

MOISTURE

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4 Water vapor is one of the gases in air. Unlike nitrogen and oxygen, water-vapor concentration can vary widely in time and space. Most people are familiar with relative humidity as a measure of water-vapor concentration because it affects our body's moisture regulation. But other humidity variables are much more useful in other contexts.

Storms get much of their energy from water vapor, because when water vapor condenses or freezes it releases latent heat. For this reason we carefully track water vapor as it rises in buoyant thermals or is carried by horizontal winds. The amount of moisture available to a storm also regulates the amount of rain or snow precipitating out.

What allows air to hold water as vapor in one case, but forces the vapor to condense in another? This depends on a concept called “saturation”.

SATURATION VAPOR PRESSURE

Vapor Pressure

All the gases in air contribute to the total pressure. The pressure associated with any one gas in a mixture is called the **partial pressure**. The partial pressure of water vapor in air is called **vapor pressure**. Symbol e is used for water-vapor pressure. Units are pressure units, such as kPa.

Saturation

Air can hold any proportion of water vapor. However, for humidities greater than a threshold called the **saturation humidity**, water vapor tends to condense into liquid faster than it re-evaporates. This condensation process lowers the humidity toward the equilibrium (saturation) value. The process is so fast that humidities rarely exceed the equilibrium value. Thus, while air can hold any portion of water vapor, the threshold is rarely exceeded by more than 1% in the real atmosphere.

Air that contains this threshold amount of water vapor is **saturated**. Air that holds less than that amount is **unsaturated**. The equilibrium (saturation) value of vapor pressure over a flat surface of



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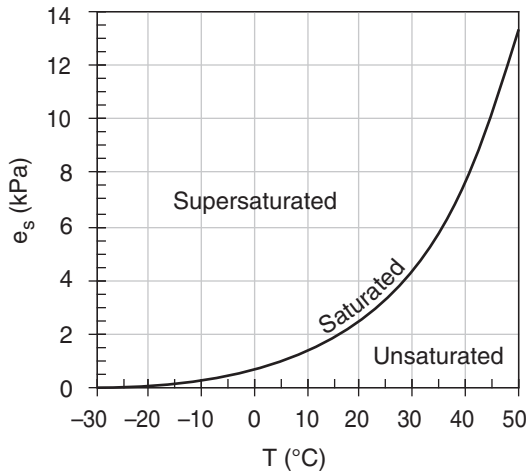


Figure 4.1
 Saturation vapor pressure over a flat surface of pure water. A blowup of the portion of this curve colder than 0°C is given in the next figure.

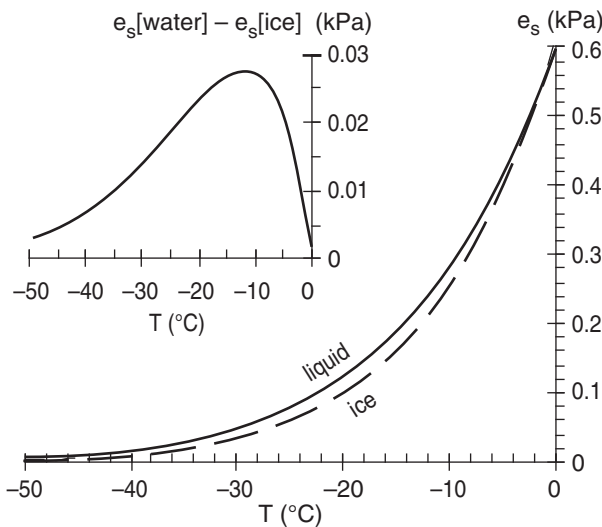


Figure 4.2
 Saturation vapor pressure over flat surfaces of pure liquid water and ice, at temperatures below 0°C. The insert shows difference between saturation vapor pressures over water and ice.

Solved Example

Find the saturation vapor pressure for $T = 21^\circ\text{C}$?

Solution

Given: $T = 21^\circ\text{C} = 294\text{ K}$

Find: $e_s = ?\text{ kPa}$. Use eq. (4.1) for liquid water.

$$e_s = (0.611\text{ kPa}) \cdot \exp\left[(5423\text{ K}) \cdot \left(\frac{1}{273\text{ K}} - \frac{1}{303\text{ K}} \right) \right]$$

$$= (0.611\text{ kPa}) \cdot \exp(1.419) = \mathbf{2.525\text{ kPa}}$$

Check: Units OK. Physics OK. Agrees with Table 4-1.

Discussion: Compared to average sea-level pressure $P = 101.3\text{ kPa}$, this is roughly 2.5% of the total.

pure liquid water is given the symbol e_s . For unsaturated air, $e < e_s$.

Air can be slightly **supersaturated** ($e > e_s$) when there are no surfaces upon which water vapor can condense (i.e., unusually clean air, with no cloud condensation nuclei, and no liquid or ice particles). Temporary supersaturation might occur when the threshold value drops so quickly that condensation does not remove water vapor fast enough. However, photographs of air flow over aircraft wings for both subsonic and supersonic flight through humid air indicate that condensation to form cloud droplets occurs almost instantly.

Even at saturation, there is a continual exchange of water molecules between the liquid water and the air. The evaporation rate depends mostly on the temperature of the liquid water. The condensation rate depends mostly on the humidity in the air. At equilibrium these two rates balance.

If the liquid water temperature increases, then evaporation will temporarily exceed condensation, and the number of water molecules in the air will increase until a new equilibrium is reached. Thus, the equilibrium (saturation) humidity increases with temperature. The net result is that warmer air can hold more water vapor at equilibrium than cooler air.

The **Clausius-Clapeyron** equation describes the relationship between temperature and saturation vapor pressure, which is approximately:

$$e_s \approx e_o \cdot \exp\left[\frac{L}{\mathfrak{R}_v} \cdot \left(\frac{1}{T_o} - \frac{1}{T} \right) \right] \quad \bullet(4.1a)$$

where $e_o = 0.611\text{ kPa}$ and $T_o = 273.15\text{ K}$ are constant parameters, and $\mathfrak{R}_v = 461\text{ J}\cdot\text{K}^{-1}\cdot\text{kg}^{-1}$ is the gas constant for water vapor. Absolute temperature in Kelvin must be used for T in eq. (4.1).

Because clouds can consist of liquid droplets and ice crystals suspended in air, you must consider saturations with respect to water and ice. Over a flat water surface, use the latent heat of vaporization $L = L_v = 2.5 \times 10^6\text{ J}\cdot\text{kg}^{-1}$ in eq. (4.1), which gives $L_v/\mathfrak{R}_v = 5423\text{ K}$. Over a flat ice surface, use the latent heat of deposition $L = L_d = 2.83 \times 10^6\text{ J}\cdot\text{kg}^{-1}$, which gives $L_d/\mathfrak{R}_v = 6139\text{ K}$.

Saturation vapor pressure is listed in Table 4-1 and plotted in Figs. 4.1 and 4.2. These were produced by solving eq. (4.1) on a computer spreadsheet. The exponential-like increase of water-vapor holding capacity with temperature has tremendous impact on clouds and storms. **Supercooled** (unfrozen) liquid water can exist between temperatures of 0 to -40°C , hence, the curve for saturation over water includes temperatures below 0°C .

The Clausius-Clapeyron equation also describes the relationship between water-vapor pressure e and dew-point temperature (T_d , to be defined later):

$$e = e_o \cdot \exp \left[\frac{L}{\mathfrak{R}_v} \cdot \left(\frac{1}{T_o} - \frac{1}{T_d} \right) \right] \quad (4.1b)$$

Table 4-1. (1) Saturation values of humidity vs. actual air temperature (T). -or- (2) Actual humidities vs. dew-point temperature (T_d).

Values are for over a flat surface of liquid water. Mixing ratio r and specific humidity q values are for sea-level pressure. Vapor-pressure e and absolute humidity ρ_v values are for any pressure. Subscript s denotes a saturation value.

For P = 101.325 kPa				
T	e_s	r_s	q_s	ρ_{vs}
or				
T_d	e	r	q	ρ_v
(°C)	(kPa)	(g/kg)	(g/kg)	(g/m ³)
-20	0.127	0.78	0.78	1.09
-18	0.150	0.92	0.92	1.28
-16	0.177	1.09	1.09	1.50
-14	0.209	1.28	1.28	1.75
-12	0.245	1.51	1.51	2.04
-10	0.287	1.77	1.76	2.37
-8	0.335	2.07	2.06	2.75
-6	0.391	2.41	2.40	3.18
-4	0.455	2.80	2.80	3.67
-2	0.528	3.26	3.25	4.22
0	0.611	3.77	3.76	4.85
2	0.706	4.37	4.35	5.57
4	0.814	5.04	5.01	6.37
6	0.937	5.80	5.77	7.28
8	1.076	6.68	6.63	8.30
10	1.233	7.66	7.60	9.45
12	1.410	8.78	8.70	10.73
14	1.610	10.05	9.95	12.17
16	1.835	11.48	11.35	13.77
18	2.088	13.09	12.92	15.56
20	2.371	14.91	14.69	17.55
22	2.688	16.95	16.67	19.76
24	3.042	19.26	18.89	22.22
26	3.437	21.85	21.38	24.94
28	3.878	24.76	24.16	27.94
30	4.367	28.02	27.26	31.27
32	4.911	31.69	30.72	34.93
34	5.514	35.81	34.57	38.96
36	6.182	40.43	38.86	43.40
38	6.921	45.61	43.62	48.27
40	7.736	51.43	48.91	53.62
42	8.636	57.97	54.79	59.47
44	9.627	65.32	61.31	65.88
46	10.717	73.59	68.54	72.87
48	11.914	82.91	76.56	80.51
50	13.228	93.42	85.44	88.84

BEYOND ALGEBRA • Clausius-Clapeyron

Historical Underpinning

To build better steam engines during the industrial revolution, engineers were working to discover the thermodynamics of water vapor. Steam engineers B.-P.-E. Clapeyron in 1834 and R. Clausius in 1879 applied the principles of S. Carnot (early 1800s), to study isothermal compression of pure water vapor in a cylinder. They measured the saturation vapor pressure at the point where condensation occurred.

Variation of Vapor Pressure with Temperature

By repeating the experiment for various temperatures, they empirically found that:

$$\frac{de_s}{dT} = \frac{L_v}{T} \left[\frac{1}{\rho_v} - \frac{1}{\rho_L} \right]^{-1}$$

where ρ_v is the density of water vapor, and ρ_L is the density of liquid water. But $\rho_L \gg \rho_v$ (see Appendix B and Table 4-1), thus $1/\rho_L \ll 1/\rho_v$, allowing us to neglect the ρ_L term:

$$\frac{de_s}{dT} \cong \frac{L_v}{T} \rho_v$$

The ideal gas law can be used to relate the water vapor pressure to the vapor density: $e_s = \rho_v \cdot \mathfrak{R}_v \cdot T$, where \mathfrak{R}_v is the gas constant for water vapor. Solving for ρ_v and plugging the result into the eq. above gives:

$$\frac{de_s}{dT} \cong \frac{L_v \cdot e_s}{\mathfrak{R}_v \cdot T^2}$$

Use the Separation of Variables method: move all e_s terms to the left, all T terms to the right:

$$\frac{de_s}{e_s} \cong \frac{L_v}{\mathfrak{R}_v} \frac{dT}{T^2}$$

Then integrate, taking care with the limits:

$$\int_{e_o}^{e_s} \frac{de_s}{e_s} \cong \frac{L_v}{\mathfrak{R}_v} \int_{T_o}^T \frac{dT}{T^2}$$

where e_o is a known vapor pressure at reference temperature T_o . The integration result is:

$$\ln \left(\frac{e_s}{e_o} \right) \cong - \frac{L_v}{\mathfrak{R}_v} \left[\frac{1}{T} - \frac{1}{T_o} \right]$$

Taking the antilog (exp) of both sides gives eq. (4.1).

$$e_s = e_o \cdot \exp \left[\frac{L_v}{\mathfrak{R}_v} \cdot \left(\frac{1}{T_o} - \frac{1}{T} \right) \right] \quad (4.1)$$

See C. Bohren and B. Albrecht [1998: *Atmospheric Thermodynamics*, Oxford, 402pp] for details, and for an interesting historical discussion.

FOCUS • Boiling**Definition:**

Boiling is the state where ambient atmospheric pressure equals the equilibrium (saturation) vapor pressure:

$$P = e_s$$

Derivation of Boiling T vs. z :

Because ambient pressure decreases roughly exponentially with height, we can use (1.9b) to replace the left hand side (LHS) of the equation above by $P = P_o \cdot \exp(-z/H_p)$, where $P_o = 101.325$ kPa is sea level pressure (at $z = 0$), and $H_p \approx 7.29$ km is the scale height for pressure. The right hand side (RHS) can be replaced with the Clausius-Clapeyron eq. (4.1), which describes how this boiling pressure is related to T . The net result is:

$$P_o \cdot \exp\left[-\frac{z}{H_p}\right] = e_o \cdot \exp\left[\frac{L_v}{\mathfrak{R}_v} \cdot \left(\frac{1}{T_o} - \frac{1}{T}\right)\right]$$

where $L_v/\mathfrak{R}_v = 5423$ K. Dividing both sides by P_o , and using the trick that $e_o/P_o = \exp[\ln(e_o/P_o)]$, gives:

$$\exp\left[-\frac{z}{H_p}\right] = \exp\left[\ln\left(\frac{e_o}{P_o}\right) + \frac{L_v}{\mathfrak{R}_v} \left(\frac{1}{T_o} - \frac{L_v}{\mathfrak{R}_v T}\right)\right]$$

But the first two terms in square brackets on the RHS are constants, and can be grouped as a dimensionless constant a . Taking the \ln of both sides gives:

$$\frac{z}{H_p} = \frac{L_v}{\mathfrak{R}_v} \frac{1}{T} - a$$

Rearranging to solve for temperature T (which equals $T_{boiling}$ because we had set $P = e_s$):

$$T_{boiling} = \frac{L_v / \mathfrak{R}_v}{a + z / H_p}$$

By definition of the Celsius temperature scale, $T = 100^\circ\text{C} = 373.15$ K for boiling at sea level $z = 0$. Using these values of z and T in the eq. above allows us solve for a , yielding $a = 14.53$ (dimensionless).

Discussion:

At $z = 1$ km, the eq. above gives $T_{boiling} = 96.6^\circ\text{C}$, and at $z = 2$ km, $T_{boiling} = 93.2^\circ\text{C}$. This is an average decrease of $3.4^\circ\text{C} / \text{km}$.

To soften vegetables to the desired tenderness or to prepare meats to the desired doneness, foods must be cooked at a certain temperature over a certain time duration. Slightly cooler cooking temperatures must be compensated with slightly longer cooking times. Thus, boiled foods take longer to cook at higher altitude because the boiling temperature is cooler.

Tetens' formula is an empirical expression for saturation vapor pressure e_s with respect to liquid water:

$$e_s = e_o \cdot \exp\left[\frac{b \cdot (T - T_1)}{T - T_2}\right] \quad (4.2)$$

It includes the variation of latent heat with temperature, where $e_o = 0.611$ kPa, $b = 17.2694$, $T_1 = 273.15$ K, and $T_2 = 35.86$ K. Although this formula depends on temperature differences (for which $^\circ\text{C}$ or K can normally be used), absolute temperature T in units of Kelvins must be used here because the parameters T_1 and T_2 are given in Kelvins.

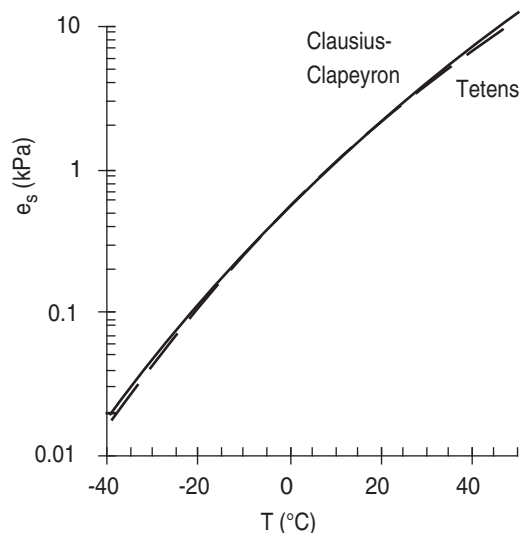
Solved Example

Compare Tetens' formula with the Clausius-Clapeyron equation for a variety of temperatures

Solution

Use eqs. (4.1) and (4.2) in a spreadsheet.

The result below on a semi-log graph shows the Clausius-Clapeyron equation as a solid line, and Tetens' formula as a dashed line.



Check: Units OK. Physics OK. Agrees with Table 4-1.

Discussion: Both formulae are extremely close over a wide range of temperatures.

Science Graffito

"I think, therefore I am." — René Descartes.

HUMIDITY VARIABLES

Mixing Ratio

The ratio of mass m of water vapor to mass of dry air is called the mixing ratio, r :

$$r = \frac{m_{\text{water vapor}}}{m_{\text{dry air}}} \quad (4.3)$$

where $m_{\text{dry air}} = m_{\text{total air}} - m_{\text{water vapor}}$. To put this into a more useful form, divide the numerator and denominator by a common volume to get a ratio of densities, substitute the ideal gas law for each gas, and assume a common temperature for all gases. The result is:

$$r = \frac{\varepsilon \cdot e}{P - e} \quad (4.4)$$

where $\varepsilon = \mathfrak{R}_d/\mathfrak{R}_v = 0.622 \text{ g}_{\text{vapor}}/\text{g}_{\text{dry air}} = 622 \text{ g/kg}$ is the ratio of gas constants for dry air to that for water vapor. Eq. (4.4) says that r is proportional to the ratio of partial pressure of water vapor (e) to partial pressure of the remaining gases in the air ($P - e$), where P is total air pressure.

The **saturated mixing ratio**, r_s , is similar to eq. (4.4), except with saturation vapor pressure e_s in place of e .

$$r_s = \frac{\varepsilon \cdot e_s}{P - e_s} \quad (4.5)$$

Examples of mixing ratio values are given in Table 4-1, for air at sea level, where the Clausius-Clapeyron equation or Tetens' formula is used to find e_s .

Although units of mixing ratio are "g/g" (i.e., grams of water vapor per gram of dry air), it is usually presented as "g/kg" (i.e., grams of water vapor per kilogram of dry air) by multiplying the "g/g" value by 1000. By convention in meteorology we don't reduce the ratio "g/g" to 1, because the numerator and denominator represent masses of different substances that must be discerned in mass budgets.

Specific Humidity

The ratio of mass m of water vapor to mass of total (moist) air is called the specific humidity, q , and to a good approximation is given by:

$$q = \frac{m_{\text{water vapor}}}{m_{\text{total air}}} \quad (4.6)$$

or

$$q = \frac{\varepsilon \cdot e}{P_d + \varepsilon \cdot e} = \frac{\varepsilon \cdot e}{P - e \cdot (1 - \varepsilon)} \quad (4.7)$$

where $\varepsilon = 0.622 \text{ g/g} = 622 \text{ g/kg}$, and P is pressure. Specific humidity q has units of g/g or g/kg.

For **saturation specific humidity**, q_s :

$$q_s = \frac{\varepsilon \cdot e_s}{P_d + \varepsilon \cdot e_s} \quad (4.8)$$

Both the saturation mixing ratio and saturation specific humidity depend on ambient pressure, but the saturation vapor pressure does not. Thus, in Table 4-1 the vapor pressure numbers are absolute numbers that can be used anywhere, while the other humidity variables are given at sea-level pressure and density. The saturation values of mixing ratio and specific humidity can be calculated for any other ambient pressure using eqs. (4.5) and (4.8).

Absolute Humidity

The concentration ρ_v of water vapor in air is called the absolute humidity, and has units of grams of water vapor per cubic meter of air (g/m^3). Absolute humidity is essentially a partial density:

$$\rho_v = \frac{m_{\text{water vapor}}}{\text{Volume}} \quad (4.9)$$

where m is mass and Vol is volume. Absolute humidity can be reformulated using the ideal gas law for water vapor:

$$\rho_v = \frac{e}{\mathfrak{R}_v \cdot T} \quad (4.10)$$

where $\mathfrak{R}_v = 4.61 \times 10^{-4} \text{ kPa} \cdot \text{K}^{-1} \cdot \text{m}^3 \cdot \text{g}^{-1}$ is the gas constant for water vapor. Alternately, by replacing temperature T using the ideal gas law for dry air:

$$\rho_v = \frac{e \cdot \varepsilon \cdot \rho_d}{P - e} \approx \frac{e}{P} \cdot \varepsilon \cdot \rho_d \quad (4.11)$$

where ρ_d is the density of dry air, e is the vapor pressure, $\varepsilon = \mathfrak{R}_d/\mathfrak{R}_v = 622 \text{ g/kg}$, and P is total air pressure. As discussed in Chapter 1, dry-air density is roughly $\rho_d = 1.225 \text{ kg/m}^3$ at sea level, and varies with altitude, pressure, and temperature according to the ideal gas law.

The **saturated** value of **absolute humidity**, ρ_{vs} , is found by using e_s in place of e in the equations above:

$$\rho_{vs} = \frac{e_s}{\mathfrak{R}_v \cdot T} \quad (4.12)$$

or

$$\rho_{vs} = \frac{e_s \cdot \varepsilon \cdot \rho_d}{P - e_s} \approx \frac{e_s}{P} \cdot \varepsilon \cdot \rho_d \quad (4.13)$$

Solved Example

Find the saturation values of mixing ratio, specific humidity, and absolute humidity for air of temperature 0°C and pressure 50 kPa.

Solution

Given: $T = 0^\circ\text{C}$, $P = 50 \text{ kPa}$

Find: $r_s = ? \text{ g/kg}$, $q_s = ? \text{ g/kg}$, $\rho_{vs} = ? \text{ g/m}^3$

$e_s = 0.611 \text{ kPa}$ was found directly from Table 4-1 because it does not depend on P .

Use eq. (4.5) for r_s :

$$r_s = [0.622 \cdot (0.611 \text{ kPa})] / [50 \text{ kPa} - 0.611 \text{ kPa}] = 0.00770 \text{ g/g} = \underline{7.70 \text{ g/kg}}$$

Use eq. (4.8) for q_s :

$$q_s = 0.622 \cdot (0.611 \text{ kPa}) / 50 \text{ kPa} = 0.00760 \text{ g/g} = \underline{7.60 \text{ g/kg}}$$

Use eq. (4.12) for ρ_{vs} :

$$\rho_{vs} = (0.611 \text{ kPa}) / [(461 \text{ J}\cdot\text{K}^{-1}\cdot\text{kg}^{-1}) \cdot (273 \text{ K})] = \underline{0.00485 \text{ kg}\cdot\text{m}^{-3}} = \underline{4.85 \text{ g}\cdot\text{m}^{-3}}$$

Check: Units OK. Physics OK.

Discussion: The first two values are roughly double those for sea-level pressure (see Table 4-1). The reason is that although the partial pressure of water vapor held in the air is the same, partial pressure of dry air is less. The last value equals the value in Table 4-1, because there is no P dependence.

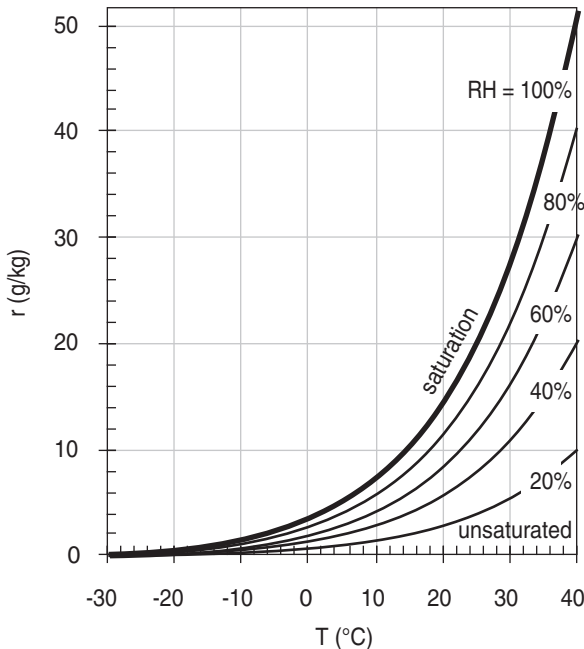


Figure 4.3 Mixing ratio at sea level vs. relative humidity and temperature.

Relative Humidity

The ratio of the actual amount of water vapor in the air compared to the equilibrium (saturation) amount at that temperature is called the relative humidity, RH :

$$\frac{RH\%}{100\%} = \frac{q}{q_s} = \frac{\rho_v}{\rho_{vs}} \approx \frac{r}{r_s} \quad \bullet(4.14)$$

where e is vapor pressure, q is specific humidity, ρ_v is absolute humidity, r is mixing ratio, and subscript s indicates saturation.

Relative humidity regulates the amount of net evaporation that is possible into the air, regardless of the temperature. At $RH = 100\%$ no net evaporation occurs because the air is already saturated. Variation of mixing ratio with RH is shown in Fig. 4.3.

Dew-Point Temperature

The temperature to which air must be cooled in order to become saturated at constant pressure is called the **dew-point temperature**, T_d , (also known as the **dew point**). It is given by eq. (4.1) or found from Table 4-1 by using vapor pressure e in place of e_s and T_d in place of T . Making those substitutions and solving for T_d yields:

$$T_d = \left[\frac{1}{T_o} - \frac{\mathfrak{R}_v}{L} \cdot \ln\left(\frac{e}{e_o}\right) \right]^{-1} \quad (4.15a)$$

with $e_o = 0.611 \text{ kPa}$, $T_o = 273 \text{ K}$, $\mathfrak{R}_v/L_v = 1.844 \times 10^{-4} \text{ K}^{-1}$.

Eq. (4.15a) can be combined with the definition of mixing ratio r to give:

$$T_d = \left[\frac{1}{T_o} - \frac{\mathfrak{R}_v}{L} \ln\left(\frac{r \cdot P}{e_o \cdot (r + \epsilon)}\right) \right]^{-1} \quad \bullet(4.15b)$$

where $\epsilon = \mathfrak{R}_d/\mathfrak{R}_v = 0.622 \text{ g}_{\text{vapor}}/\text{g}_{\text{dry air}}$.

Saturation (equilibrium) with respect to a flat surface of liquid water occurs at a slightly colder temperature than saturation with respect to a flat ice surface. With respect to liquid water, use $L = L_v$ in the equation above, where T_d is called the **dew-point temperature**. With respect to ice, $L = L_d$, and T_d is called the **frost-point temperature** (see the "Saturation" subsection for values of L).

If $T_d = T$, the air is saturated. The **dew-point depression** or **temperature-dew-point spread** ($T - T_d$) is a relative measure of the dryness of the air. T_d is usually less than T (except during supersaturation, where it might be a fraction of a degree warmer). If air is cooled below the initial dew-point temperature, then the dew-point temperature drops to remain equal to the air temperature, and

Solved Example

Find the relative humidity for air of $T = 20^\circ\text{C}$ and $e = 1$ kPa.

Solution

Given: $T = 20^\circ\text{C}$, $e = 1$ kPa

Find: $RH = ?\%$

From Table 4-1 at $T = 20^\circ\text{C}$: $e_s = 2.371$ kPa.

Use eq. (4.14):

$$RH = 100\% \cdot (1 \text{ kPa} / 2.371 \text{ kPa}) = \underline{42\%}.$$

Check: Units OK. Physics OK.

Discussion: The result does not depend on P .

the excess water condenses or deposits as dew, frost, fog, or clouds (see the Cloud and Precip. chapters).

Dew-point temperature is easy to measure and provides the most accurate humidity value, from which other humidity variables can be calculated (see Humidity Instruments section at the end of this chapter). When dew (sweat) forms on the outside of a cold beverage can or bottle, you know that the outside of the beverage container is colder than the dew-point temperature of the surrounding air.

Saturation Level or Lifting Condensation Level (LCL)

When unsaturated air is lifted, it cools at the dry adiabatic lapse rate. If lifted high enough, the temperature will drop to the dew-point temperature, and clouds will form. Dry air (air of low relative humidity) must be lifted higher than moist air. Saturated air needs no lifting at all. Hence, LCL is a measure of humidity.

When lifting non-cloudy air, the height at which saturation just occurs (with no supersaturation) is the **saturation level** or the **lifting condensation level (LCL)**. For convective (cumuliform) clouds, **cloud base** occurs there. LCL height (distance above the height where T and T_d are measured) for cumuliform clouds is very well approximated by:

$$z_{LCL} = a \cdot (T - T_d) \quad \bullet(4.16a)$$

where $a = 0.125 \text{ km}/^\circ\text{C}$.

The pressure P_{LCL} at the LCL is

$$P_{LCL} = P \cdot \left[1 - b \cdot \left(\frac{T - T_d}{T} \right) \right]^{C_p/\mathfrak{R}} \quad (4.16b)$$

where $b = a \cdot \Gamma_d = 1.225$ (dimensionless), $C_p/\mathfrak{R} = 3.5$ (dimensionless), and P is the pressure at the initial

Solved Example

Given desert conditions with a temperature of 30°C and pressure of 100 kPa, find the dew-point temperature for a relative humidity of 20%.

Solution

Given: $T = 30^\circ\text{C}$, $P = 100$ kPa, $RH = 20\%$

Find: $T_d = ?^\circ\text{C}$

From Table 4-1, $e_s = 4.367$ kPa.

Use eq. (4.14): $e = 0.2 \cdot (4.367 \text{ kPa}) = 0.8734$ kPa

We could use eq. (4.15) to solve for T_d , but it is easier to look up the answers in Table 4-1. $T_d = \underline{5^\circ\text{C}}$.

Check: Units OK. Physics OK.

Discussion: The result does not depend on P . This dry air would need to be cooled 25°C before condensation would occur.

Solved Example

Find the dew point for $r = 10 \text{ g/kg}$ and $P = 80$ kPa.

Solution

Given: $r = 0.01 \text{ g}_{\text{vapor}}/\text{g}_{\text{air}}$, $P = 80$ kPa

Find: $T_d = ?^\circ\text{C}$

Use eq. (4.15b): $T_d = [(1/(273\text{K}) - (1.844 \times 10^{-4} \text{ K}^{-1}) \cdot \ln\{ [(80\text{kPa}) \cdot (0.01 \text{ g}_{\text{vapor}}/\text{g}_{\text{air}})] / [(0.611\text{kPa}) \cdot (0.01 \text{ g}_{\text{vapor}}/\text{g}_{\text{air}}) + (0.622 \text{ g}_{\text{vapor}}/\text{g}_{\text{air}})] \})^{-1} - (1/(273\text{K}) - (1.844 \times 10^{-4} \text{ K}^{-1}) \cdot \ln(2.07))]^{-1} = 283.4\text{K}$.
Thus, $T_d = 283.4 - 273 = \text{or } \underline{10.4^\circ\text{C}}$

Check: Units OK. Magnitude OK.

Discussion: This humidity might occur in the tropics, with a surface temperature warmer than 30°C .

Solved Example

Find the LCL height and pressure, if $T = 20^\circ\text{C}$ and $T_d = 10^\circ\text{C}$ at $P = 95$ kPa.

Solution

Given: $T = 20^\circ\text{C}$, $T_d = 10^\circ\text{C}$, $P = 95$ kPa.

Find: $z_{LCL} = ? \text{ km}$, $P_{LCL} = ? \text{ kPa}$

Use eq. (4.16a)

$$z_{LCL} = (0.125 \text{ km}/^\circ\text{C}) \cdot (20 - 10^\circ\text{C}) \\ = \underline{1.25 \text{ km above the starting height}}$$

Use eq. (4.16b)

$$P_{LCL} = (95 \text{ kPa}) \cdot [1 - 1.225 \cdot (20 - 10^\circ\text{C})/293\text{K}]^{3.5} \\ = \underline{81.8 \text{ kPa}}$$

Check: Units OK. Physics OK.

Discussion: Reasonable height for cumulus cloud base. Also, since pressure decreases about 10 kPa for each increase of 1 km of altitude near the surface, the pressure answer is also reasonable.

Solved Example

At a pressure of 100 kPa, what is the mixing ratio if the dry and wet-bulb temperatures of a psychrometer are 20°C and 14°C, respectively? Use the equations here, not the look-up tables or graphs.

Solution

Given: $T = 20^\circ\text{C}$, $T_w = 14^\circ\text{C}$, $P = 100\text{ kPa}$

Find: $r = ?\text{ g/kg}$

First, solve eq. (4.18): $r_w =$

$$\frac{622\text{g/kg}}{(1.631\text{kPa}^{-1}) \cdot (90\text{kPa}) \cdot \exp\left(\frac{-17.67 \cdot 18^\circ\text{C}}{18^\circ\text{C} + 243.5^\circ\text{C}}\right) - 1}$$

$r_w = 10.13\text{ g/kg}$.

Then solve eq. (4.19):

$$r = (10.13\text{g/kg}) - [0.40224 (\text{g/kg}/^\circ\text{C}) \cdot (20 - 14^\circ\text{C})] \\ = \mathbf{7.72\text{ g/kg}}$$

Check: Units OK. Physics OK.

Discussion: Now we can compare our answer with the data in graph (Fig. 4.4). Using $T_w = 14^\circ\text{C}$ along the ordinate, and $T - T_w = 20 - 14 = 6^\circ\text{C}$ as the dew-point depression along the abscissa, we find that our answer agrees with the graph. Obviously, for every-day use, it is easier to use the psychrometric graph, or the corresponding psychrometric tables that are published in other books.

Solved Example

For the previous solved example, use its inputs and result to calculate the relative humidity. Again, don't use the look-up tables or graphs yet.

Solution

Given: $T = 20^\circ\text{C}$, $r = 7.72\text{ g/kg}$, $P = 100\text{ kPa}$

Find: $RH = ?\%$

First, use Tetens' formula (4.2), with the trick that $\Delta T = T(\text{K}) - T_1(\text{K}) = [T(^\circ\text{C}) + 273] - 273 = T(^\circ\text{C})$, and remembering that for temperature differences: $1^\circ\text{C} = 1\text{ K}$.

$$e_s = 0.611(\text{kPa}) \cdot \exp\left[\frac{17.2694 \cdot (20\text{K})}{(20 + 273.16)\text{K} - 35.86\text{K}}\right]$$

$e_s = 2.34\text{ kPa}$.

Next, use this in eq. (4.5):

$$r_s = \frac{(622\text{g/kg}) \cdot 2.34\text{kPa}}{[101.325 - 2.34]\text{kPa}} = 14.7\text{g/kg}$$

Finally, use eq. (4.14): $RH = 100\% \cdot (r/r_s)$

$$RH = 100\% \cdot (7.72/14.7) = \mathbf{52.5\%}$$

Check: Units OK. Physics OK.

Discussion: What a lot of work. If we instead had used psychrometric graph (Fig. 4.5) with $T_w = 14^\circ\text{C}$ $T - T_w = 6^\circ\text{C}$, we would have found almost the same relative humidity much more easily.

height where temperature T and dew-point T_d are measured, such as at the surface. Don't forget to use Kelvin for the T in the denominator.

These expressions do not work for stratiform (advective) clouds, because these clouds are not formed by air rising vertically from the underlying surface. Air in these clouds blows at a gentle slant angle from a surface hundreds to thousands of kilometers away.

Regardless of whether cumuliform clouds exist, the LCL or saturation level can be used as a measure of humidity. It also serves as a measure of total water content for saturated (cloudy) air. For cloudy air, the saturation level is the altitude to which one must lower an air parcel for all of the liquid or solid water to just evaporate. Eqs. (4.16) do not apply to this situation.

Wet-Bulb Temperature

When the bulb of a glass thermometer is covered by a cloth sleeve that is wet, this **wet bulb** becomes cooler than the actual (**dry bulb**) air temperature T because of the latent heat associated with evaporation of water. Drier ambient air allows more evaporation, causing the wet-bulb temperature T_w to cool significantly below the air temperature. For saturated air there is no net evaporation and the wet-bulb temperature equals that of the dry bulb. You can find the humidity from the difference between the dry and wet-bulb temperatures. This difference, called the **wet-bulb depression** ($T - T_w$), is a measure of the relative dryness of the air.

To work properly, the sleeve or wick must be wet with clean or distilled water. The wet bulb should be well ventilated by blowing air past it (**aspirated psychrometer**) or by moving the thermometer through the air by twirling it around on the end of a short chain (**sling psychrometer**). Usually psychrometers have both wet and dry-bulb thermometers mounted next to each other.

Let T_w represent the wet-bulb temperature, and r_w represent the wet-bulb mixing ratio in the air adjacent to the wet-bulb after water has evaporated into it from the sleeve. Because the latent heat used for evaporation comes from the sensible heat associated with cooling, a simple heat balance gives:

$$C_p \cdot (T - T_w) = -L_v \cdot (r - r_w) \quad (4.17)$$

where T and r are ambient (dry-bulb) air temperature and mixing ratio, respectively, C_p is the specific heat of air at constant pressure, and L_v is the latent heat of vaporization.

If the wet and dry-bulb temperatures and ambient pressure P are measured or otherwise known, the desired mixing ratio r can be found by first finding r_w :

$$r_w = \frac{\varepsilon}{b \cdot P \cdot \exp\left(\frac{-c \cdot T_w(^{\circ}\text{C})}{T_w(^{\circ}\text{C}) + \alpha}\right)} \quad (4.18)$$

Then:

$$r = r_w - \beta \cdot (T - T_w) \quad (4.19)$$

where temperatures must have units of $^{\circ}\text{C}$, and where $\varepsilon = 622 \text{ g/kg}$, $b = 1.631 \text{ kPa}^{-1}$, $c = 17.67$, $\alpha = 243.5^{\circ}\text{C}$, and $\beta = 0.40224 \text{ (g/kg)/}^{\circ}\text{C}$. Knowing r , any other moisture variable is easily found.

The easiest way for you to find humidity from dry and wet-bulb temperature is to look-up the humidity in tables called **psychrometric tables**, which are often published in meteorology books. Figures 4.4 and 4.5 present the look-up information as **psychrometric graphs**, which were computed from the equations in this Chapter.

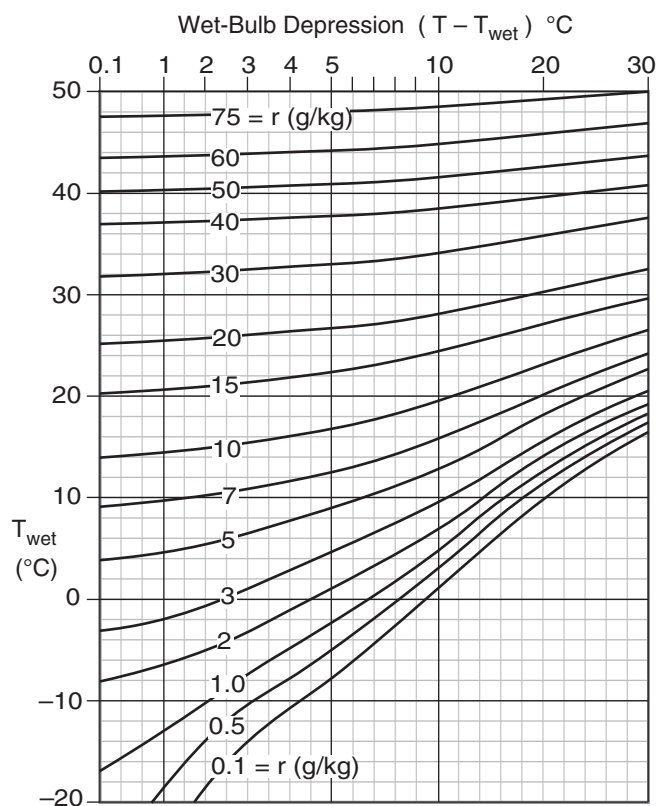


Figure 4.4

Psychrometric graph, to find mixing ratio r from wet and dry-bulb temperatures. Based on $P = 101.325 \text{ kPa}$. Caution, the darker vertical lines mark scale changes along the abscissa.

Solved Example

Find the mixing ratio and relative humidity for air temperature of 12°C and wet-bulb temperature of 10°C . Yes, you may use the graphs this time.

Solution

Given: $T = 12^{\circ}\text{C}$, $T_w = 10^{\circ}\text{C}$

Find: $r = ? \text{ g/kg}$, and $RH = ? \%$

Assume $P = 101.3 \text{ kPa}$, so we can use the Figs.

The wet-bulb depression is $12 - 10 = 2^{\circ}\text{C}$

Use Fig. 4.4. $r = \underline{7 \text{ g/kg}}$.

Use Fig. 4.5. $RH = \underline{78\%}$.

Check: Units OK. Physics OK.

Discussion: Much easier than the solved examples on the previous page. Notice that in Fig. 4.4, r depends mostly on T_w because the mixing-ratio lines are mostly horizontal. However, RH depends mostly on $T - T_w$, because the lines in Fig. 4.5 are mostly vertical.

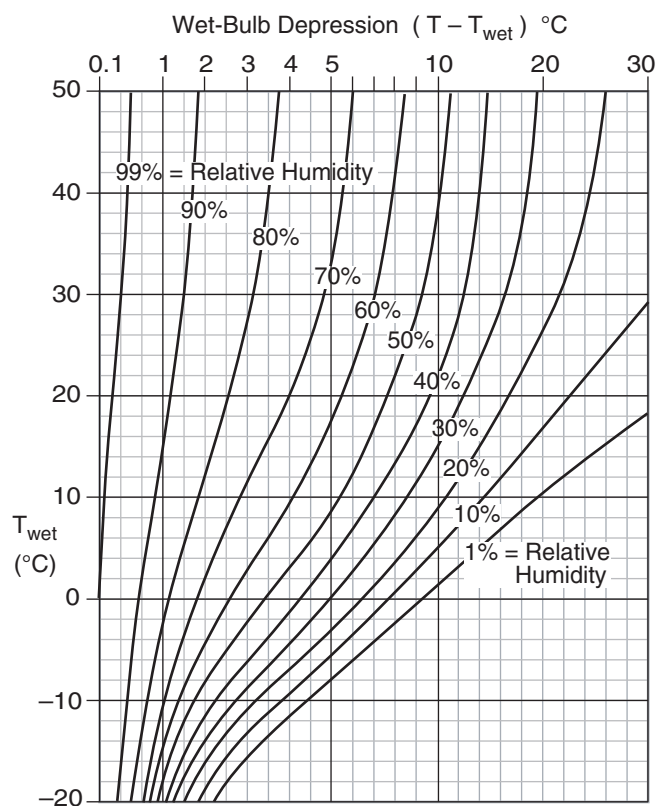


Figure 4.5

Psychrometric graph, to find relative humidity from wet and dry-bulb temperatures. Based on $P = 101.325 \text{ kPa}$. Caution, the darker vertical lines mark scale changes along the abscissa.

Solved Example

Find the wet-bulb temperature for air at sea level with temperature 30°C and dew point 24°C. Assume $\Gamma_s = 3.78 \text{ }^\circ\text{C/km}$ for this case.

Solution

Given: $T = 30^\circ\text{C}$, $T_d = 24^\circ\text{C}$, $P = 101.3 \text{ kPa}$
 Find: $T_w = ?^\circ\text{C}$

Use Normand's Rule:

Step 1, solve eq. (4.16):

$$z_{LCL} = (0.125 \text{ km}/^\circ\text{C}) \cdot (30 - 24^\circ\text{C}) = 0.75 \text{ km.}$$

Step 2, solve eq. (4.20):

$$T_{LCL} = 30 - (9.8 \text{ K/km}) \cdot (0.75 \text{ km}) = 22.65 \text{ }^\circ\text{C.}$$

Step 3, solve eq. (4.21):

$$T_w = 22.65 + (3.78 \text{ }^\circ\text{C/km}) \cdot (0.75 \text{ km}) = \underline{25.5^\circ\text{C.}}$$

Check: Units OK. Physics OK.

Discussion: The resulting wet-bulb depression is $(T - T_w) = 4.5^\circ\text{C}$.

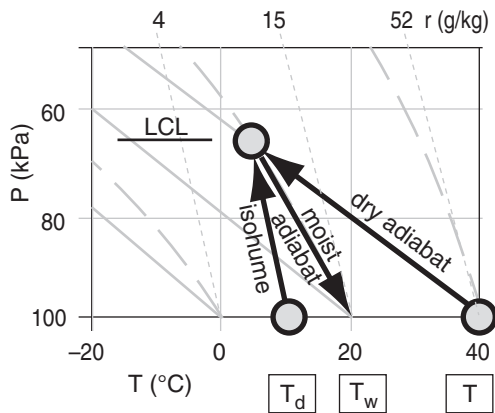


Figure 4.6
 Demonstration of Normand's rule on a thermo diagram. It shows how to find wet-bulb temperature T_w given T and T_d .

FOCUS • Why So Many Moisture Variables?

Vapor pressure, mixing ratio, specific humidity, absolute humidity, relative humidity, dew point depression, saturation level, and wet bulb temperature are different ways to quantify moisture in the air. Why are there so many?

Some variables are useful because they can be measured. Others are useful in conservation equations for water substance, for describing physical characteristics of the air, or for describing how life is affected by humidity. Often we are given or can measure one moisture variable, but must convert it to a different variable to use it.

(continues on next page)

To create your own psychrometric tables or graphs, first generate a table of mixing ratios in a spreadsheet program, using eqs. (4.18) and (4.19). I assumed a standard sea-level pressure of $P = 101.325 \text{ kPa}$ for the figures here. Then contour the resulting numbers to give Fig. 4.4. Starting with the table of mixing ratios, use eqs. (4.2), (4.5), and (4.14) to create a new table of relative humidities, and contour it to give Fig. 4.5. All of these psychrometric tables and graphs are based on Tetens' formula (eq. 4.2).

Finding T_w from T_d is a bit trickier, but is possible using **Normand's Rule**:

• Step 1: Find z_{LCL} using eq. (4.16).

• Step 2: $T_{LCL} = T - \Gamma_d \cdot z_{LCL}$ (4.20)

• Step 3: $T_w = T_{LCL} + \Gamma_s \cdot z_{LCL}$ (4.21)

where Γ_s is the moist lapse rate described later in this chapter, $\Gamma_d = 9.8 \text{ K/km}$ is the dry rate, and T_{LCL} is the temperature at the LCL. Normand's Rule tells us that $T_d \leq T_w \leq T$.

Normand's rule is easy to implement on a thermo diagram. Although isohumes and moist adiabats on thermo diagrams are not introduced until later in this chapter, I demonstrate Normand's rule here for future reference. Follow a dry adiabat up from the given dry-bulb temperature T , and follow an isohume up from the given dew point T_d (Fig. 4.6). At the LCL (where these two isopleths cross), follow a moist adiabat back down to the starting pressure to give the wet-bulb temperature T_w .

More Relationships Between Moisture Variables

Units of g/g must be used for r and q in the relationships below:

$$q = \frac{r}{1+r} \tag{4.22}$$

$$q = \frac{\rho_v}{\rho_d + \rho_v} \tag{4.23}$$

$$q = \left(\frac{\epsilon \cdot e_o}{P} \right) \cdot \exp \left[\frac{L}{\mathfrak{R}_v} \cdot \left(\frac{1}{T_o} - \frac{1}{T_d} \right) \right] \tag{4.24}$$

$$e = \frac{r}{\epsilon + r} \cdot P \tag{4.25}$$

$$r = \frac{\rho_v}{\rho_d} \tag{4.26}$$

For air that is not excessively humid:

$$r \approx q \quad \bullet(4.27a)$$

$$r_s \approx q_s \quad \bullet(4.27b)$$

TOTAL WATER

Liquid and Solid Water

In clouds, fog, or air containing falling precipitation, one measure of the amount of liquid water in the air is the **liquid water content (LWC)**. It is defined as

$$\rho_{LWC} = \frac{m_{liq.water}}{Vol} \quad (4.28)$$

where $m_{liq.water}$ is mass of liquid water suspended or falling through the air, and Vol is the air volume. Typical values in cumulus clouds are $0 \leq \rho_{LWC} \leq 5 \text{ g/m}^3$. It can also be expressed in units of $\text{kg}_{liq.water}/\text{m}^3$. LWC is the liquid-water analogy to the absolute humidity for water vapor.

Another measure is the **liquid-water mixing ratio**:

$$r_L = \frac{m_{liq.water}}{m_{dry air}} \quad (4.29)$$

where $m_{liq.water}$ is the mass of liquid water that is imbedded as droplets within an air parcel that contains $m_{dry air}$ mass of dry air. A similar **ice mixing ratio** can be defined:

$$r_i = \frac{m_{ice}}{m_{dry air}} \quad (4.30)$$

Both mixing ratios have units of $\text{kg}_{water}/\text{kg}_{air}$ or $\text{g}_{water}/\text{kg}_{air}$.

Liquid water content is related to liquid-water mixing ratio by

$$r_L = \frac{\rho_{LWC}}{\rho_{air}} \quad (4.31)$$

where ρ_{air} is air density.

Total-Water Mixing Ratio

Define **total-water mixing ratio** r_T as the mass of water of all phases (vapor, liquid, and ice) per mass of dry air:

$$r_T = r + r_L + r_i \quad \bullet(4.32a)$$

FOCUS • Why So Many Moisture Variables? (continuation)

Directly and easily measurable variables include wet-bulb temperature and dew-point temperature. Wet-bulb temperature is easy to measure by putting a wet cloth sleeve around the bulb of a thermometer, and ventilating the wet bulb. However, it is difficult to convert this into other humidity variables, which is why psychrometric tables and graphs have been developed. Dew point is measured by chilling a mirror until dew forms on it. It is one of the most accurate measurements of humidity.

Absolute humidity can be measured by shining electromagnetic radiation (infrared, ultraviolet, or microwave) across a path of humid air. The attenuation of the signal is a measure of the number of water vapor molecules along the path.

Relative humidity can be measured by: 1) expansion or contraction of organic fibers such as human hairs; 2) the change in electrical conductivity of a carbon powder emulsion on a glass slide; and 3) by how it affects the capacitance across a plastic dielectric. Relative humidity is extremely important because it affects the rate of evaporation from plants, animals, and the soil.

Mixing ratio or specific humidity are not usually measured directly, but are extremely useful because they are conserved within air parcels moving vertically or horizontally without mixing. Namely, heating, cooling, or changing the pressure of an air parcel will not change the mixing ratio or specific humidity in unsaturated air.

Lifting condensation level, or saturation level, can be used to estimate the altitude of cloud base for convective (cumulus) clouds.

Vapor pressure is neither easily measured nor directly useful. However, it is important theoretically because it describes from first principles how saturation humidity varies with temperature.

Solved Example

Find the liquid water mixing ratio in air at sea level, given a liquid water content of 3 g/m^3 .

Solution

Given: $\rho_{LWC} = 3 \text{ g}_{water}/\text{m}^3$. Sea level.

Find: $r_L = ? \text{ g}_{water}/\text{kg}_{dry air}$

Assume standard atmosphere, and use Table 1-5 from Chapter 1 to get: $\rho_{air} = 1.225 \text{ kg}_{air}/\text{m}^3$ at sea level.

Use eq. (4.31):

$$\begin{aligned} r_L &= (3 \text{ g}_{water}/\text{m}^3)/(1.225 \text{ kg}_{air}/\text{m}^3) \\ &= 2.45 \text{ g}_{water}/\text{kg}_{dry air} = \mathbf{2.45 \text{ g/kg}} \end{aligned}$$

Check: Units OK. Physics OK. Magnitude reasonable.

Discussion: Liquid, solid and water vapor might exist together in a cloud.

Solved Example

Saturated air at sea level with $T = 10^\circ\text{C}$ contains 3 g/kg liquid water. Find the total water mixing ratio.

Solution

Given: $T = 10^\circ\text{C}$, $r_L = 3 \text{ g/kg}$

Find: $r_T = ? \text{ g/kg}$

Assume: No ice.

Use Table 4-1 because it applies for sea level. Otherwise, solve equations or use thermo diagram. At $T = 10^\circ\text{C}$: $r_s = 7.66 \text{ g/kg}$ from the table.

Use eq. (4.32c):

$$r_T = r_s + r_L + r_i = 7.66 + 3 + 0 = \mathbf{10.66 \text{ g/kg}}$$

Check: Units OK. Physics OK.

Discussion: According to Table 4-1, the air would need to warm to 15°C to evaporate all liquid water

Solved Example

If an average of 5 g/kg of total water existed in the troposphere, between 100 kPa and 30 kPa, what is the precipitable water depth?

Solution

Given: $r_T = 0.005 \text{ kg/kg}$, $P_B = 100 \text{ kPa}$, $P_T = 30 \text{ kPa}$

Find: $d_W = ? \text{ m}$

Use eq. (4.33): and recall from Appendix A that $1 \text{ kPa} = 1000 \text{ kg}_{\text{air}} \cdot \text{m} \cdot \text{s}^{-2}$. Thus,

$$d_w = \left\{ (0.005 \text{ kg/kg}) / \left[(9.8 \text{ m} \cdot \text{s}^{-2}) \cdot (1000 \text{ kg} \cdot \text{m}^{-3}) \right] \right\} \cdot (100 - 30 \text{ kPa}) \cdot (1000 \text{ kg}_{\text{air}} \cdot \text{m} \cdot \text{s}^{-2} / \text{kPa})$$

$$= 0.036 \text{ m} = \mathbf{3.6 \text{ cm}} \ (\approx 1.4 \text{ inches}).$$

Check: Units OK. Physics OK.

Discussion: Why so much precipitation? Because we used an unrealistically large mixing ratio over the whole depth of the troposphere. Although 5 g/kg is reasonable near the surface, higher in the troposphere the air is much colder and can hold much less water vapor.

where r is the **water-vapor mixing ratio**, r_L is the **liquid-water mixing ratio**, and r_i is the **ice mixing ratio**. All these mixing ratios have units of $\text{kg}_{\text{water}}/\text{kg}_{\text{air}}$ or $\text{g}_{\text{water}}/\text{kg}_{\text{air}}$. Similar total-water variables are defined in terms of specific humidity and absolute humidity.

Liquid-water cloud droplets can exist unfrozen in air of temperature less than 0°C . Thus, it is possible for ice and liquid water to co-exist in the same air parcel at the same time, along with water vapor.

For negligible precipitation, eq. (4.32a) is approximately:

$$r_T = r \quad \text{for unsaturated air} \quad \bullet(4.32b)$$

$$r_T = r_s + r_L + r_i \quad \text{for saturated air} \quad \bullet(4.32c)$$

where the water-vapor mixing ratio is assumed to equal its saturation value r_s when liquid or ice is present. The words **saturated air** means cloudy air, and **unsaturated air** means non-cloudy air.

Total water is nearly conserved, in the sense that the amount of water lost or created via chemical reactions is small compared to the amounts advected with the wind or associated with precipitation. Thus, the change of total water in a volume can be calculated by the amount of water of all phases that enters or leaves the volume.

Recall that saturation water-vapor mixing ratio decreases as temperature decreases (see Table 4-1). Thus, given some initial amount of total water r_T , eq. (4.32c) says that liquid or solid water must increase as temperature and r_s decrease, in order that the sum of all water phases remain constant in saturated air.

Clouds or fog form when the total water content of the air exceeds the saturation (equilibrium) value. Because total water is an important factor in cloud and precipitation formation, budget equations for total water are presented in the next main section.

Precipitable Water

If all the vapor, liquid, and solid water within a column of air in the atmosphere were to precipitate out, the resulting depth of water on the Earth's surface (or in a rain gauge) is called the precipitable water, d_W . It is given by

$$d_W = \frac{r_T}{|g| \cdot \rho_{liq}} \cdot (P_B - P_T) \quad \bullet(4.33)$$

where r_T is average total-water mixing ratio, P_B and P_T are ambient air pressures at the bottom and top of the column segment, $|g| = 9.8 \text{ m} \cdot \text{s}^{-2}$ is gravitational acceleration magnitude, and $\rho_{liq} = 1000 \text{ kg} \cdot \text{m}^{-3}$ is density of liquid water.

If the mixing ratio varies with height, then the full column can be broken into smaller column segments of near-uniform mixing ratio. The resulting precipitable-water depths per segment can be summed to give the total precipitable-water depth.

Precipitable water is sometimes used as a humidity variable. The bottom of the atmosphere is warmer than the mid and upper troposphere, and can hold the most water vapor. In a pre-storm cloudless environment, contributions to the total-column precipitable water thus come mostly from the boundary layer. Hence, precipitable water can serve as one measure of boundary-layer total water that could serve as the fuel for thunderstorms later in the day. See the Thunderstorm chapters for a sample map of precipitable water.

In the real atmosphere, winds can advect moisture into a column to replace that lost from precipitation. Consequently, the total rainfall amount observed at the surface after storm passage is often much greater than the precipitable water at any instant. Some microwave and infrared sensors on satellites (see the Remote Sensing chapter) measure precipitable water averaged over large depths of the troposphere.

LAGRANGIAN BUDGETS

Moist air parcels have two additional properties that were unimportant for dry air. One is the amount of water in the parcel, which is important for determining cloud formation and precipitation amounts. The second is the latent heat released or absorbed when water changes phase, which is critical for determining the buoyancy of air parcels and the energy of thunderstorms.

Water Conservation

Lagrangian Water Budget

If you follow an air parcel as it moves, and assume no turbulent mixing with the environment, then the Lagrangian water budget is

$$\frac{\Delta r_T}{\Delta t} = S^{**} \quad (4.34)$$

where S^{**} is a Lagrangian net source of water. This source could be evaporation from adjacent liquid water, or it could be a loss of water falling from the parcel as precipitation (causing negative S^{**}).

If there is no source or loss, then there is no change of total water content. Phase changes must

Science Graffito

“The best way to have a good idea is to have lots of ideas.” — Linus Pauling.

Science Graffito

“Great ideas often receive violent opposition from mediocre minds.” — Albert Einstein.

Solved Example

An air parcel at sea level has a temperature of 20°C and a mixing ratio of 10 g/kg. If the air is cooled to 10°C, what is the liquid water content?

Solution

Given: $T = 20^\circ\text{C}$, $r = 10 \text{ g/kg}$
 Find: $r_L = ? \text{ g}_{\text{liq}}/\text{g}_{\text{air}}$ at $T = 10^\circ\text{C}$

Use Table 4-1 because it applies for sea level. Otherwise, solve equations or use a thermo diagram. Assume no other sources or sinks.

Initially:

$r_s = 14.91 \text{ g/kg}$ from the table, at $T = 20^\circ\text{C}$.
 Air is initially unsaturated, because $r < r_s$.
 Unsaturated air has no liquid water: $r_L = 0$.
 Assume no ice at these warm temperatures.
 $(r + r_L)_{\text{initial}} = (10 + 0) \text{ (g/kg)} = 10 \text{ g/kg}$

Finally:

After cooling the air to 10°C, Table 4-1 shows that the max amount of vapor that can be held in the air at equilibrium is $r = r_s = 7.66 \text{ g/kg}$. However, total water must be constant if it is conserved. Use eq. (4.35b):

$$10 \text{ g/kg} = (r + r_L)_{\text{final}} = 7.66 \text{ g/kg} + r_{L \text{ final}}$$

Thus,

$$r_{L \text{ final}} = 10 \text{ g/kg} - 7.66 \text{ g/kg} = \underline{2.34 \text{ g/kg}}$$

Check: Units OK. Physics OK.

Discussion: Some liquid water has formed in the air parcel, so it is now a cloud or fog.

Solved Example

Use Fig. 4.7 to answer (A) these questions (Q).

Solution

Q: What is the saturation mixing ratio for air of temperature 0°C and pressure 40 kPa?

A: Follow the $T = 0^\circ\text{C}$ isotherm vertically, and the $P = 40 \text{ kPa}$ isobar horizontally, to find where they intersect. The saturation mixing ratio (dotted diagonal) line that crosses through this intersection is the one labeled: $r_s \approx \underline{10 \text{ g/kg}}$.

Q: What is the actual mixing ratio for air of dew point 0°C and pressure 40 kPa?

A: This is the same intersection point yielding the same numerical answer, but because dew-point temperature was used, the answer represents the actual mixing ratio. $r \approx \underline{10 \text{ g/kg}}$.

Q: What is the dew-point temperature for air of mixing ratio 20 g/kg and pressure 80 kPa?

A: From the intersection of the diagonal dotted line for $r = 20 \text{ g/kg}$ and the horizontal isobar for $P = 80 \text{ kPa}$, go vertically straight down to find $T_d \approx \underline{21^\circ\text{C}}$.

compensate each other to maintain constant total water. Thus, for no net source:

$$(r + r_i + r_L)_{\text{initial}} = (r + r_i + r_L)_{\text{final}} \quad \bullet(4.35a)$$

For the special case of no ice, this reduces to:

$$(r + r_L)_{\text{initial}} = (r + r_L)_{\text{final}} \quad \bullet(4.35b)$$

On a thermodynamic diagram, conservation of total water corresponds to following an isohume of constant mixing ratio as a parcel rises or descends. Thus, it is total water mixing ratio that is conserved, not dew-point temperature.

Thermo Diagrams – Part 2: Isohumes

Two classes of information are included on thermo diagrams: **states** and **processes**. State of the air is defined by **temperature**, **pressure**, and **moisture**. Two processes are **unsaturated (dry)** and **saturated (moist)** adiabatic vertical movement, such as experienced by air parcels rising through the atmosphere. Processes can change the state.

The chart background of vertical **isotherms** and horizontal **isobars** was introduced in Chapters 1 (Fig. 1.4) and 3 (Fig. 3.3). Here we will overlay **isohumes** of constant saturation-mixing-ratio state over the background (Fig. 4.7).

To use the thermo-diagram background of P vs. T for isohumes, we need to describe r_s as a function of P and T [abbreviate as $r_s(P, T)$]. Eq. (4.5) gives $r_s(P, e_s)$ and the Clausius-Clapeyron eq. (4.1) gives $e_s(T)$. So combining these two equations gives $r_s(P, T)$.

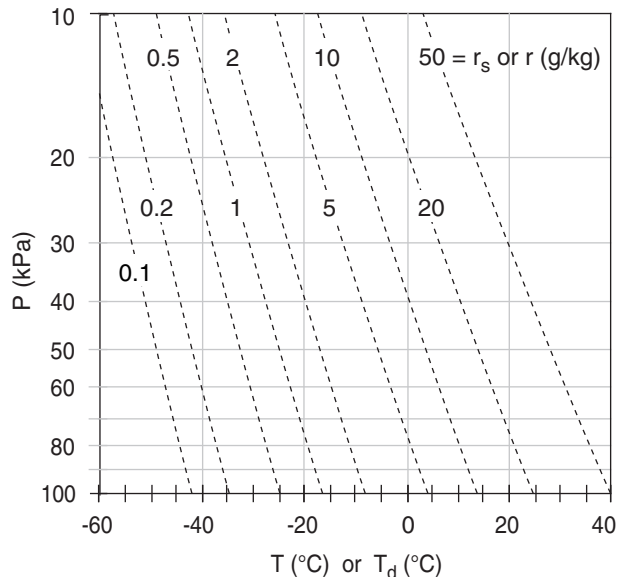


Figure 4.7
 Thermodynamic diagram with isohumes of mixing ratio.

But to draw any one isohume (i.e., for any one value of r_s), we need to rearrange the result to give $T(P, r_s)$:

$$T = \left[\frac{1}{T_0} - \frac{\mathfrak{R}_v}{L_v} \cdot \ln \left\{ \frac{r_s \cdot P}{e_0 \cdot (r_s + \epsilon)} \right\} \right]^{-1} \quad (4.36)$$

where $e_0 = 0.611$ kPa, $T_0 = 273$ K, $\epsilon = 0.622$ g/g, and $\mathfrak{R}_v/L = 0.0001844$ K⁻¹. Be sure to use g/g and not g/kg for r_s . The answer is in Kelvin.

Thus, pick any fixed r_s to plot. Then, for a range of P from the bottom to the top of the atmosphere, solve eq. (4.36) for the corresponding T values. Plot these T vs. P values as the isohume line on a thermo diagram. Use a spreadsheet to repeat this calculation for other values of r_s to plot the other isohumes.

Eqs. (4.36) with (4.15b) are similar. Thus, you can use isohumes of $T(P, r_s)$ to also represent isohumes of $T_d(P, r)$. Namely, you can use isohumes to find the saturation state r_s of the air at any P and T , and you can also use the isohumes to describe the process of how T_d changes when an air parcel of constant r rises or descends to an altitude of different P .

Solved Example

Calculate the dew point of air at pressure 80 kPa and mixing ratio 1 g/kg.

Solution

Given: $P = 80$ kPa, $r = 1$ g/kg

Find: $T_d = ?^\circ\text{C}$

Use eq. (4.15b):

$$\begin{aligned} T_d &= \left[\frac{1}{273\text{K}} - 0.000184\text{K}^{-1} \cdot \right. \\ &\quad \left. \ln \left\{ \frac{(0.001\text{g/g}) \cdot (80\text{kPa})}{0.611\text{kPa} \cdot (0.001 + 0.622\text{g/g})} \right\} \right]^{-1} \\ &= [0.003663 - 0.0001844 \cdot \ln(0.210)]^{-1} \\ &= 253.12 \text{ K} = \underline{\underline{-20^\circ\text{C}}} \end{aligned}$$

Check: Units OK. Physics OK. Agrees with Fig. 4.7.

Discussion: We would have found the same numerical answer if we asked for the temperature corresponding to air at 80 kPa that is saturated with $r_s = 1$ g/kg with $r_L = 0$. For this situation, we would have used eq. (4.36).

Heat Conservation for Saturated Air

Saturated Adiabatic Lapse Rate

The unsaturated (dry) adiabatic lapse rate, $\Gamma_d = 9.8$ K/km, was discussed earlier. It is the temperature decrease ($-\Delta T/\Delta z = \Gamma_d$) an air parcel would experience as it moves vertically in the atmosphere into regions of lower pressure. No mixing or heat transfer across the skin of the parcel can occur for an adiabatic process. Although the word “dry” appears, this lapse rate applies to moist as well as dry air, but only if the air is unsaturated (i.e., not cloudy). Air cools adiabatically as it rises, and warms as it descends.

Saturated (cloudy) air also cools as it rises and warms as it descends, but at a lesser rate than a dry parcel. The reason is that, by definition, a saturated parcel is in equilibrium with the liquid drops. If the parcel then rises and cools, it is no longer in equilibrium, and some of the water-vapor molecules condense as a new equilibrium is approached. During condensation, latent heat is released into the air, partially compensating the adiabatic cooling.

The opposite is true during descent. Some of the cloud droplets evaporate and remove heat from the air, partially compensating the adiabatic warming.

For saturated air, the decrease of temperature with height ($-\Delta T/\Delta z = \Gamma_s$) in clouds and fog is given by the **saturated-adiabatic lapse rate**, Γ_s :

Solved Example

Find the moist adiabatic lapse rate near sea level for $T = 26^\circ\text{C}$, using (a) the actual specific heat, and (b) the specific heat for dry air. (c) Also find $\Delta T/\Delta P$ for a rising saturated air parcel.

Solution

Given: $T = 26^\circ\text{C} = 299\text{ K}$, $P = 101.325\text{ kPa}$
 Find: $\Gamma_s = ?\text{ }^\circ\text{C/km}$

In general, we would need to calculate the saturation mixing ratio using eqs. (4.5) and (4.1). However, for this scenario, we can use Table 4-1 for simplicity, because sea-level pressure was specified. Thus: $r_s = 21.85\text{ g/kg} = 0.0219\text{ g/g}$.

(a) Using eq. (3.2), the actual specific heat is:

$$C_p = 1023\text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$$

Thus: $|g|/C_p = 9.58\text{ K/km}$

$$L_v/C_p = 2444\text{ K}$$

$$L_v/\mathfrak{R}_d = 8711\text{ K}$$

Plugging these into eq. (4.37a) gives:

$$\Gamma_s = \frac{(9.58 \frac{\text{K}}{\text{km}}) \cdot \left[1 + \frac{(0.0219\text{g/g}) \cdot (8711\text{K})}{299\text{K}} \right]}{\left[1 + \frac{(2444\text{K}) \cdot (8711\text{K}) \cdot (0.0219\text{g/g}) \cdot 0.622}{(299\text{K})^2} \right]}$$

$$= (9.58\text{ K/km}) \cdot [1.638] / [4.307] = \underline{\underline{3.70\text{ K/km}}}$$

(b) Eq. (4.37b) uses C_p for dry air:

$$\Gamma_s = \frac{(9.8 \frac{\text{K}}{\text{km}}) \cdot \left[1 + \frac{(0.0219\text{g/g}) \cdot (8711\text{K})}{299\text{K}} \right]}{\left[1 + \frac{(1.35 \times 10^7 \text{K}^2) \cdot (0.0219\text{g/g})}{(299\text{K})^2} \right]}$$

$$= (9.8\text{ K/km}) \cdot [1.638] / [4.307] = \underline{\underline{3.73\text{ K/km}}}$$

(c) Use eq. (4.38b):

$$\frac{\Delta T}{\Delta P} = \frac{[0.28571 \cdot (299\text{K}) + (2488.4\text{K} \cdot (0.0219\text{g/g}))]}{P \cdot \left[1 + [1.35 \times 10^7 \text{K}^2 \cdot (0.0219\text{g/g}) / (299\text{K})^2] \right]}$$

$$= (32.49\text{K}) / P$$

Assume $P = 101.325\text{ kPa}$ at sea level:

$$\Delta T/\Delta P = \underline{\underline{0.32\text{ K/kPa}}}$$

Check: Units OK. Physics OK.

Discussion: Don't forget for answers (a) and (b) that lapse rates are the rate of cooling with altitude. As expected the rate of cooling of a rising cloudy air parcel is less than the 9.8 K/km of a dry parcel. Also, method (b) is accurate enough for many situations.

Normally, pressure decreases as altitude increases, thus answer (c) also gives cooling for negative ΔP .

$$\Gamma_s = \frac{|g|}{C_p} \cdot \frac{\left(1 + \frac{r_s \cdot L_v}{\mathfrak{R}_d \cdot T} \right)}{\left(1 + \frac{L_v^2 \cdot r_s \cdot \epsilon}{C_p \cdot \mathfrak{R}_d \cdot T^2} \right)} \quad (4.37a)$$

This is sometimes called the **moist adiabatic lapse rate**. The value of specific heat C_p is a function of humidity, and can be found using eq. (3.2).

If the variation of specific heat with humidity is neglected for simplicity, then (4.37a) simplifies to:

$$\Gamma_s = \Gamma_d \cdot \frac{[1 + (a \cdot r_s / T)]}{[1 + (b \cdot r_s / T^2)]} \quad \bullet(4.37b)$$

where $\Gamma_d = 9.8\text{ K/km}$, $a = 8711\text{ K}$, and $b = 1.35 \times 10^7\text{ K}^2$. As shown in the solved example, eq. (4.37b) differs from (4.37a) by roughly 1%, so it is often accurate enough for most applications. In both equations the saturation mixing ratio r_s varies with temperature, and must be used in units of g/g . Also, temperature must be Kelvin.

Near the ground in humid conditions, the value of the moist adiabatic lapse rate is near 4 K/km , and often changes to 6 to 7 K/km in the middle of the troposphere. At high altitudes where the air is colder and holds less water vapor, the moist rate nearly equals the dry rate of 9.8 K/km .

Instead of a change of temperature with height, the moist lapse rate can be written as a change of temperature ΔT with change of pressure ΔP :

$$\frac{\Delta T}{\Delta P} = \frac{\left[(\mathfrak{R}_d / C_p) \cdot T + (L_v / C_p) \cdot r_s \right]}{P \cdot \left(1 + \frac{L_v^2 \cdot r_s \cdot \epsilon}{C_p \cdot \mathfrak{R}_d \cdot T^2} \right)} \quad (4.38a)$$

where saturation mixing ratio r_s varies with T .

The corresponding simplified form is:

$$\frac{\Delta T}{\Delta P} = \frac{[a \cdot T + c \cdot r_s]}{P \cdot [1 + (b \cdot r_s / T^2)]} \quad \bullet(4.38b)$$

where $a = 0.28571$, $b = 1.35 \times 10^7\text{ K}^2$, $c = 2488.4\text{ K}$.

Thermo Diagrams – Part 3: Saturated Adiabats

Recall that two processes shown in thermo diagrams are unsaturated (**dry**) and saturated (**moist**) adiabatic vertical movement, such as experienced by air parcels rising through the atmosphere. Un-

saturated processes were previously indicated by overlaying isentropes (**dry adiabats**) on the background (Chapter 3, Fig. 3.3). Here we will add **saturated adiabats**, also known as **moist adiabats**.

Moist adiabats show how cloudy air changes temperature as it rises or sinks. Unfortunately, eq. (4.38) does not directly give the temperature of a parcel at any pressure height. Instead, one must solve the equation recursively, stepping upward in altitude (toward decreasing P) from an initial temperature at $P = 100$ kPa. Each step must take a small-enough pressure-height increment that the moist lapse rate does not change significantly from one step to the next. For each step, compute:

$$T_2 = T_1 + \frac{\Delta T}{\Delta P} \cdot (P_2 - P_1) \tag{4.39}$$

where $(\Delta T/\Delta P)$ is from eq. (4.38).

For example, use pressure height increments of $\Delta P = (P_2 - P_1) = -0.1$ kPa all the way from $P = 100$ kPa up to $P = 10$ kPa. Repeat the calculations starting from five initial temperatures at $P = 100$ kPa: $T_1 = 40, 20, 0, -20,$ and -40°C . Fig. 4.8 shows the results of such calculations on a spreadsheet. The calculations were done in units of Kelvin, and then converted to Celsius for plotting. The moist adiabats are labeled with their starting temperature at $P = 100$ kPa, as discussed next. [The solved example at right uses coarser vertical increments of $\Delta P = -2$ kPa. This is too coarse, and can cause 5°C errors at 20 kPa.]

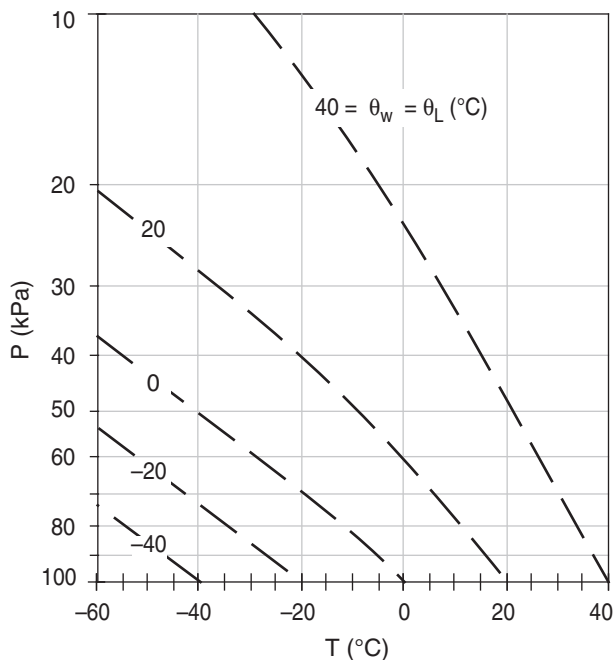


Figure 4.8
Thermodynamic diagram with moist adiabats, labeled by wet-bulb potential temperature θ_w or by liquid-water potential temperature θ_L .

FOCUS • Spreadsheet Thermodynamics

Saturated (Moist) Adiabatic Lapse Rate

The iteration process of computing the $\theta_w = 20^\circ\text{C}$ moist adiabat is demonstrated here.

First, set up a spreadsheet with the following row and column headers:

	A	B	C	D	E
1	Moist Adiabat Example				
2	$\Delta P(\text{kPa})=$	2			
3					
4	P (kPa)	T($^\circ\text{C}$)	$e_s(\text{kPa})$	$r_s(\text{g/g})$	$\Delta T/\Delta P$
5	100	20.0			

where the starting pressure and temperature have been entered as numbers, and where the pressure increment of $\Delta P = 2$ kPa has also been entered.

Because $\Delta T/\Delta P$ in eq. (4.38b) depends on r_s , which in turn depends on e_s , there are the two extra columns C and D in the spreadsheet above. Two typical mistakes are to forget to convert T into Kelvins before using it, or to forget to use g/g for r .

In cell C5, you can enter eq. (4.1), which is:

$$=0.611*\text{EXP}(5423*((1/273.15)-(1/(B5+273.15))))$$

Then, in cell D5, enter eq. (4.5): $=0.622*C5/(A5-C5)$

Next, in cell E5, enter eq. (4.38b): $=0.28571*(B5+273.15) + (2488.4*D5) / (A5*(1+(13500000*D5)/((B5+273.15)^2)))$

In cell A6, increment the pressure: $=A5-\$B\2

Finally, in cell B6 enter eq. (4.39): $=B5-(E5*\$B\$2)$

where the previous 2 eqs referred to B2 as an absolute reference ($\$B\2 on some spreadsheets).

Finally, select cells C5:E5 and fill them down one row. Then, select all of row 6; namely cells A6:E6, and fill down about 45 rows. The result is shown below, where we have not shown rows 8 to 43 to save space:

	A	B	C	D	E
1	Moist Adiabat Example				
2	$\Delta P(\text{kPa})=$	2			
3					
4	P (kPa)	T($^\circ\text{C}$)	$e_s(\text{kPa})$	$r_s(\text{g/g})$	$\Delta T/\Delta P$
5	100	20.00	2.368	0.0151	0.360
6	98	19.28	2.262	0.0147	0.369
7	96	18.54	2.158	0.0143	0.379
...
44	22	-52.28	0.006	0.0002	2.766
45	20	-57.82	0.003	0.0001	3.007

Discussion: Columns A and B can be plotted as the 20°C moist adiabat. For other moist adiabats, duplicate these cells, but with different starting T in cell B5.

The vertical pressure increment $\Delta P = 2$ kPa is much too coarse to give an accurate value near the top of the graph (i.e., at low pressures). When I repeat the calculation with $\Delta P = 0.1$ kPa, I find $T = -60.93$ at $P = 20$ kPa. Even smaller increments would be better.

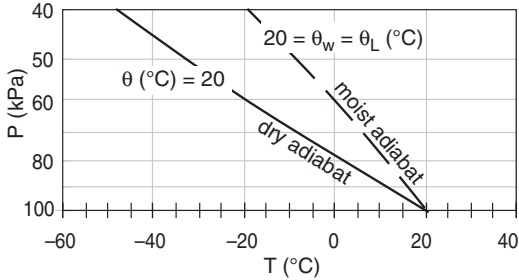
Solved Example

Compare on a thermo diagram dry and moist adiabats starting at $T = 20^\circ\text{C}$ at $P = 100 \text{ kPa}$.

Solution

Given: $T = 20^\circ\text{C}$ at $P = 100 \text{ kPa}$.
 Plot: $\theta = 20^\circ\text{C}$ and $\theta_w = 20^\circ\text{C}$.

Use curves from Fig. 4.8 and Fig. 3.3.



Check: Slopes look reasonable. Labels match.

Discussion: Moist-adiabat temperature decreases more slowly than dry, because condensational latent heating partially compensates for adiabatic cooling.

Solved Example

Air at pressure 80 kPa and temperature 0°C is saturated, & holds 2 g/kg of liquid water. Find θ_e and θ_L .

Solution:

Given: $P = 80 \text{ kPa}$, $T = 0^\circ\text{C} = 273\text{K}$, $r_L = 2 \text{ g/kg}$.
 Find: $\theta_e = ?^\circ\text{C}$, $\theta_L = ?^\circ\text{C}$

First, do preliminary calculations shared by both eqs:
 Rearrange eq. (3.12) to give:

$$(\theta/T) = (P_o/P)^{0.28571} = (100\text{kPa}/80\text{kPa})^{0.28571} = 1.066$$

$$\text{Thus, } \theta = 291\text{K} = 18^\circ\text{C}$$

At 80 kPa and 0°C , solve eq. (4.5) for $r_s = 4.7 \text{ g/kg}$

Then use eq. (3.2):

$$C_p = C_{pd}(1 + 1.84 \cdot r) = (1004.67 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}) \cdot [1 + 1.84 \cdot (0.0047 \text{ g/g})] = 1013.4 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$$

$$\text{Thus, } L_v/C_p = 2467 \text{ K}/(\text{g}_{\text{water}}/\text{g}_{\text{air}}) \text{ and } (L_v/C_p) \cdot (\theta/T) = 2630 \text{ K}/(\text{g}_{\text{water}}/\text{g}_{\text{air}})$$

Use eq. (4.40):

$$\theta_e = (18^\circ\text{C}) + (2630 \text{ K}/(\text{g}_{\text{water}}/\text{g}_{\text{air}})) \cdot (0.0047) = \underline{30.4^\circ\text{C}}$$

Use eq. (4.41):

$$\theta_e = (18^\circ\text{C}) - (2630 \text{ K}/(\text{g}_{\text{water}}/\text{g}_{\text{air}})) \cdot (0.002) = \underline{12.7^\circ\text{C}}$$

Check: Units OK. Physics OK. Magnitudes OK.

Discussion: The answers are easier to find using a thermo diagram (after you've studied the Stability chapter). For θ_e , find the θ value for the dry adiabat that is tangent at the diagram top to the moist adiabat. For θ_L , follow a moist adiabat down to where it crosses the $(2 + 4.7 = 6.7 \text{ g/kg})$ isohume, and from there follow a dry adiabat to $P = 100 \text{ kPa}$.

Liquid-Water, Equivalent, and Wet-Bulb Potential Temperatures

Recall from the Heat chapter that potential temperature θ is conserved during unsaturated adiabatic ascent or descent. However, if an air parcel containing water vapor is lifted above its LCL, then condensation will add latent heat, causing θ to increase. Similarly, if the air contains liquid water such as cloud drops, when it descends some of the drops can evaporate, thereby cooling the air and reducing θ .

However, we can define new variables that are conserved for adiabatic ascent or descent, regardless of any evaporation or condensation that might occur. One is the **equivalent potential temperature** θ_e :

$$\theta_e \approx \theta + \left(\frac{L_v \cdot \theta}{C_p \cdot T} \right) \cdot r \approx \theta + \frac{L_v}{C_{pd}} \cdot r \quad \bullet(4.40)$$

Another is **liquid water potential temperature**, θ_L :

$$\theta_L \approx \theta - \left(\frac{L_v \cdot \theta}{C_p \cdot T} \right) \cdot r_L \approx \theta - \frac{L_v}{C_{pd}} \cdot r_L \quad \bullet(4.41)$$

where $L_v = 2.5 \times 10^6 \text{ J}\cdot\text{kg}^{-1}$ is the latent heat of vaporization, C_p is the specific heat at constant pressure for air (C_p is not constant, see the Heat chapter), T is the absolute temperature of the air, and mixing ratios (r and r_L) have units of $(\text{g}_{\text{water}}/\text{g}_{\text{air}})$. The last approximation in both equations is very rough, with $L_v/C_{pd} = 2.5 \text{ K}/(\text{g}_{\text{water}}/\text{g}_{\text{air}})^{-1}$.

Both variables are conserved regardless of whether the air is saturated or unsaturated. Consider unsaturated air, for which θ is conserved. In eq. (4.40), water-vapor mixing ratio r is also conserved during ascent or descent, so the right side of eq. (4.40) is constant, and θ_e is conserved. Similarly, for unsaturated air, liquid water mixing ratio $r_L = 0$, hence θ_L is also conserved in eq. (4.41).

For saturated air, θ will increase in a rising air parcel due to latent heating, but r will decrease as some of the vapor condenses into liquid. The two terms in the right side of eq. (4.40) have equal but opposite changes that balance, leaving θ_e conserved. Similarly, the two terms on the right side of eq. (4.41) balance, due to the minus sign in front of the r_L term. Thus, θ_L is conserved.

By subtracting eq. (4.41) from (4.40), we can see how θ_e and θ_L are related:

$$\theta_e \approx \theta_L + \left(\frac{L_v \cdot \theta}{C_p \cdot T} \right) \cdot r_T \quad (4.42)$$

where the total water mixing ratio (g/g) is $r_T = r + r_L$. Although θ_e and θ_L are both conserved, they are not equal to each other.

We can use θ_e or θ_L to identify and label moist adiabats. Consider an air parcel starting at $P = 100$ kPa that is saturated but contains no liquid water ($r = r_s = r_T$). For that situation θ_L is equal to its initial temperature T (which also equals its initial potential temperature θ at that pressure). A rising air parcel from this point will conserve θ_L , hence we could label the moist adiabat with this value (Fig. 4.9).

An alternative label starts from same saturated air parcel at $P = 100$ kPa, but conceptually lifts it to the top of the atmosphere ($P = 0$). All of the water vapor will have condensed out at that end point, heating the air to a new potential temperature. The potential temperature of the dry adiabat that is tangent to the top of the moist adiabat gives θ_e (Fig. 4.9). [CAUTION: On some thermo diagrams, equivalent potential temperature is given in units of Kelvin.]

In other words, θ_L is the potential temperature at the bottom of the moist adiabat (more precisely, at $P = 100$ kPa), while θ_e is the potential temperature at the top. Either labeling method is fine — you will probably encounter both methods in thermo diagrams that you get from around the world.

Wet-bulb potential temperature (θ_w) can also be used to label moist adiabats. For θ_w use Normand’s rule on a thermo diagram (Fig. 4.10). Knowing temperature T and dew-point T_d at initial pressure P , and plot these points on a thermo diagram. Next, from the T point, follow a dry adiabat up, and from the T_d point, follow an isohume up. Where they cross is the lifting condensation level LCL.

From that LCL point, follow a saturated adiabat back down to the starting altitude, which gives the **wet-bulb temperature** T_w . If you continue to follow the saturated adiabat down to a reference pressure ($P = 100$ kPa), the resulting temperature is the **wet-bulb potential temperature** θ_w (see Fig. 4.10). Namely, θ_w equals the θ_L label of the moist adiabat that passes through the LCL point. Labeling moist adiabats with values of wet-bulb potential temperature θ_w is analogous to the labeling scheme for the dry adiabats, which is why I use θ_w here.

To find θ_e (K) for a moist adiabat if you know its θ_w (K), use:

$$\theta_e = \theta_w \cdot \exp(a_3 \cdot r_s / \theta_w)_o \quad (4.43a)$$

where $a_3 = 2491$ K kg_{air}/kg_{vapor}, and r_s is initial saturation mixing ratio (kg_{vapor}/kg_{air}) at $T = \theta_w$ and $P = 100$ kPa (as denoted by subscript “o”). You can approximate (4.43a) by

$$\theta_e(\text{K}) \approx a_0 + a_1 \theta_w(\text{°C}) + a_2 [\theta_w(\text{°C})]^2 \quad (4.43b)$$

where $a_0 = 282$, $a_1 = 1.35$, and $a_2 = 0.065$, for θ_w in the range of 0 to 30°C (see Fig. 4.11). Also the θ_L label for

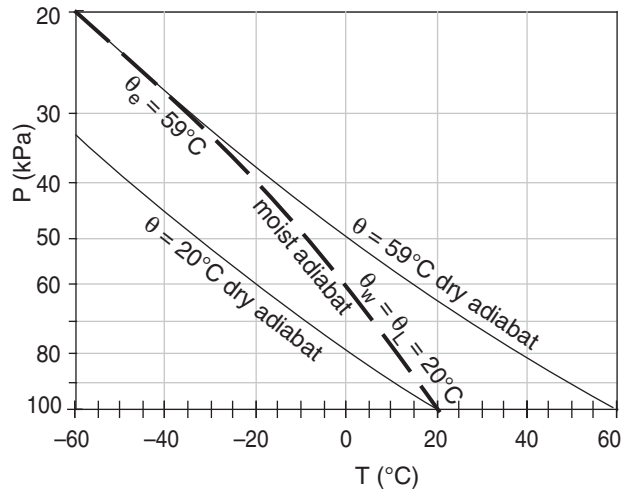


Figure 4.9
Comparison of θ_L , θ_w and θ_e values for the same moist adiabat.

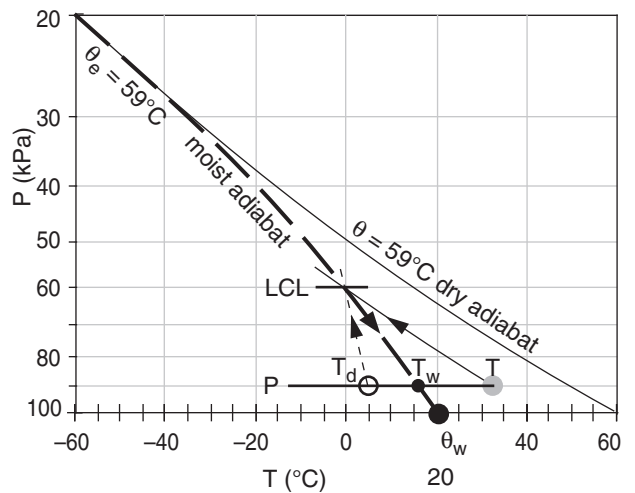


Figure 4.10
Thermo diagram showing how to use Normand’s Rule to find wet-bulb potential temperature θ_w , and how it relates to θ_e .

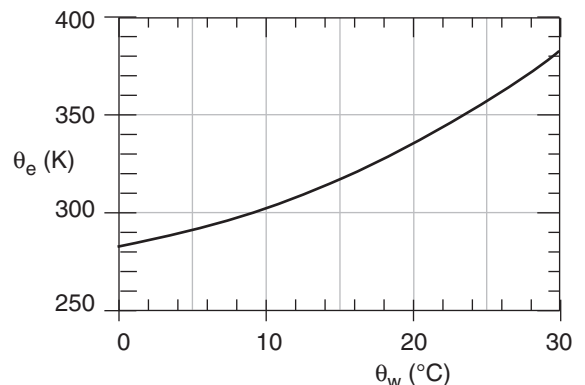


Figure 4.11
Approximate relationship between θ_w and θ_e .

Solved Example

Verify the labels on the moist adiabat that passes through the LCL in Fig. 4.10, given starting conditions $T = 33^\circ\text{C}$ and $T_d = 5.4^\circ\text{C}$ at $P = 90\text{ kPa}$.

Solution

Given: $T = 33^\circ\text{C} = 306\text{K}$, $T_d = 5.4^\circ\text{C} = 278.4\text{K}$
 Find: θ_e , θ_w , and θ_L labels ($^\circ\text{C}$) for the moist adiabat

First, find the initial θ , using eq. (3.12)
 $\theta = T(P_0/P)^{0.28571} = (306\text{K}) \cdot (100\text{kPa}/90\text{kPa})^{0.28571} = 315.4\text{K} = 42.4^\circ\text{C}$
 Next, find the mixing ratio using eq. (4.1b) & (4.4):
 $e = 0.611\text{kPa} \cdot \exp[5423 \cdot (1/273.15 - 1/278.4)] = 0.8885\text{kPa}$
 $r = (622\text{g/kg}) \cdot (0.8885\text{kPa}) / [90 - 0.8885\text{kPa}] = 6.2\text{g/kg}$
 Next, use eq. (3.2) to find $C_p = C_{pd} \cdot (1 + 1.84 \cdot r)$
 $C_p = (1004.67\text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}) \cdot [1 + 1.84 \cdot (0.0062\text{ g/g})]$
 $C_p = 1016.1\text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$

Solve the more accurate version of eq. (4.40):
 $\theta_e = (42.4^\circ\text{C}) + \{ (2500\text{ J/g}_{\text{water}}) \cdot (315.4\text{K}) / [(1016.1\text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}) \cdot (306\text{K})] \} \cdot (6.2\text{ g}_{\text{water}}/\text{kg}_{\text{air}})$
 $= (42.4^\circ\text{C}) + (2.536\text{ K} \cdot \text{kg}_{\text{air}}/\text{g}_{\text{water}}) \cdot (6.2\text{ g}_{\text{water}}/\text{kg}_{\text{air}})$
 $= \mathbf{58.1^\circ\text{C}} = 331.1\text{ K}$

The approximate version of eq. (4.40) gives almost the same answer, and is much easier:
 $\theta_e = (42.4^\circ\text{C}) + (2.5\text{ K} \cdot \text{kg}_{\text{air}}/\text{g}_{\text{water}}) \cdot (6.2\text{ g}_{\text{water}}/\text{kg}_{\text{air}})$
 $= \mathbf{57.9^\circ\text{C}}$

Eq. (4.43b) is a quadratic eq. that can be solved for θ_w . Doing this, and then plugging in $\theta_e = 331.1\text{ K}$ gives:
 $\theta_w \approx \mathbf{19^\circ\text{C}}$, which is the label on the moist adiabat.
 Using $\theta_L \approx \theta_w$:
 $\theta_L \approx \mathbf{19^\circ\text{C}}$.

Check: Units OK. Physics OK.
Discussion: These values are within a couple degrees of the labels in Fig. 4.10. Disappointing that they aren't closer, but the θ_e results are very sensitive to the starting point.

Solved Example

For a moist adiabat of $\theta_w = 14^\circ\text{C}$, find its θ_e .

Solution

Given: $\theta_w = 14^\circ\text{C} = 287\text{ K}$
 Find: $\theta_e = ?\text{ K}$

First get r_s from Fig. 4.7 at $P = 100\text{ kPa}$ and $T = 14^\circ\text{C}$:
 $r_s = 10\text{ g/kg} = 0.010\text{ kg/kg}$.
 Next, use eq. (4.43a):
 $\theta_e = (287\text{K}) \cdot \exp[2491(\text{K} \cdot \text{kg}_{\text{air}}/\text{kg}_{\text{vapor}}) \cdot (0.010\text{ kg}_{\text{vapor}}/\text{kg}_{\text{air}}) / (287\text{K})] = \mathbf{313\text{K}}$

Check: Units OK. Agrees with Fig. 4.11.
Discussion: This $\theta_e = 40^\circ\text{C}$. Namely, if a saturated air parcel started with $\theta_w = T = 14^\circ\text{C}$, and then if all the water vapor condensed, the latent heat released would warm the parcel to $T = 40^\circ\text{C}$.

Solved Example

Use Normand's rule (in reverse) on a thermodynamic diagram to find the mixing ratio and dew point temperature, given dry and wet-bulb observations of 40°C and 20°C , respectively, from a psychrometer near the surface (where $P = 100\text{ kPa}$).

Solution

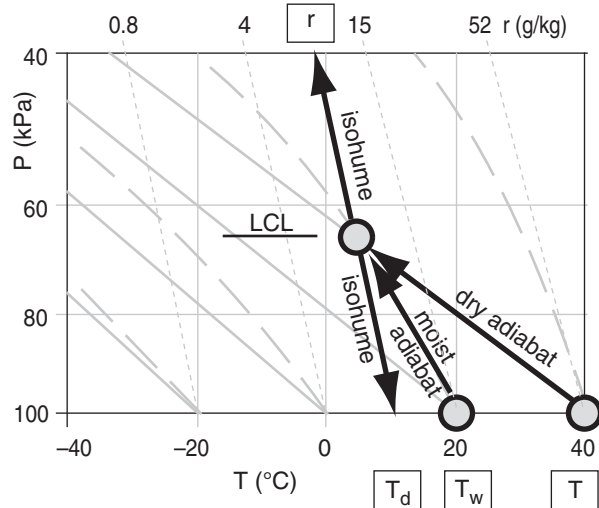
Given: $T = 40^\circ\text{C}$, $T_w = 20^\circ\text{C}$, $P = 100\text{ kPa}$.
 Find: $T_d = ?^\circ\text{C}$, $r = ?\text{ g/kg}$

Combining all the components of the thermo diagram (which will be covered in more detail in the Stability chapter) yields the figure below, for which all the background lines are gray. Thick diagonal lines on this diagram are dry adiabats, and curved dashed lines are moist adiabats, while horizontal and vertical lines are isobars and isotherms, respectively. Isohumes are the nearly-vertical dotted lines.

First, the given values of T and T_w at the given P are plotted as the two circles at the bottom of the Figure. Starting from the T observation, follow a dry adiabat upward. Simultaneously, starting from the T_w observation, follow a moist adiabat upward. They intersect at the LCL at about 65 kPa , which is marked by another circle.

From the LCL, follow (or go parallel to) the isohumes back down to the starting pressure to find the T_d . Alternately, from the intersection point, follow the isohumes upward to find r (interpolating if necessary between the printed values).

The results are roughly $r = \mathbf{7.5\text{ g/kg}}$, and $T_d = \mathbf{10^\circ\text{C}}$. (These were actually found from a more precise thermo diagram, as provided in the Stability chapter.)



Check: Units OK. Physics OK.
Discussion: Using $r_s = 52\text{ g/kg}$ from the starting T , the relative humidity is $RH = 8/52 = 15\%$.

the moist adiabat passing through the LCL equals this θ_w .

EULERIAN WATER BUDGET

Any change of total water content within a fixed Eulerian volume such as a cube of moist air must be explained by transport of water through the sides. Transport processes include advection by the mean wind, turbulent transport, and precipitation of liquid and solid water.

As was done for the heat budget, the moisture budget can be simplified for many atmospheric situations. In particular, the following terms are usually small except in thunderstorms, and will be neglected: vertical advection by the mean wind, horizontal turbulent transport, and conduction (except for effective moisture fluxes near the Earth's surface). The net Eulerian total water (r_T) budget is:

$$\frac{\Delta r_T}{\Delta t} = - \left[U \cdot \frac{\Delta r_T}{\Delta x} + V \cdot \frac{\Delta r_T}{\Delta y} \right] + \left(\frac{\rho_L}{\rho_d} \right) \frac{\Delta Pr}{\Delta z} - \frac{\Delta F_{z \text{ turb}}(r_T)}{\Delta z} \quad (4.44)$$

storage horiz. advection precipitation turbulence

where (U , V) are the horizontal wind components in the (x , y) Cartesian directions, z is height, Pr is precipitation rate, and $F_{z \text{ turb}}(r_T)$ is the kinematic turbulent flux of total water.

Most of these terms are similar to those in the heat budget eq. (3.51). The ratio of liquid-water density to dry-air density is $\rho_L/\rho_d = 836.7 \text{ kg}_{\text{liq}}/\text{kg}_{\text{air}}$ at STP. **Standard Temperature and Pressure (STP)** is $T = 0^\circ\text{C}$ and $P = 101.325 \text{ kPa}$. The liquid water density is close to $\rho_L = 1000 \text{ kg}_{\text{liq}}/\text{m}^3$ (see Appendix B).

Each term is examined next. Surface moisture flux is also discussed because it contributes to the turbulence term for a volume of air on the surface.

Horizontal Advection

Horizontal advection of total water is similar to that for heat. Within a fixed volume, moisture increases with time if air blowing into the volume contains more water than air leaving. The advected water content includes water vapor (i.e., humidity), liquid, and solid cloud particles and precipitation.

Precipitation

Liquid water equivalent is the depth of liquid water that would occur if all solid precipitation were melted. For example, 10 cm depth of snow accumulated in a cylindrical container might be equivalent to only 1 cm of liquid water when melted.

ON DOING SCIENCE • Look for Patterns

Suppose you know that $A = C$, and $C = E$. You can use this knowledge to yield new understanding, by various methods of reasoning.

Deductive reasoning combines known relationships in new ways to allow new applications. For example, using the knowns from the previous paragraph, you could deduce that $A = E$.

Inductive reasoning goes from the particular to the general. It does this by looking for patterns. By such reasoning, you could state that $E = G$. The pattern is as follows: A equals a value that is two letters later in the alphabet, and C equals a value two letters later. By extension, you might hope that any letter of the alphabet equals a value that is two letters later in the alphabet.

Inductive reasoning is riskier than deductive. If the knowns you use as a starting point are unrepresentative of the whole situation, then you will get a biased (i.e., wrong) result. For example: given that I have a sister, and that my best friend has a sister, therefore by inductive reasoning everyone has a sister. To avoid such blunders, it helps to get a large sample of knowns before you extend the pattern into the unknown.

Inductive reasoning usually gives the greatest advances in science, while deductive reasoning gives the greatest utility to engineering. Both forms of reasoning are encouraged.

Regardless of the form of reasoning, you should test your conclusions against independent data. Isaac Newton used inductive reasoning to develop his concepts of motion and gravity, based on observations he made near his home in England. To test his theories with independent data, he built telescopes to watch the motion of planets. Because his theories were found to be **universal** (i.e., they worked elsewhere in the universe), Newton's theories were accepted as the Laws of Motion.

Similarly, engineers might use deductive reasoning to design a bridge, dam, or airplane. However, they first test their design with prototypes, scale models, or computer simulations to support their deductions before they construct the final products.

As an example, we can use reasoning with the Eulerian budgets for heat and moisture. From the Heat chapter, the Eulerian heat budget (eq. 3.51) was:

$$\left. \frac{\Delta T}{\Delta t} \right|_{x,y,z} = - \left[U \cdot \frac{\Delta T}{\Delta x} + V \cdot \frac{\Delta T}{\Delta y} \right] - \frac{\Delta F_{z \text{ turb}}}{\Delta z} + \frac{L_v}{C_p} \cdot \frac{m_{\text{condensing}}}{m_{\text{air}} \cdot \Delta t} - 0.1 \frac{\text{K}}{\text{h}}$$

where the first term on the right describes the effects of advection, the second is turbulence, the third term is latent heating, and the last is radiation. The last

(continues in page after next)

Solved Example

Consider a 9 km deep thunderstorm, and examine the bottom 1 km layer of the cloud. Rain from higher in the thunderstorm falls into the top of this layer at rate 0.9 cm/h. From the bottom of the layer, the precipitation rate is 1 cm/h. What is the rate of total water change in the layer?

Solution

Given: $Pr_{top} = 0.9 \text{ cm/h} = 2.5 \times 10^{-5} \text{ m/s}$,

$Pr_{bot} = 1.0 \text{ cm/h} = 2.78 \times 10^{-5} \text{ m/s}$,

$\Delta z = 1 \text{ km} = 1000 \text{ m}$

Find: $\Delta r_T / \Delta t = ? \text{ (g/kg)/s}$.

Assume: No advection or turbulence. Assume the average air density is 1 kg/m^3 in lower troposphere.

Use eq. (4.44):

$$\begin{aligned} \Delta r_T / \Delta t &= (\rho_L / \rho_d) \cdot (Pr_{top} - Pr_{bot}) / (z_{top} - z_{bot}) = \\ &= \frac{(1.0 \times 10^6 \text{ g}_{\text{water}} / \text{m}^3)}{(1 \text{ kg}_{\text{air}} / \text{m}^3)} \cdot \frac{[(2.5 - 2.78) \times 10^{-5} \text{ m/s}]}{1000 \text{ m}} \\ &= \underline{-0.0028 \text{ (g/kg)/s}} = -10.1 \text{ (g/kg)/h} \end{aligned}$$

Check: Units OK. Physics OK.

Discussion: Thus, water is lost from this part of the cloud. If there is no advection of moisture to replace that lost as rain, then this part of the cloud will rain itself dry in several hours if loss continues at a constant rate.

Solved Example

Given a latent heat flux of $300 \text{ W} \cdot \text{m}^{-2}$, find the evaporation rate.

Solution

Given: $F_E = 300 \text{ W} \cdot \text{m}^{-2}$

Find: $Evap = ? \text{ mm/day}$

$$\begin{aligned} \text{Use eq. (4.49): } Evap &= a \cdot F_E \\ &= [0.0346 \text{ (m}^2/\text{W)} \cdot (\text{mm/day})] \cdot (300 \text{ W/m}^2) \\ &= \underline{10.38 \text{ mm/day}} \end{aligned}$$

Check: Units OK. Physics OK.

Discussion: Evaporation of about 1 cm of water per day could cause significant drying of soil and lowering of water levels in lakes unless replenished by precipitation.

Precipitation rate Pr has units of depth of liquid water equivalent per time (e.g., mm/h, or m/s). It tells us how quickly the depth of water in a hypothetical rain gauge would change.

Precipitation rate is a function of altitude, as if the rain gauge could be hypothetically mounted at any altitude in the atmosphere. Some precipitation might evaporate when falling through the air, or condensation might increase the precipitation rate on the way down. If more precipitation falls into the top of a volume than falls out of the bottom, then total water within the volume must increase. At the Earth's surface, precipitation rate equals **rainfall rate** RR by definition.

Surface Moisture Flux

The latent heat flux described in the Heat chapter is, by definition, associated with a flux of water. Hence, the heat and moisture budgets are coupled. Methods to calculate the latent heat flux were described in that chapter under the topic of the surface heat budget. Knowing the **surface latent heat flux**, you can use the equations below to transform it into a water flux.

The **vertical flux of water vapor** F_{water} ($\text{kg}_{\text{water}} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$) is related to the latent heat flux F_E (W/m^2) by:

$$F_{\text{water}} = F_E / L_v \quad \bullet(4.45)$$

$$F_{\text{water}} = \rho_{\text{air}} \cdot (C_p / L_v) \cdot F_E \quad (4.46)$$

$$= \rho_{\text{air}} \cdot \gamma \cdot F_E \quad (4.47)$$

where the latent heat of vaporization is $L_v = 2.5 \times 10^6 \text{ J/kg}$, $\gamma = C_p / L_v = 0.4 \text{ (g}_{\text{water}}/\text{kg}_{\text{air}}) \cdot \text{K}^{-1}$ is the **psychrometric constant**, and where the kinematic latent heat flux F_E has units of $\text{K} \cdot \text{m} \cdot \text{s}^{-1}$.

Eq. (4.45) can be put into kinematic form by dividing by air density, ρ_{air}

$$F_{\text{water}} = F_E / (\rho_{\text{air}} \cdot L_v) = \gamma \cdot F_E \quad (4.48)$$

where F_{water} is like a mixing ratio times a vertical velocity, and has units of $(\text{kg}_{\text{water}}/\text{kg}_{\text{air}}) \cdot (\text{m} \cdot \text{s}^{-1})$.

Sometimes, this moisture flux is expressed as an **evaporation rate** $Evap$ in terms of depth of liquid water that is lost per unit time (e.g., mm/day):

$$Evap = F_{\text{water}} / \rho_L = F_E / (\rho_L \cdot L_v) = a \cdot F_E \quad (4.49)$$

$$Evap = (\rho_{\text{air}} / \rho_L) \cdot F_{\text{water}} = (\rho_{\text{air}} / \rho_L) \cdot \gamma \cdot F_E \quad \bullet(4.50)$$

where $a = 4.0 \times 10^{-10} \text{ m}^3 \cdot \text{W}^{-1} \cdot \text{s}^{-1}$ or $a = 0.0346 \text{ (m}^2/\text{W)} \cdot \text{(mm/day)}$, and $\rho_L = 1000 \text{ kg}_{\text{liq}}/\text{m}^3$.

If the latent heat flux is not known, you can estimate F_{water} from the mixing-ratio difference between the surface (r_{sfc}) and the air (r_{air}). For windy, overcast conditions, use:

$$F_{\text{water}} = C_H \cdot M \cdot (r_{\text{sfc}} - r_{\text{air}}) \quad (4.51)$$

where C_H is the dimensionless **bulk transfer coefficient** (moisture and heat are nearly identical, see the Heat chapter), and M is wind speed. Both r_{air} and M are measured at $z = 10 \text{ m}$. C_H varies from 2×10^{-3} over smooth surfaces to 2×10^{-2} over rough surfaces.

During free convection of sunny days and light or calm winds:

$$F_{\text{water}} = b_H \cdot w_B \cdot (r_{\text{sfc}} - r_{ML}) \quad (4.52)$$

where r_{ML} is the mixing ratio in the middle of the mixed layer, and $b_H = 5 \times 10^{-4}$ is the **convective transport coefficient** for heat (assumed to be identical to that for moisture, see the Heat chapter). The buoyancy velocity scale w_B was given in the Heat chapter for convective conditions.

These formulas are difficult to use because r_{sfc} is not usually measured. Over oceans, lakes, rivers, and saturated ground, you can assume that the air touching the surface is nearly saturated; thus, $r_{\text{sfc}} \approx r_s(T_{\text{sfc}})$, where T_{sfc} is the water-surface (skin) temperature. Over drier ground the value is less than saturated, but is not precisely known.

Turbulent Transport

Turbulent transport tends to homogenize moisture by mixing moist and dry regions of air to create a larger region of average humidity. $\Delta F_z \text{ turb}(r_T)/\Delta z$ is the change of turbulent flux of total water across distance Δz , and is analogous to the turbulence heat flux discussed in the Heat chapter. $\Delta F/\Delta z$ is called the **flux divergence**.

In the fair-weather, daytime, convective boundary layer (i.e., for $0 < z < z_i$), turbulence causes a flux divergence in the vertical:

(4.53)

$$\frac{\Delta F_z \text{ turb}(r_T)}{\Delta z} = \frac{F_z \text{ turb at } z_i(r_T) - F_z \text{ turb at surface}(r_T)}{z_i - z_{\text{surface}}}$$

or

$$\frac{\Delta F_z \text{ turb}(r_T)}{\Delta z} = - \left[0.2 \cdot (\Delta z_i r_T / \Delta z_i \theta) \cdot F_H \right] + F_{\text{water}} \quad (4.54)$$

Solved Example

Find the kinematic water flux over a lake with saturation mixing ratio of $r_{\text{sfc}} = 15 \text{ g/kg}$. The air is overcast with $M = 10 \text{ m/s}$ and $r = 8 \text{ g/kg}$ at height 10 m

Solution

Given: $r_{\text{sfc}} = 15 \text{ g/kg}$, $M = 10 \text{ m/s}$,

$r_{\text{air}} = 8 \text{ g/kg}$

Find: $F_{\text{water}} = ? \text{ (kg}_{\text{water}}/\text{kg}_{\text{air}}) \cdot (\text{m} \cdot \text{s}^{-1})$

Assume: smooth lake surface: $C_H = 2 \times 10^{-3}$ (from paragraph after eq. 3.35).

Use eq. (4.51): $F_{\text{water}} = C_H M (r_{\text{sfc}} - r_{\text{air}})$

$$\begin{aligned} F_{\text{water}} &= (2 \times 10^{-3}) \cdot (10 \text{ m/s}) \cdot (15 - 8 \text{ g/kg}) \\ &= 0.14 \text{ (g}_{\text{water}}/\text{kg}_{\text{air}}) \cdot (\text{m} \cdot \text{s}^{-1}) \\ &= \underline{\underline{0.00014}} \text{ (kg}_{\text{water}}/\text{kg}_{\text{air}}) \cdot (\text{m} \cdot \text{s}^{-1}) \end{aligned}$$

Check: Units OK. Physics OK.

Discussion: Eq. (4.50) says this is equivalent to $1.72 \times 10^{-7} \text{ m/s}$ of evaporation, or 14.8 mm/day , which is a strong evaporation rate.

ON DOING SCIENCE (continuation)

two terms are source and sink terms of heat.

The Eulerian moisture budget (eq. 4.44) is:

$$\begin{aligned} \frac{\Delta r_T}{\Delta t} = - \left[U \cdot \frac{\Delta r_T}{\Delta x} + V \cdot \frac{\Delta r_T}{\Delta y} \right] - \frac{\Delta F_z \text{ turb}(r_T)}{\Delta z} \\ + \left(\frac{\rho_L}{\rho_d} \right) \frac{\Delta Pr}{\Delta z} \end{aligned}$$

Again, the first term on the right describes advection, the second gives turbulence, and the last is a source term associated with precipitation.

Both equations above have the same pattern; namely, a time-tendency term on the left. On the right are advection, turbulence, and source/sink terms.

Inductive reasoning would allow us to state that Eulerian conservation or budget equations in general should be of the form:

$$\frac{\Delta \Psi}{\Delta t} = - \left[U \cdot \frac{\Delta \Psi}{\Delta x} + V \cdot \frac{\Delta \Psi}{\Delta y} \right] - \frac{\Delta F_z \text{ turb}(\Psi)}{\Delta z} + S_\Psi$$

where Ψ is any variable, and S_Ψ represents the source and sink terms. Mean vertical motion and horizontal turbulence flux divergence were neglected.

Given the equation above, we could use **deductive reasoning** to apply it to a new variable, such as the U component of velocity:

$$\frac{\Delta U}{\Delta t} = - \left[U \cdot \frac{\Delta U}{\Delta x} + V \cdot \frac{\Delta U}{\Delta y} \right] - \frac{\Delta F_z \text{ turb}(U)}{\Delta z} + S_U$$

Before accepting this, it should be tested and verified against observations. (This was already done by others. The new equation indeed works, as will be shown in the Dynamics chapter.)

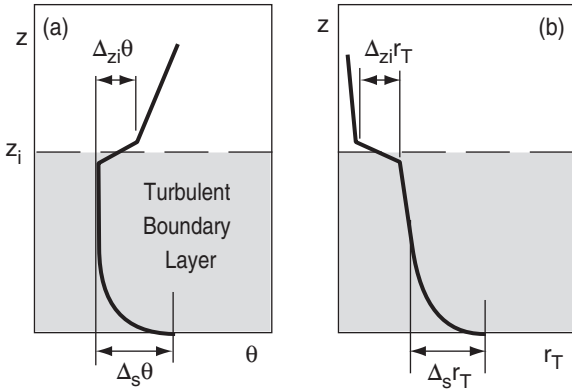


Figure 4.12 Method of estimating jumps in (a) potential temperature $\Delta\theta$, and in (b) total water Δr_T at the top and bottom of a turbulent boundary layer of depth z_i , given the sounding (thick lines). For moisture, the jump $\Delta_{z_i} r_T$ can be positive or negative depending on whether the air aloft is moister or drier than the boundary layer. Only the case of drier air aloft is shown in (b).

where F_H is the kinematic effective surface heat flux (units of $K \cdot m/s$), and F_{water} is the kinematic effective surface water-vapor flux [units of $(kg_{water}/kg_{air}) \cdot (m/s)$]. The operator $\Delta_{z_i} = ()_{z_{i+}} - ()_{z_{i-}}$ is the difference between the value of any variable $()$ just above $(+)$ the top of the boundary layer, minus the value just below $(-)$ the top (Fig. 4.12), where the average top of the boundary layer is at height z_i .

Because surface moisture flux F_{water} is directly related to the surface latent heat flux F_E (eq. 4.48), rewrite (4.54) as:

$$\frac{\Delta F_{z \text{ turb}}(r_T)}{\Delta z} = - \frac{0.2 \cdot (\Delta_{z_i} r_T / \Delta_{z_i} \theta) \cdot F_H + \gamma \cdot F_E}{z_i} \quad \text{for } 0 < z < z_i \quad (4.55)$$

for the convective boundary layer, where $\gamma = 0.4 (g_{water}/kg_{air}) \cdot K^{-1}$ is the psychrometric constant.

Another version can be written using the effective surface heat-flux parameterization of the Heat chapter:

$$\frac{\Delta F_{z \text{ turb}}(r_T)}{\Delta z} = - \frac{b_H \cdot w_B}{z_i} \left\{ \left[0.2 \cdot (\Delta_{z_i} r_T) \cdot \left| \frac{\Delta_s \theta}{\Delta_{z_i} \theta} \right| \right] + |\Delta_s r_T| \right\} \quad \text{for } 0 < z < z_i \quad (4.56)$$

where b_H is the convective transport coefficient (see the Heat chapter), w_B is the buoyancy velocity scale, and operator $\Delta_s() = ()_{surface} - ()_{mid \text{ mixed-layer}}$. The four equations above are equivalent — use the version that is most convenient for the values on hand.

Above the fair-weather boundary layer (i.e., for $z > z_i$), turbulence is often negligible:

$$\frac{\Delta F_{z \text{ turb}}(r_T)}{\Delta z} \approx 0 \quad (4.57)$$

Also, within the nighttime boundary layer, the equation above is valid at all heights.

For some situations, the moisture flux (F_{water}) at the Earth's surface into or out of the bottom of a volume of air might be known. A cube of air sitting on a lake surface might gain total water by evaporation from the lake, resulting from an upward or positive flux of water at the surface. Alternately, a cube of humid air sitting over a snow surface might lose water by condensation onto the snow, corresponding to a negative or downward flux of water at the surface. If the flux at the top of the volume is near zero, then the vertical flux divergence is approximately

Solved Example

Given measurements of a 1 km thick boundary layer as shown in Fig. 4.12, with $|\Delta_s \theta| = 8^\circ C$, $|\Delta_{z_i} \theta| = 3^\circ C$, $|\Delta_s r_T| = 5 \text{ g/kg}$, and $\Delta_{z_i} r_T = -4 \text{ g/kg}$. The convective transport coefficient times buoyancy velocity is 0.025 m/s on this calm day. What is the vertical flux divergence of total water due to turbulence, and how does that contribute to moistening the boundary layer?

Solution

Given: $|\Delta_s \theta| = 9^\circ C$, $|\Delta_{z_i} \theta| = 3^\circ C$,
 $|\Delta_s r_T| = 5 \text{ g/kg}$, $\Delta_{z_i} r_T = -4 \text{ g/kg}$,
 $z_i = 1 \text{ km} = 1000 \text{ m}$, $b_H \cdot w_B = 0.025 \text{ m/s}$.
 Find: $\Delta r_T / \Delta t = -\Delta F_{z \text{ turb}}(r_T) / \Delta z = ? \text{ (g/kg)/s}$
 Assume: No advection or precipitation.

Use eq. (4.56):

$$\begin{aligned} \frac{\Delta F_{z \text{ turb}}(r_T)}{\Delta z} &= - \frac{b_H \cdot w_B}{z_i} \left[0.2 \cdot (\Delta_{z_i} r_T) \cdot \left| \frac{\Delta_s \theta}{\Delta_{z_i} \theta} \right| + |\Delta_s r_T| \right] \\ &= - \frac{(0.025 \text{ m/s})}{(1000 \text{ m})} \left[0.2 \cdot \left(-4 \frac{\text{g}}{\text{kg}} \right) \cdot \left| \frac{9^\circ C}{3^\circ C} \right| + \left(5 \frac{\text{g}}{\text{kg}} \right) \right] \\ &= -0.000065 \text{ (g/kg)/s} \end{aligned}$$

Thus:

$$\Delta r_T / \Delta t = -\Delta F_{z \text{ turb}}(r_T) / \Delta z = +0.000065 \text{ (g/kg)/s}$$

Check: Units OK. Physics OK.

Discussion: (continues on next page)

$\Delta F_{z \text{ turb}}(r_T)/\Delta z = -F_{\text{bottom}}(r_T)/d$, where d is depth of the volume.

Turbulent mixing of moisture by thunderstorms is vigorous and complex. Prior to the storm, a humid boundary layer underlies a dry layer of air in the middle troposphere. The thunderstorm lifts and consumes the humid air from the boundary layer, and replaces it with drier air from aloft. However, much of the moisture from the former boundary layer is squeezed out as precipitation, and is accounted for in the precipitation term of the Eulerian moisture budget. The net result is that the humid, prestorm boundary layer becomes drier after the storm.

HUMIDITY INSTRUMENTS

Instruments that measure humidity are called **hygrometers**. Don't confuse the word with "hydrometers", which are used to measure specific gravity of fluids such as battery acid.

Dew-point hygrometers, also known as **chilled-mirror hygrometers**, reflect a beam of light off a tiny metal mirror. When the mirror is cooled to the dew-point temperature, dew forms on the mirror and the light beam scatters instead of reflecting into a detector. Electronics in the instrument cool or heat the mirror to maintain the surface precisely at the dew point temperature, which is provided as an output. These are accurate instruments with relatively slow response. For cold temperatures and low humidities, **frost-point hygrometers** are used instead.

Hair hygrometers use organic fibers such as long hairs, anchored at one end and attached at the other end by amplifying levers to a dial that reads out relative humidity. As the RH increases, the hairs get longer, causing the dial to turn. These are inaccurate, but are inexpensive and are the most common hygrometers for home use. Other hygrometers use other materials that also change their dimensions when they absorb water molecules.

Psychrometers are instruments with two liquid-in-glass thermometers attached to a board or frame. The bulb of one thermometer is surrounded by a sleeve or wick of cloth that is saturated with distilled water, while the other bulb remains dry. After both thermometers are actively ventilated [by whirling the instrument through the air on a hand-held axle (**sling psychrometer**), or by using a spring or electrically driven fan to blow air past the thermometers (**aspirated psychrometer**)], the two thermometers are read to give the **wet and dry-bulb temperatures**. The wet-bulb is cooler than the dry, because of the latent heat absorbed when

Solved Example (continuation)

Discussion: This is equivalent to a rate of +0.234 (g/kg)/h increase of water in the boundary layer. The air is getting juicier with time.

Water evaporates from the surface to increase the humidity, but dry air entrained into the top of the boundary layer tends to decrease humidity. For this example, the moistening from the surface was greater than the drying from aloft, but it could differ from day to day in real life. Also, there can be other situations where moistening happens from both the top and bottom of the boundary layer, such as when a dry boundary layer is growing into a cloud layer aloft.

water evaporates. This thermodynamic information can be used with psychrometric tables or charts (Figs. 4.4 & 4.5) to determine the humidity. These instruments are extremely slow response, but relatively simple. Modern psychrometers replace the liquid-in-glass thermometers with electronic thermometers such as thermistors.

In old **radiosondes** (balloon-borne weather instruments), the electrical resistance across a carbon-coated glass slide was measured. In more humid air, this **carbon-film hygrometer** becomes more resistive. Modern radiosondes often measure the capacitance across a very thin dielectric plastic that is coated on both sides with a porous metallic grid. Both approaches are small and light enough to be carried aloft, but both sensors can be easily contaminated by chemical vapors that change their electrical properties.

Microwave refractometers draw air into a small chamber filled with microwaves. The refraction (bending) of these microwave beams depends on humidity (see the radar section of the Remote Sensing chapter), and can be measured. These are very fast-response sensors.

Spectral absorption hygrometers, also known as **optical hygrometers**, transmit frequencies of electromagnetic radiation that are strongly absorbed by water vapor. By passing the beam of radiation across a short path of air to a detector, the amount of attenuation can be measured to allow calculation of the absolute humidity. One such instrument, the **Lyman-alpha hygrometer**, uses ultraviolet light of wavelength 0.121567 μm , corresponding to an absorption/emission line of hydrogen. Another, the **krypton hygrometer**, uses emissions at 0.12358 μm , generated by glow tube filled with the noble gas krypton. Other instruments use absorption of infrared light (**infrared hygrometers**). These are

all fast-response instruments. See the Remote Sensing chapter for absorption spectra across the atmosphere.

Some **lidars** (laser radars) have been developed to transmit two neighboring wavelengths of electromagnetic radiation, one of which is affected by water vapor and the other which is not. Such **differential absorption lidars (DIAL)** can remotely measure humidity along vertical or slant paths, and can scan the atmosphere to measure the humidity in a volume or in a plane.

Weather radars and other microwave profilers can be used to measure profiles of humidity in the atmosphere, because the speed and/or polarity of microwaves through air depends on humidity.

Some sensors measure path-averaged humidity. One example is the **water-vapor channel on weather satellites**, which measures infrared emissions from water vapor in the air. As discussed in the Remote Sensing chapter, such emissions come from a layer of air several kilometers thick in the top third of the troposphere. These instruments have the advantage of remotely sampling the atmosphere at locations that are difficult to reach otherwise, such as over the oceans. A disadvantage is that they have difficulty seeing through clouds.

Transmissions from **Global Positioning System (GPS)** satellites are slightly delayed or refracted by humidity along the path of the beam through the atmosphere. However, data from many such crossing beams from the constellation of GPS satellites can be computationally inverted to yield vertical profiles of humidity, similar to the medical X-ray tomography methods used for brain scans.

can be drawn on thermodynamic diagrams. Using such diagrams is quicker than calculating complex recursive equations for the moist lapse rate.

Isohumes can also be drawn on thermodynamic diagrams, to show the actual or maximum amount of moisture in the air. For many formulae and for thermodynamic diagrams, actual air temperature is related to equilibrium (saturation) amount of water vapor. Alternately, dew-point temperature is related to actual amount of water vapor in the air.

Threads

Isohumes and moist adiabats are used in Chapters 5 and 6 to create more detailed thermodynamic diagrams to help understand cloud growth. Saturation mixing ratio and vapor pressure determine the growth of individual cloud particles leading to precipitation (Chapter 7). Some of the energy for cyclones (Chapter 13), thunderstorms (Chapters 14 and 15) and hurricanes (Chapter 16) comes from latent heat release, which again depends on the humidity and saturation values in the air. Global warming associated with climate change (Chapter 21) might affect the cloud-cover amount and albedo via changes in humidity.

Temperature and pressure of the standard atmosphere of Chapter 1 were used to find boiling temperatures. The thermodynamic diagram of Chapter 3 was extended here to include isohumes and moist adiabats. Routine forecasts of humidity are made using numerical weather prediction (Chapter 20).

SUMMARY

The saturation (equilibrium) vapor pressure of warm air is greater than that of cool air. The Clausius-Clapeyron equation and Tetens' formula quantify this relationship.

Some relative moisture variables compare the actual amount of water vapor in the air to the saturation amount for that temperature: relative humidity, temperature dew-point spread, wet-bulb depression, and condensation level. Other variables measure the concentration of water in the air: vapor pressure, mixing ratio, specific humidity, and absolute humidity.

The amount of water in the air is described by a total water budget. Budgets can be formed following a cloud or air parcel (Lagrangian), or for a fixed volume (Eulerian). As saturated (cloudy) air rises, its temperature cools at the moist adiabatic lapse rate, which is less than the dry rate. These moist adiabats

EXERCISES
Numerical Problems

N1. Compare the saturation vapor pressure (with respect to liquid water) calculated with the Clausius-Clapeyron equation and Tetens' formula, for T (°C):

- a. 45 b. 40 c. 35 d. 30 e. 25 f. 20 g. 15
h. 10 i. 5 j. 0 k. -5 l. -10 m. -15 n. -20

N2. Calculate the saturation vapor pressures with respect to both liquid water and flat ice, for T (°C) =

- a. -3 b. -6 c. -9 d. -12 e. -15 f. -18
g. -21 h. -24 i. -27 j. -30 k. -35 l. -40

N3. Find the boiling temperature (°C) of pure water at altitudes (km) of:

- a. 0.2 b. 0.4 c. 0.6 d. 0.8 e. 1.2 f. 1.4
g. 1.6 h. 1.8 i. 2.2 j. 2.4 k. 2.6 l. 2.8

N4. Find e_s (kPa), T_d (°C), r and r_s (g/kg), q and q_s (g/kg), ρ_v and ρ_{vs} (g/m³), RH (%), LCL (km), and T_w (°C) for the situations below:

	T (°C)	P (kPa)	e (kPa)
a.	30	101.3	1.5
b.	10	80.0	1.0
c.	0	90.0	0.5
d.	5	50.0	0.4
e.	8	101.3	1.0
f.	40	70.0	5.0
g.	10	85.0	0.5
h.	20	90.0	2.0

N5(S). Generate a table similar to Table 4-1, but for an altitude where P (kPa) =

- a. 95 b. 90 c. 85 d. 80 e. 75 f. 70 g. 65
h. 60 i. 55 j. 50 k. 45 l. 40 m. 35 n. 30

N6. Saturated outside air of $T = 0^\circ\text{C}$ is drawn into your house to replace hot air going up the chimney. If this induced air is heated in the house to 20°C , what is the relative humidity inside?

N7. Find the mixing ratio at $P = 100$ kPa?

- a. $T = -10^\circ\text{C}$ with $RH = 100\%$
b. $T = +20^\circ\text{C}$ with $RH = 20\%$
c. $T = +5^\circ\text{C}$ with $RH = 50\%$
d. $T = -10^\circ\text{C}$ with $RH = 20\%$
e. $T = +20^\circ\text{C}$ with $RH = 50\%$
f. $T = +5^\circ\text{C}$ with $RH = 100\%$
g. $T = +50^\circ\text{C}$ with $RH = 100\%$
h. $T = +50^\circ\text{C}$ with $RH = 2\%$

N8. What is the LCL height for air with the following conditions at the surface. This is the cloud-base height for convective (cumuliform) clouds.

	T (°C)	T_d (°C)		T (°C)	T_d (°C)
a.	30	10	e.	20	14
b.	10	10	f.	40	35
c.	0	-5	g.	-20	-20
d.	30	0	h.	-10	-40

N9. Psychrometer measurements give the dry-bulb temperatures, T , and wet-bulb temperatures, T_w listed below. What is the mixing ratio, dew point and relative humidity? Use the equations for this exercise.

	T (°C)	P (kPa)	T_w (°C)
a.	30	101.3	20
b.	10	80.0	10
c.	0	90.0	-5
d.	30	100.0	15
e.	20	90.0	14
f.	0	95.0	-5
g.	20	100.0	6
h.	40	100.0	20

N10. Same as the previous exercise, but you may use the psychrometric graphs (Figs. 4.4 or 4.5).

N11. Given the following temperatures and dew-point temperatures, use the equations of Normand's Rule to calculate the wet-bulb temperature.

	T (°C)	T_d (°C)
a.	30	10
b.	10	10
c.	0	-5
d.	30	0
e.	20	14
f.	40	35
g.	-20	-20
h.	-10	-40

N12. Same as the previous exercise, but you may use a thermo diagram

N13. What is the total-water mixing ratio at sea level, given:

- a. $r = 5$ g/kg, $r_L = 1$ g/kg
b. $r = 5$ g/kg, $r_L = 3$ g/kg
c. $T = 10^\circ\text{C}$, $r_L = 2$ g/kg
d. $r = 10$ g/kg, $r_L = 5$ g/kg
e. $T = 5^\circ\text{C}$, $r_L = 5$ g/kg
f. $T = 3^\circ\text{C}$, $r_L = 3$ g/kg
g. $T = 0^\circ\text{C}$, $r_L = 4$ g/kg
h. $T = 20^\circ\text{C}$, $r_L = 5$ g/kg

- N14. What is the liquid water mixing ratio at sea level for:
- a. $T = 30^\circ\text{C}$, $r_T = 35 \text{ g/kg}$
 - b. $T = 20^\circ\text{C}$, $r_T = 25 \text{ g/kg}$
 - c. $T = 30^\circ\text{C}$, $r_T = 30 \text{ g/kg}$
 - d. $T = 20^\circ\text{C}$, $r_T = 30 \text{ g/kg}$
 - e. $T = 10^\circ\text{C}$, $r_T = 20 \text{ g/kg}$
 - f. $T = 10^\circ\text{C}$, $r_T = 10 \text{ g/kg}$
 - g. $T = 0^\circ\text{C}$, $r_T = 5 \text{ g/kg}$
 - h. $T = 0^\circ\text{C}$, $r_T = 7 \text{ g/kg}$

- N15. Using the appropriate equation, calculate the dew-point temperature for air at $P = 50 \text{ kPa}$, given a mixing ratio (g/kg) of
- a. 30 b. 25 c. 20 d. 15 e. 10 f. 5 g. 3
 - h. 2 i. 1 j. 0.5 k. 0.3 l. 0.2 m. 0.1 n. 0.05

- N16. Same as the previous exercise, but you may use Fig. 4.7.

- N17. Using Fig. 4.7, what is the dew point temperature of air parcels ending at a pressure height of 50 kPa, if they start at 100 kPa with dew-point temperatures ($^\circ\text{C}$) of
- a. 40 b. 35 c. 30 d. 25 e. 20 f. 15 g. 10
 - h. 5 i. 0 j. -5 k. -10 l. -15 m. -20 n. -25

- N18. Using Fig. 4.7, find the relative humidity for air at
- | | $P \text{ (kPa)}$ | $T \text{ (}^\circ\text{C)}$ | $T_d \text{ (}^\circ\text{C)}$ |
|----|-------------------|------------------------------|--------------------------------|
| a. | 70 | 20 | 0 |
| b. | 70 | 10 | 10 |
| c. | 70 | 30 | -10 |
| d. | 90 | 30 | 30 |
| e. | 90 | 30 | 20 |
| f. | 90 | 30 | 10 |
| g. | 100 | 20 | -20 |
| h. | 100 | -20 | -30 |

- N19. Using the equations, what is the value of the moist adiabatic lapse rate ($^\circ\text{C/km}$) at $P = 100 \text{ kPa}$ for the following temperatures ($^\circ\text{C}$)?
- a. 40 b. 30 c. 20 d. 10 e. 0 f. -10 g. -20
- Same, but at $P = 50 \text{ kPa}$:
- h. 20 i. 15 j. 10 k. 5 l. 0 m. -5 n. -10

- N20. Same as the previous exercise, but find the moist adiabatic lapse rate as a change of temperature with pressure ($^\circ\text{C/kPa}$).

- N21. Using Fig. 4.8, what is the final temperature of a saturated air parcel at $P = 70 \text{ kPa}$ if lifted moist adiabatically from $P = 100 \text{ kPa}$ with the following initial temperatures ($^\circ\text{C}$):
- a. -30 b. -25 c. -20 d. -15 e. -10 f. -5 g. 0
 - h. 5 i. 10 j. 15 k. 20 l. 25 m. 30

- N22. Same as the previous exercise, but starting at $P = 50 \text{ kPa}$ and lowering moist adiabatically to $P = 80 \text{ kPa}$.

- N23. Identify the liquid-water and equivalent potential temperatures for the air parcels in the previous two exercises, using Fig. 4.8 and other figures for dry adiabats in the Heat chapter.

- N24. At height 1 km, the temperature is 5°C , the water vapor mixing ratio is 6 g/kg , and liquid-water mixing ratio is 2 g/kg . Use the equations to find the:
- a. liquid water potential temperature
 - b. equivalent potential temperature

- N25. An unsaturated air parcel from sea level has $r = 15 \text{ g/kg}$ and $T = 30^\circ\text{C}$. If it rises to a height where it is saturated with $r_L \text{ (g/kg)}$ as given below, what is its new value of r ?
- a. 0.5 b. 1 c. 1.5 d. 2 e. 2.5 f. 3 g. 3.5
 - h. 4 i. 4.5 j. 5 k. 5.5 l. 6 m. 6.5 n. 7

- N26. A horizontally uniform wind given below is advecting air of $r_T = 10 \text{ g/kg}$ at the west side of a region. The other side of the region is 100 km further east, and has a humidity of $r_T = 15 \text{ g/kg}$. How does this advection alter the total water mixing ratio across that 100 km-wide region? $U \text{ (m/s)} =$
- a. 2 b. 4 c. 5 d. 7 e. 10 f. 12 g. 15
 - h. 18 i. 20 j. 21 k. 23 l. 25 m. 27 n. 30

- N27. Precipitation of rate 4 mm/h enters a volume of air at altitude $z = 100 \text{ m}$, but only the amount given below (mm/h) reaches the ground at $z = 0$. How does this precipitation gradient alter the total water mixing ratio within that 100 m-thick layer?
- a. 6 b. 5.5 c. 5 d. 4.5 e. 4.3 f. 4.2 g. 4.1
 - h. 4 i. 3.9 j. 3.8 k. 3.7 l. 3.5 m. 3 n. 2.5

- N28. Given below the value of latent heat flux ($\text{W}\cdot\text{m}^{-2}$) at the surface. Find the kinematic value of latent flux ($\text{K}\cdot\text{m/s}$), the vertical flux of water vapor ($\text{kg}_{\text{water}}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$), the vertical flux of water vapor in kinematic form ($\text{kg}_{\text{water}}/\text{kg}_{\text{air}}\cdot\text{m/s}$), and the evaporation rate (mm/d).
- a. 100 b. 150 c. 200 d. 250 e. 300 f. 350
 - g. 80 h. 75 i. 70 j. 60 k. 50 l. 40 m. 25

N29. For windy, overcast conditions, estimate the kinematic latent heat flux at the surface, assuming $C_H = 5 \times 10^{-3}$.

	M (m/s)	r_{sfc} (g/kg)	r_{air} (g/kg)
a.	2	25	10
b.	2	20	10
c.	2	15	10
d.	5	25	10
e.	5	20	10
f.	5	15	10
g.	12	25	10
h.	12	20	10
i.	12	15	10

N30. For sunny, free-convective conditions, estimate the kinematic latent heat flux at the surface.

	w_B (m/s)	r_{sfc} (g/kg)	r_{ML} (g/kg)
a.	5	25	10
b.	5	20	10
c.	5	15	10
d.	12	25	10
e.	12	20	10
f.	12	15	10
g.	25	25	10
h.	25	20	10
i.	25	15	10

N31. A radiosonde balloon launched during daytime fair-weather shows vertical profiles in a 1 km thick boundary layer as shown in Fig. 4.12, with $|\Delta_s \theta| = 10^\circ\text{C}$, $|\Delta_{zi} \theta| = 2^\circ\text{C}$, $|\Delta_s r_T| = 4 \text{ g/kg}$, and $\Delta_{zi} r_T$ (g/kg) as given below. The convective transport coefficient times buoyancy velocity is 0.025 m/s on this calm day. What is the vertical flux divergence of total water due to turbulence, and how does that contribute to moistening the air?

- a. -10 b. -8 c. -5 d. -3 e. -2 f. -1 g. 0
h. +1 i. 2 j. 4 k. 6 l. 7 m. 9 n. 12

Understanding & Critical Evaluation

U1. Vapor pressure e represents the partial pressure of water vapor in air. Namely, the total pressure in air can be thought of as consisting as the sum of partial pressures from each of the individual gases within the air.

If total air pressure remains constant, but if evaporation of liquid water into the air causes the humidity to increase (i.e., the vapor pressure increases), then what happens to the other gases in air? Where do they go, if total pressure remains constant?

U2(S). a. Use a spreadsheet to calculate saturation vapor pressure over ice (not over water) for temperatures between -50 and 0°C . Plot the resulting ice curve together with the water curve on the same

graph, as in Fig. 4.2. Also, plot a curve of the difference between the ice and water curves, as shown in the inset in Fig. 4.2.

b. Replot the results as semi-log and log-log.

c. Why are the saturation vapor pressures different over water vs. ice?

U3. The Clausius-Clapeyron equation is based on an assumption of air over a flat surface of pure liquid water, and assumes that the liquid water and the air are of the same temperature. However, sometimes in the atmosphere there is warm rain falling into a region of colder air. Explain what would happen in this situation. (Neglect droplet surface curvature.)

U4. In Table 4-1, why can the same columns of numbers be used for actual temperature with saturation values of humidity (the top set of column labels), and for the dew point temperature with actual values of humidity (the bottom set of column labels)?

U5(S). a. Plot a curve of boiling temperature vs. height over the depth of the troposphere.

b. The purpose of pressure cookers is to cook boiled foods faster. Plot boiling temperature vs. pressure for pressures between 1 and 2 times $P_{sea\ level}$.

U6(S). It is often assumed that specific humidity is approximately equal to mixing ratio. How good is that assumption? Create a graph comparing the two values over a range of temperatures. Do this for a few different pressures.

U7(S). On a graph of T vs. T_d , plot curves for different values of RH .

U8. Why is it important to keep thermometers dry (i.e., placing them in a ventilated enclosure such as a **Stevenson screen**) when measuring outside air temperature?

U9(S). Check whether eq. (4.16a) or (4.16b) is a reasonable approximation to the lifting condensation level (LCL). The exact definition is the height of a rising air parcel where temperature and dew point become equal.

a. Do this check using the thermo diagram, for a variety of T and T_d values at the same starting height. For T , follow a dry adiabat upward from the starting temperature and height. For T_d , follow an line of constant mixing ratio up from the starting T_d and height. Where the two lines cross is the LCL.

b. Do the check using the equations for adiabatic temperature change from an earlier chapter, and using the equation for dew point temperature as a function of height or pressure.

U10. Create a table that lists the max and min values allowed for vapor pressure, mixing ratio, specific humidity, absolute humidity, relative humidity, dew-point temperature, LCL, and wet-bulb temperature.

U11. In arid regions (hot, dry), an inexpensive form of air conditioning (sometimes known as a **swamp cooler**) is possible where liquid water is evaporated into the dry air. The result is cooler air (due to latent heat effects) that is more humid.

a. For air of initial temperature $T = 40^\circ\text{C}$, how effective are swamp coolers for different initial relative humidities?

b. At what initial humidity would a swamp cooler not cause the air to be more comfortable? (Hint, use the heat index info from the Heat chapter.)

U12. a. Rearrange the T_w equations to find the T_w value corresponding to $r = 0$, for arbitrary T and P .

b. Test these values from part (a) for a variety of temperatures at $P = 100$ kPa.

U13(§). Use a spreadsheet program to generate a table that gives dew point temperatures from measurements of wet and dry bulb temperatures. Check that your results are consistent with Figs. 4.4 & 4.5.

U14(§). For **Normand's Rule**, verify that eqs. (4.20) and (4.21) describe the same process as shown graphically in Fig. 4.6, and compare the numerical value of T_w using thermo-diagram vs. equation methods, for several different starting values of T and T_d .

U15. Use the definitions of mixing ratio eq. (4.4), specific humidity eq. (4.7), and absolute humidity eq. (4.10) to verify relationships (4.22) - (4.27).

U16. Consider a Lagrangian, cloudy air parcel.

a. Suppose precipitation falls into the top of the air parcel, passes through the air parcel, and falls out of the bottom at the same rate that it entered. Discuss what happens with respect to eqs. (4.35).

b. What if no precipitation enters the top or sides of the air parcel, but some falls out of the bottom. Discuss what happens with respect to eqs. (4.35).

U17(§). Use a spreadsheet program to draw dry adiabats and mixing ratio lines on the same thermodynamic diagram, and plot as a semi-log graph. That is, combine Figs. 3.3 and 4.7.

U18. a. Derive eq. (4.36) from the Clausius-Clapeyron equation.

b. Derive a similar equation for dew-point temperature, but using Tetens' formula.

U19. What is the behavior of eq. (4.37b) in the limit of zero absolute temperature, and in the limit of very high temperature?

U20(§). Use a spreadsheet to draw dry and moist adiabats on the same thermodynamic diagram, and plot as a semi-log graph similar to Fig. 4.8.

U21(§). (Very time-consuming but important problem, resulting in a complete thermodynamic diagram.)

a. Use a spreadsheet program to draw dry and moist adiabats and mixing ratio lines on the same thermodynamic diagram. If you can draw graphs in color, use red lines for the dry adiabats, blue lines for the moist adiabats, and green lines for the mixing ratios. The background isotherm and isobar grid lines can be black. Plot the isotherms and adiabats for $(T, \theta, \theta_w) = -60, -40, -20, 0, 20, 40^\circ\text{C}$. Plot the isohumes for $r = 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50$ g/kg.

b. (Optional) Also, compute and plot on the same graph lines of constant altitude (m) vs. T_{surface} and P , using the hypsometric equation. If you have color, draw these lines in brown or tan. Assume a dry adiabatic lapse rate from the surface.

U22. In eqs. (4.40) and (4.41) is a factor θ/T . Why is it in those equations? Do they cancel because they are both absolute temperatures?

U23. Let $r_T = r + r_L$ be the total water mixing ratio, which is the sum of the water vapor r plus any liquid water r_L (cloud droplets) present in the air. Consider a rising air parcel with initially no liquid water that is unsaturated, starting from near the surface. Thus the total water equals the initial water vapor mixing ratio.

a. As the air parcel rises toward its LCL, is θ_L conserved? Explain.

b. Above the LCL, a rising air parcel does not conserve θ , because condensation of water releases latent heat into the air and increases the liquid water content. Is θ_L conserved for this rising air parcel that is above its LCL? Explain.

U24. If air starting from near the surface (at $P = 100$ kPa) is initially just saturated, find the values of equivalent potential temperature θ_e for each of the moist adiabats drawn in Fig. 4.8.

Explanation: $\theta_e > \theta_L$, because θ_e represents the potential temperature of the air after being heated by condensation of ALL the water vapor in the air, while θ_L represents the potential temperature of the air after being cooled by evaporation of all the liquid water present in the air.

U25. Rewrite eq. (4.44) to include vertical advection by mean wind W , and to include evaporation from the ocean. Assume that you are considering a fixed Eulerian volume (such as a hypothetical cube of air) that rests on the sea surface.

U26. Without the simplifying assumptions, write the full Eulerian water budget equation (i.e., like eq. 4.44, but including the missing terms). (Hint, look at the full heat budget equation from the Heat chapter.)

U27. In the Eulerian budget eq. for total water (4.44), there is a precipitation term. In the Eulerian budget eq. for heat (in the Heat chapter), there is a latent heat term that depends on condensation. Condensation and precipitation are related, yet the terms in those two eqs. are different. Why must they be different?

U28(S). Plot curves kinematic latent flux vs. evaporation rate for different altitudes.

U29. Derive eq. (4.54), assuming that $F_{water\ z_i} = w_e \cdot \Delta_{z_i} r_T$ and $F_{H\ z_i} = 0.02 \cdot F_H$, and $F_{H\ z_i} = w_e \cdot \Delta_{z_i} \theta$, where w_e is entrainment velocity.

U30. Derive eq. (4.56) from (4.55).

U31. Which humidity sensors would be best suited for measuring the rapid fluctuations of humidity in the turbulent boundary layer? Why?

Web-Enhanced Questions

W1. Access the surface weather data for a weather station close to your location (or at some other location specified by your instructor). What is the current humidity (value and units)?

W2. Find a weather map on the web that shows either colored or contoured values of humidity over your country for a recent observation time. Where are the humidities the greatest; the smallest?

W3. Do the humidity formulas presented in the El Paso NWS web page agree with those in this textbook? Compare and discuss. <http://www.srh.noaa.gov/epz/?n=wxcalc>

W4. Search the web for weather calculator tools (other than the one listed above from the NWS) for calculating or converting humidities, temperatures, and/or pressures between various units.

W5. Search the web to create a catalog or table of the different types of instruments that can be used for measuring humidity. Don't give brand names here, but instead list the type of instrument (such as sling psychrometer). For each type of instrument, briefly explain the physical process that it utilizes to measure humidity, and state which humidity variable it measures.

W6. Search the web for tables or graphs or definitions of various indices that measure discomfort or heat-stress danger for humans and/or livestock as a function of hot air temperature and high humidity. (Hint, Two traditional sources of info are: Houghton, D.D., 1985: Handbook of Applied Meteorology, Wiley Interscience, p786. Weiss, M. H., 1983: Quantifying summer discomfort, Bulletin American Meteor. Soc., 64, 654-655.)

There are a variety of such indices:

- **apparent temperature,**
- **discomfort index,**
- **effective temperature,**
- **heat index** (also known as the R. G. Steadman Heat Index),
- **humidex,**
- **humisery,**
- **humiture,**
- **index of thermal stress.**
- **livestock weather safety index,**
- **summer simmer index,**
- **temperature-humidity index (THI),**
- **wet-bulb globe temperature,**

W7. Search the web to find a current weather map analysis that shows isohumes (surface, or at 85 kPa, or at 70 kPa) over your state, province, or country (as assigned by the instructor). Locate your town or school on this map. Search for related current weather info that gives the wind speed and direction at that same altitude over your town.

a. Compute the moisture advection term of the Eulerian moisture budget, and discuss whether the result will cause the air at your location to become more or less humid with time.

b. What precipitation or evaporation rate would be needed to compensate for the advection from part (a), which would result in no change to total water mixing ratio (neglecting turbulence)?

W8. Access the current surface weather data near you every hour, and plot humidity vs. time. Alternately, a web site might already have a plot of humidity vs. time that you could use directly. Plots of weather variables vs. time are called **meteograms**.

W9. Search the web for web sites that plot upper-air soundings on thermodynamic diagrams. In particular, look for different types of thermo diagrams (not for different soundings on the same type of diagram). Print out a sample of each different type of thermo diagram.

In almost all thermo diagrams, the moist adiabats are curved lines that gradually become tangent to the dry adiabats at high altitudes. Use this behavior to identify the moist and dry adiabats in each type of diagram. Then, also identify the isotherms, isobars, and isohumes. You should be able to interpret any type of thermo diagram.

Synthesis Questions

S1. Suppose that saturation vapor pressure decreases as air temperature increases. Assume that clouds, fog, and precipitation can still form in air that becomes saturated. How would cloudiness and fog-giness near the ground vary during a daily cycle? How would it vary seasonally?

S2. Suppose that dew-point temperature did not depend on pressure or altitude. Would the lines in a thermo diagram be different? If so, describe them.

S3. Suppose you were to design a new humidity sensor that measured absorption of radiation across a path of humid air. Assume that the path length through which the radiation propagates, and the absorption coefficient for water vapor for the wavelength of interest, are both known. Which principle or law would you utilize to convert radiation absorption into a humidity, and which humidity variable would you get from this instrument?

S4. What if the amount of latent heat released in a rising, saturated air parcel was greater than the adiabatic cooling of the parcel as it expands into lower pressure.

a. What would be different in a thermo diagram?

b. How would this difference affect the weather and climate, if at all?

S5. What if the evaporation rate of water from the surface was constant, and did not depend on surface humidity, air humidity, wind speed, or solar heating (i.e., convection). How would the weather and climate be different, if at all?

S6. Suppose the whole troposphere was saturated with water vapor (100% relative humidity), but there were no cloud droplets or ice crystals in the air.

a. Assuming a standard atmosphere temperature profile, how many kg of water vapor are there over each square meter of Earth surface?

b. How many kg of water vapor would there be in the troposphere total, when summed around the whole surface of the Earth?

c. If all this water were to rain out (leaving the atmosphere completely dry) and flow into the oceans, which cover roughly 70.8% of the Earth's surface, how many meters would sea level rise?

d. Repeat questions a-c, but for the stratosphere.

S7. a. Rewrite the Clausius-Clapeyron equation in the form $e_s = C \cdot \exp[-(L/\mathfrak{R}_v) \cdot (1/T)]$, where C is a constant. What is C equal to?

b. The gas constant for water vapor is $\mathfrak{R}_v \approx k_B/m_v$, where m_v is the mass of a water molecule, and $k_B = 1.3806 \times 10^{-23} \text{ J} \cdot \text{K}^{-1} \cdot \text{molecule}^{-1}$ is the **Boltzmann constant**. Rewrite the eq. from part (a) after substituting for the water-vapor gas constant.

c. The result from (b) contains an exponent of an argument, where that argument is a ratio. The numerator represents the energy needed to separate one molecule from liquid water to allow it to become a vapor molecule in the neighboring air, while the denominator represents the average energy available (as a kinetic energy associated with temperature). Discuss the implications of this interpretation, and why it is relevant to determining equilibrium saturation vapor pressure.

S8. Devise an equation to estimate surface water flux that works for a sunny windy day, which reduces to eqs. (4.51 & 4.52) in the limits of zero convection and zero mean wind, respectively.