

STABILITY

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5 A **sounding** is the vertical profile of temperature and other variables in the atmosphere over one geographic location. **Stability** refers to the ability of the atmosphere to be turbulent. Stability is determined by temperature, humidity, and wind profiles. Turbulence and stability vary with time and place because of the corresponding variation of the soundings.

We notice the effects of stability by the: wind gustiness, dispersion of smoke, refraction of light and sound, strength of thermal updrafts, size of clouds, and intensity of thunderstorms.

Thermodynamic diagrams have been devised to help us plot soundings and determine stability. They look complicated at first, but with a bit of practice they can make thermodynamic analysis much easier than solving sets of coupled equations. In this chapter, we first discuss the different types of thermodynamic diagrams, and then use them to determine stability and turbulence.

BUILDING A THERMO-DIAGRAM

Components

In previous chapters, we learned how to compute **isohumes** (the Moisture chapter), **dry adiabats** (the Heat chapter), and **moist adiabats** (the Moisture chapter). We plotted these isopleths on a background graph having temperature along the abscissa and log of pressure along the ordinate. These are reproduced here as Figs. 5.1a-d.

When these isopleths are combined on a single graph, the result is called a **thermodynamic diagram**, or **thermo diagram** for short (Fig. 5.1e). At first glance, Fig. 5.1e looks like a confusing nest of lines; however, you can use the pattern-recognition capability of your mind to focus on the components as shown in Fig. 5.1a-d. Your efforts to master thermo diagrams now will save you time in the future.

Several types of thermo diagrams are used in meteorology. They all can show the same information, and are used the same way. The thermo diagram we learned so far is called an **Emagram**. We learned this one first because it was easy to create using a computer spreadsheet.

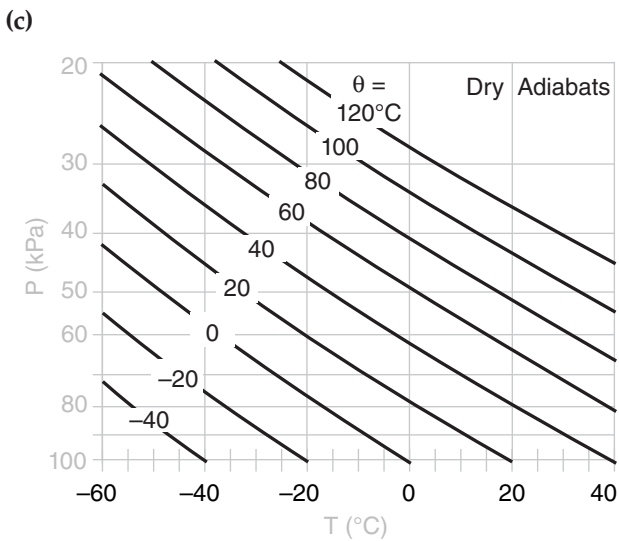
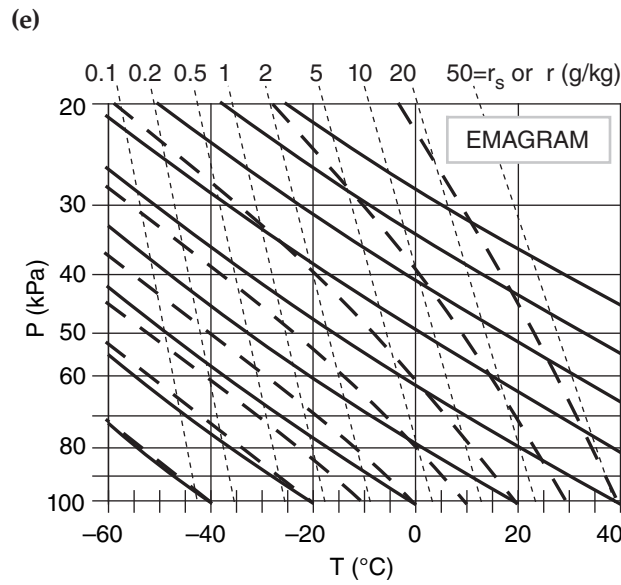
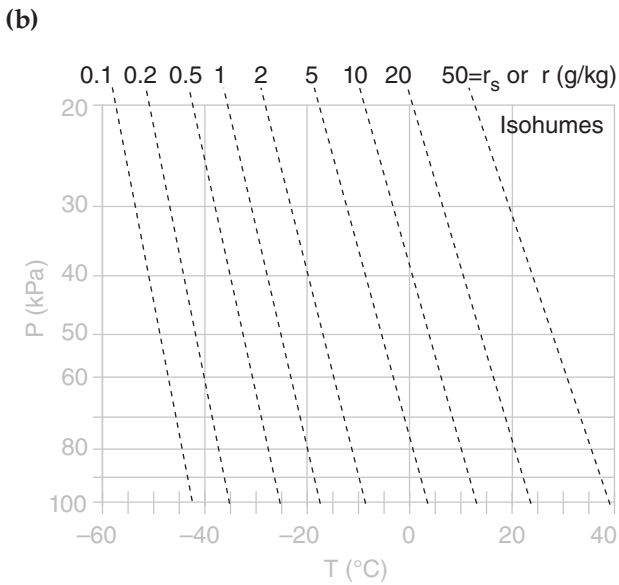
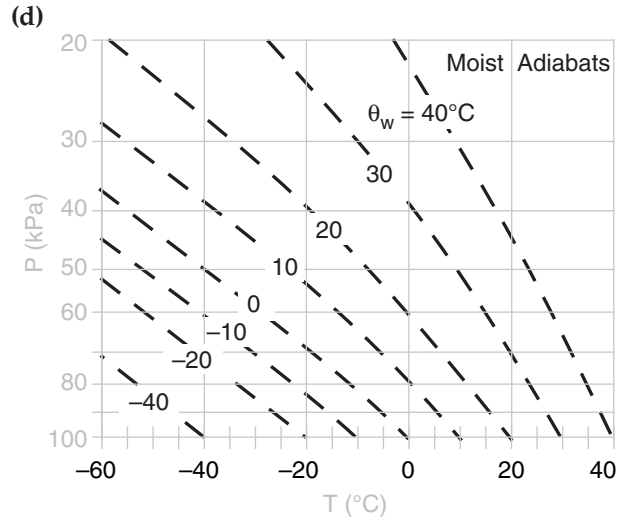
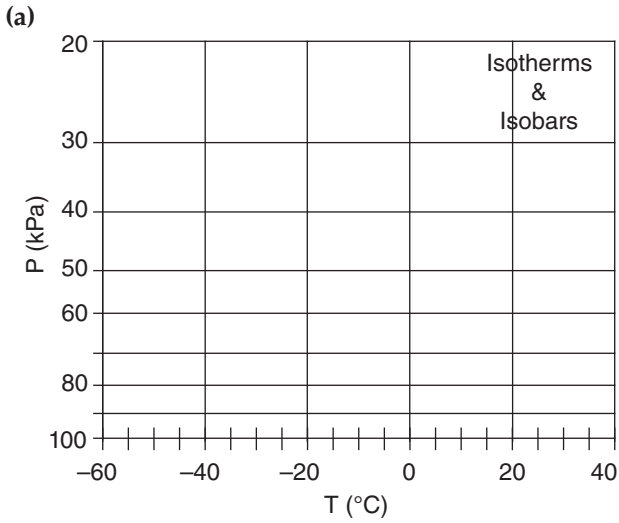


Figure 5.1

Components of an Emagram thermo diagram. (a) The background on all these charts are isotherms (vertical thin solid lines), and isobars (horizontal thin solid lines) plotted with logarithmic spacing. (b) Isohumes (from the Moisture chapter) are nearly vertical dotted lines. (c) Dry adiabats (from the Heat chapter) are diagonal solid lines; (d) Moist adiabats (from the Moisture chapter) are curved dashed lines. (e) Thermo diagram formed by combining parts (a) through (d). P is pressure, T is temperature, r is mixing ratio, r_s is saturated mixing ratio, θ is potential temperature, and θ_w is wet-bulb potential temperature.

On every thermo diagram, including the Emagram in Fig. 5.1e, are plotted at least five types of lines: three give the **state** of the air (**isobars**, **isotherms**, and **isohumes**) and two show how the state changes by a **process** of vertical movement of air (**dry** and **moist adiabats**). Additional state lines for **height contours** are left out of this introductory thermo diagram to avoid clutter, but are included in full thermo diagrams at the end of this chapter.

Pseudoadiabatic Assumption

In the Moisture chapter, we assumed an adiabatic process (no heat transfer or mixing to or from the air parcel) when computing the moist adiabats. However, for any of the thermo diagrams, the moist adiabats can be computed assuming either:

- **adiabatic processes** (i.e., **reversible**, where all liquid water is carried with the air parcels), or
- **pseudoadiabatic processes** (i.e., **irreversible**, where all condensed water is assumed to fall out immediately).

Air parcels in the real atmosphere behave between these two extremes, because small droplets and ice crystals are carried with the air parcel while larger ones precipitate out.

When liquid or solid water falls out, it removes from the system some of the sensible heat associated with the temperature of the droplets, and also changes the heat capacity of the remaining air because of the change in relative amounts of the different constituents. The net result is that an air parcel lifted pseudoadiabatically from 100 kPa to 20 kPa will be about 3°C colder than one lifted adiabatically. This small difference between adiabatic and pseudoadiabatic can usually be neglected compared to other errors in measuring soundings.

Complete Thermo Diagrams

Color thermo diagrams printed on large-format paper were traditionally used by weather services for hand plotting of soundings, but have become obsolete and expensive compared to modern plots by computer. The simplified, small-format diagrams presented so far in this chapter are the opposite extreme – useful for education, but not for plotting real soundings. Also, some weather stations have surface pressure greater than 100 kPa, which is off the scale for the simple diagrams presented so far.

As a useful compromise, full-page, black-and-white thermo diagrams in several formats are included at the end of this chapter. They are optimized for you to reproduce on a copy machine.

[Hint: Keep the original thermo diagrams in this book clean and unmarked, to serve as master copies.]

FOCUS • Why so many thermo diagrams?

Meteorological thermo diagrams were originally created as optimized versions of the P vs. α diagrams of classical physics (thermodynamics), where α is the **specific volume** (i.e., $\alpha = \text{volume per unit mass} = 1/\rho$, where ρ is air density). A desirable attribute of the P vs. α diagram is that when a cyclic process is traced on this diagram, the area enclosed by the resulting curve is proportional to the specific work done by or to the atmosphere. The disadvantage of P vs. α diagram is that the angle between any isotherm and adiabat is relatively small, making it difficult to interpret atmospheric soundings.

Three meteorological thermo diagrams have been devised that satisfy the “area = work” attribute, and are optimized for meteorology to have greater angles between the isotherms and adiabats:

- Emagram,
- Skew-T Log-P Diagram
- Tephigram.

Meteorologists rarely need to utilize the “area = work” attribute, so they also can use any of three additional diagrams:

- Stüve Diagram,
- Pseudoadiabatic (Stüve) Diagram,
- Theta-Height (θ - z) Diagram.

Why are there so many diagrams that show the same things? Historically, different diagrams were devised somewhat independently in different countries. Nations would adopt one as the “official” diagram for their national weather service, and teach only that one to their meteorologists. For example, the tephigram is used in British Commonwealth countries (UK, Canada, Australia, New Zealand). To this day, many meteorologists feel most comfortable with the diagram they learned first.

For many readers, this myriad of diagrams might make an already-difficult subject seem even more daunting. Luckily, all the diagrams show the same thermodynamic state (T , P , r_s) and process lines (θ , θ_w), but in different orientations. So once you have learned how to read one diagram, it is fairly easy to read the others.

The skill to read diverse thermo diagrams will serve you well when acquiring weather data via the internet, because they can come in any format. The internet is the main reason I cataloged the different thermo diagrams in this book.

Of all these diagrams, the Skew-T and Tephigram have the greatest angle between isotherms and adiabats, and are therefore preferred when studying soundings and stability. These two diagrams look similar, but the Skew-T is growing in popularity because it is easier to create on a computer.

TYPES OF THERMO DIAGRAMS

Emagram

“Emagram” is a contraction for “Energy-per-unit-mass diagram.” This semi-log diagram (Fig. 5.3a) has temperature (T) along the abscissa, and pressure (P) decreasing logarithmically upward along the ordinate. The isotherms and isobars are straight lines that are perpendicular to each other, and form the orthogonal basis for this diagram.

Dry adiabats (θ) are nearly-straight, diagonal lines slanted up towards the left, and curve slightly concave upward. Moist adiabats (θ_w) curve concave downward, but become parallel to the dry adiabats at cold temperatures and high altitudes. Isohumes (r) are nearly-straight, nearly-vertical lines (with a slight tilt upward to the left).

Height contours (on the large Emagram at the end of this chapter) are approximate, and are provided as a rough guide to users. These contours are nearly horizontal (tilting downward slightly to the right) and nearly straight (slightly concave upward). They are often not drawn in thermo diagrams, and indeed are not shown in Fig. 5.3.

Stüve & Pseudoadiabatic Diagrams

The Stüve diagram (Fig. 5.3b) has T along the abscissa, and pressure decreasing upward along the ordinate according to $(P/P_0)^{\mathfrak{R}_d/C_{pd}}$, where $\mathfrak{R}_d/C_{pd} = 0.28571$ (dimensionless) and $P_0 = 100$ kPa. Stüve diagrams look virtually identical to Emagrams, except that the dry adiabats are perfectly straight. These dry adiabats converge to a point at $P = 0$ kPa and $T = 0$ K, which is usually well off of the upper left corner of the graph.

Any thermo diagram can be computed using the pseudoadiabatic assumption for the moist adiabats. However, the particular diagram that is known as a “Pseudoadiabatic Diagram” is often a Stüve plot.

Skew-T Log-P Diagram

Fig. 5.3c is a Skew-T. Although T is labeled linearly along the abscissa, the isotherms are parallel, straight, diagonal lines tilting upward to the right. The Skew-T gets its name because the isotherms are not vertical, but skewed. Pressure decreases logarithmically upward along the ordinate, and the isobars are parallel, horizontal, straight lines.

Dry adiabats are diagonal lines slanted up towards the left, with a pronounced curve concave upward. Moist adiabats are more sharply curved concave left near the bottom of the diagram, changing to less curved, concave to the right, as they merge

into the dry adiabats at higher altitudes and colder temperatures. Isohumes are almost straight lines, tilting upward to the right.

This diagram is designed so that the isotherms and dry adiabats are nearly (but not perfectly) perpendicular to each other.

Tephigram

The name Tephigram is a contraction of Tee-Phi Diagram. The logarithm of potential temperature θ physically represents the **entropy** ($\phi =$ Greek letter Phi) change, and is plotted along the ordinate of Fig. 5.2. Temperature T is along the abscissa. Thus, it is a temperature-entropy diagram, or T - ϕ diagram. Isobars are curved lines in Fig. 5.2.

The range of meteorological interest is shown with the bold rectangle in this figure. When this region is enlarged and tilted horizontally, the result is the tephigram as plotted in Fig. 5.3d.

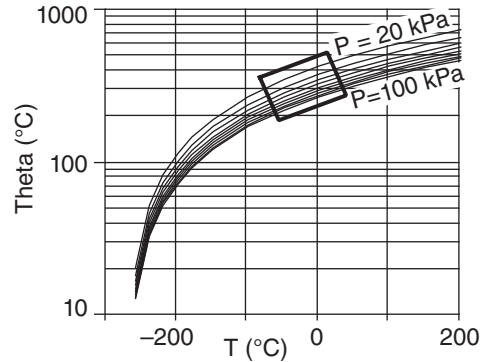


Fig. 5.2
Rectangle shows portion of thermo diagram used for tephigram.

The tephigram looks similar to the Skew-T, with isotherms as straight diagonal lines tilting upward to the right. Dry adiabats are perfectly straight diagonal lines tilting upward left, and are exactly perpendicular to the isotherms. Moist adiabats have strong curvature concave left, and become parallel to the dry adiabats at cold temperatures and high altitudes. Isohumes are gently curved (concave down) nearly-horizontal lines. Isohumes are nearly straight diagonal lines tilting upward right.

Theta-Height (θ - z) Diagrams

The orthogonal basis for this diagram are heights (z) plotted along the ordinate and potential temperature (θ) along the abscissa (Fig. 5.3e). Moist adiabats curve to the right of the dry adiabats, but become parallel to the dry adiabats at high altitudes and cold temperatures. Heights are accurate in this diagram, but the isobars are only approximate. Isohumes tilt steeply upward to the right. This diagram is used extensively in boundary-layer meteorology.

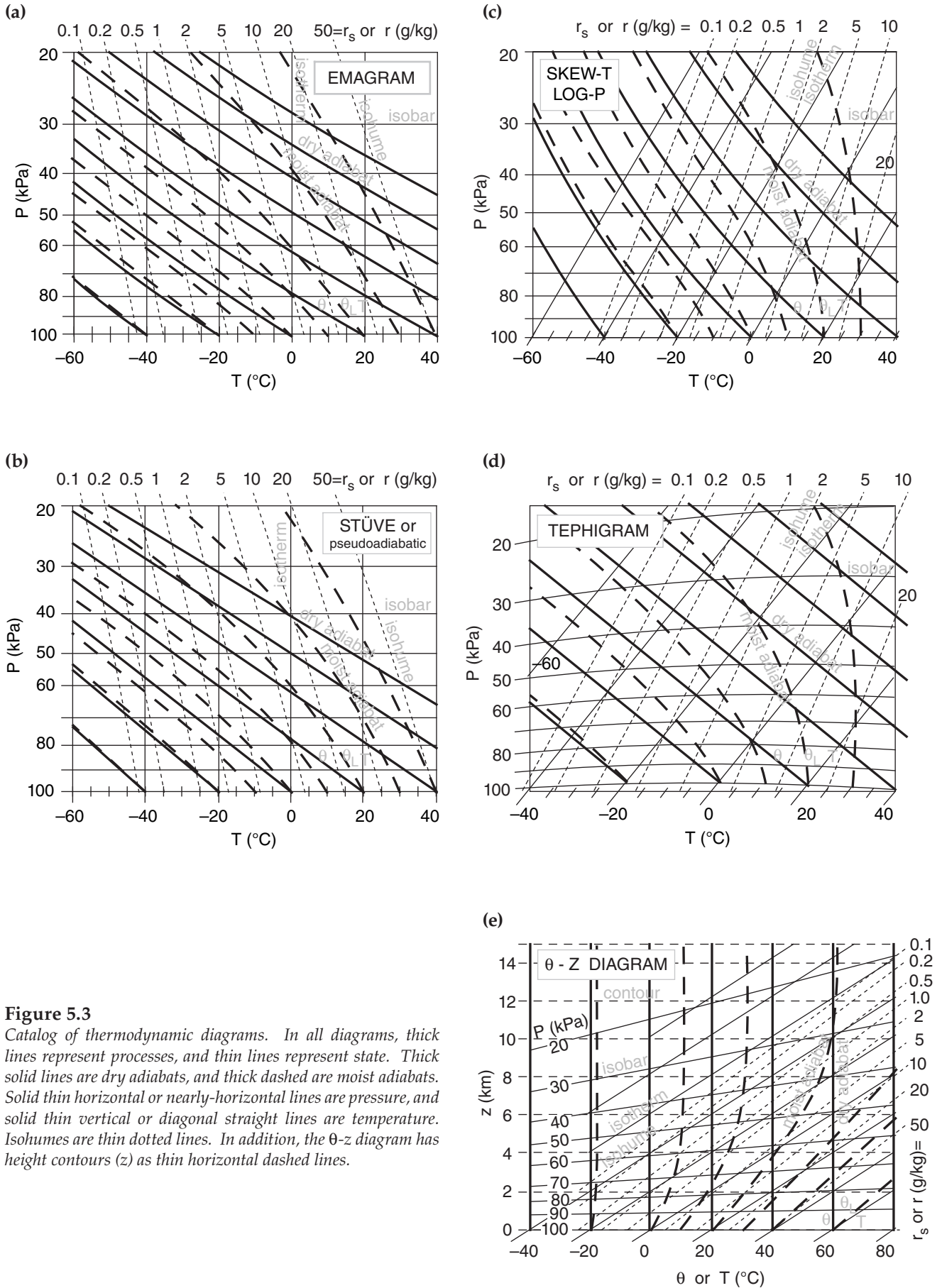
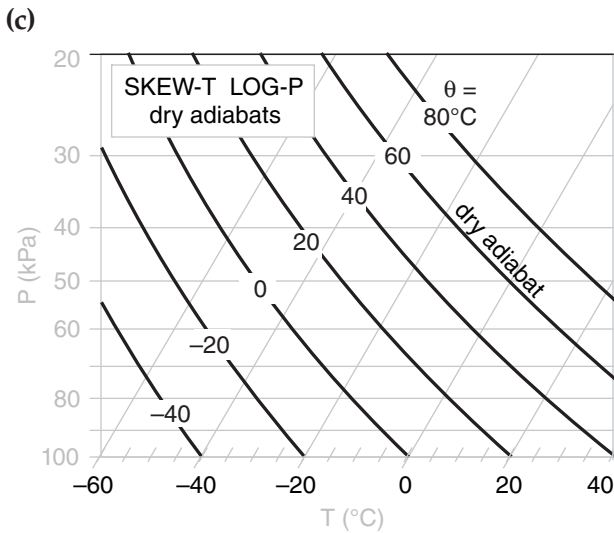
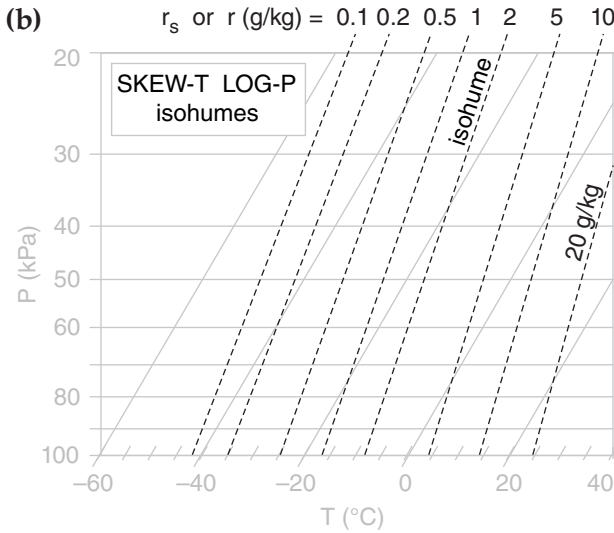
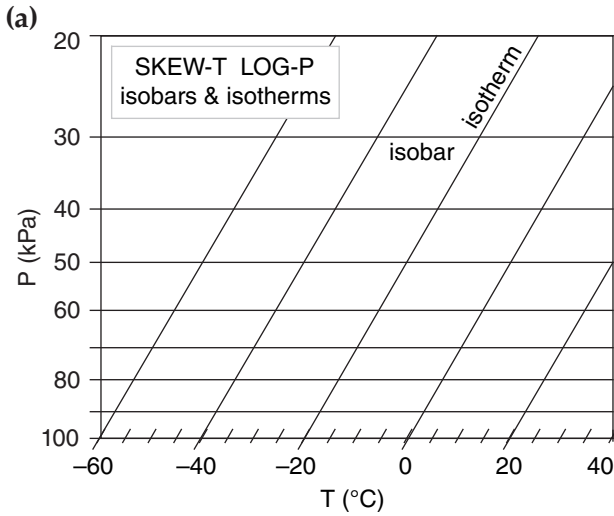


Figure 5.3
 Catalog of thermodynamic diagrams. In all diagrams, thick lines represent processes, and thin lines represent state. Thick solid lines are dry adiabats, and thick dashed are moist adiabats. Solid thin horizontal or nearly-horizontal lines are pressure, and solid thin vertical or diagonal straight lines are temperature. Isohumes are thin dotted lines. In addition, the θ -z diagram has height contours (z) as thin horizontal dashed lines.



MORE ON THE SKEW-T

Because of the popularity of the Skew-T, we dissect it in Fig. 5.4, to aid in its interpretation. Tephigram users should find this equally useful, because the only noticeable difference is that the isobars are slightly curved, and increase in slope higher in the tephigram.

The Skew-T diagram is trivial to create, once you have already created an Emagram. The easiest way is to start with a graphic image of an Emagram, and skew it into a parallelogram (Fig. 5.5) using a graphics or drawing program.

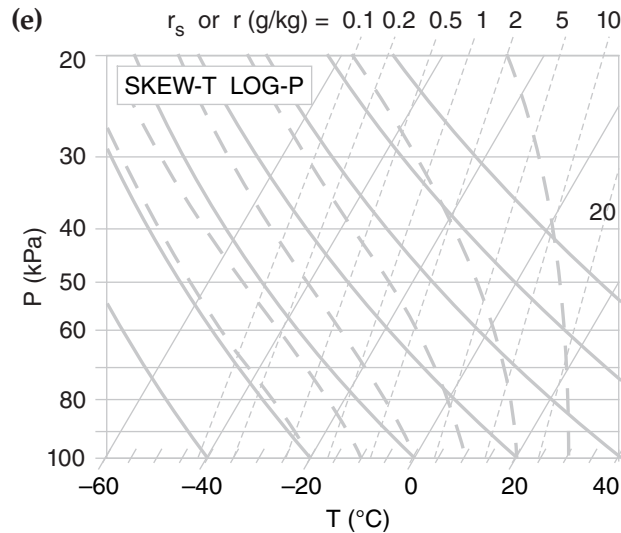
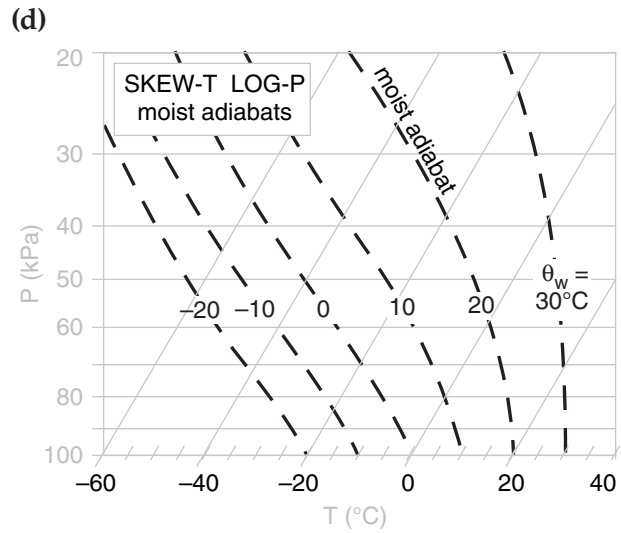


Fig. 5.4

(a)-(d) Components of Skew-Ts, drawn to the same scale so they can be superimposed as a learning aid. The state of the air: (a) isobars and isotherms; (b) isohumes as thin dotted lines. Processes: (c) dry adiabats; (d) moist adiabats. (e) Full Skew-T Log-P, shaded grey to make it easier to write on. The “Skew-T Log-P” diagram is often called a “Skew-T” for short.

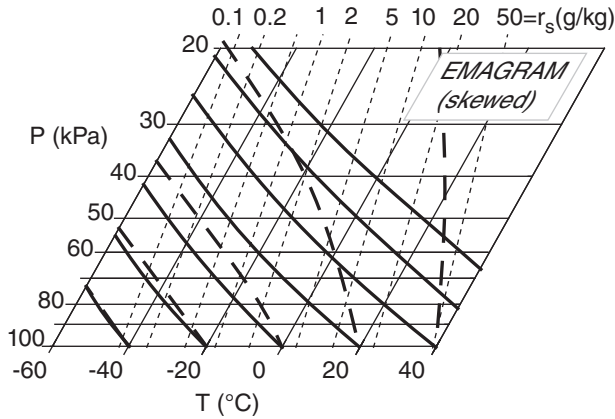


Fig. 5.5
A Skew-T is nothing more than a skewed Emagram.

It is equally simple to draw a Skew-T with a spreadsheet program, especially if you start with the numbers you already calculated to create an Emagram. Let X ($^{\circ}\text{C}$) be the abscissa of a Cartesian graph, and Y (dimensionless) be the ordinate. Let K be the skewness factor, which is arbitrary. A value of $K = 35^{\circ}\text{C}$ gives a nice skewness (such that the isotherms and adiabats are nearly perpendicular), but you can pick any skewness you want.

The coordinates for any point on the graph are:

$$Y = \ln\left(\frac{P_0}{P}\right) \tag{5.1a}$$

$$X = T + K \cdot Y \tag{5.1b}$$

To draw isotherms, set T to the isotherm value, and find a set of X values corresponding to different heights Y . Plot this one line as an isotherm. For the other isotherms, repeat for other values of T . [Hint: it is convenient to set the spreadsheet program to draw a logarithmic vertical axis with the max and min reversed. If you do this, then use P , not Y , as your vertical coordinate. But you still must use Y in the equation above to find X .]

For the isohumes, dry adiabats, and moist adiabats, use the temperatures you found for the Emagram as the input temperatures in the equation above. For example, the $\theta = 20^{\circ}\text{C}$ adiabat has the following temperatures as a function of pressure (recall Thermo Diagrams-Part 1, in the Heat chapter), from which the corresponding values of X can be found.

$P(\text{kPa}) =$	100	, 80	, 60	, 40	, 20
$T(^{\circ}\text{C}) =$	20	, 1.9	, -19.8	, -47.5	, -88
$X =$	20	, 9.7	, -1.9	, -15.4	, -31.7

Solved Example

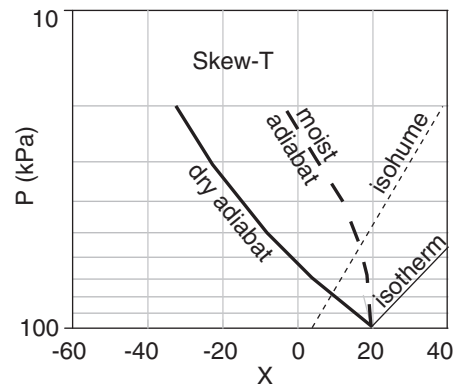
Create a Skew-T diagram with one isotherm ($T = 20^{\circ}\text{C}$), one dry adiabat ($\theta = 20^{\circ}\text{C}$), one moist adiabat ($\theta_w = 20^{\circ}\text{C}$), and one isohume ($r = 5 \text{ g/kg}$). Use as input the temperatures from the Emagram (as calculated in earlier chapters). Use $K = 35^{\circ}\text{C}$, and $P_0 = 100 \text{ kPa}$.

Solution

Given: The table of Emagram numbers below. Use eq. (5.1) in a spreadsheet to find X and Y .

	A	B	C	D	E
1	Create Skew-T from Emagram Numbers				
2					
3		Iso-	Dry	Moist	Iso-
4		therm	Adiabat	Adiabat	hume
5		$T=20^{\circ}\text{C}$	$\theta=20^{\circ}\text{C}$	$\theta_w=20^{\circ}\text{C}$	$r=5\text{g/kg}$
6	Emagram (Given):				
7	P (kPa)	T ($^{\circ}\text{C}$)	T ($^{\circ}\text{C}$)	T ($^{\circ}\text{C}$)	T ($^{\circ}\text{C}$)
8	100	20	20	20	3.7
9	90	20	11.3	16	2.3
10	80	20	1.9	11.4	0.7
11	70	20	-8.4	6	-1.2
12	60	20	-19.8	-0.3	-3.2
13	50	20	-32.6	-8.8	-5.6
14	40	20	-47.5	-20	-8.5
15	30	20	-65.3	-37	-12
16	20	20	-88	-60	-17
17					
18	Skew-T:				
19	Let K=	35 $^{\circ}\text{C}$		Let $P_0 =$	100kPa
20	P	Y	X	X	X
21	100	0.00	20	20	3.7
22	90	0.11	23.7	15.0	6.0
23	80	0.22	27.8	9.7	8.5
24	70	0.36	32.5	4.1	11.3
25	60	0.51	37.9	-1.9	14.7
26	50	0.69	44.3	-8.3	18.7
27	40	0.92	52.1	-15.4	23.6
28	30	1.20	62.1	-23.2	30.1
29	20	1.61	76.3	-31.7	39.3

Using the spreadsheet to plot the result gives:



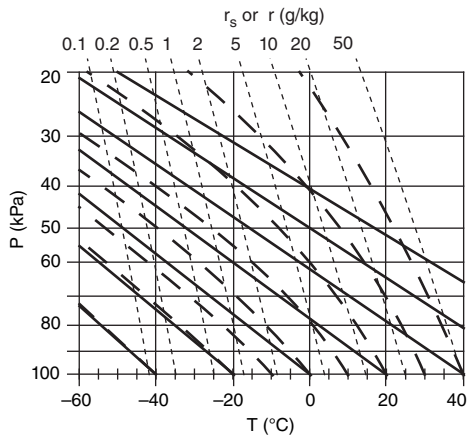
Check: Sketch OK. Matches Fig. 5.4e.

Discussion: Experiment with different values of K starting with $K = 0^{\circ}\text{C}$, and see how the graph changes from an Emagram to a Skew-T.

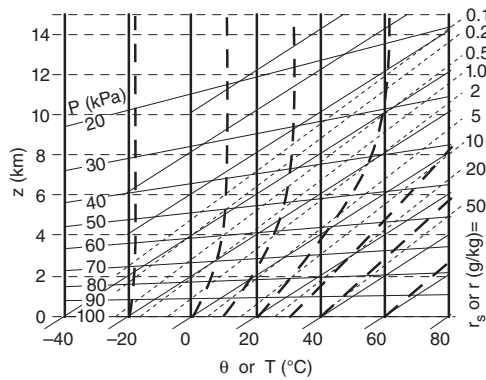
Solved Example

For each thermo diagram numbered below, identify its name.

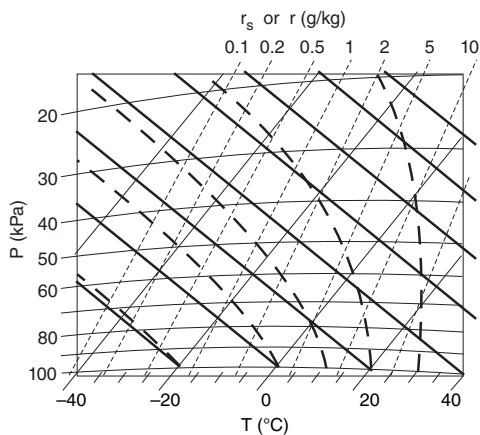
1)



2)



3)



(continued next page)

GUIDE FOR QUICK IDENTIFICATION OF THERMO DIAGRAMS

If you receive a thermo diagram via the internet without any labels identifying the diagram type, use the following procedure to figure it out:

- 1) if there are some lines from lower left to upper right, then either Skew-T, Tephigram, or θ -Z .
 - a) if the isobars are exactly horizontal and straight, then Skew-T,
 - b) else if the strongly curved lines (moist adiabats) asymptotically approach vertical near the top of the diagram, then θ -Z,
 - c) else Tephigram,
- 2) else either Emagram or Stüve .
 - a) if dry adiabats are slightly curved, then Emagram,
 - b) else Stüve (including Pseudoadiabatic version of Stüve).

When you encounter an unfamiliar thermo diagram, you can take steps to help segregate and identify the various lines. First, identify the isobars, which are horizontal and straight, or nearly so, on virtually all diagrams. Isobars are usually spaced roughly logarithmically with height, and are labeled as either 100, 90, 80, 70 kPa etc., or as 1000, 900, 800, 700 mb or hPa etc. These serve as a surrogate measure of height.

Next, identify the dry and moist adiabats. These are the lines that converge and become parallel at cold temperatures and high altitudes. In all the diagrams, the moist adiabats are the most curved. The adiabats are spaced and labeled roughly linearly, such as 0, 10, 20, 30 °C etc, or often in Kelvin as 273, 283, 293, 303 K, etc. While the dry adiabats are always labeled as potential temperature θ , the moist adiabats can be labeled as either equivalent potential temperature θ_e , liquid-water potential temperature θ_L , or wet-bulb potential temperature θ_w .

An alternative for identifying adiabats is to look at any labeled temperature at the bottom of the graph. Usually at 100 kPa (1000 mb or hPa), you will find that T , θ_w , and θ lines all radiate from the same

Hint: For viewing thermo diagrams: hold the page so you view it from near its edge, rather than perpendicular to the page as is usually done for reading. This allows the curvature of lines to be more apparent. This is the same trick used by carpenters, sighting along a piece of lumber to detect warps and other imperfections.

points. For all diagrams, the T line is rightmost, θ is leftmost, and θ_w is in the middle.

Finally, identify the isotherms and isohumes. These lines cross each other at a small angle, and extend from the bottom to the top of the graph. The isohumes always tilt to the left of the isotherms with increasing height. The isotherms increment linearly (e.g., 0, 10, 20, 30, °C etc.), while the isohumes usually increment logarithmically (e.g., 1, 2, 5, 10, 20 g/kg, etc.). Isotherms are either vertical, or tilt upward to the right.

THERMO-DIAGRAM APPLICATIONS

Fig. 5.6 shows a complete, but simplified, Emagram. We use this as the common background for the demonstrations that follow.

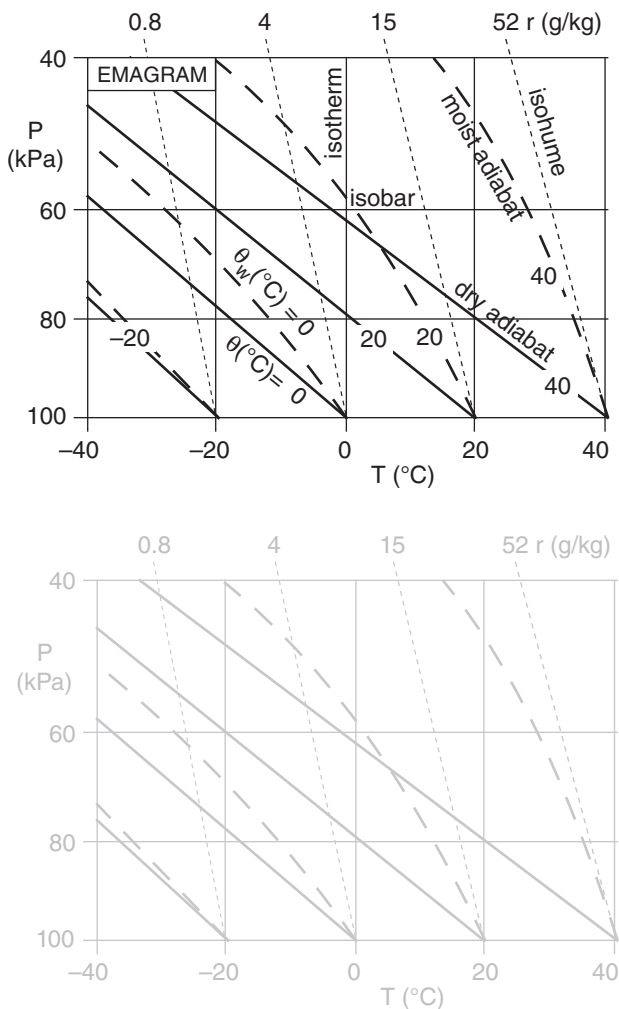
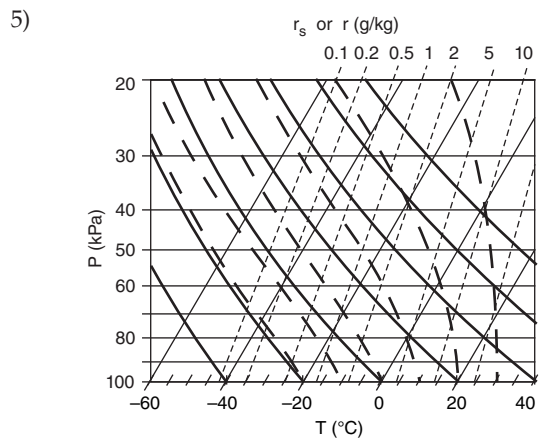
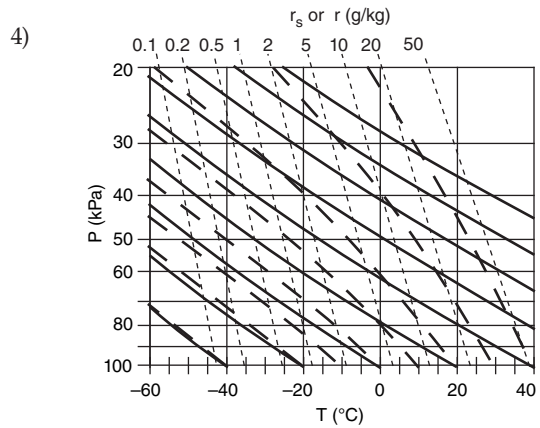


Figure 5.6
Simplified Emagrams. Top: with isopleths labeled. Bottom: with isopleths shaded grey to serve as a background for future discussions.

Solved Example (continuation)



Solution: Use the quick guide.

- 1) **Stüve**, because satisfies item 2b in the guide.
- 2) **θ -Z diagram**, because satisfies item 1b.
- 3) **Tephigram**, because satisfies item 1c.
- 4) **Emagram**, because satisfies item 2a.
- 5) **Skew-T**, because satisfies item 1a.

Check: OK, agrees with Fig. 5.3.

Discussion: For practice, pick any one of the diagrams above, and without looking back in Fig. 5.3, try to identify which lines are which.

[Hint: in this book, all thermo diagrams use the same format for the same types of lines: isotherms and isobars are thin solid, isohumes are thin dotted, dry adiabats are thick solid, moist adiabats are thick dashed, and heights are thin dashed (if plotted at all). **CAUTION:** Thermo diagrams printed or published by others do NOT follow any standard format of line style or color.]

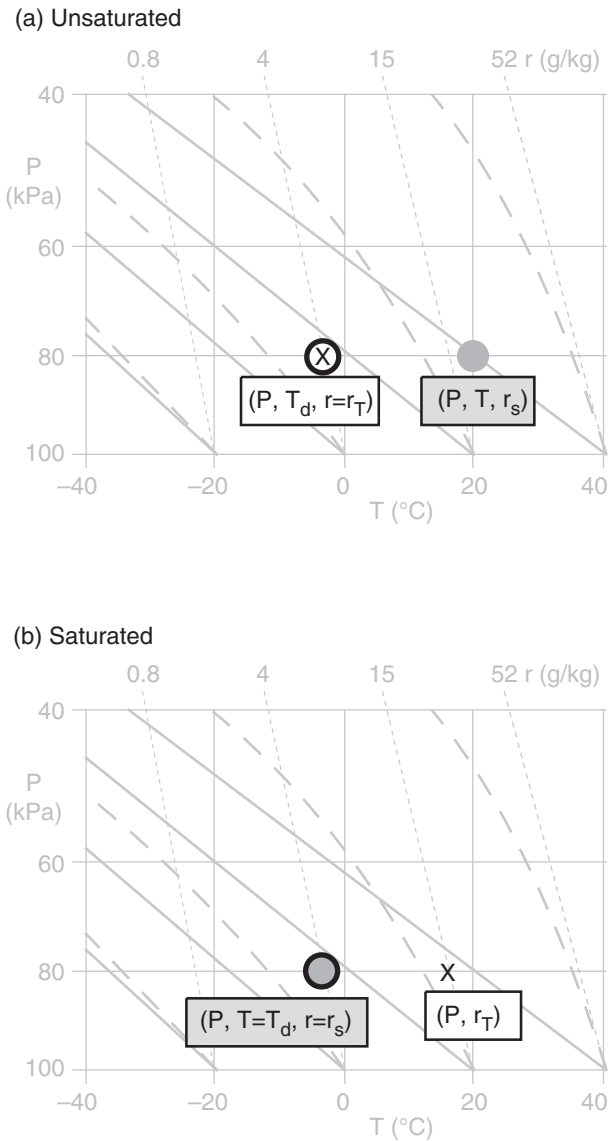


Figure 5.7
 In general, three points are needed to represent one air parcel: the right circle (filled with grey) indicates air temperature, and the left circle (black outline) indicates dew-point temperature (as a measure of the amount of water vapor in the air). The X indicates total water content. (a) Unsaturated air. (b) Saturated air. (See the Solved Example for details)

Thermodynamic State

For any one air parcel, three points are needed to describe its state (i.e., its temperature, pressure, and humidity) on a thermo diagram. For this one air parcel at one height, all three points are always plotted on the same isobar (P).

One point represents T , the actual air temperature (grey filled circle in Fig. 5.7). Its coordinates in the thermo diagram are given by (P, T, r_s) . Note that r_s is redundant, because it can be determined from P and T . But we include it for convenience, because we use the r_s value for many calculations.

Another point represents T_d , the dew-point temperature (black circle outline). Its coordinates are (P, T_d, r) . Again, r is redundant.

The third point gives r_T , the total water mixing ratio in the air parcel (designated by “X” on the thermo diagram). Its coordinates are (P, r_T) .

Often, two of these three points coincide. One example is for unsaturated air (Fig. 5.7a) with no rain falling through it. For this case, the temperature and dew-point circles are plotted as before. However, the X is plotted inside the dew-point circle because the total water in the air parcel equals the water-vapor mixing ratio ($r_T = r$), which is fully specified by the dew point. Namely, the isohume that passes through the dew-point circle indicates the actual amount of water vapor in the air, and there are no other phases of water in that air parcel.

The isohume that passes through the actual air temperature gives the saturation mixing ratio (r_s , the maximum equilibrium amount of moisture that could be held as vapor). A separate symbol is NOT plotted on the thermo diagram for r_s for two reasons. One, it does not indicate the actual state of the air parcel. Two, it is fully specified as the isohume that passes through the temperature point.

For saturated (cloudy or foggy) air parcels (Fig. 5.7b), the two temperature points coincide ($T = T_d$ and $r = r_s$). In this figure, the grey-filled circle still represents air temperature, and the black circle outline still represents the dew-point temperature (for water vapor). However, the total water X is usually to the right of the temperature circles, because clouds contain liquid water and/or ice in addition to water vapor. This is explained later.

[Tip: On these figures, we use large size circles and X’s to make them easier to see. However, when you plot them on your own thermo diagram, use smaller circles and X’s that you write very precisely on the diagram. Also, you may use different colors for each symbol, such as red for temperature and blue for dew-point.]

Processes

Dry (Unsaturated) Lifting

Suppose that an unsaturated air parcel rises without mixing with the surrounding environment, and with no radiative or other heat transfer to or from it. For this idealized situation, the temperature cools dry adiabatically due to the decreasing pressure, and any molecules (water and air) in the parcel stay in the parcel. Hence, both potential temperature θ and mixing ratio r are conserved.

On the thermodynamic diagram, this corresponds to moving the temperature point (grey filled circle) along the “dry” adiabat that passes through its initial position, and moving the moisture points (black outline circle and “X”) along the isohume of constant r (NOT along a T_d line). Always move all three points together to the same pressure (see Fig. 5.8).

If the points don't lie directly on a drawn isopleth (which is usually the case), then imagine a new isopleth that goes through your point, and which is parallel to (and maintains its relative distance from) the neighboring drawn isopleths. You can do this by eye, or you may use a straight-edge to draw on the diagram a new isopleth that goes through your point of interest. Then, move the air parcel along this imaginary isopleth. Namely, move the temperature point in a direction parallel to the dry adiabats, and move the humidity point parallel to the isohumes.

Likewise, for descending air parcels that are unsaturated (and no mixing with the environment and no liquid water present), the temperature point follows the “dry” adiabat and the humidity points follow the isohume.

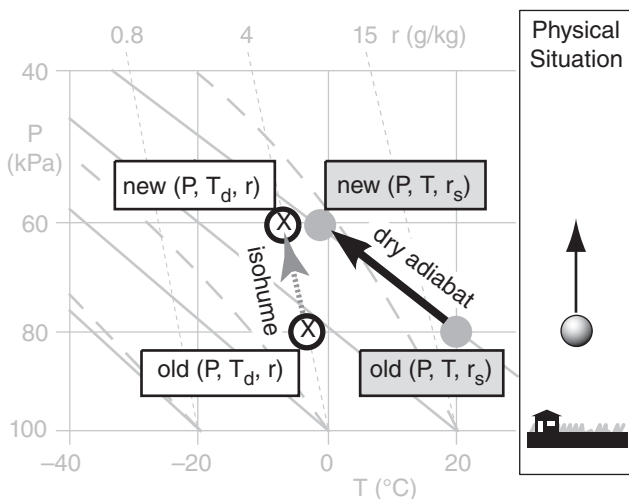


Figure 5.8

Potential temperature and mixing ratios are constant for rising unsaturated air. The new points still represent the one parcel. The new r equals the old r , although new r_s does not equal old r_s .

Solved Example

For the air parcel of Fig. 5.7a, find its temperature, dew point, pressure, mixing ratio, total water mixing ratio, saturation mixing ratio, and relative humidity.

Solution

Given: Fig. 5.7a. Estimate points by eye.

Find: $T = ?$ °C, $T_d = ?$ °C, $P = ?$ kPa

$r = ?$ g/kg, $r_T = ?$ g/kg, $r_s = ?$ g/kg,

$RH = ?$ %

The grey circle on the right is directly on the $T = 20^\circ\text{C}$ isotherm. The temperature for the white circle on the left must be interpolated between the 0°C and -20°C isotherms. I estimate by eye roughly $T_d = -2.5^\circ\text{C}$. The pressure, which must be the same for both points, is $P = 80$ kPa based on the isobar that passes through them.

Interpolating the right circle between isohumes gives roughly $r_s = 19$ g/kg, while the actual mixing ratio using the left circle is $r = 4$ g/kg. For this unsaturated air parcel, the total water mixing ratio is $r_T = r = 4$ g/kg. Relative humidity is $RH = (100\%) \cdot (r/r_s) = (100\%) \cdot (4/19) = 21\%$.

Check: Units OK. Physics OK.

Discussion: I actually used the larger thermo diagram at the end of this chapter to get these answers, because it is more accurate. The air is fairly dry.

If the two temperature points would have coincided, such as on a different day with fog (Fig. 5.7b), then $T - T_d = 0$ °C, and $r_s - r = 0$ g/kg.

Solved Example

Given Fig. 5.8, what is the new state of the air after being lifted dry adiabatically to $P = 60$ kPa?

Solution

Given: Fig. 5.8. Estimate points by eye.

Find: T & $T_d = ?$ °C,

r , r_s & $r_T = ?$ g/kg, and $RH = ?$ %

The temperature, when moved along (or parallel to) the dry adiabats, has cooled to about $T = -3^\circ\text{C}$ at $P = 60$ kPa. The dew point temperature (X), when moved along (or parallel to) the isohumes, decreases slightly to $T_d = -6.5^\circ\text{C}$.

Mixing ratio and total-water mixing ratio are still $r_T = r = 4$ g/kg, but saturation mixing ratio has decreased to $r_s = 5$ g/kg because the actual air temperature decreased. The new relative humidity is: $RH = (100\%) \cdot (4/5) = 80\%$.

Check: Units OK. Physics OK.

Discussion: Although not yet saturated (i.e., not cloudy), the RH is increasing as the air parcel rises.

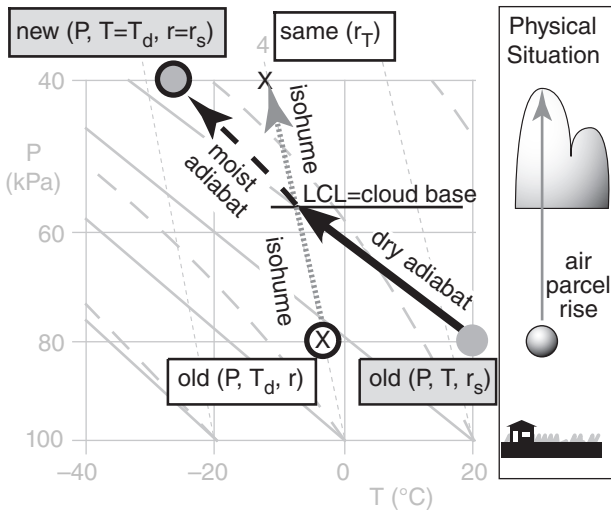


Figure 5.9
 The height where the two points converge is the lifting condensation level (LCL). This is cloud base. As the saturated parcel continues to rise, the temperature points coincide and follow the moist adiabat. If no precipitation falls out of the parcel, then total water continues to follow the original isohume (indicated by X at same height as parcel).

Moist (Saturated) Lifting & Liquid Water

Adiabats and isohumes converge with increasing height. Thus, for a rising air parcel there will be some height where the temperature and dew-point coincide. At this altitude or pressure, the air is just saturated. Hence this altitude corresponds to convective (cumulus) cloud base. It is called the **lifting condensation level (LCL)**, (Fig. 5.9). At this point, all three marks (grey filled circle, black outline circle, and "X") representing the air parcel coincide. Namely, $T_d = T$, and $r = r_s = r_T$ at the LCL.

For air rising above its LCL, T and T_d cool together moist adiabatically, maintaining $T_d = T$. Thus, on the thermo diagram we do not need two circles because there is only one temperature. In Fig. 5.9, this one temperature is represented by the black circle filled with grey. However, we still need to keep track of total water, as indicated by the X in Fig. 5.9. If all water molecules stay in the parcel (i.e., if cloud droplets move with the air and do not precipitate out), then total water mixing ratio r_T remains constant.

The difference between the actual mixing ratio in the cloudy air (using the isohume associated with the parcel's temperature) and the original mixing ratio before the parcel was lifted (indicated by the total water) gives the amount of liquid or solid water that has condensed or deposited. Namely, $r_L = r_T - r_s$. In Fig. 5.9, this is the difference between the X and the grey circle representing the new state of the parcel.

For descending saturated air, follow the total water mixing ratio (from point X) down the isohume, and follow the temperature state down the moist adiabat until they intersect. This intersection point indicates cloud base (LCL) for descending air. At the LCL, split the parcel temperature point into two points representing temperature and dew point, and follow them down the dry adiabat and isohume respectively. The total water (X) coincides with the dew-point below the LCL. The result is exactly as sketched in Fig. 5.9 except all arrows would be in the opposite direction. The reason is that below the LCL, the descending air has warmed sufficiently to hold all the water as vapor.

Precipitation

When liquid or solid water precipitates out, the X point representing total water moves to the left along an isobar on the thermo diagram (Fig. 5.10).

If all the liquid or solid water falls out, then the X on the thermo diagram would move all the way to the circle representing the parcel temperature. If the parcel were to then descend, it would do so dry adiabatically, with T following a dry adiabat, and total water and T_d following the isohume.

If only some of the rain or ice falls out, then the X would move only part way toward the parcel tem-

Solved Example

Given the example in Fig. 5.9:

- (a) At what pressure height is the LCL, and what is the state of the air parcel at that pressure?
- (b) What is the state of the air parcel when it reaches a pressure height of 40 kPa?
- (c) How much liquid water has been condensed out at that final height?

Solution

Given: Fig. 5.9. Estimate points by eye.

- Find: (a) $P_{LCL} = ?$ kPa, $T = T_d = ?$ °C,
 $r = r_s = r_T = ?$ g/kg
 (b) $T = T_d = ?$ °C, $r = r_s = ?$ g/kg, $r_T = ?$ g/kg
 (c) $r_L = ?$ g/kg

Using the diagram:

- (a) The LCL is at roughly $P_{LCL} = 57$ kPa. The parcel is just barely saturated, with temperature and dew points of roughly $T = T_d = -7$ °C. The mixing ratio is still $r = r_s = r_T = 4$ g/kg.
- (b) Moving parallel to the moist adiabat, from the LCL up to $P = 40$ kPa, the final state is $T = T_d = -25.5$ °C. The saturation mixing ratio is now only $r = r_s = 1.2$ g/kg. If no precipitation, then total water is unchanged at $r_T = 4$ g/kg.
- (c) $r_L = r_T - r_s = 4 - 1.2 = 2.8$ g/kg.

Check: Units OK. Physics OK.

Discussion: Liquid water amount not only indicates the amount of rainfall possible, but for conditions below freezing it indicates the rate of ice formation on aircraft wings, a hazard to flight. Most commercial jet aircraft heat the leading edge of the wing to prevent ice formation.

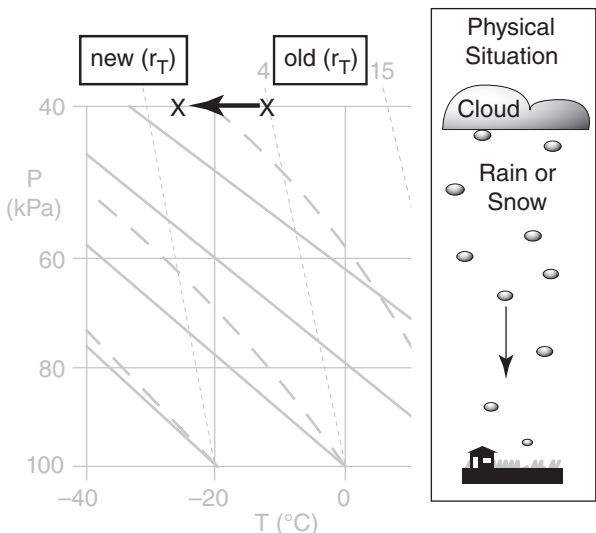


Figure 5.10
Reduction of total water due to precipitation. The new X point cannot be further left than the temperature of the air parcel.

perature. The new X must then be used as the starting point for total water if the parcel rises further or descends. Namely, the X-point stays on its new isohume during vertical parcel movement. If this parcel descends, it will reach its new LCL at a height greater than its original LCL.

The rain or snow that falls out of the parcel implies net heating. It is an irreversible process. Namely, the latent heating of condensation that occurred in the rising air is not balanced by evaporation in the descending air, because there is now less liquid or solid water to evaporate.

Radiative Heating/Cooling

Clouds are opaque to IR radiation, and translucent to visible light. This can warm or cool the cloudy air, but it does not alter the total-water mixing ratio. Such warming or cooling is said to be **diabatic**, because there is heat transfer across the “skin” of the cloudy air parcel. (Recall that “adiabatic” means no heat transfer.)

There is net heat loss by IR radiation upward from cloud top toward space, both day and night. Also, cloud base will exchange IR radiation with the ground and with the underlying air. If the ground is warmer than cloud base, then cloud base will warm if the cloud is low enough.

Sunlight can penetrate into the cloud top (and sides of cumuliform clouds) during daytime, with an e-folding extinction distance on the order of meters. The absorption of sunlight warms the cloudy air parcel. Not all the sunlight is absorbed by the cloud droplets. Some scatters and rescatters off the

Solved Example

From the previous solved example, suppose that 1 g/kg of liquid water falls out, leaving the remaining liquid water as cloud droplets that stay with the parcel. If this parcel descends, what is its new LCL and thermo state back at $P = 80$ kPa?

Solution

Given: Fig. 5.10, $r_{T\ old} = 4$ g/kg, and $r_{precip} = 1$ g/kg
Find: $P_{LCL} = ?$ kPa, $r = ?$ g/kg, T & $T_d = ?$ °C.

The amount of total water remaining in the cloud is $r_{T\ new} = r_{T\ old} - r_{precip} = 4 - 1 = 3$ g/kg. Follow the $r_T = 3$ g/kg isohume down until it crosses the moist adiabat of the parcel. The intersection pressure height is the new LCL: about $P_{LCL} = 52$ kPa.

Starting from this intersection, create two points, where the temperature point follows the dry adiabat down to $P = 80$ kPa, and the dew-point point follows the mixing ratio down. This gives: $r \approx 3$ g/kg, $T_d \approx -6.5$ °C, and $T \approx 23$ °C.

Check: Units OK. Physics OK.

Discussion: The parcel is warmer and drier than when it started (in the first solved example), because the precipitation removed some water, and caused net latent heating due to condensation.

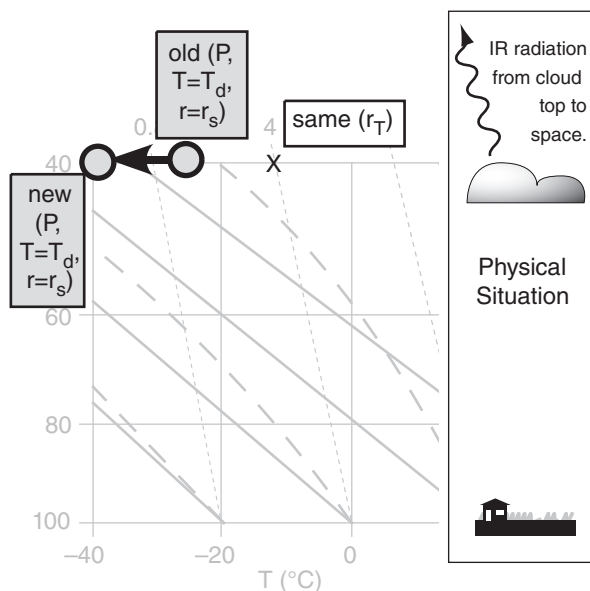


Figure 5.11
Radiative cooling of cloud top. The total water mixing ratio (X) does not change, but temperature and dew point decrease. As less of the total water can be held as vapor at equilibrium, the remainder condenses into additional liquid water. Thus, $r_{L\ new} = r_T - r_{s\ new}$

Solved Example

Given the example in Fig. 5.11 with initial $T = -25^\circ\text{C}$ and $r_T = 4 \text{ g/kg}$ at cloud top ($P = 40 \text{ kPa}$). What is the new state of the air in cloud top after IR radiation has cooled the air to -40°C ? Also find the liquid water mixing ratio.

Solution

Given: Fig. 5.11, $T = -25^\circ\text{C}$, $r_T = 4 \text{ g/kg}$

Find: T & $T_d = ?^\circ\text{C}$, r & $r_s = ? \text{ g/kg}$, $r_L = ? \text{ g/kg}$

Initially:

Because the air is saturated, $T_d = T = -25^\circ\text{C}$.

Also, $r_T = 4 \text{ g/kg}$.

Using a large thermo diagram from the end of this chapter, $r = r_s = 1.3 \text{ g/kg}$.

Thus, $r_L = r_T - r_s = 4 - 1.3 = 2.7 \text{ g/kg}$.

Finally:

The air is still saturated, $T_d = T = -40^\circ\text{C}$.

Total water is unchanged: $r_T = 4 \text{ g/kg}$.

Using a large thermo diagram, $r = r_s = 0.3 \text{ g/kg}$.

Thus, $r_L = r_T - r_s = 4 - 0.3 = 3.7 \text{ g/kg}$.

Check: Units OK. Physics OK.

Discussion: Liquid water mixing ratio has increased as temperature decreased at cloud top.

At $T = -40^\circ\text{C}$, any supercooled liquid water will spontaneously freeze into ice particles (see the Precipitation chapter). When this happens, first set $r_{ice\ new} = r_{L\ new}$ and then reset $r_{L\ new} = 0$.

droplets, eventually leaving the cloud. This is why clouds look white during daytime.

At night, when IR cooling from cloud top dominates, cloud top becomes colder, and the cold parcels of cloudy air sink through the cloud and cause turbulence and mixing. The whole cloud becomes colder this way, even though only the top few centimeters experience the direct IR net radiative loss to space.

Fig. 5.11 illustrates net IR cooling at cloud top. As temperature decreases, so must dew point, because the air is saturated and $T_d = T$. However, the colder air can hold less water vapor at equilibrium than before, thus $r = r_s$ decrease. Although total water remains constant, the liquid water mixing ratio increases: $r_{L\ new} = r_T - r_{s\ new}$.

During daytime, the top few centimeters of the cloud are still cooled by net IR radiative loss, but the top few meters are warmed by sunlight. The sum could be either positive or negative when averaged over the whole cloud. However, with cooling happening at the top of the cloud while warming occurs lower in the cloud, turbulence is set up by the cold sinking air and warm rising air.

In short-lived cumulus clouds, the buoyancy of the thermal updraft rising from the ground often dominates, allowing radiative effects to be neglected. However, for stratiform clouds, and longer-lived thunderstorms and hurricanes, radiative effects can be important.

FOCUS • LeChatelier's Principle

LeChatelier examined a few reacting chemicals in equilibrium with each other in a beaker. By adding more of one of the chemicals, he disturbed the equilibrium. Reactions then occurred to partially eliminate the excess chemical, thereby bringing the reagents in the beaker to a new equilibrium.

LeChatelier's notions of chemical equilibria are easily generalized to describe circulations in the atmosphere. When solar heating induces static instabilities in the atmosphere, the atmosphere generates circulations (i.e., convection) to eliminate the instability and reach a new equilibrium.

For situations where the atmosphere is continuously being destabilized by solar heating of the ground, the atmosphere reacts by creating continuous convection (thermals, or cumulus). Similarly, continuous dynamic destabilization creates continuous turbulence, as a reaction that reduces wind shear by mixing.

Science Graffito

"If you come to a fork in the road, take it."
– Yogi Berra.

Science Graffito

"...the purpose of knowledge is to appreciate wonders [of nature] even more."
– Richard Feynman, 1999: *The Pleasure of Finding Things Out: The Best Short Works of Richard Feynman*.

Solved Example

Air initially at 100 kPa has temperature 40°C and dew-point temperature of 20°C. It rises to a height where the pressure is 50 kPa. Precipitation reduces the total water by 5 g/kg, and the parcel radiatively cools by 11°C while at cloud top. Finally, the parcel descends back to 100 kPa. What is the final relative humidity?

Solution

Given: $T = 40^\circ\text{C}$ and $T_d = 20^\circ\text{C}$ at $P = 100$ kPa.

Find: $RH = ?\%$ at $P = 100$ kPa after all processes.

Method: Use a large thermo diagram from the end of this chapter, for better accuracy. (Fig. 5.6 is shown below to illustrate what was actually accomplished with the larger thermo diagram.)

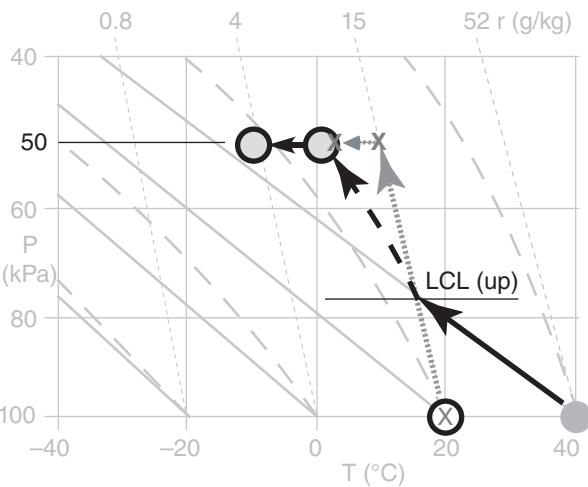
The initial condition is plotted as the two points initially at $P = 100$ kPa, in the Fig below. The initial temperature happens to be on the $\theta = 40^\circ\text{C}$ dry adiabat, and the initial dew point happens to be on the $r = 15$ g/kg isohume.

Follow the dry adiabat (black arrow) and isohume (dotted arrow) up from their starting points, until those isopleths cross. This crossing point is the upward-bound LCL (i.e., cloud base) and is at $P_{LCL} = 75$ kPa and $T_{LCL} = 15^\circ\text{C}$ for this example.

This LCL point happens to lie on the $\theta_w = 25^\circ\text{C}$ moist adiabat. Above the LCL, follow this moist adiabat up to the pressure of 50 kPa. The parcel temperature is roughly $T = 1^\circ\text{C}$. Also, continue tracking the total water (indicated by the X) as it follows the $r_T = 15$ g/kg isohume up.

At the cloud top, the temperature drops by 11°C, from 1°C to -10°C. This puts the temperature near the $\theta_w = 18^\circ\text{C}$ moist adiabat (interpolating by eye between the $\theta_w = 15$ and 20°C moist adiabats) on the large thermo diagram. Precipitation removes 5 g/kg, leaving the total water content at $r_T = 15 - 5 = 10$ g/kg (see gray X's).

(continued next column)



Solved Example (continuation)

Then the parcel descends. From the new temperature point at the cloud top ($T = -10^\circ\text{C}$ at $P = 50$ kPa), follow the new moist adiabat down (see Fig. below). From the new total water point ($r_T = 10$ g/kg at $P = 50$ kPa), follow the isohume down.

Where they cross is the LCL for the descending air. $P_{LCL} = 80$ kPa, which is different than the initial LCL because of the irreversible diabatic processes that acted on the parcel (i.e., radiative cooling, and loss of water). The temperature at this new LCL is roughly $T = 10^\circ\text{C}$.

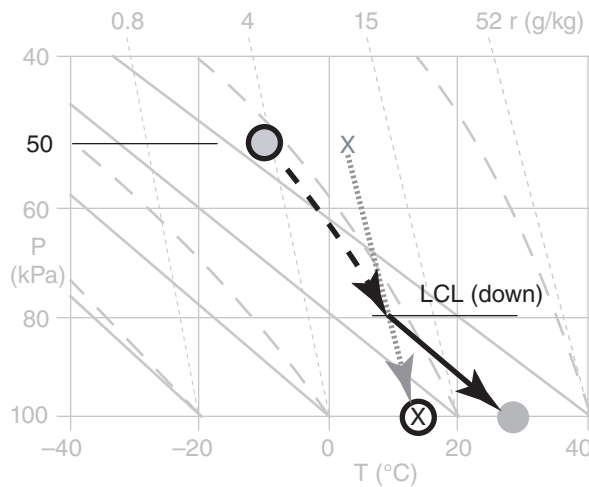
Continuing down below the LCL, temperature follows a dry adiabat down to $P = 100$ kPa. The final temperature is roughly $T = 28^\circ\text{C}$. Mixing ratio follows the 10 g/kg isohume, which gives a final dew point of $T_d = 14^\circ\text{C}$.

At the final temperature, the saturation mixing ratio is roughly $r_s = 25$ g/kg (interpolating by eye between the 20 and 30 g/kg isohumes on the large thermo diagram). Thus, the final relative humidity is:

$$RH = r/r_s = (10 \text{ g/kg}) / (25 \text{ g/kg}) = 40\%$$

Check: Units OK. Physics OK.

Discussion: It helps to use different colors for the temperature and the mixing ratio lines, to avoid confusion. The initial humidity was $RH = 15/52 = 29\%$. So the final air parcel is cooler, with larger RH but smaller T_d .



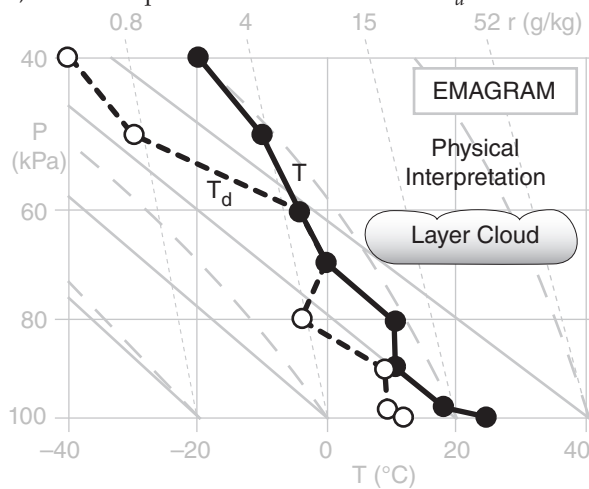
Solved Example

Plot the following sounding on (a) an emagram; and (b) a Skew-T, and discuss the differences.

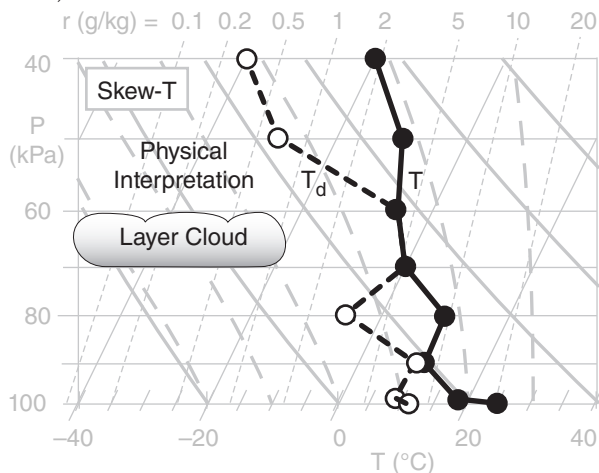
P (kPa)	T_d (°C)	T (°C)
40	-40	-20
50	-30	-10
60	-5	-5
70	0	0
80	-5	10
90	9	10
99	9	19
100	11	25

Solution

a) Emagram: Using black points and solid lines for T, and white points and dashed lines for T_d :



b) Skew-T.



Check: Sketches OK. It is always true that $T_d \leq T$.

Discussion: The kinks in the Skew-T are more apparent than in the Emagram, making it easier to interpret the layers. Both indicate the layer of air between $P = 70$ and 60 kPa is cloudy, because the temperature and dew points are equal. These are mid-level clouds, and would be identified as altostratus or altocumulus. Air aloft near 40 kPa is very dry, because the temperature - dewpoint spread is large. Near 90 kPa the air is almost saturated; namely, the relative humidity is very high because the temperature - dewpoint spread is small.

PARCELS VS. ENVIRONMENT

Air parcels do not move through a vacuum. They are moving through air that is already present at all heights and locations in the atmosphere. We call this omnipresent air the **environment**. Of course much of the air in the environment got there by moving from somewhere else; however, for simplicity we will assume in this chapter that environmental movement is small compared to the movement of parcels through it.

Soundings

As stated at the start of this chapter, the vertical profile of environmental conditions is called an **upper-air sounding**, or just a **sounding**. The word "sounding" is used many ways. It is the:

- activity of collecting the environment data (as in "to make a sounding").
- data that was so collected (as in "to analyze the sounding data").
- resulting plot of these data on a thermo diagram (as in "the sounding shows a deep unstable layer").

Traditionally, a sounding is collected from an expendable instrument (called a **sonde**) hanging on a **weather balloon** that rises through the ambient environment and radios back its observations to a receiving station. In addition to such **radiosondes** (and **rawinsondes** that also give winds) there are **rocketsondes** lofted by small sounding rockets, and **dropsondes** dropped by parachute from aircraft. **Remote sensors** are also used to observe the atmosphere from the ground or from satellite (see the Remote Sensing chapter).

Regardless of the mechanics of acquiring a sounding, the purpose is to determine the state of the environmental air at various heights at some instant in time. The end result is a **vertical profile** of temperature, pressure, humidity, and winds in the environment. Density is rarely measured directly, but can easily be computed using the ideal gas law. Constituents such as ozone and other pollutants can also be measured. At any height, the decrease of temperature with height is called the **environmental lapse rate**.

Plotting an environmental sounding is similar to that for individual parcels. Each measurement of environmental temperature and humidity at any one height (pressure) is plotted as two points on the thermo diagram for unsaturated air. For cloudy air, the T_d points coincide with the T points. Usually, straight lines are drawn connecting the temperature points (see Focus box on **mandatory** and **significant levels**), and separate straight lines are drawn for the humidity points for unsaturated air. Liquid

or ice mixing ratios are not usually measured by the radiosondes, but can be obtained by **research aircraft** flying slant ascent or descent soundings.

The “standard atmosphere” (see Chapter 1) is an example of a hypothetical environmental sounding.

Buoyancy

As Archimedes discovered, an object that is immersed in a fluid displaces some of the fluid, resulting in buoyant forces on the object. The gravitational force minus the buoyant force gives the net specific force F (i.e., force/mass) acting on an object in a fluid:

$$\frac{F}{m} = \frac{\rho_o - \rho_f}{\rho_o} \cdot g = g' \quad (5.2)$$

where m is the mass of the object, ρ_o and ρ_f are the densities of the object and fluid, respectively, and $g = -9.8 \text{ m}\cdot\text{s}^{-2}$ is the gravitational acceleration, where the negative sign implies a downward acceleration. The right hand side of eq. (5.2) is called the **reduced gravity**, g' , where the sign of g' can be different than the sign of g , depending on whether the net buoyant force is up or down.

The force F is the weight of the object, and is defined to be positive in the same direction that g is positive. Negative F implies a downward force (sinking), and positive implies an upward force (floating). Zero force is said to be **neutrally buoyant**.

Air parcels can also be considered as objects within a fluid of the surrounding environmental air. Plugging the ideal gas law into eq. (5.2) and rearranging gives the reduced gravity for an air parcel:

$$\frac{F}{m} = \frac{T_{ve} - T_{vp}}{T_{ve}} \cdot g = g' \quad (5.3a)$$

where T_{ve} and T_{vp} are the absolute virtual temperatures (Kelvin) of the environment and parcel at the same altitude or same pressure. For relatively dry air, the approximation that $T_v \approx T$ is often used.

$$\frac{F}{m} \approx \frac{T_p - T_e}{T_e} \cdot |g| = g' \quad (5.3b)$$

Warm air rises, as given in eq. (5.3). More precisely, a parcel that is warmer than its environment experiences an upward buoyant force.

On a thermodynamic diagram, buoyancy is determined by comparing the temperature of an air parcel against the temperature of the environment at the same height or pressure. An air parcel is assumed to move vertically in the direction of the buoyant force. Of course, as it moves its temperature changes according to the appropriate moist (cloudy) or dry (non-cloudy) process (Fig. 5.12).

Thus, at each new height, the new parcel temperature must be compared to the environment temperature at its new altitude, and the new buoy-

FOCUS • Mandatory & Significant Levels

Rising rawinsondes record the weather at $\Delta z \approx 5$ m increments, yielding data at about 5000 heights. To reduce the amount of data transmitted to weather centers, straight-line segments are fit to the sounding, and only the end points of the line segments are reported. These points, called **significant levels**, are at the kinks in the sounding (as plotted in the previous solved example). Additional **mandatory levels** at the surface and 100, 92.5, 85, 70, 50, 40, 30, 25, 20, 15, 10, 7, 5, 3, 2, & 1 kPa are also reported, to make it easier to analyze upper-air charts at these levels.

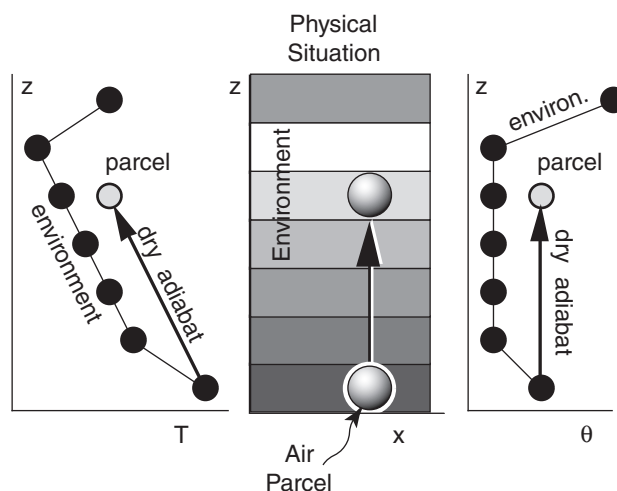


Figure 5.12

Comparison of temperature $T(z)$ methods and potential-temperature $\theta(z)$ methods of representing parcel movement through the environment. Darker shades in the center sketch represent warmer temperatures T . Horizontal distance is x . Thin lines in the left and right plots show the environmental sounding. Thick lines show the adiabatic process for the rising air parcel.

Solved Example

Find the buoyant force per mass acting on an air parcel of temperature $T_p = 20^\circ\text{C}$ in an environment of temperature $T_e = 15^\circ\text{C}$.

Solution

Given: $T_p = 20 + 273 = 293 \text{ K}$, $T_e = 15 + 273 = 288 \text{ K}$
Find: $F/m = ? \text{ m}\cdot\text{s}^{-2}$

Assume dry air, so $T_v = T$. Use eq. (5.3a):
 $F/m = (288 \text{ K} - 293 \text{ K}) \cdot (-9.8 \text{ m}\cdot\text{s}^{-2}) / 288 \text{ K}$
 $= \underline{0.17 \text{ m}\cdot\text{s}^{-2}}$

Check: Physics OK. Units don't look like a force per unit mass, but using conversions from Appendix A, the answer can also be written as 0.17 N/kg . Thus, acceleration is equivalent to force per unit mass in classical physics.

Discussion: The positive sign of the answer means an upward buoyant force. Hot air indeed rises.

Solved Example

- a) Convert the following sounding to potential temperatures, and plot on a linear graph.
- b) If a piece of the environment at $P = 100$ kPa separates from the rest and rises as an air parcel, what is the resulting potential temperature difference between the parcel and environment at $P = 90$ kPa?

P (kPa)	T (°C)
80	10
90	10
99	19
100	25

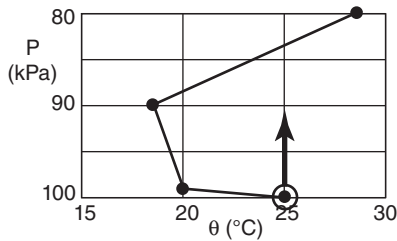
Solution

Given: Sounding above
 Find: $\theta(z)$ and plot it.
 $\Delta\theta$ at $P = 90$ kPa

Method

a) You could use equations from the Heat chapter to convert from T to θ , but it is easier to pick off these numbers from the large thermo diagram at the end of this Chapter.

P (kPa)	θ (°C)
80	28.5
90	18.5
99	20
100	25



b) Air parcel retains $\theta = 25^\circ\text{C}$ as it rises. Thus at $P = 90$ kPa: $\Delta\theta = 25 - 18.5^\circ\text{C} = 6.5^\circ\text{C}$.

Check: Units OK. Physics OK.

Discussion: You would find the same temperature difference using a thermo diagram, without any calculation of potential temperature. Namely, $\Delta T = \Delta\theta$.

ant force can then be estimated to determine if the parcel continues to rise or not. Similar procedures must be used for downward moving parcels.

Virtual potential temperatures (see eqs. 3.13 and 3.14) can be used instead of virtual temperature for plotting soundings and determining buoyancy.

$$\frac{F}{m} = \frac{\theta_{ve} - \theta_{vp}}{T_{ve}} \cdot g = g' \quad (5.3c)$$

Do not forget that g is negative in eqs. (5.3). Thus, parcels that are warmer than their environment cause a negative ratio on the RHS, which when multiplied by the negative gravity gives a positive (upward) force (i.e., warm air wants to move upward). If you use gravitational acceleration as a magnitude:

$$\frac{F}{m} = \frac{\theta_{vp} - \theta_{ve}}{T_{ve}} \cdot |g| = g' \quad (5.3d)$$

An example is sketched in Fig. 5.12. The left diagram represents the parcel and environment as it would be plotted on a thermo diagram such as the Emagram of Fig. 5.3a. The air parcel cools as it rises. The right diagram represents the same situation, but in terms of potential temperature θ vs. height z (Fig. 5.3e). The center sketch shows the parcel rising through the environment, where the darker shading represents warmer temperatures T .

Thermo diagrams such as Fig. 5.3 also give θ , so instead of solving equations we can pick-off θ from the graph. The buoyancy of an air parcel at any height is proportional to the potential temperature difference between the parcel and the environmental sounding at the same height or pressure.

The advantage of using potential temperature is that the parcel maintains constant θ as it moves dry adiabatically. That is, it moves vertically on a θ - Z graph. It is for this reason that the θ - Z thermo diagram of Fig. 5.3e is designed. Also, as a conserved variable it is convenient to use in numerical weather-prediction models (see the NWP chapter).

Brunt-Väisälä Frequency

In a statically stable environment, an air parcel behaves like a mass hanging between two springs (Fig. 5.13a). When displaced vertically from its rest position, the mass experiences a restoring force. As it accelerates back toward its equilibrium point, inertia causes it to overshoot. During this overshoot, the net spring force is in the opposite direction. The mass decelerates and changes direction. It overshoots again and again, giving an oscillation.

When an air parcel is lifted from its starting equilibrium height, it becomes colder than the environment due to adiabatic cooling (Fig. 5.13b & c), and

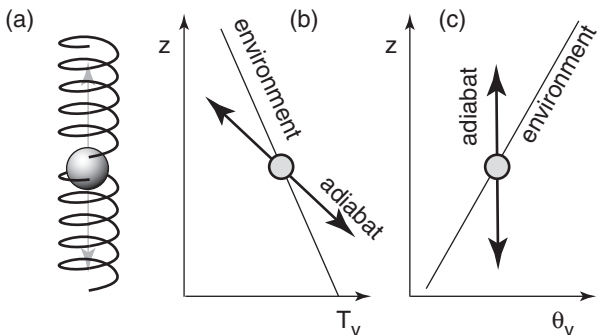


Figure 5.13

(a) A mass constrained between two springs will oscillate up and down if displaced from its equilibrium height. The same will happen for an air parcel displaced vertically in a statically stable environment, regardless of whether it is plotted as (b) virtual temperature T_v or (c) virtual potential temperature θ_v .

the buoyancy force acts downwards. The parcel accelerates downward but overshoots below its starting point, adiabatically warming during its descent to become hotter than the environment, thereby causing a buoyancy force that acts upward. Thus, buoyancy force acts to restore the parcel to its starting point, but inertia causes it to overshoot, the net result being an oscillation.

The frequency of oscillation is called the Brunt-Väisälä frequency, N_{BV} :

$$N_{BV} = \sqrt{\frac{|g|}{T_v} \cdot \left(\frac{\Delta T_v}{\Delta z} + \Gamma_d \right)} \quad (5.4a)$$

or

$$N_{BV} = \sqrt{\frac{|g|}{T_v} \cdot \frac{\Delta \theta_v}{\Delta z}} \quad (5.4b)$$

where virtual temperature must be in Kelvin in the denominator, and where the frequency has units of radians/s. For relatively dry air, it is often assumed that $T_v \approx T$, and $\theta_v \approx \theta$. Frequency increases as static stability increases, analogous to stiffening of the spring. The frequency is undefined for unstable air.

The period of oscillation (time to complete one full oscillation) is

$$P_{BV} = \frac{2\pi}{N_{BV}} \quad (5.5)$$

As the static stability approaches neutral, the period of oscillation approaches infinity. In the real atmosphere, frictional drag on the parcel rapidly damps the oscillation.

Solved Example

Find the period of oscillation of a displaced air parcel in an isothermal environment of 300 K.

Solution

Given: $\Delta T/\Delta z = 0$, $T = 300$ K.

Find: $P_{BV} = ?$ s

Assume dry air. Combine eqs. (5.4a) and (5.5):

$$P_{BV} = \frac{2\pi}{\sqrt{\frac{|g|}{T} \cdot \left(\frac{\Delta T}{\Delta z} + \Gamma_d \right)}}$$

$$P_{BV} = \frac{2\pi}{\sqrt{\frac{(9.8\text{m} \cdot \text{s}^{-2})}{300\text{K}} \cdot (0 + 0.0098\text{K/m})}} = 351\text{ s}$$

Check: Units OK. Physics OK.

Discussion: This period equals 5.9 minutes, which is greater than most water-surface wave periods you might see from a ship. Atmospheric oscillations are so slow that you usually cannot see them by eye. However, a time-lapse camera captures these oscillations quite easily, using smoke as a tracer. Wave clouds (lenticular) are caused by these oscillations (Chapters 6 & 17).

BEYOND ALGEBRA

• Brunt-Väisälä Frequency

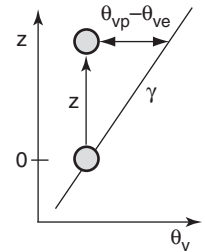
Setting up the Problem

An air parcel that moves up and down during an oscillation is governed by Newton's second law, $F = m \cdot a$, which will be covered in the Dynamics chapter. F is force, m is mass, and a is acceleration. Dividing both sides by m , and using the definition of acceleration as change of velocity w with time t gives:

$$F/m = dw/dt$$

Buoyant F/m is given by eq. (5.3d), and vertical velocity is defined as change of height z with time; namely, $w = dz/dt$. Thus:

$$\frac{\theta_{vp} - \theta_{ve}}{T_{ve}} \cdot |g| = \frac{d^2z}{dt^2}$$



For simplicity, consider the environment shown at right, where environmental virtual potential temperature θ_{ve} increases linearly with height at rate $\gamma = \partial\theta_v/\partial z$. Define $z = 0$ at the initial

(rest) height of the air parcel. From the geometry shown in the diagram, a parcel displaced upward by distance z will be cooler than the environment by:

$\theta_{vp} - \theta_{ve} = -z \cdot \gamma$. Using this in the eq. above gives:

$$-|g| \cdot \gamma \cdot z / T_{ve} = d^2z / dt^2$$

Define $|g| \cdot \gamma / T_{ve} = N_{BV}^2$ (see eq. 5.4b). Thus:

$$d^2z / dt^2 = -N_{BV}^2 \cdot z \quad (a)$$

This is a simple 2nd-order differential equation.

The Solution

From the bag of tricks learned in Diff. Eqs. class, assume a solution of the form:

$$z = A \cdot \sin(f \cdot t) \quad (b)$$

where A and f are unknown amplitude (in units of m) and oscillation frequency (in radians/s). Plugging (b) into (a) gives:

$$-f^2 \cdot A \cdot \sin(f \cdot t) = -N_{BV}^2 \cdot A \cdot \sin(f \cdot t)$$

Canceling the A and sine terms leaves: $f = N_{BV}$. Thus, the variation of air-parcel height with time is:

$$z = A \cdot \sin(N_{BV} \cdot t) \quad (c)$$

Discussion

N_{BV} is the frequency of air-parcel oscillation in radians/s. Use (5.5) to rewrite (c) as:

$$z = A \cdot \sin(2\pi \cdot t / P_{BV})$$

As t increases from 0 to P_{BV} , the argument of the sine goes from 0 to 2π , causing one full cycle.

FLOW STABILITY

Flow stability indicates whether the atmosphere will develop turbulence or will allow waves to grow. We will focus on turbulence of all scales here, from the smallest eddies near the ground to deep convection and thunderstorms filling the troposphere.

- **Unstable** air becomes, or is, turbulent (irregular, gusty).
- **Stable** air becomes, or is, **laminar** (non-turbulent, smooth).
- **Neutral** air has no tendency to change (i.e., is neither stable nor unstable).

Flow stability is controlled by ALL the processes acting on the flow. However, to simplify our understanding of flow, we sometimes focus on just a subset of processes. **Static stability** considers only buoyancy to estimate flow stability, and ignores shears in the mean wind. “Static” means not-moving. **Dynamic stability** considers both wind shears and buoyancy.

Static Stability

One way to estimate static stability is by hypothetically moving a small air parcel through a horizontally-large quasi-stationary environment. Another is to lift whole environmental layers. We will look at both methods.

Parcel Method for Static Stability

To determine static stability, a small portion of the environmental air can be conceptually displaced from its starting point and tracked as an air parcel, such as sketched in the middle part of Fig. 5.12. When conceptually moving this air parcel, follow a dry adiabat while the air is unsaturated (i.e., below its LCL), otherwise follow a moist adiabat. At any height, the buoyant force is based on the temperature difference between the parcel and the environment surrounding it.

• Rule for Determining Static Stability •
 First, find unstable domains using the nonlocal method described next. Then find neutral and stable layers using the local lapse rate, but only in the remaining regions that were not unstable.

Statically Unstable (U): If the buoyant force on a displaced parcel is in the same direction as the displacement, then the air between the initial parcel height and the displaced height is unstable (see

Fig. 5.14). Because of this instability, the parcel will continue accelerating in the direction of its initial movement. This results in **convective** circulations (vertical motions driven by buoyancy), and perhaps convective clouds.

In some environments, it is possible for such an unstable parcel to move across large vertical distances before reaching a height where there is no net buoyancy force. In such a case, the whole vertical domain of buoyant movement of the air parcel is statically unstable. Thus, statically unstable air must be determined by **nonlocal** air parcel movement.

• Rule for Determining Unstable Regions •
 Parcels from each relative θ maximum in the environmental sounding should be conceptually lifted adiabatically until they hit the sounding, and those from each relative minimum should be lowered adiabatically, until they hit the sounding or hit the ground (Fig. 5.14). The domains spanned by these conceptual parcel movements define the statically unstable regions.

For this rule, remember to follow a dry adiabat below the LCL of the conceptual parcel, but follow a moist adiabat above the LCL. Each parcel from each relative max or min will have its own LCL.

A **relative maximum** is a height with warmer θ than the neighboring environmental points just above and below it. Similarly, a **relative minimum** is a height with cooler θ than the neighboring points just above and below it. Using large size thermo diagrams, it is easy to identify by eye the relative max and min θ points, and easy to conceptually raise or lower air parcels until they hit the sounding.

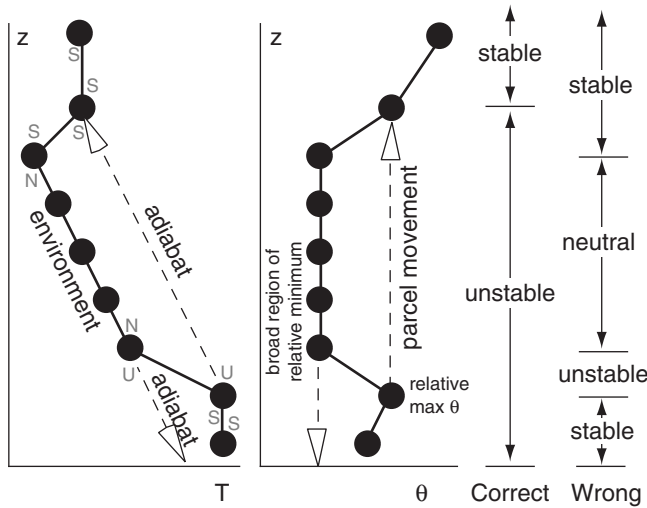


Figure 5.14
 Example of correct static stability determination using parcel displacement. Incorrect determination utilizes lapse rates only. If the displaced parcel becomes saturated, follow a moist adiabat instead of a dry adiabat. Grey U, N, S are for the “apex” method of determining nonlocal static stability.

Statically Neutral (N): Those portions of the sounding where the environmental lapse rate approximately equals the adiabatic lapse rate, *and which are not otherwise nonlocally unstable*, are approximately statically neutral. A parcel displaced in this environment will feel no buoyant forces. The equations describing this lapse rate (in terms of either temperature T or potential temperature θ), are:

$$\frac{\Delta T}{\Delta z} \approx - \begin{cases} \Gamma_d & \text{if unsaturated} \\ \Gamma_s & \text{if saturated} \end{cases} \quad \bullet(5.6a)$$

or

$$\frac{\Delta \theta}{\Delta z} \approx \begin{cases} 0 & \text{if unsaturated} \\ \Gamma_d - \Gamma_s & \text{if saturated} \end{cases} \quad \bullet(5.6b)$$

where $\Gamma_d = 9.8 \text{ K/km}$ is the dry adiabatic lapse rate, and Γ_s is the saturated (or moist) adiabatic lapse rate (which varies, see the Moisture chapter), but which is always a positive number.

Statically Stable (S): Those portions of the sounding where the temperature does not decrease with height as fast as the adiabatic rate (including isothermal layers, and **temperature inversion** layers where temperature increases with height), *and which are not otherwise nonlocally unstable*, are statically stable. A parcel displaced in this environment will experience buoyant forces opposite to the direction of displacement.

$$\frac{\Delta T}{\Delta z} > - \begin{cases} \Gamma_d & \text{if unsaturated} \\ \Gamma_s & \text{if saturated} \end{cases} \quad \bullet(5.7a)$$

or

$$\frac{\Delta \theta}{\Delta z} > \begin{cases} 0 & \text{if unsaturated} \\ \Gamma_d - \Gamma_s & \text{if saturated} \end{cases} \quad \bullet(5.7b)$$

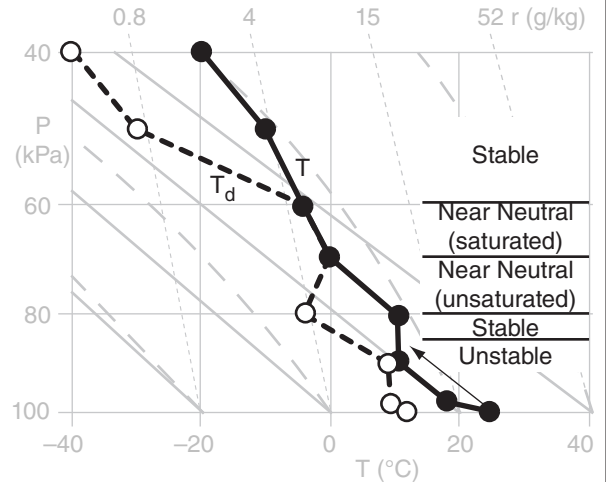
Equation (5.7b) states that unsaturated air is statically stable if potential temperature increases with height (unless it was previously determined to be nonlocally unstable).

The Apex Method for Determining Nonlocal Static Stability: (1) At every apex (kink in the sounding), write a small U, N, or S above that apex if an air parcel displaced upward a small distance from that point is unstable, neutral, or stable, respectively. Similarly, below each apex, write U, N, or S for air displaced downward. (2) For each U above an apex, extend it adiabatically upward until it hits the sounding. Similarly, extend each U below an apex downward. (3) All domains spanned by U are unstable. (4) Any non-U domains are stable or neutral.

Solved Example

Determine the static stability of the sounding in the solved example from the “Parcel vs. Environment” section, earlier in this chapter.

Solution



Check: Sketch OK.

Discussion: The unstable layer is turbulent. The near-neutral layers could become turbulent given the slightest wind shear.

FOCUS • A holistic approach to stability

Consider an environmental air parcel in the middle of the mixed layer (at a middle height in Fig. 5.14). If you raise it or lower it a small amount, it will have the same temperature and potential temperature as its new environment. Therefore, you might incorrectly think that the flow is statically neutral, implying only moderate amounts of turbulence and smoke-plume envelopes that cone (see the Air-Pollution chapter).

However, something else is ALSO happening in the middle of the mixed layer that is NOT associated with the middle air parcel. Namely, there are buoyant air parcels rising through the mid-mixed layer from the warmer surface-layer air below. These rising air parcels are creating strong convective turbulence. The vigorous up- and down-drafts in these thermals cause smoke plumes to loop up and down in the mid-mixed layer.

Such a holistic approach is not new. Thunderstorm forecasters usually use the whole sounding, and consider air parcels rising from the surface to the top of the storm to estimate the potential severity of storms. They don't focus on only the local lapse rate in the mid troposphere to estimate thunderstorm intensity.

“Lapse rate” (subadiabatic, adiabatic, superadiabatic) is **not** synonymous with “static stability” (stable, neutral, unstable). If the purpose of a stability definition is to determine whether flow becomes turbulent, then you must consider the whole sounding.

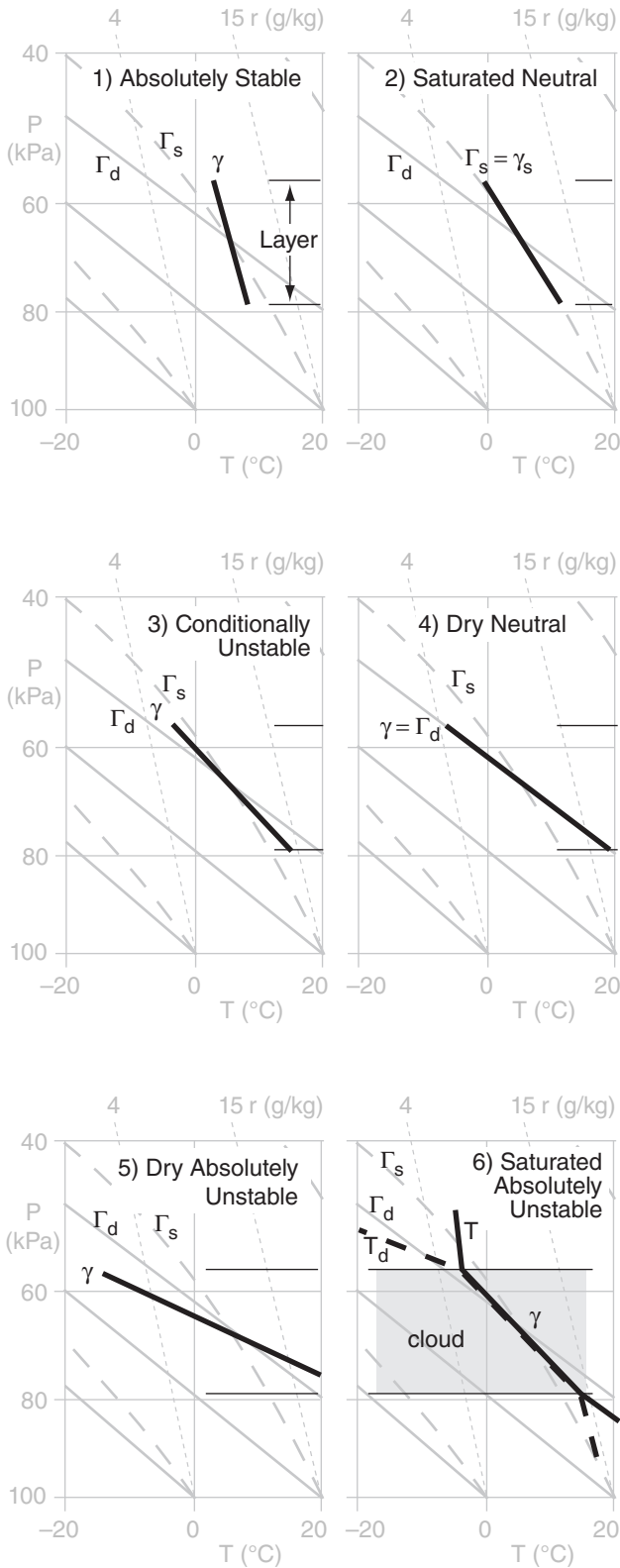


Figure 5.15
 Layer stabilities, shown on a portion of a thermo diagram. The thick solid line is environmental air temperature within one layer of air (i.e., a line segment from a complete sounding), and the thick dotted line is environmental dew-point temperature. Γ_d and Γ_s are the dry and saturated adiabatic lapse rates.

Layer Method for Static Stability

Sometimes a whole layer of environmental air is lifted or lowered by an outside process. Synoptic-scale warm and cold fronts (see the Airmasses and Fronts chapter) and coherent clusters of thunderstorms (**mesoscale convective complexes**, see the Thunderstorm chapters) are examples of layer-lifting processes. In these cases we cannot use the parcel method to determine static stability, because it assumes a quasi-stationary environment.

The lapse rate γ of an environmental layer is defined as the temperature decrease with height:

$$\gamma = -\Delta T / \Delta z \tag{5.8}$$

Lapse rates are named as shown in Table 5-1, where isothermal and inversion are both subadiabatic.

Table 5-1. Names of environmental lapse rates, where Γ_d is the dry adiabatic lapse rate.

Name	Layer Lapse Rate	Fig.
adiabatic	$\gamma = \Gamma_d = 9.8^\circ\text{C}/\text{km}$	5.15(4)
superadiabatic	$\gamma > \Gamma_d$	5.15(5)
subadiabatic	$\gamma < \Gamma_d$	5.15(1)
• isothermal	$T = \text{constant with } z$	
• inversion	$T = \text{increases with } z$	

Layer Stability Classes: Six classes of layer stability are listed in Table 5-2. For a layer of air that is already saturated (namely, it is a layer of clouds, with $T = T_d$), then the environmental lapse rate γ , which is still defined by the equation above, is indicated as γ_s to remind us that the layer is saturated. As before, the word “dry” just means unsaturated here, so there could be moisture in the air.

Table 5-2. Layer static stability, where $\Gamma_d = 9.8^\circ\text{C}/\text{km}$ is the dry adiabatic lapse rate, and Γ_s is the saturated adiabatic lapse rate. Γ_s is always $\leq \Gamma_d$.

Name	Layer Lapse Rate
1) absolutely stable	$\gamma < \Gamma_s$
2) saturated neutral	$\gamma_s = \Gamma_s$
3) conditionally unstable	$\Gamma_s < \gamma < \Gamma_d$
4) dry neutral	$\gamma = \Gamma_d$
5) dry absolutely unstable	$\gamma > \Gamma_d$
6) saturated absolutely unstable	$\gamma_s > \Gamma_s$

Fig. 5.15 illustrates these stabilities. The layer of air between the thin horizontal lines has a linear lapse rate shown by the thick line. The slope of this thick line relative to the slopes of the dry Γ_d and moist Γ_s adiabats determines the layer stability class.

Conditional Stability: One of the types of layer stability listed in Table 5-2 is conditional stability. A layer of conditionally unstable air has lapse rate between the dry and moist adiabatic lapse rates. This layer is stable if the air is unsaturated, but is unstable if the air is saturated (cloudy). Hence, the “condition” refers to whether the air is cloudy or not.

The 6th stability class is also just a special case of conditional instability. Namely, it satisfies the condition of being cloudy, so it is unstable.

Conditionally unstable air, if over a deep enough layer, is a favorable environment for thunderstorms to grow. A special set of **thunderstorm stability indices** has been developed to determine if thunderstorms will form, and how intense they might be. These are discussed in the Thunderstorm chapters of this book. The presence of thunderstorms indicate violently unstable air that extends throughout the depth of the troposphere.

When a whole layer is lifted, the temperature at each level within that layer will change according to whether those levels are saturated or not. Fig. 5.16 shows a sample sounding, and how temperatures change following dry adiabats until reaching their local LCL, above which they follow the moist adiabat. This particular example shows how a layer that starts as conditionally unstable can become absolutely unstable if the bottom of the layer reaches saturation sooner than the top.

Dynamic Stability

Flow can become turbulent in statically stable air if the **wind shear** (change of wind speed or direction with height) is strong enough. Such dynamic stability is indicated by the dimensionless **bulk Richardson number, Ri**:

$$Ri = \frac{|g| \cdot (\Delta T_v + \Gamma_d \cdot \Delta z) \cdot \Delta z}{T_v \cdot [(\Delta U)^2 + (\Delta V)^2]} \quad \bullet(5.9a)$$

or

$$Ri = \frac{|g| \cdot \Delta \theta_v \cdot \Delta z}{T_v \cdot [(\Delta U)^2 + (\Delta V)^2]} \quad \bullet(5.9b)$$

where ΔT_v , ΔU , and ΔV represent virtual temperature and wind differences across a height $\Delta z = z_2 - z_1$. The dry adiabatic lapse rate is $\Gamma_d = 9.8 \cdot K/km$. Temperature in the denominator must be in Kelvin. For relatively dry air, it is often assumed that $T_v \approx T$, and $\theta_v \approx \theta$.

For the special case of statically stable air:

$$Ri = \frac{N_{BV}^2 \cdot (\Delta z)^2}{(\Delta U)^2 + (\Delta V)^2} \quad (5.9c)$$

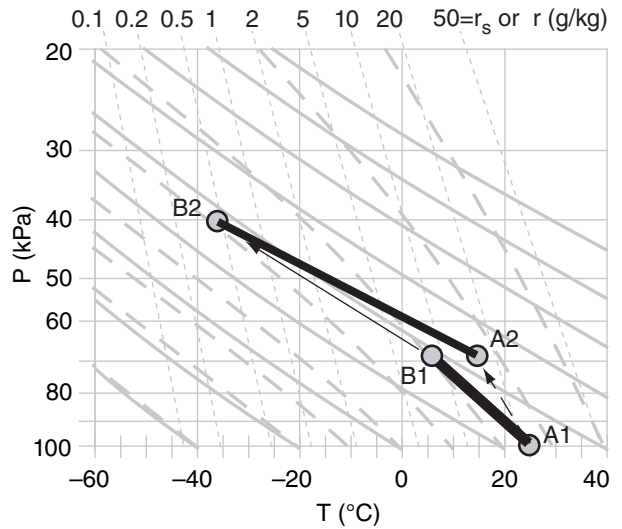


Figure 5.16
Example of lifting a layer to determine layer stability. Conditionally unstable layer (thick black line between points A1 and B1) is lifted 30 kPa. Suppose that the bottom of the layer is already saturated, but the top isn't. Upon lifting, the bottom of the layer will follow a moist adiabat from A1 to A2, while the top follows a dry adiabat from B1 to B2. The layer at its new height (black line A2 to B2) has become absolutely unstable.

Solved Example

Given air at $z = 2 \text{ km}$ with $U = 13 \text{ m/s}$, $V = 0$, and $T = 10^\circ\text{C}$, while air at 1.8 km has $U = 5 \text{ m/s}$, $V = 0$, and $T = 10^\circ\text{C}$. a) Find the bulk Richardson number. b) Is the air turbulent? Assume relatively dry air.

Solution

Given: $\Delta T = 0$, $\Delta V = 0$, $\Delta U = 8 \text{ m/s}$, $\Delta z = 0.2 \text{ km}$
Find: $Ri = ?$ (dimensionless), and turbulence.

a) Remembering to take all differences (Δ) in the same direction, use eq. (5.9a):

$$Ri = \frac{(9.8 \text{ m} \cdot \text{s}^{-2}) \cdot [0 + (9.8 \text{ K/km}) \cdot (0.2 \text{ km})] \cdot (200 \text{ m})}{(283 \text{ K}) \cdot [(8 \text{ m/s})^2 + (0)^2]} = \underline{0.21}$$

b) The flow is **dynamically unstable** and **turbulent** because $Ri < 0.25$.

Check: Units OK. Physics OK.

Discussion: Isothermal air is strongly statically stable. However, the wind shear is strong enough to overpower the stability, giving a small Richardson number, dynamic instability, and turbulence.

As turbulence causes mixing of temperature and winds, ΔT , ΔU , and ΔV all approach zero. Thus, the denominator of Ri becomes smaller faster than the numerator, because velocities are squared in the denominator. As a result, Ri increases above the critical value, and turbulence ceases. Thus, turbulence tends to eliminate itself (per **LeChatelier's Principle**, see Focus Box), unless other processes continue to strengthen the wind shear to maintain turbulence.

Solved Example

For the following sounding, determine the regions of turbulence.

z (km)	T (°C)	U (m/s)	V (m/s)
2	0	15	0
1.5	0	12	0
1.2	2	6	4
0.8	2	5	4
0.1	8	5	2
0	12	0	0

Solution:

Given: The sounding above.

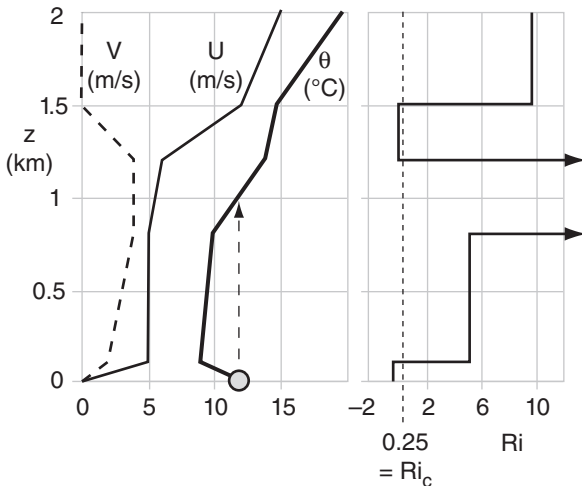
Find: a) Static stability (parcel method),
b) dynamic stability, & (c) identify turbulence.

Assume dry air, so $T = T_v$.

Method: Use spreadsheet to compute θ and Ri . Note that Ri applies to the layers between sounding levels.

z (km)	θ (°C)	z_{layer} (km)	Ri
2	19.6		
1.5	14.7	1.5 to 2.0	9.77
1.2	13.8	1.2 to 1.5	0.19
0.8	9.8	0.8 to 1.2	55.9
0.1	9.0	0.1 to 0.8	5.25
0	12	0 to 0.1	-0.358

Next, plot these results:



- a) Static Stability: Unstable & turbulent for $z = 0$ to 1 km, as shown by air-parcel rise in the θ sounding.
- b) Dynamic Stability: Unstable & turbulent for $z = 0$ to 0.1 km, & for $z = 1.2$ to 1.5 km, where $Ri < 0.25$.
- c) Turbulence exists where the air is statically OR dynamically unstable, or both. Therefore:
Turbulence at 0 - 1 km, and 1.2 to 1.5 km.

Check: Sketch OK. Physics OK. Units OK.

Discussion: At $z = 0 - 1$ km is the mixed layer. At $z = 1.2 - 1.5$ km is **clear-air turbulence (CAT)** and K-H waves.

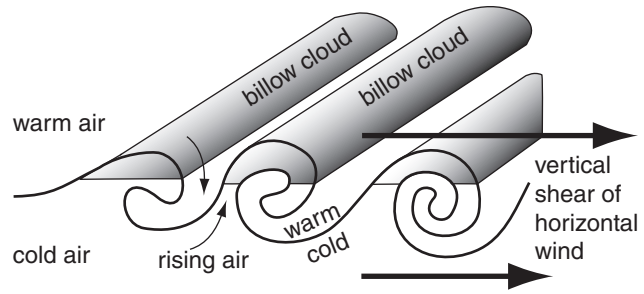


Figure 5.17

Kelvin-Helmholtz (K-H) waves with sufficient moisture to create rows of billow clouds perpendicular to the wind shear.

N_{BV} is undefined for air that is not statically stable.

To a good approximation when Δz is small, the flow is **dynamically unstable** and **turbulent** whenever

$$Ri < Ri_c \quad \bullet(5.10)$$

The **critical Richardson number** is $Ri_c = 0.25$.

When condition (5.10) is met, the flow is called **dynamically unstable**. Otherwise, for larger Ri , the flow is dynamically stable. Statically unstable air results in negative Richardson numbers, which are also dynamically unstable.

Flow that is just becoming dynamically unstable often forms **Kelvin-Helmholtz (K-H) waves** in the air, which look like ocean waves breaking on shore (Fig. 5.17). When there is sufficient moisture in the air to make these waves visible, they appear as parallel bands of narrow clouds called **billow clouds**, filling the crest of each wave. Non-thunderstorm turbulence at altitudes above the top of the atmospheric boundary layer is called **clear-air turbulence (CAT)**.

Turbulence Determination

Both dynamic instability and static instability give incomplete measures of the existence of turbulence. Static stability does not include the effects of shear in turbulence generation. Dynamic stability does not include the nonlocal processes that can generate turbulence. Thus, to determine whether the flow in any region is turbulent, you must compute both the static stability (using the nonlocal parcel methods given in this chapter), and the dynamic stability (using the Richardson number).

• Turbulence Determination Rule •

- 1) To be turbulent, the flow must be either statically OR dynamically unstable.
- 2) To be **laminar** (non-turbulent), the flow must be both statically AND dynamically stable.

FINDING THE TROPOPAUSE HEIGHT AND MIXED-LAYER DEPTH

Tropopause

Recall from Chapter 1 (Fig 1.10) that the standard-atmosphere temperature decreases with height within the troposphere, but is isothermal with height in the bottom part of the stratosphere. Isothermal layers are very strongly statically stable. The strong and deep statically stable layer in the stratosphere discourages vertical motion through it, which is why most of our weather (including thunderstorms and hurricanes) is trapped below it. The base of the stratospheric isothermal layer is the tropopause.

Real soundings are more complex than the simple standard atmosphere, and can have many different stable layers (and isothermal layers) at different heights (Fig. 5.18). Nonetheless, if the radiosonde balloon rises high enough before bursting, then it often can enter the stratosphere. To locate the tropopause, we just need to look for the bottom of the very thick isothermal layer that is near a pressure altitude of 40 to 20 kPa.

Fig. 5.19 illustrates a hypothetical vertical slice through the atmosphere, showing the **isentropes** (lines of equal potential temperature, θ). The spacing between isentropes is a measure of the static stability. Tight packing (i.e., small spacing) between isentropes indicates strong stability. Large spacing between isentropes indicates weak stability. Thus, you can locate the tropopause at the bottom of the region of tight isentropes, as sketched in Fig. 5.19. [CAUTION: The tropopause usually does NOT follow any single isentrope.]

To illustrate the relationship between stability and isentrope spacing, let the circle in Fig. 5.19 represent a weather balloon that is filled with helium, and which is rising through the environment. On the way up, its attached radiosonde measures temperature and pressure, from which potential temperature can be calculated.

The radiosonde would have first risen through relatively cold air with large spacing between isentropes, indicating weak static stability (near neutral). However, as it rises across the 280 K and the 290 K isentropes, the tight packing there indicates stronger static stability, perhaps marking the top of a frontal surface or the inversion at the top of the mixed layer. Higher still in the troposphere, the medium spacing between isentropes indicates moderately stable air. Finally the balloon enters the stratosphere, where very tight packing of isentropes indicates very strong static stability.

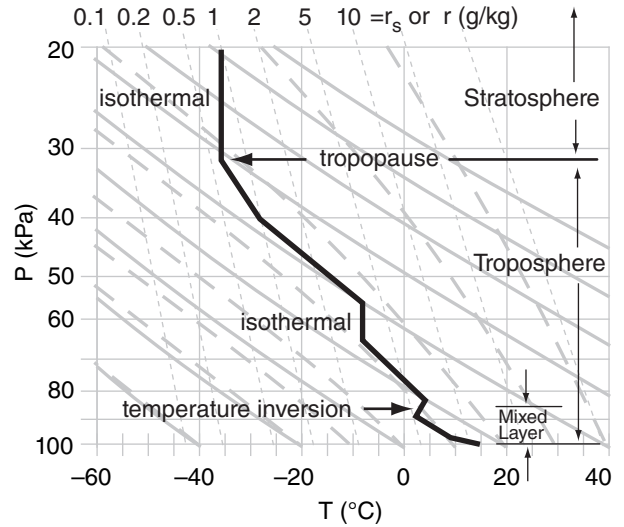


Figure 5.18

Sample atmospheric sounding having two isothermal layers and one temperature inversion.

Solved Example

What is the tropopause height (or pressure) in the sounding of Fig. 5.18, and how is it identified?

Solution

Given: the plotted sounding.

Find: $P_{Trop} = ?$ kPa

The base of the deep isothermal layer near the top of this sounding is at $P_{Trop} = \underline{32 \text{ kPa}}$.

The other isothermal and inversion layers are too low, and too thin.

Check: Sketch OK.

Discussion: The tropopause is lower near the poles and higher near the equator. It is lower in winter, higher in summer (see Table 6-1 in the Cloud chapter), but varies considerably from day to day.

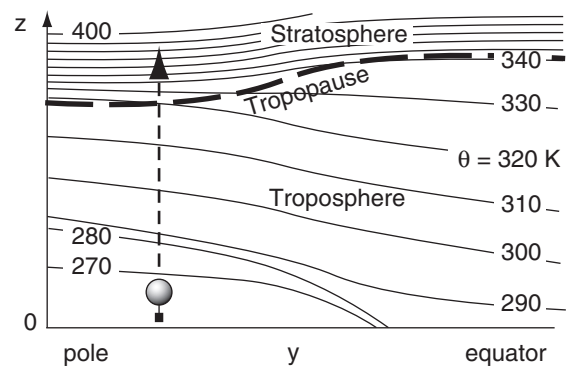


Figure 5.19

Simplified vertical slice through the atmosphere, showing isentropes (lines of equal potential temperature, θ). Heavy dashed line is the tropopause. Grey circle represents a radiosonde balloon rising through the environment.

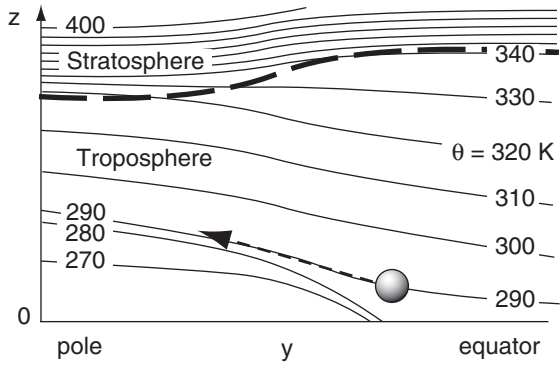


Figure 5.20
Same as Fig. 5.19, but now the grey circle represents an air parcel of potential temperature 290 K, which rides along the environmental 290 K isentrope as the parcel is advected poleward. Thick dashed line indicates the tropopause. Thin dashed line shows the path of the air parcel.

Because of buoyant forces, air parcels tend to follow isentropic surfaces as they advect. In Fig. 5.20, suppose that the circle represents an air parcel, not a weather balloon. Further suppose that the potential temperature of that parcel is 290 K. Because that air parcel is surrounded by an environment of the same potential temperature (as indicated by the 290 K isentrope), the parcel experiences no buoyant forces on it. Namely, it is at its equilibrium height.

Let the parcel be blown horizontally toward the pole. If we assume (incorrectly) that it maintains a constant altitude above sea level, then it finds itself surrounded by an environment of colder air (of $\theta = 270$ K). Thus, the parcel of $\theta = 290$ K is buoyant, and rises through the environment until it reaches its equilibrium 290 K isentrope. In the real atmosphere, such rise happens concurrently with its horizontal motion, so that the parcel path (thin dashed line in Fig. 5.20) follows the sloping surface of the 290 K isentrope. Similarly, if the parcel were blown equatorward, it would follow the same isentrope and descend. The study of forces, motions, and thermodynamics along surfaces of constant potential temperature is called **isentropic analysis**.

Solved Example

Given the following potential temperature measurements. Plot them on the T vs. $\log P$ diagram, and on the θ vs. z diagram. Indicate the static stability of each layer, and find the location of z_i using the exact method.

z (km)	θ ($^{\circ}\text{C}$)
3.0	35
2.0	28
1.5	28
1.3	25
1.0	24
0.2	24
0	26

Solution

$z_i \approx 1.37$ km

Check: Physics OK. Sketch reasonable.
Discussion: θ vs. z is often easier to use than T vs. $\log(P)$. Note that z_i is not at the base of the subadiabatic region, but slightly above it due to nonlocal air-parcel rise.

Mixed-Layer

When the ground is warmer than the air, a convective mixed layer is created that is nonlocally unstable. The height z_i of the top of this mixed layer is defined as the top of the nonlocally unstable region. The easiest way to determine this height is to use a thermo diagram. The θ vs. z thermo diagram is ideal for mixed-layer studies.

First, obtain a sounding of temperature or potential temperature vs. height. By plotting the sounding on a thermo diagram and conceptually lifting an air parcel from the surface, z_i is found as the height where this rising parcel hits the sounding.

A less accurate, but quick approach, is to look for the temperature inversion (or similar layer that is statically stable) that is near the ground, but not touching it. A **temperature inversion** is where T increases with height. The top of the mixed layer is usually somewhere within this temperature inversion. In Fig. 5.18, the average height of the temperature inversion is about $z_i = 85$ kPa.

Sometimes thermodynamic information is needed for the boundary layer, to study thermal rise, turbulence, low-cloud and fog formation, and smoke dispersion. Enlargements of the bottom portion of thermo diagrams, focusing on the boundary layer, are provided at the end of this chapter.

Other measures of stability, and applications of the mixed layer, are given in the Atmospheric Boundary Layer and Air-Pollution Dispersion chapters.

SUMMARY

You can use the thermodynamic diagram as a tool to simplify analyses of stability and thermodynamics. There are many different types of thermo diagrams: Emagram, Skew-T, Stüve (or Pseudoadiabatic Diagram), Tephigram, and θ -Z diagram. They all serve the same purpose, and all have isotherms, isobars, isohumes, dry and moist adiabats, and height contours (in some cases).

The state of the air (i.e., its temperature, pressure, and water content), as well as processes that can change the state, can be described via thermo diagrams. Processes include rising or sinking air, condensation/evaporation, precipitation, and radiative heating/cooling. With a thermo diagram you can find the saturation characteristics of air, the amount of liquid water in a cloud, the height of cloud base, potential temperatures, and cloud depth.

Thermo diagrams are often used to plot upper-air soundings. You will also find them valuable to analyze thunderstorms and hurricanes, and to help anticipate their intensity, as will be described in later chapters.

As winds and turbulence move the air from place to place, an air parcel of one temperature can become immersed in an environment of different temperatures. Such a parcel experiences buoyant forces, which causes it to rise or sink. Parcels displaced from their starting location will accelerate further away in a statically unstable environment, but will tend to return to their starting height in a statically stable environment.

In a statically stable environment, air parcels can oscillate vertically at the Brunt-Väisälä frequency. When wind shear, the change of wind speed or direction with height, is considered, statically stable air can be dynamically unstable and turbulent, as estimated using the Richardson number.

Threads

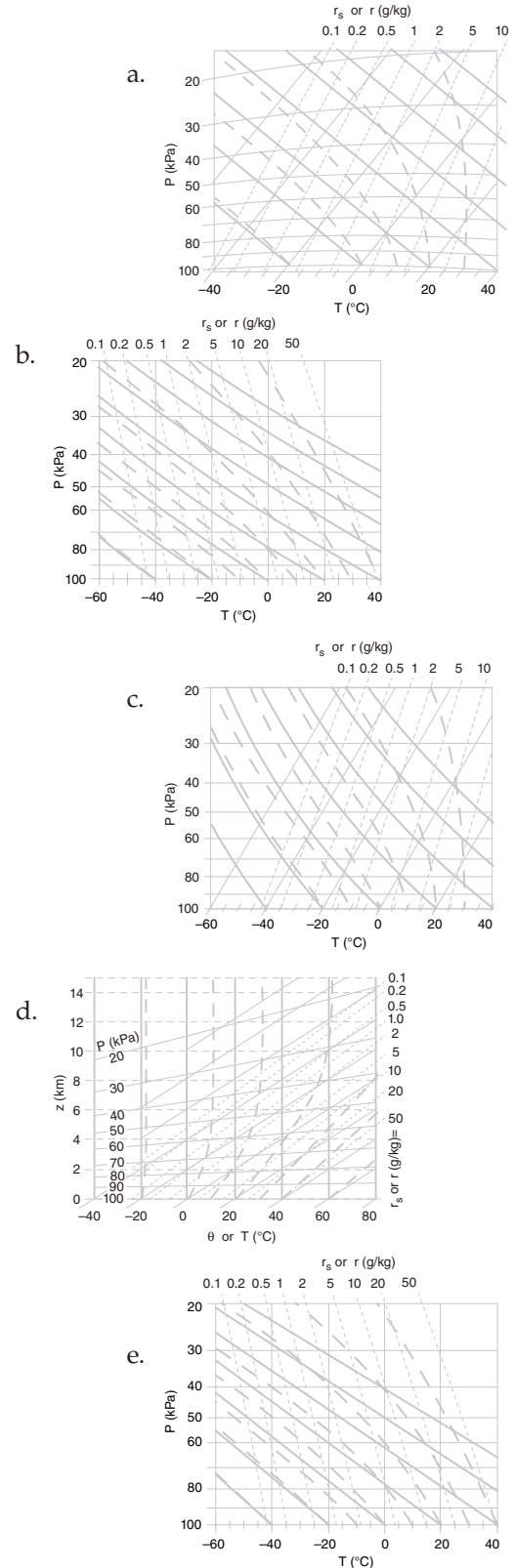
The growth of thunderstorms (Chapters 14 and 15) fueled by warm humid air, and the energy available to hurricanes (Chapter 16) are analyzed using thermo diagrams. Thermo diagrams are used to study the vertical structure of the atmosphere (Chapter 1), the boundary layer (Chapter 18), dispersion of air pollutants (Chapter 19), and frontogenesis (Chapter 12).

The wavelength of mountain waves (Chapter 17) and formation of lenticular clouds depend on the Brunt-Väisälä frequency. Anabatic and katabatic winds (Chapter 17) are driven by buoyant forces.

EXERCISES

Numerical Problems

N1. Identify the thermo diagram type.



N2. On copies of ALL 5 thermo diagrams from N1, find and label one of the:

- a. dry adiabats b. isohumes c. isobars
 d. isentropes e. saturated adiabats
 f. isotherms g. moist adiabats

N3(S). Use a spreadsheet to create the Skew-T, as given in the Solved Example in the Skew-T section of this Chapter, except do it for isopleths listed below. Describe the results of your experiments with different values of the coefficient K , which determines the amount of skewness.

	$T_r \theta$, & θ_w (°C)	r (g/kg)
a.	30	50
b.	25	20
c.	15	10
d.	10	2
e.	5	1
f.	0	0.5
g.	-5	0.2
h.	-10	0.1
i.	-15	15
j.	-20	25

N4. On copies of ALL 5 thermo diagrams from N1, plot the following air-parcel state. Hint: The data below represent individual air parcels, NOT a sounding, NOT a vertical profile.

	P (kPa)	T (°C)	T_d (°C)
a.	40	0	-20
b.	90	-5	-5
c.	45	-20	-40
d.	85	20	18
e.	50	10	-5
f.	80	15	15
g.	55	-30	-45
h.	75	-5	-40
i.	60	8	3
j.	70	17	12
k.	65	-7	-13

N5. Repeat N4, but on copies of ALL 5 of the LARGE thermo diagrams from the end of this chapter (but do not use the special ABL diagrams).

N6. For one assigned air-parcel state from question N4, find each of the following using only a Skew-T or Tephigram to answer these questions. Do not solve equations (except for relative humidity).

- i) potential temperature
 ii) wet-bulb potential temperature
 iii) mixing ratio
 iv) saturation mixing ratio
 v) relative humidity vi) LCL
 vii) equivalent potential temperature
 viii) wet-bulb temperature

N7. Same as N6, but use only an Emagram or Stüve.

N8. Starting with one assigned air-parcel with state from question N4:

i) lower it to a pressure of 100 kPa, and find its new temperature and dew point. Use only a tephigram or Skew-T.

ii) lower it to a pressure of 100 kPa, and find its new temperature and dew point. Use only a Stüve or Emagram.

iii) raise it to a pressure of 20 kPa, and find its new temperature and dew point. Use only a tephigram or Skew-T.

iv) raise it to a pressure of 20 kPa, and find its new temperature and dew point. Use only a Stüve or Emagram.

N9. Which thermo diagrams from Fig. 5.3 have

- a. straight isobars?
 b. straight dry adiabats?
 c. straight moist adiabats?
 d. have the greatest angle between isotherms and dry adiabats?
 e. moist adiabats that asymptotically approach dry adiabats near the top of the diagram?
 f. moist adiabats that asymptotically approach dry adiabats near the left of the diagram?

N10. Given the initial state of an air parcel below, lift it to a final pressure of 30 kPa, and determine its final values of T , T_d , r_D , r_S , and r_L . Use a copy of the large thermo diagram from the end of this chapter, as specified below.

	P (kPa)	T (°C)	T_d (°C)	Thermo Diagram
a.	100	35	20	Skew-T
b.	100	30	20	Stüve
c.	100	25	10	Tephigram
d.	100	20	-5	Emagram
e.	90	30	25	Skew-T
f.	90	30	15	Stüve
g.	90	25	15	Tephigram
h.	90	20	5	Emagram
i.	80	20	15	Skew-T
j.	80	20	10	Stüve
k.	80	15	0	Tephigram
l.	80	10	-5	Emagram

N11. For the air parcel specified for exercise N10, find its LCL pressure, using the specified thermo diagram.

N12. Starting with the FINAL state of the air parcel specified from exercise N10, allow all of the liquid water to rain out (except if the liquid water from the

final state of N10 was greater than 1 g/kg, then allow only 1 g/kg of liquid water to rain out). Then lower the air parcel back to its starting pressure, and determine its new temperature and dew point. Use the specified thermo diagram from N10.

N13. Starting with the FINAL state of the air parcel specified from exercise N10, allow radiative cooling to change the air parcel temperature by $\Delta T = -8^\circ\text{C}$. Then lower the air parcel back to its starting pressure, and determine its new temperature and dew point. Use the specified thermo diagram from N10.

Use the following environmental sounding to answer the exercises below. Assume $V = 0$ everywhere.

P(kPa)	T(°C)	T_d (°C)	U (m/s)
20	-30	-60	40
40	-30	-30	40
60	-5	-30	40
70	-5	-20	40
80	2	0	10
87	7	4	5
98	16	6	5
100	20	9	0

Assume that these pressures are **significant levels** (i.e., altitudes where there is a kink or bend in the profile, with straight line segments between each significant level). Use the word “**layer**” to represent the air between two adjacent significant levels.

N14. Plot the above sounding on a large thermodynamic diagram from the end of this chapter. Use solid dots (of red color if possible) for the temperature and mark an open circle (blue color if available) for the dew points. Draw a solid (red, if possible) line to connect the temperature dots, starting from the bottom up, and draw a dashed (blue, if possible) line for the dew points. Use a/an:

- a. Emagram b. Skew-T c. Stüve
d. Tephigram e. θ -Z diagram

N15. Using the sounding above:

- a. Find the potential temperature at each height, and plot it on a linear graph of θ vs P .
b. Calculate and plot θ_v vs P .

N16. If an air parcel from the sounding above moves up one level, what buoyancy force per mass acts on it in this new layer. Use a starting air parcel from the layer at P (kPa) =

- a. 100 b. 98 c. 87 d. 80 e. 70 f. 60 g. 40

N17. Calculate the Brunt-Väisälä frequency and the corresponding period of oscillation for one layer

from the sounding above, where the base of the layer is at level P (kPa) =

- a. 100 b. 98 c. 87 d. 80 e. 70 f. 60 g. 40

N18. Using the sounding above as plotted for exercise N14, determine the static stability using the parcel method for the one environmental layer, where the bottom of that one layer is at P (kPa) =

- a. 100 b. 98 c. 87 d. 80 e. 70 f. 60 g. 40

Hints: Neglect moisture & condensation. Also, the stability in any one layer could depend on nonlocal effects from other layers.

N19. Same as N18, but find the layer stability.

N20. Using the sounding above, determine the Richardson number and the dynamic stability for the layer that has a layer base at level P (kPa) =

- a. 100 b. 98 c. 87 d. 80 e. 70 f. 60 g. 40

N21. Indicate which portions of the sounding above are likely to be turbulent, and explain why.

N22. Using the sounding above:

- a. Determine if there is a mixed layer. If so, find the mixed-layer depth z_i .
b. Determine the height of the tropopause.
c. Locate any layer (stratiform) clouds that might be present in the environment.

N23. With the sounding above:

a. Use the thermo diagram to determine the pressure-height of the LCL for an air parcel rising from 100 kPa. How does this compare to the height using a formula (see the Moisture chapter) for the LCL?

b. For a parcel rising from 100 kPa, at what pressure-heights are cloud base and cloud top? What is the mixing ratio of water vapor in the cloud at cloud top? What is the value of liquid water mixing ratio in the cloud at cloud top?

c. Starting with the answer from part (b), assume that all of the liquid water falls out as precipitation from the air parcel at cloud top. Then, if the cloud-top parcel is lowered to 100 kPa, what is its temperature, dew point, and relative humidity? How does that relative humidity compare to the initial relative humidity of the parcel at 100 kPa?

N24. At $P = 100$ kPa, $z = 0$ km, an air parcel has initial values of $T = 30^\circ\text{C}$ and $T_d = 0^\circ\text{C}$. Use each of the large thermo diagrams at the end of this chapter to find the LCL. How do these LCL values compare to the theoretical value from the equation in the Moisture chapter?

Understanding & Critical Evaluation

U1. Why does the humidity of an unsaturated, rising air parcel follow a line of constant mixing ratio in a thermo diagram, rather than a line of constant dew-point temperature?

U2. Under what conditions would a rising, unsaturated air parcel not follow a dry adiabat in a thermo diagram?

U3. In the full-size Emagram, why do the lines of constant height tilt up to the left, rather than up to the right? If warmer air has greater thickness than colder, then wouldn't the height lines be pushed up on the right side of the diagram? Explain.

U4. In the thermodynamic diagram, the labels at the top of the diagram for mixing ratio are not linear from left to right. Namely, as you look from left to right, the mixing ratio increment between neighboring lines gets larger and larger. Why?

U5. On the full-size Emagram, notice that the dry adiabats are slightly curved. Why are they not straight?

U6. a. Why do you need two points to represent one air parcel for unsaturated air?
 b. Why do you need two points to represent one air parcel for saturated air?
 c. If only two points are needed for parts (a) and (b), then why do we track three points for each one air parcel as it rises adiabatically?

U7. If you know the starting temperature and pressure of an unsaturated air parcel, and you know its LCL instead of its mixing ratio, is that sufficient information to represent its full thermodynamic state (regarding its temperature and humidity)?

U8. The method of moving air parcels up and down in a thermo diagram assumes that the background environment is steady. Is that a good assumption? When might that assumption be violated? Also, when an air parcel moves from its starting point, what happens to the hole it leaves behind?

U9. Suppose an air parcel is surrounded by an environment that has the same density. The net buoyant force on the parcel would be zero. Yet gravity is still acting on the parcel. Why doesn't gravity pull the parcel downward?

U10. Metal ships float in water. Is it possible for metal to float in air, even if it is not moving? Discuss.

U11. Eq. (5.2) gives buoyant force in terms of object density minus surrounding fluid density. Yet eq. (5.3a) gives the force in terms of fluid (environment) minus object (the air parcel). Why the sign change? Is it a mistake? Derive (5.3a) from (5.2).

U12. On the θ - Z thermo diagram, why do the moist adiabats tilt to the right with increasing altitude, while the same moist adiabats tilt to the left on the Emagram?

U13. Start with $T = 25^\circ\text{C}$ and $T_d = 20^\circ\text{C}$ for an air parcel at $P = 100$ kPa (assumed to be at $z = 0$). Plot this starting condition on copies of all versions of the boundary-layer thermodynamic diagram.

a. Lift the parcel to the LCL on all diagrams, and compare the LCL heights. At this point, compare the values of T , r , and θ on all diagrams.

b. Continue lifting to a height of 2 km, and compare the values of T , r , r_T , and θ on both diagrams. Discuss.

U14. Fig. 5.14 shows correct and incorrect methods of determining static stability. Why is the wrong method wrong?

U15. The Brunt-Väisälä frequency is undefined for statically unstable air. Why? If θ decreases with increasing z , calculate the Brunt-Väisälä frequency.

U16. Why is the factor 2π in the numerator of eq. (5.5), rather than a factor of 1?

U17. a. Verify that eqs. (5.9) are dimensionless.

b. For zero wind shear, the denominator of eqs. (5.9) are zero, causing Ri to be infinite. Does that mean that there can never be any turbulence when the wind shear is zero? Explain.

U18. The small thermo diagrams in this chapter indicate a surface pressure of 100 kPa for demonstration purposes. However, the standard atmospheric pressure at sea-level is 101.325 kPa, and pressures on any given day can be greater than average. How can you plot a pressure greater than 100 kPa on the thermo diagram?

U19. Explain the last sentence in the Fig. 5.8 caption.

U20(§). (This exercise is lengthy.)

Use a computer spreadsheet to create a:

a. Stüve diagram. [Hint: use $(P_o/P)^{\kappa_d/C_{pd}}$ as vertical coordinate.]

b. Skew-T diagram.

c. Tephigram. [Hint: plot it in tilted form, as in Fig. 5.2, but with moist adiabats and isohumes add-

ed to the other lines in that figure. This is a very difficult exercise.]

d. θ - Z diagram. [Hint, plot all the isopleths except the isobars.]

U21(§). Use a computer spreadsheet to create an Emagram, but utilize pressures in the range of 110 kPa to 10 kPa.

U22. Suppose the parcel in Fig. 5.12 gradually mixes with its environment as it rises. Plot the path of such a parcel on a T vs. z graph similar to that figure.

U23. What does it mean if the numerator of the Richardson number is negative?

U24. Compare the bulk Richardson number with the flux Richardson number from the Atmospheric Boundary Layer chapter. Hint, consider K-theory, as explained in that chapter.

U25. Create a new conceptual algorithm different from the "Guide" in this chapter to help you identify different thermo diagram types. Test it to ensure that it doesn't falsely identify some diagrams.

U26(§). Use a spreadsheet to reproduce Fig. 5.2, using the relationship for potential temperature as a function of T and P as given in the Heat chapter.

U27. All the dry adiabats in a Stüve diagram converge to a point above the top left of the diagram. Explain why or why not the dry adiabats in an Emagram converge to a point.

U28. Suppose that turbulent mixing in a dynamically unstable, but statically stable, environment causes both $\Delta T/\Delta z$ and $\Delta U/\Delta z$ to be reduced by the same fraction, b . Show how the Richardson number increases as b decreases. Use this to explain why turbulence acts to reduce the dynamic instabilities that caused it, analogous to LeChatelier's principle. (Hint: Assume $V = 0$ for simplicity.)

U29. Fig. 5.10 indicates state changes due to fallout of precipitation.

a. Can the opposite happen? Namely, can precipitation fall INTO an air parcel, with all of the water staying in the air? If so, give an example and show how that process would be plotted on a thermo diagram.

b. What happens if rain from above falls through an air parcel with zero or partial evaporation? Namely, the rainfall does not change during its passage through the air parcel. Indicate this on a thermo diagram.

Web-Enhanced Questions

W1. Search the web for a map of the locations of all the rawinsonde (i.e., upper-air) sites in your country. What are the three sites closest to your location, and what are the station identifiers for those sites?

W2. Search the web for a site that allows you to find the name, abbreviation, latitude and longitude, altitude, and other characteristics of rawinsonde sites.

a. Type in the name of a major city, and determine if it has a rawinsonde launch site.

b. Compare the **WMO (World Meteorological Organization)** and **ICAO (International Civil Aviation Organization)** identification codes for several stations.

W3. Search the web for soundings plotted on various types of thermodynamic diagrams (e.g., pseudoadiabatic diagrams, tephigrams, Stüve diagrams, Skew-T Log-P diagrams, etc.). Instead of focusing on the particular sounding, look at the background diagram, and identify each type of line in it (dry adiabats, moist adiabats, isobars, isotherms, isohumes, height contours, etc.). How many different types of thermo diagrams can you find on the web? Indicate the URLs for these sites.

W4. Get from the web the current surface T and T_d for the weather station closest to you (or to a station specified by your instructor).

a. Plot those values on a thermo diagram, and determine the LCL height and pressure.

b. If there are cumuliform clouds at the time of this data observation, also get cloud-base height from the web, and compare with the computed LCL. (Hint: search the web for METAR reports of cloud-base altitudes for the airport closest to you.)

W5. Get from the web the current sounding in text form for the rawinsonde (upper-air) site closest to you (or to an upper-air site specified by your instructor). Plot this by hand on one of the large thermo diagrams copied from the end of this chapter, using different colors for T and T_d . For air lifted from the surface, find the LCL. If this lifted air at the LCL is buoyant, continue lifting it until it becomes neutrally buoyant, and mark this height as cloud-top height.

W6. Get from the web a sequence of soundings (once every 12 h if available, otherwise every 24 h) from the past few days for a rawinsonde (upper-air) site near you (or a site specified by your instructor).

a. Compare these soundings, to see how the weather evolves.

b. If you have appropriate software, put each sounding as a frame in a movie, and play the movie

to see the soundings evolve.

c. Check the weather maps to see if frontal passage will happen during the next several days at your rawinsonde site. If so, download and save soundings from just before, during, and after frontal passage, and compare and discuss the results.

W7. Get from the web a sounding from just before a rain event in your area (or for a region specified by your instructor). From this sounding, conceptually lift an air parcel from the boundary layer to a reasonable height for cloud top. Calculate the liquid-water mixing ratio for water that would have condensed during the rise of an air parcel from the surface. Compare this to the amount of precipitation that fell, and discuss why they are different.

W8. On some soundings plotted on the web, other information is also plotted (such as winds). Also, sometimes tables of information such as LCL and many stability indices are given. The web sites that serve such detailed soundings will often have an information page that you can select to learn the definitions of the other terms in such a table. Access and print those definitions, and use them to interpret the info table that is given with the sounding.

W9. Get the most recent sounding from the rawinsonde site closest to you (or a site specified by your instructor).

a. Determine the domains of different parcel static stability in that sounding, and indicate them on or next to the sounding similar to that shown in Fig. 5.14 (using the correct version only, please).

b. For the statically stable regions of the sounding, determine the Brunt-Väisälä frequency for each of those regions. (For simplicity, you can neglect the moisture contribution to virtual temperature for this exercise, and just use the air temperature in the calculation. However, normally you should not neglect the moisture contribution to virtual temperature when you calculate stabilities and Brunt-Väisälä frequencies, especially in the tropics.)

c. For each layer in the sounding, calculate the Richardson number, using the winds and temperatures at the top and bottom of each layer. The resulting Richardson numbers would apply across the whole layer between the top and bottom heights from which temperature and winds were obtained. But each different layer probably has a different Richardson number.

d. Using both (b) and (c), determine the ranges of heights that are likely to be turbulent.

W10. Get the most recent early morning sounding from the rawinsonde site closest to you (or a site

specified by your instructor). How warm must the surface temperature be later in the day in order to allow cumuliform cloud tops to reach an altitude where the pressure is 50 kPa? Do you think that such warming is possible on this day?

W11. Get the most recent sounding from the rawinsonde site closest to you (or a site specified by your instructor), and determine the depth of the boundary layer and the height of the tropopause.

W12. Search for images of Kelvin-Helmholtz waves and billow clouds. Print two examples of each, along with their URLs.

Synthesis Questions

S1. The denominator of eq. (5.9c) looks something like a **kinetic energy** per unit mass (ignoring the missing factor of 1/2). Could the numerator be interpreted as a potential energy per unit mass? Explain.

S2. Why did **Archimedes** run naked through the streets of Syracuse when he discovered the principle of displacement and buoyancy? [Note: Archimedes also devised city defenses using lenses to blind the enemy and cranes to capsize their ships, but the Romans conquered Syracuse anyway and killed him.]

S3. Sketch lines in a thermo diagram for a world where condensation of water causes cooling of the air instead of warming. How would the weather and climate be different, if at all? Why?

S4. What if buoyancy depended on the temperature of an air parcel instead of the temperature difference between the parcel and its environment. How would weather and climate be different, if at all? Why?

S5. The Brunt-Väisälä frequency implies oscillations, and oscillations imply waves in the air. Could there really be waves in the air? If so, where do they “come ashore” and “break”?

S6. Why isn't vertical velocity included in the definition of the Richardson number?

S7. What if **global warming** caused the atmosphere to warm, but the Earth's surface stayed about the same temperature. How would static stability and vertical cloud growth be affected, and how would the resulting weather and climate be different, if at all?

S8. In a **nuclear winter** scenario (after nuclear bombs have destroyed the Earth, and had sprayed bits of Earth into the atmosphere), the stratosphere would be filled with fine dust and dark soot particles. Given the short and long wave radiation characteristics that you have studied in earlier chapters, how might the static stability of the troposphere change, and what effect would it have on weather and climate? (Hint: Assume that you are a cockroach, because all other life forms would have been destroyed, so you are the only species left to do this important calculation – as if you would care.)

S9. An isentropic chart shows the altitude (either z or P) of a constant θ surface. Describe how you would use upper-air soundings from different weather stations to get the data needed to draw an isentropic chart.

S10. Devise a new type of thermo diagram that has $\log(r)$ along the abscissa, and $\log(P)$ along the ordinate with scale reversed (max and bottom and min at top). Plot the isotherms and dry adiabats in this diagram. Name the diagram after yourself, and explain the virtues and utility of your diagram.

S11. What if T_d could be greater than T in a sounding. Explain how that might be possible, and describe how convective clouds might be different, if at all.

S12. Suppose the tropopause was touching the ground everywhere on Earth. How would the weather or climate be different, if at all?

LARGE-SIZE THERMO DIAGRAMS

You have our permission to freely reproduce the following seven thermo diagrams for your own personal use, or for education (but not for commercial resale), so long as you retain the author, title, and publisher citations in the copies. We recommend that you do NOT write on the following seven pages, so that they remain as clean master copies.

These diagrams share common line formats: thin lines representing state of the air; thick lines representing adiabatic processes that change the state.

- Isotherms (temperature: T) and isobars (pressure: P) are thin solid,
- Isohumes (mixing ratio: r) are thin dotted,
- Dry adiabats (potential temperature: θ) are thick solid,
- Moist adiabats (wet-bulb potential temperature: θ_w) are thick dashed, and
- Contours (height: z) are thin dashed (if plotted at all).

The height contours are only approximate. Potential temperatures use a reference pressure of $P = 100$ kPa, and heights are set to zero at that pressure.

The line labels also follow a common format (except for the θ - Z diagram):

- Isobars are labeled at left. Units are kPa.
- Isohumes are labeled at top (and right in one case). Units are g/kg.
- Heights are labeled at right. Units are km.
- Isotherms and both dry and moist adiabats are labeled at the bottom. Units are °C.

For this last item, the three isopleths sprout from each temperature label like branches from a bush. The branches are always in the same order, although their angles differ from graph to graph. Dry adiabats are the left branch, moist adiabats are the center branch, and isotherms are the right branch.

Two versions of the Skew- T and θ - Z Diagram are included. The standard version spans the range $P = 105$ to 20 kPa. The other version [with “(ABL)” added to the title] is designed for **atmospheric boundary-layer** and air-pollution work, and gives more detail in the bottom third of the troposphere.

On the next pages (152-158) are these thermo diagrams:

- Emagram
- Stüve Diagram
- Skew- T Log- P Diagram
- Tephigram
- Theta- Z Diagram
- Skew- T Log- P Diagram (ABL)
- Theta- Z Diagram (ABL).