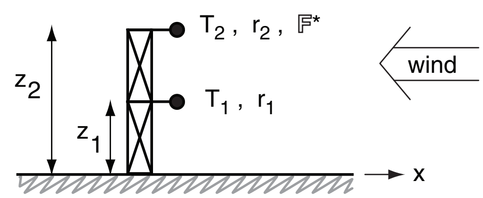

Figure 3.10
Instrumented tower for measuring temperature T and humidity r at two heights, z_1 and z_2. Net radiation is $\mathbb{F}^*$.

Solved Example

Given the following measurements from an instrumented tower, find the sensible and latent heat fluxes. Also, $\mathbb{F}^* = -500\ \mathrm{W{\cdot}m^{-2}}$.

index	z (m)	T (°C)	r (g_{vapor}/kg_{air})
2	10	15	8
1	2	18	10

Solution

Given: the table above.
Find: $\mathbb{F}_H$ and $\mathbb{F}_E$ = ? W·m^{-2}

First step is to find $\Delta\theta$:

$$\Delta\theta = T_2 - T_1 + (0.0098\ \mathrm{K/m})\cdot(z_2 - z_1)$$
$$= 15\ \mathrm{K} - 18\ \mathrm{K} + (0.0098\ \mathrm{K/m})\cdot(10\mathrm{m} - 2\mathrm{m})$$
$$= -3\ \mathrm{K} + 0.0784\ \mathrm{K} = -2.92\ \mathrm{K}$$

Use eq. (3.58)

$$\mathbb{F}_H = \frac{-0.9\cdot(-500\mathrm{W\cdot m^{-2}})}{\dfrac{(-2 g_{vap}/kg_{air})}{[0.4(g_{vap}/kg_{air})\cdot \mathrm{K^{-1}}]\cdot(-2.92\mathrm{K})} + 1}$$

$\mathbb{F}_H$ = **165.91 W·m**$^{-2}$

Next, use eq. (3.62):

$$\mathbb{F}_E = -0.9\mathbb{F}^* - \mathbb{F}_H$$
$$= -0.9\cdot(-500\ \mathrm{W{\cdot}m^{-2}}) - 165.91\ \mathrm{W{\cdot}m^{-2}}$$
$$= \underline{\mathbf{284.09\ W{\cdot}m^{-2}}}$$

Check: Units OK. Physics OK.
Discussion: Sensible heat flux is roughly 50% of the latent heat flux (i.e., $B = 0.5$). Thus, we can guess that these observations were made over moist grassland.

Solved Example

What is the wind chill index for air temperature of –50°C and wind of 70 km/h.

Solution

Given: $T_{air} = -50°\mathrm{C}$, $M = 70$ km/h
Find: $T_{wind\ chill}$ = ? °C.

Use eq. (3.64a):

$$T_{wind\ chill} = [0.62\cdot(-50°\mathrm{C}) + 13.1°\mathrm{C}] + [0.51\cdot(-50°\mathrm{C}) - 14.6°\mathrm{C}]\cdot\left(\frac{70\mathrm{km/h}}{4.8\mathrm{km/h}}\right)^{0.16}$$

$$= [-17.9°\mathrm{C}] + [-40.1°\mathrm{C}]\cdot(1.535) = \underline{\mathbf{-79.5°C}}$$

Check: Units OK. Physics OK.
Discussion: This is well in the danger zone. Stay indoors in front of the fireplace and read a good book. I can recommend the book "Meteorology for Scientists and Engineers" — it will put you right to sleep.

In the equations above, recall that kinematic fluxes F have units of (K·m/s), while dynamic fluxes $\mathbb{F}$ have units of (W/m^2).

Measurements of temperature and humidity at two heights (Fig. 3.10) along with measurements of the net radiation near the surface are thus sufficient to determine the vertical effective fluxes of heat and moisture. In the Moisture chapter are equations for converting latent-heat fluxes into water fluxes.

APPARENT TEMPERATURES

Humans and livestock are warm blooded (**homeothermic**). Our metabolism generates heat, while our perspiration evaporates to keep us cool. Our bodies attempt to regulate our metabolism and perspiration to maintain a relatively constant internal temperature of about 37°C (= 98.6°F). Our skin is normally cooler — about 33.9°C (= 93°F).

Whether we feel warm or cool depends not only on air temperature, but on wind speed and humidity. During winter, faster wind makes the air feel colder, because it removes heat from our bodies faster. Wind chill is a measure of this effect.

During summer, higher humidity makes the air feel hotter, because it reduces evaporation of our perspiration. The humidex and heat index measure this effect. These indices are given as **apparent temperatures**; namely, how warm or cold it feels.

Wind Chill

The **wind-chill temperature index** is a measure of how cold the air feels to your exposed face. The official formula, as revised in 2001 by the USA and Canada, for wind chill in °C is:

$$T_{wind\ chill} = (a\cdot T_{air} + T_1) + (b\cdot T_{air} - T_2)\cdot\left(\frac{M}{M_o}\right)^{0.16} \quad \text{for } M > M_o \qquad (3.64a)$$

$$T_{wind\ chill} = T_{air} \quad \text{for } M \le M_o \qquad (3.64b)$$

where $a = 0.62$, $b = 0.51$, $T_1 = 13.1°\mathrm{C}$, $T_2 = 14.6°\mathrm{C}$, and where $M_o = 4.8$ km/h is the average speed that people walk. M is the wind speed measured at the official anemometer height of 10 m. For $M < M_o$, the wind chill equals the actual air temperature. This index applies to non-rainy air.

The air feels colder with colder temperatures and with stronger winds. Table 3-1 shows wind chills computed using this equation on a computer spreadsheet, with the results plotted in Fig. 3.11. The data used to create eq. (3.64) was from volunteers in Canada who sat in refrigerated wind tunnels, wearing warm coats with only their face exposed.

At wind chills colder than –27°C, exposed skin can freeze in 10 to 30 minutes. At wind chills colder than –48°C: WARNING, exposed skin freezes in 2 to 5 min. At wind chills colder than –55°C: DANGER, exposed skin freezes in less than 2 minutes. In this danger zone is an increased risk of **frostbite** (fingers, toes, ears and nose numb or white), and **hypothermia** (drop in core body temperature).

Heat Index and Humidex

More humid air feels warmer and more uncomfortable than the actual temperature. **Heat index** (or **apparent temperature** or **temperature-humidity index**) is one measure of heat discomfort and heat-stress danger.

The equation-set below approximates Steadman's temperature-humidity index of sultriness:

$$T_{heat\ index}(°C) = T_R + [T - T_R] \cdot \left(\frac{RH \cdot e_s}{100 \cdot e_R} \right)^p \tag{3.65a}$$

where e_R = 1.6 kPa is reference vapor pressure, and

$$T_R(°C) = 0.8841 \cdot T + (0.19°C) \tag{3.65b}$$

$$p = (0.0196°C^{-1}) \cdot T + 0.9031 \tag{3.65c}$$

$$e_s(\text{kPa}) = 0.611 \cdot \exp\left[5423 \left(\frac{1}{273.15} - \frac{1}{(T + 273.15)} \right) \right] \tag{3.65d}$$

The two input variables are T (dry bulb temperature in °C), and RH (relative humidity percentage, with values of 0 to 100). Also, T_R (°C), and p are parameters, and e_s is the saturation vapor pressure, discussed in the Moisture chapter. Eqs. (3.65) assume that you are wearing a normal amount of clothing for warm weather, are in the shade or indoors, and a gentle breeze is blowing.

Table 3-2 shows the solution to this equation. For low relative humidities (at and below the bold-face underlined numbers in the table), the air feels cooler than the actual air temperature because perspiration evaporates effectively, keeping humans cool. However, for high relative humidity, the apparent temperature is warmer than the actual air temperature.

In Canada, a **humidex** is defined as

$$T_{humidex}(°C) = T(°C) + a \cdot (e - b) \tag{3.66a}$$

where T is air temperature, a = 5.555 (°C/kPa), b = 1 kPa, and

Table 3-1. Wind-chill temperatures (°C) approx.

Wind Speed		Air Temperature (°C)					
km/h	**m/s**	**–40**	**–30**	**–20**	**–10**	**0**	**10**
60	**16.7**	–64	–50	–36	–23	–9	5
50	**13.9**	–63	–49	–35	–22	–8	6
40	**11.0**	–61	–48	–34	–21	–7	6
30	**8.3**	–58	–46	–33	–20	–6	7
20	**5.6**	–56	–43	–31	–18	–5	8
10	**2.8**	–51	–39	–27	–15	–3	9
0	**0**	–40	–30	–20	–10	0	10

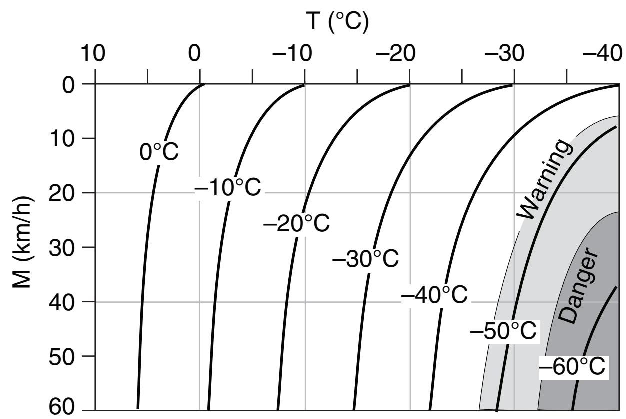

Figure 3.11
Curves are wind chill temperature index (°C), as a function of actual air temperature (T) and wind speed (M).

Table 3-2. Heat Index Apparent Temperature (°C)

RH	Actual Air Temperature (°C)						
(%)	**20**	**25**	**30**	**35**	**40**	**45**	**50**
100	21	29	41	61			
90	21	29	39	57			
80	21	28	37	52			
70	20	27	35	48			
60	20	26	34	45	62		
50	**<u>19</u>**	25	32	41	55		
40	19	**<u>24</u>**	30	38	49	66	
30	19	24	**<u>29</u>**	36	44	56	
20	18	23	28	**<u>33</u>**	40	48	59
10	18	23	27	32	**<u>37</u>**	**<u>42</u>**	**<u>48</u>**
0	18	22	27	31	36	40	44

Solved Example

Use the equations to find the heat index and humidex for an air temperature of 38°C and a relative humidity of 75% (which corresponds to a dew-point temperature of about 33°C).

Solution

Given: $T = 38°C$, $RH = 75\%$, $T_d = 33°C$

Find: $T_{heat\ index} = ?\ °C$, $T_{humidex} = ?\ °C$

For heat index, use eqs. (3.65):

$T_R = 0.8841 \cdot (38) + 0.19 = 33.8°C$ (3.65b)

$p = 0.0196 \cdot (38) + 0.9031 = 1.65$ (3.65c)

$e_s = 0.611 \cdot \exp[5423 \cdot (\{1/273.15\} - \{1/(38+273.15)\})]$

$= 6.9$ kPa (3.65d)

$T_{heat\ index} = 33.8 + [38-33.8] \cdot (0.75 \cdot 6.9/1.6)^{1.65}$

= **<u>62.9°C</u>** (3.65a)

For humidex, use eqs. (3.66):

$e = 0.611 \cdot \exp[5418 \cdot (\{1/273.16\} - \{1/(33+273.16)\})]$

$= 5.18$ kPa (3.66b)

$T_{humidex} = 38 + 5.555 \cdot (5.18-1) =$ **<u>61.2°C</u>** (3.66a)

Check: Units OK. Values agree with extrapolation of Table 3-2.

Discussion: These values are in the danger zone, meaning that people are likely to suffer heat stroke. The humidex and heat index values are nearly equal for this case.

Table 3-3. **Humidex** apparent temperature (°C)

T_d	Actual Air Temperature T (°C)						
(°C)	20	25	30	35	40	45	50
50							118
45						96	101
40					77	82	87
35				62	67	72	77
30			49	54	59	64	69
25		37	42	47	52	57	62
20	28	33	38	43	48	53	58
15	24	29	34	39	44	49	54
10	21	26	31	36	41	46	51
5	19	24	29	34	39	44	49
0	18	23	28	33	38	43	48
-5	17	22	27	32	37	42	47
-10	16	21	26	31	36	41	46

$$e(\text{kPa}) = 0.611 \cdot \exp\left[5418\left(\frac{1}{273.16} - \frac{1}{[T_d(°C) + 273.16]}\right)\right] \quad (3.66b)$$

T_d is dew-point temperature, a humidity variable discussed in the Moisture chapter.

Humidex is also an indicator of summer discomfort due to heat and humidity (Table 3-3). Values above 40°C are uncomfortable, and values above 45°C are dangerous. **Heat stroke** is likely for humidex ≥ 54°C.

TEMPERATURE SENSORS

Temperature sensors are generically called **thermometers**. Anything that changes with temperature can be used to measure temperature. Many materials expand when warm, so the size of the material can be calibrated into a temperature. Classical **liquid-in-glass thermometers** use either mercury or a dyed alcohol or glycol fluid that can expand from a reservoir or **bulb** up into a narrow tube.

House **thermostats** (temperature controls) often use a **bimetalic strip**, where two different metals are sandwiched together, and their different expansion rates with temperature causes the metal to bend as the temperature changes. Car thermostats use a wax that expands against a valve to redirect engine coolant to the radiator when hot. Some one-time use thermometers use wax that melts onto a piece of paper at a known temperature.

Many electronic devices change with temperature, such as resistance of a wire, capacitance of a capacitor, or behavior of various transistors (**thermistors**). These changes can be measured electronically and displayed. **Thermocouples** (such as made by a junction between copper and constantan wires, where constantan is an alloy of roughly 60% copper and 40% nickel) generate a small amount of electricity that increases with temperature. **Liquid crystals** change their orientation with temperature, and can be designed to display temperature.

Sonic thermometers measure the speed of sound through air between closely placed transmitters and receivers of sound. Radio Acoustic Sounder Systems (**RASS**) transmit a loud pulse of sound upward from the ground, and then infer temperature vs. height via the speed that the sound wave propagates upward, as measured by a radio or microwave profiler.

Warmer objects emit more radiation, particularly in the infrared wavelengths. An **infrared thermometer** measures the intensity of these emissions to infer the temperature. Satellite remote sensors also detect emissions from the air upward into space, from which temperature profiles can be calculated (see the Remote Sensing chapter).

Even thick layers of the atmosphere expand when they become warmer, allowing the **thickness** between two different atmospheric pressure levels to indicate average temperature in the layer.

SUMMARY

Air parcels that move through the atmosphere while conserving heat are said to be adiabatic. When parcels rise adiabatically, they cool due to the change of pressure with height. The change of temperature with height is called the adiabatic lapse rate, and indicates a physical process. The ambient environmental air through which the parcel moves might have a different lapse rate. Thermodynamic diagrams are convenient for determining how the temperature varies with height or pressure, and for comparing different lapse rates.

Once heat from the sun is in the Earth-atmospheric system, it is redistributed by advection, radiation, turbulence, and conduction. Some of the sensible heat is converted to/from latent heat by water phase change. Eventually, heat is lost from the system as infrared radiation to space.

At the Earth's surface, net radiation is balanced by turbulent fluxes into the atmosphere and conduction into the ground. Turbulent fluxes consist of sensible heat flux, and latent heat flux associated with evaporation of water. The ratio of sensible to latent heat flux is the Bowen ratio. All of these fluxes vary with the diurnal cycle.

Warm blooded animals and humans feel heat loss rather than temperature. On cold, windy days, the heat loss is quantified as a wind chill temperature. During humid hot days, the apparent temperature can be warmer than actual.

Threads

The conversion between latent and sensible heat provides energy that drives thunderstorms (Chapters 14 and 15) and hurricanes (Chapter 16). These thunderstorms stabilize the atmosphere by moist convective adjustment. The transport of moisture by the wind is thus equivalent to a transport of energy, and affects the global energy budget (Chapters 11 and 21).

Latent heat flux (i.e., evaporation driven by the incoming sunlight or other heat source) provides the moisture (Chapter 4) for clouds (Chapter 6) and precipitation (Chapter 7). Surface heat and moisture fluxes drive atmospheric boundary-layer evolution (Chapter 18) and contribute to the Eulerian budget of moisture (Chapter 4).

The Eulerian heat budget is one of the governing equations that is solved as part of a numerical weather forecast (Chapter 20). The Lagrangian heat budget associated with rising air parcels is used to determine cloud formation (Chapter 6), atmospheric turbulence (Chapter 18) and air-pollutant dispersion (Chapter 19). Thermodynamic diagrams (Chapter 5) combine potential temperature, latent heat, and the Lagrangian heat budget. Moist convective adjustment is caused by thunderstorms (Chapters 14 and 15).

EXERCISES

Numerical Problems

N1. Find the change in sensible heat (enthalpy) (J) possessed by 3 kg of air that warms by __°C.
a. 1 b. 2 c. 3 d. 4 e. 5 f. 6 g. 7
h. 8 i. 9 j. 10 k. 11 m. 12

N2. Find the specific heat C_p of humid air having water-vapor mixing ratio ($g_{vapor}/g_{dry\ air}$) of:
a. 0.010 b. 0.012 c. 0.014 d. 0.016 e. 0.018
f. 0.020 h. 0.022 i. 0.024 j. 0.026 k. 0.028
m. 0.030

N3. Find the change in latent heat (J) for condensation of ___ kg of water vapor.
a. 0.2 b. 0.4 c. 0.6 d. 0.8 e. 1.0 f. 1.2 g. 1.4
h. 1.6 i. 1.8 j. 2.0 k. 2.2 m. 2.4

N4. Find the temperature change (°C) of air given the following values of heat transfer and pressure change, assuming air density of 1.2 kg/m^3.

	Δq (J/kg)	ΔP (kPa)
a.	500	5
b.	1000	5
c.	1500	5
d.	2000	5
e.	2500	5
f.	3000	5
g.	500	10
h.	1000	10
i.	1500	10
j.	2000	10
k.	2500	10
m.	3000	10

N5. Find the change in temperature (°C) if an air parcel rises the following distances while experiencing the heat transfer values given below.

	Δq (J/kg)	Δz (km)
a.	500	0.5
b.	1000	0.5
c.	1500	0.5
d.	2000	0.5
e.	2500	0.5
f.	3000	0.5
g.	500	1
h.	1000	1
i.	1500	1
j.	2000	1
k.	2500	1
m.	3000	1

N6. Given the following temperature change ΔT (°C) across a height difference of Δz = 4 km, find the lapse rate (°C/km):
a. 2 b. 5 c. 10 d. 20 e. 30 f. 40 g. 50
h. –2 i. –5 j. –10 k. –20 m. –30

N7. Find the final temperature (°C) of an air parcel with the following initial temperature and height change, for an adiabatic process.

	$T_{initial}$ (°C)	Δz (km)
a.	15	0.5
b.	15	–1.0
c.	15	1.5
d.	15	–2.0
e.	15	2.5
f.	15	–3.0
g.	5	0.5
h.	5	–1.0
i.	5	1.5
j.	5	–2.0
k.	5	2.5
m.	5	–3.0

N8. Using the equations (not using the thermo diagram), find the final temperature (°C) of dry air at a final pressure, if it starts with the initial temperature and pressure as given. (Assume adiabatic)

	$T_{initial}$ (°C)	$P_{initial}$ (kPa)	P_{final}(kPa)
a.	5	100	80
b.	5	100	50
c.	5	80	50
d.	5	80	100
e.	0	60	80
f.	0	60	50
g.	0	80	40
h.	0	80	100
i.	–15	90	80
j.	–15	90	50
k.	–15	70	50
m.	–15	70	100

N9. Same as previous question, but use the thermo diagram Fig. 3.3.

N10. Using equations (and show your work) rather than a thermo diagram, what is the potential temperature of air at:

	z (m)	T (°C)
a.	200	20
b.	1,000	5
c.	200	5
d.	1,000	20
e.	300	10
f.	6,000	–50
g.	10,000	–90
h.	–30	35

i.	700	3
j.	1,300	–5
k.	400	5
m.	2,000	–20

N11. Same as the previous exercise, but find the virtual potential temperature for humid air. Use a water-vapor mixing ratio of 0.01 $g_{vapor}/g_{dry\ air}$ if the air temperature is above freezing, and use 0.002 $g_{vapor}/g_{dry\ air}$ if air temperature is below freezing. Assume no liquid water or ice suspended in the air.

N12. Using equations (and show your work) rather than a thermo diagram, what is the potential temperature of air at:

	P (kPa)	T (°C)
a.	90	20
b.	80	5
c.	110	5
d.	70	20
e.	85	10
f.	40	–50
g.	20	–90
h.	105	35
i.	75	3
j.	60	–5
k.	65	5
m.	50	–20

N13. Same as previous exercise, but use the thermo diagram Fig. 3.3.

N14. Use the thermodynamic diagram (Fig. 3.3) to answer the following.

	Given:		Find:
a.	T = 0°C,	P = 70 kPa	θ (°C)
b.	T = 20°C,	P = 100 kPa	θ (°C)
c.	T = –20°C,	P = 40 kPa	θ (°C)
d.	θ = 40°C,	P = 40 kPa	T (°C)
e.	θ = –20°C,	P = 90 kPa	T (°C)
f.	θ = 40°C,	P = 80 kPa	T (°C)
g.	θ = 40°C,	T = 0 °C	P (kPa)
h.	θ = 60°C,	T = –20 °C	P (kPa)

N15(§). Create a thermodynamic diagram on a spreadsheet like the one plotted in Fig. 3.3, only print it with more lines. Namely, draw isotherm grid lines every 10°C, and draw dry adiabats for every 10°C from –50°C to 80°C.

N16. Find the rate of temperature change (°C/h) in an Eulerian coordinate system with no internal heat source, given the kinematic flux divergence values below. Assume $\Delta x = \Delta y = \Delta z = 1$ km.

	ΔF_x (K·m/s)	ΔF_y (K·m/s)	ΔF_z (K·m/s)
a.	1	2	3
b.	1	2	–3
c.	1	–2	3
d.	1	–2	–3
e.	–1	2	3
f.	–1	2	–3
g.	–1	–2	3
h.	–1	–2	–3

N17. Given the wind and temperature gradient, find the value of the kinematic advective flux gradient (°C/h).

	V (m/s)	$\Delta T/\Delta y$ (°C/100 km)
a.	5	–2
b.	5	2
c.	10	–5
d.	10	5
e.	–5	–2
f.	–5	2
g.	–10	–5
h.	–10	5

N18. Given the wind and temperature gradient, find the value of the kinematic advective flux gradient (°C/h).

	W (m/s)	$\Delta T/\Delta z$ (°C/km)
a.	5	–2
b.	5	2
c.	10	–5
d.	10	–10
e.	–5	–2
f.	–5	2
g.	–10	–5
h.	–10	–10

N19. Find the value of the conductive flux $\mathbb{F}_{z\ cond}$ (W/m^2) given a vertical temperature gradient of __ °C per meter.

a. –1 b. –2 c. –3 d. –4 e. –5 f. –6 g. –7
h. 1 i. 2 j. 3 k. 4 m. 5 n. 6 o. 7

N20. Find the effective surface turbulent heat flux (°C·m/s) over a forest for wind speed of 10 m/s, air temperature of 20°C, and surface temperature (°C) of

a. 21 b. 22 c. 23 d. 24 e. 25 f. 26 g. 27
h. 19 i. 18 j. 17 k. 16 m. 15 n. 14 o. 13

N21. Find the effective kinematic heat flux at the surface on a calm day, for a buoyant velocity scale of 50 m/s, a mixed-layer potential temperature of 25°C, and with a surface potential temperature (°C) of:

a. 26 b. 28 c. 30 d. 32 e. 34 f. 36 g. 38
h. 40 i. 42 j. 44 k. 46 m. 48 n. 50

N22. Find the effective kinematic heat flux at the surface on a calm day, for a Deardorff velocity of 2

m/s, a mixed-layer potential temperature of 24°C, and with a surface potential temperature (°C) of:
a. 26 b. 28 c. 30 d. 32 e. 34 f. 36 g. 38
h. 40 i. 42 j. 44 k. 46 m. 48 n. 50

N23. For dry air, find the buoyancy velocity scale, given a mixed-layer potential temperature of 25°C, a mixed-layer depth of 1.5 km, and with a surface potential temperature (°C) of:
a. 27 b. 30 c. 33 d. 36
e. 40 f. 43 g. 46 h. 50

N24. For dry air, find the Deardorff velocity w_* for an effective kinematic heat flux at the surface of 0.2 K·m/s, air temperature of 30°C, and mixed-layer depth (km) of:
a. 0.4 b. 0.6 c. 0.8 d. 1.0
e. 1.2 f. 1.4 g. 1.6 h. 1.8

N25. Find the value of vertical divergence of kinematic heat flux, if the flux at the top of a 200 m thick air layer is 0.10 K·m/s, and flux (K·m/s) at the bottom is:
a. 0.2 b. 0.18 c. 0.16 d. 0.14
e. 0.12 f. 0.10 g. 0.08 h. 0.06

N26. Find the vertical turbulent flux gradient, for the following surface effective flux values and ABL depths, assuming fair-weather during daytime:

	F_H (K·m·s^{-1})	z_i (km)
a.	0.2	1.0
b.	0.1	2.0
c.	0.3	1.5
d.	0.05	0.2
e.	0.08	0.3
f.	0.12	0.8
g.	0.15	1.0
h.	0.25	1.5

N27. Just before thunderstorms form in a stormy troposphere, the air is 10°C warmer than standard near the ground, and is 5°C cooler than standard near the top of the troposphere (11 km), with a linear temperature variation in between. Find the vertical turbulent flux gradient (K/s) at height (km):
a. 0 b. 0.5 c. 1 d. 1.5 e. 2 f. 2.5 g. 3
h. 3.5 i. 4 j. 5 k. 6 m. 7 n. 8 o. 11

N28. Find the mid-tropospheric maximum value of heat flux (K·m/s) for a stormy atmosphere, where the troposphere is 11 km thick, and the air temperature at the top of the troposphere equals the air temperature of a standard atmosphere. But the air temperature (°C) at the ground is:
a. 16 b. 17 c. 18 d. 19 e. 20 f. 21 g. 22
h. 23 i. 24 j. 25 k. 26 m. 27 n. 28 o. 29

N29. Find the latent-heating rate (°C/h) averaged over the troposphere for a thunderstorm when the rainfall rate (mm/h) is:
a. 0.5 b. 1 c. 1.5 d. 2 e. 2.5 f. 3 g. 3.5
h. 4 i. 4.5 j. 5 k. 5.5 m. 6 n. 6.5 o. 7

N30. Given below the net radiative flux (W/m^2) reaching the surface, find the sum of sensible and latent heat fluxes (W/m^2) at the surface. (Hint: determine if it is day or night by the sign of the radiative flux.)
a. –600 b. –550 c. –500 d. –450 e. –400
f. –350 g. –300 h. –250 i. –200 j. –150
k. –100 m. –50 n. 50 o. 100 p. 150

N31. Same as the previous problem, but estimate the values of the sensible and latent heat fluxes (W/m^2) assuming a Bowen ratio of:
(1) 0.2 (2) 5.0

N32. Given the following measurements of temperature, and specific humidity, find the sensible and latent heat-flux values. Net radiation is –400 W/m^2.

index	z(m)	T(°C)	r (g_{vap}/kg_{air})
2	20	T_2	8
1	10	14	10

where T_2 (°C) is:
a. 13.5 b. 13 c. 12.5 d. 12 e. 11.5 f. 11
g. 10.5 h. 10 i. 9.5 j. 9 k. 8.5 m. 8

N33. Assume you ride your bicycle at a speed given below, during a calm day of temperature given below. What wind chill temperature to your feel?
a. 10 m/s, 20°C b. 5 m/s, 20°C c. 15 m/s, 20°C
d. 10 m/s, –5°C e. 5 m/s, –5°C f. 15 m/s, –5°C

N34(§). Use eqs. (3.64) to create a wind-chill table or graph similar to Table 3-1 and Fig. 3.11, but for wind speeds in miles per hour and temperatures in °F.

N35. Find the heat index apparent temperature (°C) for an actual air temperature of 33°C and a relative humidity (%) of:
a. 5 b. 10 c. 20 d. 30 e. 40 f. 50 g. 60
h. 70 i. 75 j. 80 k. 85 m. 90 n. 90

N36. Find the humidex apparent air temperature (°C) for an actual air temperature of 33°C and a dew-point temperature (°C) of:
a. 32.5 b. 32 c. 31 d. 30 e. 29 f. 28 g. 27
h. 26 i. 25 j. 23 k. 20 m. 15 n. 10 o. 5

Understanding & Critical Evaluation

U1. Suppose you are given 1 kg of ice at 0°C. If the ice is placed in 1 kg of liquid water initially at 20°C, what will be the final temperature after all the ice melts?

U2. Explain in your own words why the units for specific heat C_p (J·kg^{-1}·K^{-1}) are slightly different than the units for the latent heat factor L (J·kg^{-1}). (Hint: read the Focus box on Internal Energy.)

U3. Explain in your own words why the magnitude of C_p should be larger than the magnitude of C_v. (Hint: read the Focus box on C_p vs. C_v).

U4. Consider the Focus box on C_p vs. C_v, with Fig. 3F.3c representing an initial state at equilibrium. Suppose you add some weight to the piston in Fig (c) causing the piston to become lower to reach a new equilibrium. Describe what would happen to: (a) the molecules on average, (b) the gas temperature in the cylinder, (c) the air density in the cylinder, and (d) the air pressure in the cylinder.

U5. For the First Law of Thermodynamics (eq. 3.4) which term(s) is/are zero for a process that is:
a. adiabatic b. isothermal c. isobaric

U6. Derive eq. (3.5) from eq. (3.4), showing your steps. (Calculus is not needed to do this.) Show all your work and state any assumptions and limitations of the result.

U7. For Fig. 3.1, speculate on other processes not listed that might affect the air-parcel temperature.

U8. Using Fig. 3.2, explain in your own words the difference between a process lapse rate and an environmental lapse rate. Can both exist with different values at the same height? Why?

U9. Eq. (3.7) tells us that temperature of an adiabatically rising air parcel will decrease linearly with increasing height. In your own words, explain why you would NOT expect the same process to cause temperature to decrease linearly with decreasing pressure.

U10. If an air parcel rises isothermally (namely, heat is added or subtracted to maintain constant temperature), then what would happen to the potential temperature of the air parcel as it rises?

U11. Chinook winds (also known as foehn winds) consist of air descending down the lee slope of a mountain and then continuing some distance across the neighboring valley or plain. Why are Chinook winds usually warm when they reach the valley? (Hint: consider adiabatic descent of an air parcel.)

U12. In the definition of virtual potential temperature, why do liquid water drops and ice crystals cause the air to act heavier (i.e., colder virtual potential temperature), even though these particles are falling through the air?

U13. On a copy of Fig. 3.3 plot the standard-atmosphere temperature profile. If an air parcel from this standard atmosphere at P = 100 kPa is lifted dry adiabatically to a height where P = 50 kPa, how does its final (process) temperature compare with the standard (environmental) temperature at the same height?

U14(§). Use the equations for the standard atmosphere from Chapter 1, and convert temperatures into potential temperatures every km within the troposphere and stratosphere. Plot the resulting θ vs z.

U15(§). Create a semi-log graph of log P vs. T similar to Fig. 3.3, however do not plot the dry adiabats. Instead, solve the hypsometric equation to plot curves corresponding to the following heights:
z = 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 km. In other words there will be one curve corresponding to z = 1 km, and another for z = 2 km, etc.

U16. Explain why a positive flux gradient in the presence of a positive wind is associated with cooling and not warming.

U17. Given a 1 km thick mixed layer in the air over the ocean, with sea surface temperature 30°C and air temperature 25°C at z = 10 m, and calm winds. If the mixed layer potential temperature is 20°C and the water-vapor mixing ratio is 0.015 g_{vapor}/g_{air}, then what is the convective effective surface heat flux?

U18. Light travels faster in warm air than in cold. Use this info, along with Fig. 3.6, to explain why **inferior mirages** (reflections of the sky) are visible on hot surfaces such as asphalt roads. (Hint: Consider a wave front that is moving mostly horizontally, but also slightly downward at a small angle relative to the road surface, and track the forward movement of each part of this wave front — an optics method known as Huygens' Principle. See details in the atmospheric Optics chapter.)

U19. Under what conditions would eqs. (3.34 - 3.35) be expected to fail? Why?

U20. Use eqs. (3.37) and (3.39) to solve for the heat flux as a function of the temperature difference.

U21. In Fig. 3.7, the heat flux is greatest at the height where there is no change in the vertical temperature profile from before to after a storm. Why should that be the case?

U22. If no other heating process was acting except radiative divergence in the vertical, what would be the heating rate of air. This typically occurs on calm nights at altitudes roughly 500 m above ground.

U23. In a thunderstorm, the amount of water condensation in the troposphere is often much greater than the amount of rain reaching the ground. Why is that, and how might it affect the heat budget averaged over the whole thunderstorm depth?

U24. What are the limitations of the net heat budget equation (3.51)?

U25. Given a situation where a wind speed of $U = 10$ m/s is blowing air with horizontal (east-west) temperature gradient of 2°C/100 km. Also suppose that 2 g of water per 1 kg of air is condensing every 15 minutes. Neglect turbulence. Compare the magnitudes of each term in the net heat budget equation (3.51).

U26. Sketch and discuss a figure such as Fig. 3.8, except for nighttime over a desert.

U27. Speculate why the X factor for ground flux is different from day to night. Discuss limitations of that expression for ground flux.

U28. Given a layer of air 1 km thick advecting over a sea surface of 20°C at wind speed $U = 20$ m/s. The initial air temperature is 10°C. The layer of air experiences bulk turbulent heat transfer into the bottom of it, but no heat transfer across the top, and no radiative heating. Assume the heat from the sea surface is mixed instantaneously in the vertical across the whole air layer. For a steady-state situation with only advection and the effective turbulent surface heat flux, derive an equation for the change of temperature with horizontal distance $\Delta T/\Delta x$.

U29. Use the flux estimates from eqs. (3.58 - 3.61) in the surface heat budget equation to show that they do indeed balance the budget. Also, what are the limitations of those equations?

U30. (§). An alternative equation for wind chill temperature index, based on a modified version of the heat transfer equation (3.35) is:

$$T_{wind\ chill} = T_s + \left(T_{air} - T_s\right) \cdot \left[b + a \cdot \left(\frac{M + M_o}{M_o} \right)^{0.16} \right] + T_c \qquad (3.67)$$

where T_s = 34.6°C is an effective skin temperature, and where, $a = 0.5$, $b = 0.62$, T_c = 4.2°C, and $M_o = 4.8$ km/h. Create a table or graph of wind chills using this formula, and compare to wind chills from Table 3-1 or Fig. 3.11.

U31. In Fig. 3.11, where does a small increase in wind speed cause the greatest decrease in wind-chill temperature? Why might you expect this to be the case?

U32. Here is an alternative expression for heat index (HI) apparent temperature. Create tables or curves of heat index, and compare with the table in this chapter (after converting the results to Celsius). This expression is designed for use in a spreadsheet, where the (Tf) and (RH) factors must be replace with the appropriate cell references for temperature in Fahrenheit and relative humidity (0 - 100) in percent.

HI (°F) = -42.379
+ 2.04901523*(*Tf*)
+ 10.14333127*(*RH*)
– 0.22475541*(*Tf*)*(*RH*)
– (6.83783*10^(-3))*(*Tf*^(2))
– (5.481717*10^(-2))*(*RH*^(2))
+ (1.22874*10^(-3))*(*Tf*^(2))*(*RH*)
+ (8.5282*10^(-4))*(*Tf*)*(*RH*^(2))
– (1.99*10^(-6))*(*Tf*^(2))*(*RH*^(2))

where "^" means "raised to the power of". This eq. is adapted from web site http://www.usatoday.com/weather/whumcalc.htm.

Web-Enhanced Questions

W1(§). Access today's actual temperature sounding from the web for a rawinsonde station close to you (or for another sounding station specified by your instructor). Convert the resulting temperatures to potential temperatures, and plot the resulting θ vs. z.

W2. Access upper-air soundings from the web for a rawinsonde launch site near you (or for another sounding station specified by your instructor). What type of thermodynamic diagram is it plotted on? Can you identify which lines are the isobars; the isotherms, the dry adiabats? There are many different types of thermodynamic diagrams in use around the world, so it is important to learn how to identify the isopleths in each type.

W3. Access the current temperature and wind at your town or location (or for another place as specified by your instructor). Then access the temperature from a town upwind of you. Calculate the portion of heating or cooling rate at your location due only to advection.

W4. Access from the web a weather map or other weather report that shows the observed near-surface air temperature just before sunrise at your location (or at another location specified by your instructor). For the same location, find a map or report of the temperature in mid afternoon. From these two observations, calculate the rate of temperature change over that time period. Also, qualitatively describe which terms in the Eulerian heat budget might be largest. (Hint: if windy, then perhaps advection is important. If clear skies, then heat transfer from the solar-heated ground might be important. Access other weather maps as needed to determine which physical process is most important for the temperature change.)

W5. During winter, access a web report of wind chill for your location (or a different location as specified by your instructor). Can you find on the web a weather map that plots the wind chill? In addition to checking official weather agencies, also check local and network TV web pages.

W6. During summer, access a web report of heat index, temperature-humidity index, or apparent temperature for your location (or a different location specified by your instructor). Can you find on the web a weather map that plots any of these? In addition to checking official weather agencies, also check local and network TV web pages.

Synthesis Questions

S1. What if the sign of the latent heat of condensation or vaporization were opposite. Namely, when liquid water evaporates it heats the air, and when it condenses it cools the air. What would happen to the Earth's oceans as water evaporates from the surface?

S2. Suppose that zero latent heat was associated with the phase changes of water. How would the weather and climate be different, if at all?

S3. Suppose the adiabatic lapse rate had the opposite sign, so that air parcels become warmer as they rise adiabatically. Assuming a standard atmosphere in the troposphere, what would happen to an air parcel from the middle of the troposphere that would be displace slightly up or down?

S4. Suppose that for each 1 km rise of an air parcel, the parcel mixes with an equal mass of surrounding environmental air. How would the process lapse rate for this rising air parcel be different (if at all) from the lapse rate of an adiabatically rising air parcel (having no mixing).

S5. Macro thermodynamics (the kind we've used in this chapter) considers the statistical state of a large collection of molecules that frequently collide with each other, and how they interact on average with their surroundings. Can this same macro thermodynamics be used in the exosphere, where individual air molecules are very far apart (i.e., have a large mean-free path) and rarely interact? Why? Also, explain if/how heat budgets can be used in the exosphere.

S6. Could there be situations where environmental and process lapse rates are equal? If so, give some examples.

S7. Suppose that the virtual potential temperature was not affected by the amount of solid or liquid water in the air. How would weather and climate change, if at all?

S8. The background of the thermo diagram of Fig. 3.3 is an orthogonal grid, where the isotherms are plotted perpendicular to the isobars. Suppose you were to devise a new thermo diagram with the dry adiabats perpendicular to the isobars. On such a diagram, how would the isotherms be drawn? To answer this, draw a sketch of this new diagram, showing the isobars, adiabats, and isotherms. (Do this as a conceptual exercise, not by solving equations to get numbers.)

S9. What if the Earth's crust and surface were made of a metal such a aluminum many hundreds of meters thick. Speculate about the importance of conduction on the surface heat budget, and other implications about the weather.

S10. Lagrangian frameworks follow air parcels as they move. Eulerian frameworks remain fixed relative to the Earth's surface.

What if you were on a cruise ship moving at constant speed from east to west across the ocean.

Devise a heat budget equation relative to this framework. You may name this equation after yourself. Check that it works for any wind direction, and in the limits of zero ship speed.

S11. For the real Earth during cloud-free conditions, most visible light is absorbed by the Earth's surface, while the troposphere is mostly transparent. Suppose that the troposphere were translucent to visible light, but opaque enough that the absorption of sunlight was spread uniformly over the depth of the troposphere, with no sunlight reaching the surface. How would atmospheric structure and weather be different, if at all?

S12. Suppose that the thermometers and hygrometers on the instrumented tower sketched in Fig. 3.10 had random errors of 10% of their magnitudes. Would this error affect the surface flux estimate using the Bowen ratio method? If so, what is the sensitivity (what percentage error does the flux have, compared to the percentage error that the input measurements had)?

S13. The wind-chill concept shows how it feels colder when it is winder. For situations where the wind chill is much colder than the actual air temperature, to what temperature will an automobile engine cool after it is turned off? Why? (Assume the car is parked outside and is exposed to the wind.)