EXTRATROPICAL CYCLONES

CONTENTS

13 **Cyclones** are synoptic-scale regions of **low pressure** near the surface (Fig. 13.1), with horizontal winds turning **cyclonically** (counterclockwise in N. Hemisphere) and **upward vertical velocity** in the mid troposphere. The updrafts support clouds, precipitation, and sometimes thunderstorms.

Tropical cyclones are covered separately, in the chapter on hurricanes. **Extratropical cyclones** (cyclones outside of the tropics) are covered here, and include transient **mid-latitude cyclones** and **polar cyclones**. Other names for extratropical cyclones are **lows** or **low-pressure centers**. Table 13-1 compares names for cyclones.

Lows can cause floods, blizzards, strong winds and hazardous travel conditions. They are the large regions of bad weather that come from the west every two to seven days in mid-latitudes.

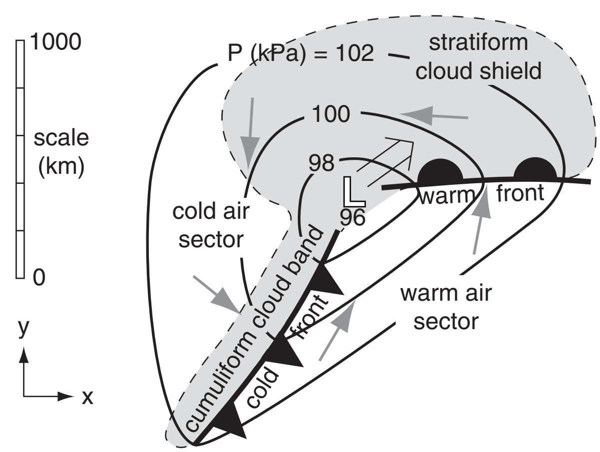


Figure 13.1
Components of a typical extratropical cyclone in the N. Hemisphere. Light grey shows clouds, dark grey arrows are near-surface winds, thin black lines are isobars (kPa), thick black lines are fronts, and the double-shaft arrow shows movement of the low center L.

Table 13-1. Cyclone names. "Core" is storm center. *T* is relative temperature.

Common Name in N. Amer.	Formal Name	Other Common Names	T of the Core	Map Symbol
low	extra-tropical cyclone	mid-latitude cyclone	cold	L
		low-pressure center		
		storm system*		
		cyclone *(in N. America)*		
hurricane	tropical cyclone	typhoon *(in W. Pacific)*	warm	
		cyclone *(in Australia)*		

(Often used by TV meteorologists.)*

FOCUS • Southern Hemisphere Lows

Some aspects of mid-latitude cyclones in the Southern Hemisphere are similar to those of N. Hemisphere cyclones. They have low pressure at the surface, rotate cyclonically, form east of upper-level troughs, propagate from west to east and poleward, and have similar stages of their evolution. They often have fronts and bad weather.

Different are the following: warm tropical air is to the north and cold polar air to the south, and the cyclonic rotation is clockwise due to the opposite Coriolis force. The figure below shows an idealized extra-tropical cyclone in the S. Hemisphere.

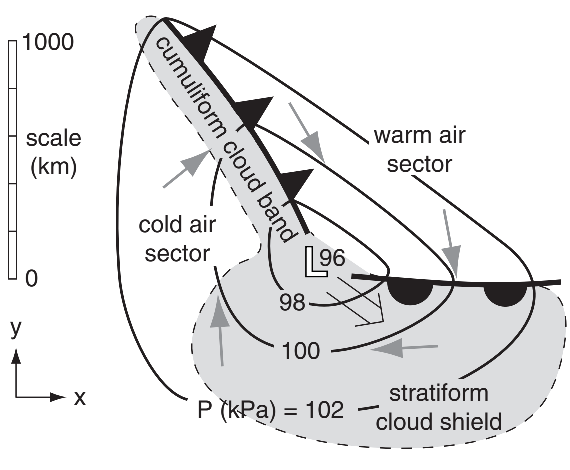


Figure a.
Sketch of mid-latitude cyclone in the Southern Hemisphere.

CYCLONE CHARACTERISTICS

Cyclogenesis & Cyclolysis

Cyclogenesis is the birth and growth of cyclones. Such intensification can be defined by the:

- sea-level pressure decrease,
- upward-motion increase, and
- vorticity increase.

These characteristics are not independent; for example, upward motion can reduce surface pressure, which draws in lower-tropospheric air that begins to rotate due to Coriolis force. However, you can gain insight into the workings of the storm by examining the dynamics and thermodynamics that govern each of these characteristics. Each of these will be explored in detail later in this chapter.

The following conditions favor <u>rapid</u> cyclogenesis:

- strong baroclinicity – a strong horizontal temperature gradient, such as a frontal zone
- weak static stability – temperature decreasing with height faster than the tropospheric standard atmosphere
- mid- or high-latitude location – Earth's contribution to vorticity increases toward the poles
- large moisture input – latent heating due to cloud condensation adds energy and reduces static stability
- large-amplitude wave in the jet stream – a trough to the west and ridge to the east of the surface low enhance horizontal divergence aloft, which strengthens updrafts
- terrain elevation decrease toward east – cyclogenesis to the lee of mountains

A cyclone that develops extremely fast is called a **cyclone bomb**, and the process is called **explosive cyclogenesis**. To be classified as a bomb, the central pressure of a cyclone must decrease at a rate of 0.1 kPa per hour for at least 24 hours. Explosive cyclogenesis often occurs when cold air moves over the northern edge of a very warm ocean current, such as over the Gulf Stream off the Northeast USA. In these regions, strong evaporation and heat transfer from the sea surface enhances many of the factors listed above for rapid cyclogenesis. Cyclone bombs can cause sudden major winter storms, with high seas, snow and freezing rain that are hazards to shipping and road travel.

The opposite of cyclogenesis is **cyclolysis**. This is literally death of a cyclone.

Cyclone Evolution

Most cyclones go through a life cycle of formation, growth, weakening, and death over a period of about a week at mid-latitudes. However, lifetimes less than a day and greater than two weeks have been observed. While cyclones exist, they are moved by the jet stream and by other large-scale components of the global circulation.

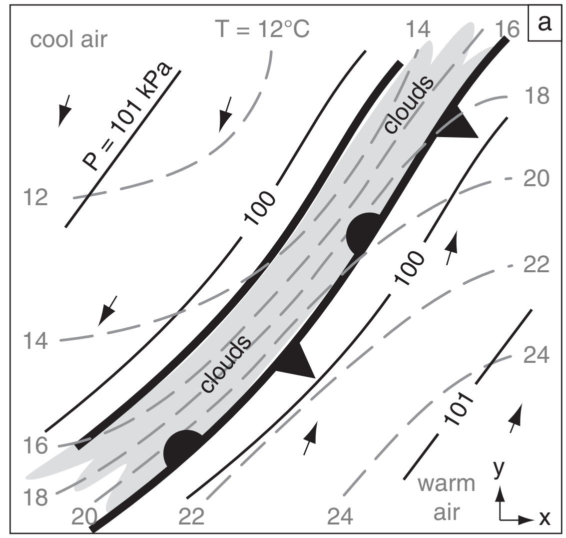
One condition that favors cyclogenesis is a **baroclinic** zone; namely, a region of large temperature change across a short horizontal distance near the surface. Frontal zones such as stationary fronts (Fig. 13.2a) are regions of strong baroclinicity.

Above (near the tropopause) and parallel to this baroclinic zone is often a strong jet stream (Fig. 13.2b), driven by the thermal wind effect (see the chapters on Global Circulation, and Airmasses & Fronts). If conditions are right (as discussed later in this chapter), the jet stream can remove air molecules from a column of air above the front, at location "D" in Fig. 13.2b. This lowers the surface pressure under location "D", causing **cyclogenesis** at the surface. Namely, location "D" is where you would expect a surface low-pressure center to form.

The resulting pressure gradient around the surface low starts to generate lower-tropospheric winds that circulate around the low (Fig. 13.3a, again near the Earth's surface). This is the **spin-up** stage — so named because vorticity is increasing as the cyclone intensifies. The winds begin to advect the warm air poleward on the east side of the low and cold air equatorward on the west side, causing a kink in the formerly stationary front near the low center. The kinked front is wave shaped, and is called a **frontal wave**. Parts of the old front advance as a warm front, and other parts advance as a cold front. Also, these winds begin to force some of the warmer air up over the colder air, thereby generating more clouds.

If jet-stream conditions continue to be favorable, then the low continues to intensify and mature (Fig. 13.3b). As this cyclogenesis continues, the central pressure drops (namely, the cyclone **deepens**), and winds and clouds increase as a **vortex** around the **low center**. Precipitation begins if sufficient moisture is present in the regions where air is rising.

The advancing <u>cold front often moves faster than the warm front</u> for three reasons. (1) The Sawyer-Eliassen circulation tends to push near-surface cold air toward warmer air at both fronts. (2) Circulation around the vortex tends to deform the frontal boundaries and shrink the warm-air region to a smaller wedge shape east and equatorward of the low center. This wedge of warm air is called the **warm-air sector** (Fig. 13.1). (3) Evaporating precipitation cools both fronts (enhancing the cold front



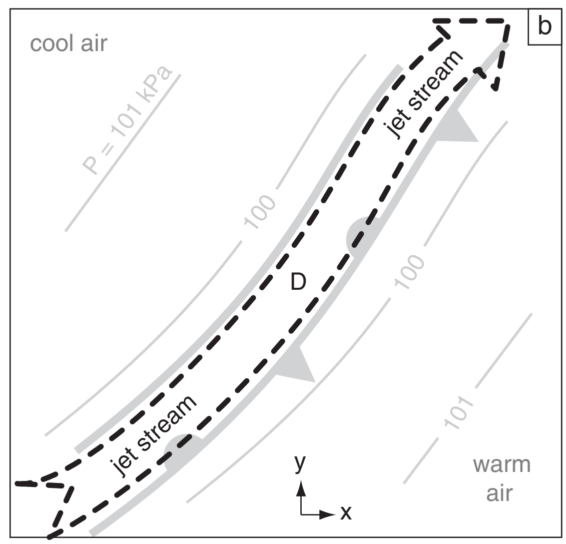


Figure 13.2
Initial conditions favoring cyclogenesis in N. Hemisphere. (a) Surface weather map. Solid thin lines are isobars. Dashed lines are isotherms. The thick black lines mark the leading and trailing edges of the frontal zone. Grey shading indicates clouds. Fig. 13.3 shows subsequent evolution. (b) Upper-air map over the same frontal zone, where the dashed black arrow indicates the jet stream near the tropopause. The grey lines are a copy of the surface isobars and frontal zone from (a), to help you picture the 3-D nature of this system.

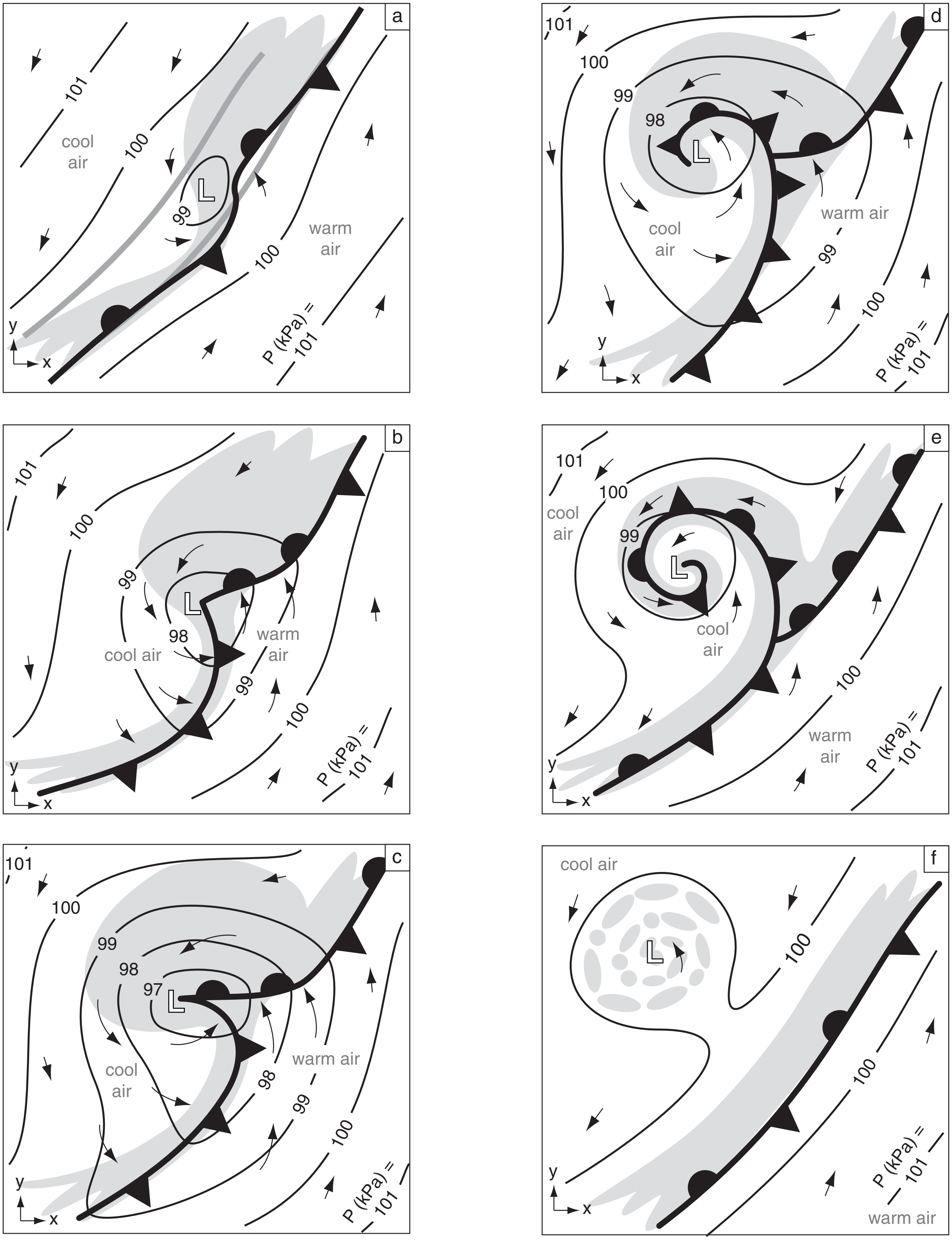


Figure 13.3
Extratropical cyclone evolution in the N. Hemisphere, including cyclogenesis (a - c), and cyclolysis (d - f). These idealized surface weather maps move with the low center. Grey shading indicates clouds, solid black lines are isobars (kPa), thin arrows are near-surface winds, 𝕃 is at the low center, and medium grey lines in (a) bound the original frontal zone. Fig. 13.2a shows the initial conditions.

but diminishing the warm front). These combined effects amplify the frontal wave.

At the peak of cyclone intensity (lowest central pressure and strongest surrounding winds) the cold front often catches up to the warm front near the low center (Fig. 13.3c). As more of the cold front overtakes the warm front, an occluded front forms near the low center (Fig. 13.3d). The cool air is often drier, and is visible in satellite images as a **dry tongue** of relatively cloud-free air that begins to wrap around the low. This marks the beginning of the **cyclolysis** stage. During this stage, the low is said to **occlude** as the occluded front wraps around the low center.

As the cyclone occludes further, the low center becomes surrounded by cool air (Fig. 13.3e). Clouds during this stage spiral around the center of the low — a signature that is easily seen in satellite images. But the jet stream, still driven by the thermal wind effect, moves east of the low center to remain over the strongest baroclinic zone (over the warm and cold fronts, which are becoming more stationary).

Without support from the jet stream to continue removing air molecules from the low center, the low begins to **fill** with air due to convergence of air in the boundary layer. The central pressure starts to rise and the winds slow as the vorticity **spins down**.

As cyclolysis continues, the low center often continues to slowly move further poleward away from the baroclinic zone (Fig. 13.3f). The central pressure continues to rise and winds weaken. The tightly wound spiral of clouds begins to dissipate into scattered clouds, and precipitation diminishes.

But meanwhile, along the stationary front to the east, a new cyclone might form if the jet stream is favorable (not shown in the figures).

In this way, cyclones are born, evolve, and die. While they exist, they are driven by the baroclinicity in the air (through the action of the jet stream). But their circulation helps to reduce the baroclinicity by moving cold air equatorward, warm air poleward, and mixing the two airmasses together. As described by **Le Chatelier's Principle**, the cyclone forms as a response to the baroclinic instability, and its existence partially undoes this instability. Namely, cyclones help the global circulation to redistribute heat between equator and poles.

Figures 13.3 are in a moving frame of reference following the low center. In those figures, it is not obvious that the warm front is advancing. To get a better idea about how the low moves while it evolves over a 3 to 5 day period, Figs. 13.4 show a superposition of all the cyclone locations relative to a fixed frame of reference. In these idealized figures, you can more easily see the progression of the low center, the advancement of the warm fronts and the advancement of cold air behind the cold fronts.

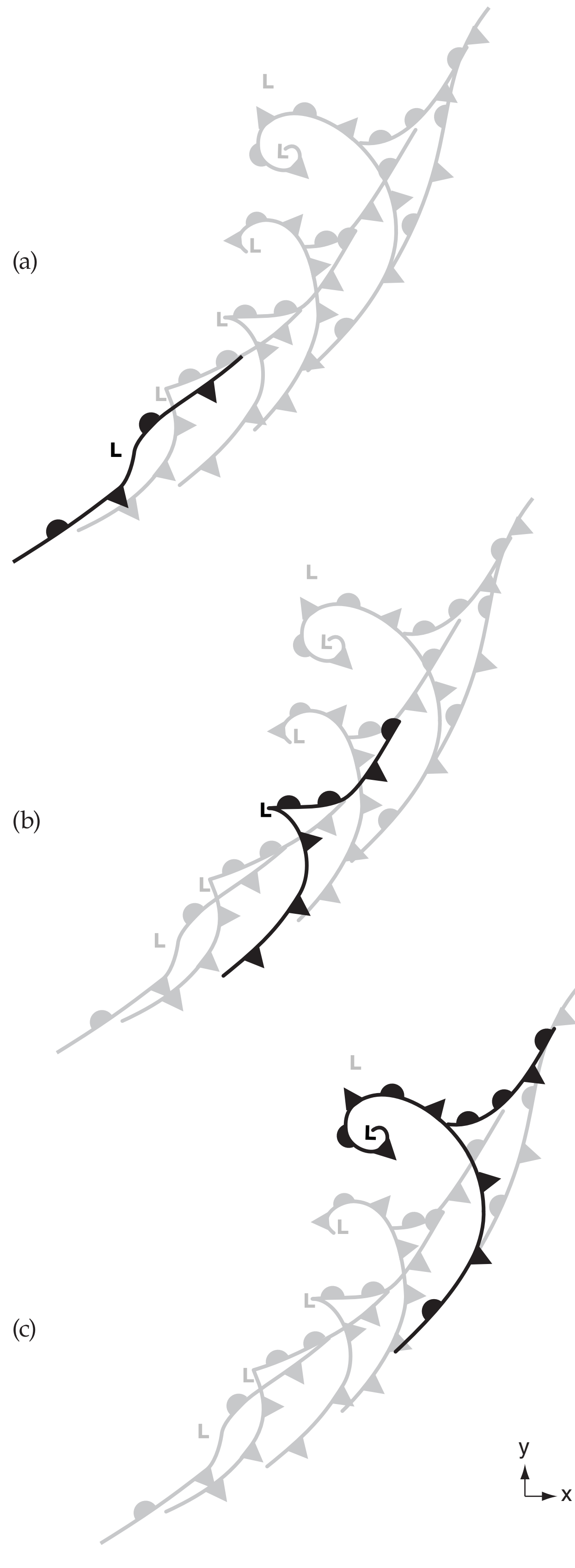
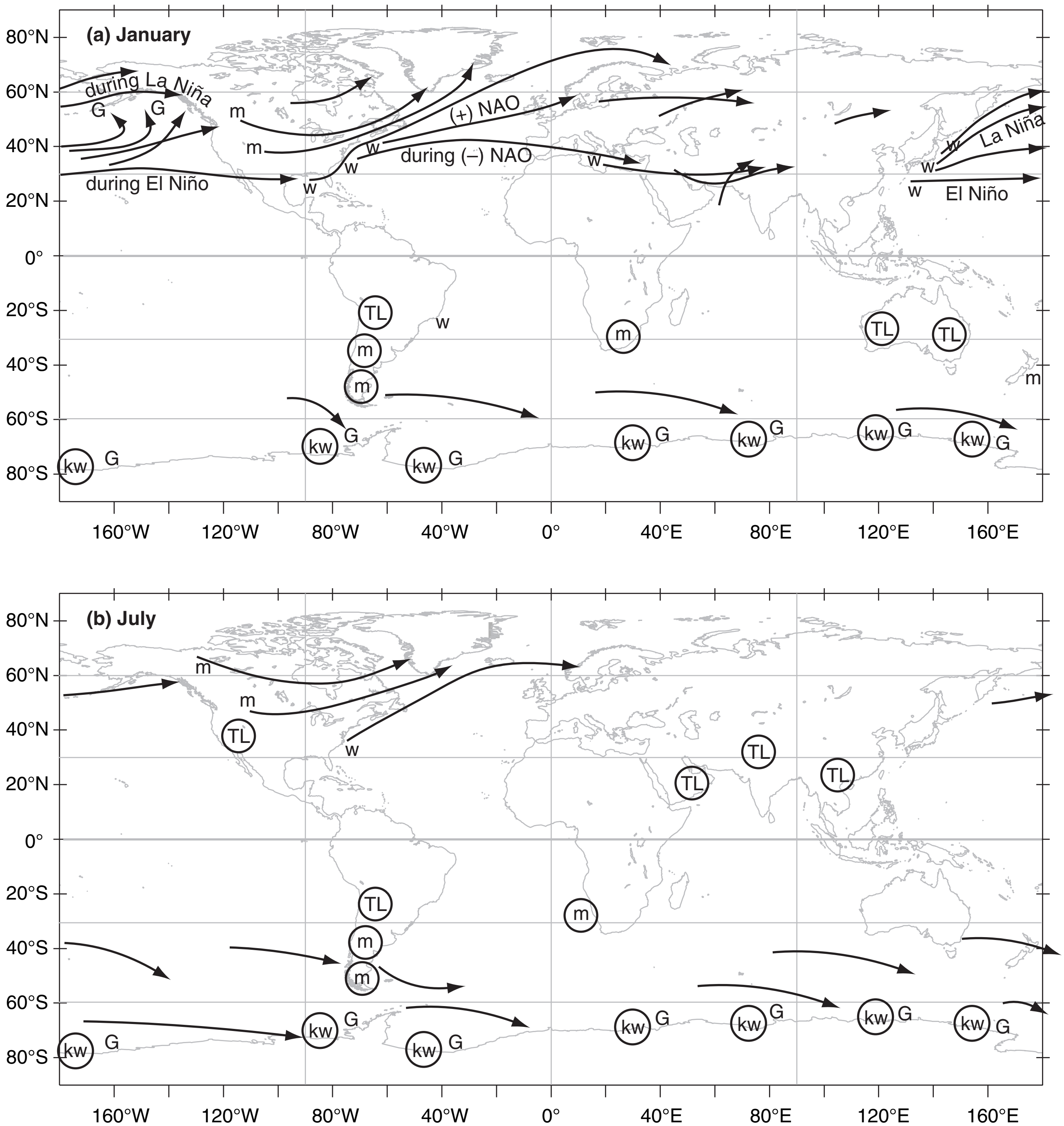


Figure 13.4
Illustration of movement of a low while it evolves. Fronts and low centers are from Figs. 13.3. Every second cyclone location is highlighted in a through c.

Cyclone Tracks

Extratropical cyclones are steered by the global circulation, including the prevailing westerlies at mid-latitudes and the meandering Rossby-wave pattern in the jet stream. Typical **storm tracks** (cyclone paths) of low centers are shown in Fig. 13.5. Multi-year climate variations (see the Climate chapter) in the global circulation, such as associated with the El Niño / La Niña cycle or the North Atlantic Oscillation (NAO), can alter the cyclone tracks. Mid-latitude cyclones are generally stronger, translate faster,

Figure 13.5 (below)
Climatology of extratropical cyclone tracks (lines with arrows) for (a) January and (b) July. Other symbols represent genesis and decay regions, as explained in the text. Circled symbols indicate stationary cyclones.



and are further equatorward during winter than in summer.

One favored cyclogenesis region is just east of large mountain ranges (shown by the "m" symbol in Fig. 13.5; see **Lee Cyclogenesis** later in this chapter). Other cyclogenesis regions are over warm ocean **boundary currents** along the western edge of oceans (shown by the symbol "w" in the figure), such as the **Gulf Stream** current off the east coast of N. America, and the **Kuroshio Current** off the east coast of Japan. During winter over such currents are strong sensible and latent heat fluxes from the warm ocean into the air, which adds energy to developing cyclones. Also, the strong wintertime contrast between the cold continent and the warm ocean current causes an intense baroclinic zone that drives a strong jet stream above it due to thermal-wind effects.

Cyclones are often strengthened in regions under the jet stream just east of troughs. In such regions, the jet stream steers the low center toward the east and poleward. Hence, cyclone tracks are often toward the northeast in the N. Hemisphere, and toward the southeast in the S. Hemisphere.

Cyclones in the Northern Hemisphere typically evolve during a 2 to 7 day period, with most lasting 3 - 5 days. They travel at typical speeds of 12 to 15 m/s (43 to 54 km/h), which means they can move about 5000 km during their life. Namely, they can travel the distance of the continental USA from coast to coast or border to border during their lifetime. Since the Pacific is a larger ocean, cyclones that form off of Japan often die in the Gulf of Alaska just west of British Columbia (BC), Canada — a cyclolysis region known as a **cyclone graveyard** (G).

Quasi-stationary lows are indicated with circles in Fig. 13.5. Some of these form over hot continents in summer as a monsoon circulation. These are called **thermal lows** (TL), as was explained in the Global Circulation chapter in the section on Hydrostatic Thermal Circulations. Others form as quasi-stationary lee troughs just east of mountain (m) ranges.

In the Southern Hemisphere (Fig. 13.5), cyclones are more uniformly distributed in longitude and throughout the year, compared to the N. Hemisphere. One reason is the smaller area of continents in Southern-Hemisphere mid-latitudes and subpolar regions. Many propagating cyclones form just north of 50°S latitude, and die just south. The region with greatest cyclone activity (cyclogenesis, tracks, cyclolysis) is a band centered near 60°S.

These Southern Hemisphere cyclones last an average of 3 to 5 days, and translate with average speeds faster than 10 m/s (= 36 km/h) toward the east-south-east. A band with average translation speeds faster than 15 m/s (= 54 km/h; or > 10° lon-

FOCUS • North American Geography

To help you interpret the weather maps, the map and tables give state and province names.

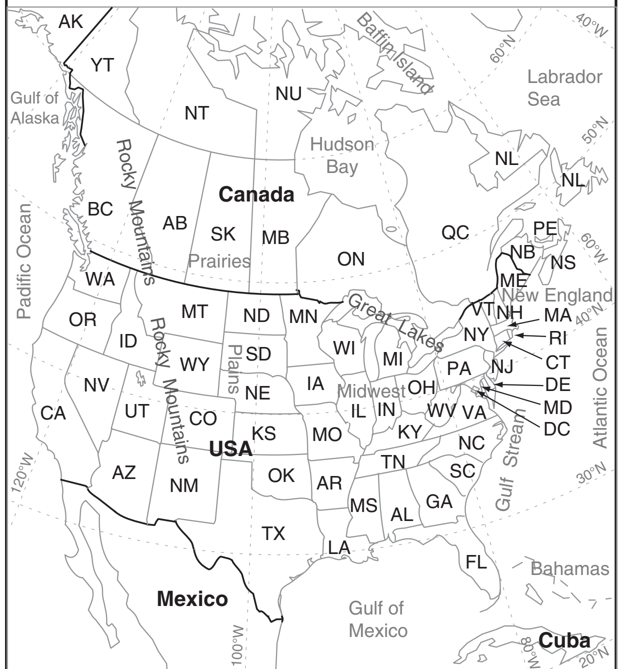


Figure b.

Canadian Postal Abbreviations for Provinces:

AB	Alberta	NT	Northwest Territories
BC	British Columbia	NU	Nunavut
MB	Manitoba	ON	Ontario
NB	New Brunswick	PE	Prince Edward Isl.
NL	Newfoundland & Labrador	QC	Quebec
NS	Nova Scotia	SK	Saskatchewan
		YT	Yukon

USA Postal Abbreviations for States:

AK	Alaska	MD	Maryland	OK	Oklahoma
AL	Alabama	ME	Maine	OR	Oregon
AR	Arkansas	MI	Michigan	PA	Pennsylvania
AZ	Arizona	MN	Minnesota	RI	Rhode Isl.
CA	California	MO	Missouri	SC	South Carolina
CO	Colorado	MS	Mississippi	SD	South Dakota
CT	Connecticut	MT	Montana	TN	Tennessee
DE	Delaware	NC	North Carolina	TX	Texas
FL	Florida	ND	North Dakota	UT	Utah
GA	Georgia	NE	Nebraska	VA	Virginia
HI	Hawaii	NH	New Hampshire	VT	Vermont
IA	Iowa	NJ	New Jersey	WA	Washington
ID	Idaho	NM	New Mexico	WI	Wisconsin
IL	Illinois	NV	Nevada	WV	West Virginia
IN	Indiana	NY	New York	WY	Wyoming
KS	Kansas	OH	Ohio	DC	Wash. DC
KY	Kentucky				
LA	Louisiana				
MA	Massachusetts				

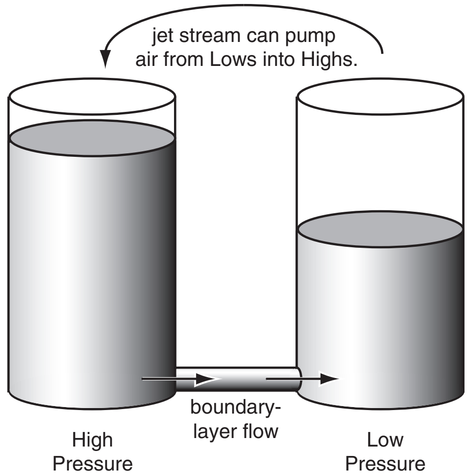


Figure 13.6
Two tanks filled with water to different heights are an analogy to neighboring high and low pressure systems in the atmosphere.

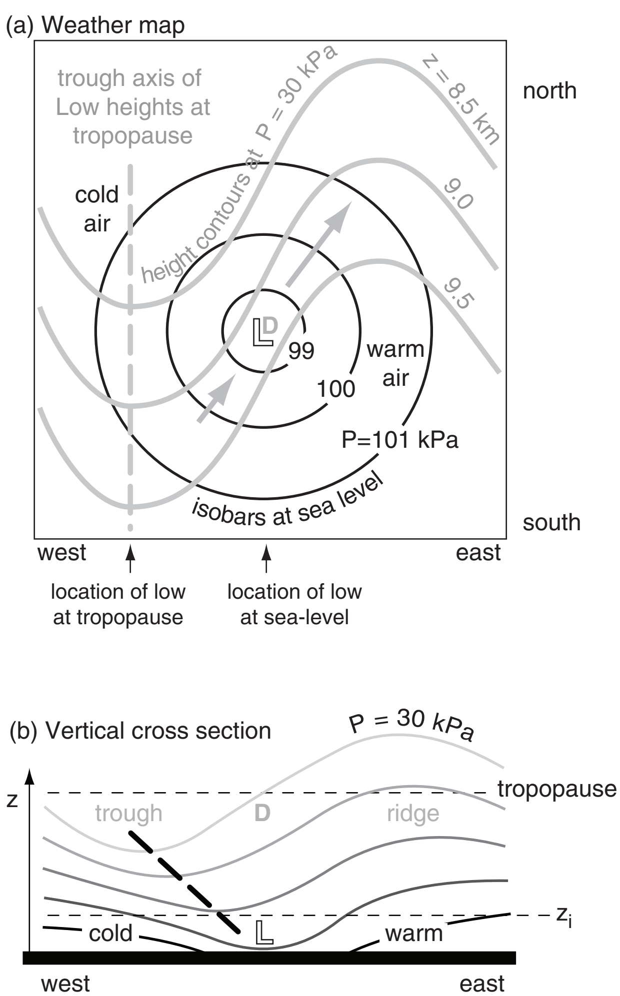


Figure 13.7
(a) Two N. Hemisphere weather maps superimposed: (thin black lines) sea-level pressure, and (grey lines) 30 kPa heights. Jet-stream winds (thick grey arrows) follow the height contours (b) East-west vertical cross section through middle of (a). Heavy dashed line is trough axis. L indicates low center at surface, and D indicates divergence aloft. (Pressure and height variations are exaggerated.)

gitude/day) extends from south of southwestern Africa eastward to south of western Australia. The average track length is 2100 km. The normal **cyclone graveyard** (G, cyclolysis region) in the S. Hemisphere is in the **circumpolar trough** (between 65°S and the Antarctic coastline).

Seven stationary centers of enhanced cyclone activity occur around the coast of Antarctica, during both winter and summer. Some of these are believed to be a result of fast **katabatic** (cold downslope) winds flowing off the steep Antarctic terrain (see the Airmasses & Fronts chapter). When these very cold winds reach the relatively warm unfrozen ocean, strong heat fluxes from the ocean into the air contribute energy into developing cyclones. Also the downslope winds can be channeled by the terrain to cause cyclonic rotation. But some of the seven stationary centers might not be real — some might be caused by improper reduction of surface pressure to sea-level pressure. These seven centers are labeled with "kw", indicating a combination of **k**atabatic winds and relatively **w**arm sea surface.

Stacking & Tilting

Lows at the bottom of the troposphere always tend to kill themselves. The culprit is the boundary layer, where turbulent drag causes air to cross isobars at a small angle from high toward low pressure. By definition, a low has lower central pressure than the surroundings, because fewer air molecules are in the column above the low. Thus, boundary-layer flow will always move air molecules toward surface lows (Fig. 13.6). As a low fills with air, its pressure rises and it stops being a low. Such **filling** is quick enough to eliminate a low in less than a day, unless a compensating process can remove air more quickly.

Such a compensating process often occurs if the axis of low pressure **tilts** westward with increasing height (Fig. 13.7). Recall from the **gradient-wind** discussion in the Dynamics chapter that the jet stream is slower around troughs than ridges. This change of wind speed causes divergence aloft; namely, air is leaving faster than it is arriving. Thus, with the upper-level trough shifted west of the surface low (L), the divergence region (D) is directly above the surface low, supporting cyclogenesis. Details are explained later in this Chapter. But for now, you should recognize that a westward tilt of the low-pressure location with increasing height often accompanies cyclogenesis.

Conversely, when the trough aloft is **stacked** vertically above the surface low, then the jet stream is not pumping air out of the low, and the low fills due to the unrelenting boundary-layer flow. Thus, vertical stacking is associated with cyclolysis.

Other Characteristics

Low centers often move parallel to the direction of the isobars in the warm sector (Fig. 13.1). So even without data on upper-air steering-level winds, you can use a surface weather map to anticipate cyclone movement.

Movement of air around a cyclone is three-dimensional, and is difficult to show on two-dimensional weather maps. Fig. 13.8 shows the main streams of air in one type of cyclone, corresponding to the snapshot of Fig. 13.3b. Sometimes air in the **warm-air conveyor belt** is moving so fast that it is called a low-altitude **pre-frontal jet**. When this humid stream of air is forced to rise over the cooler air at the warm front (or over a mountain) it can dump heavy precipitation and cause flooding.

Behind the cold front, cold air often descends from the mid- or upper-troposphere, and sometimes comes all the way from the lower stratosphere. This dry air **deforms** (changes shape) into a **diffluent** (horizontally spreading) flow near the cold front.

To show the widespread impact of a winter mid-latitude cyclone, a case-study is introduced next.

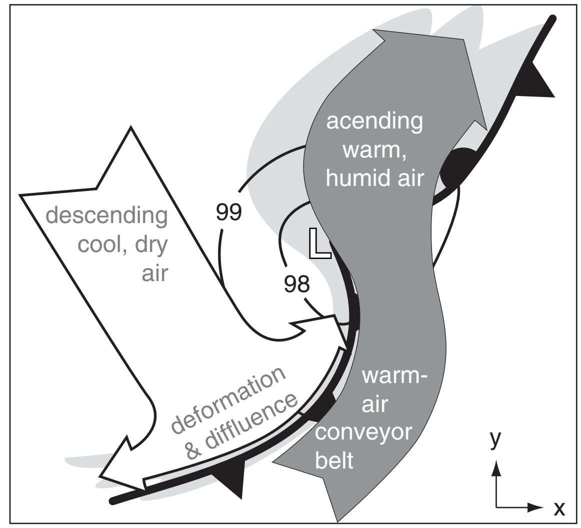


Figure 13.8
Ascending and descending air in a cyclone. Thin black lines with numbers are isobars (kPa). Thick black lines are fronts.

CASE STUDY OF AN EXTRATROPICAL CYCLONE OVER NORTH AMERICA

Overview and Storm Track

On 22 February 1994, an upper-level trough is over the east side of the Rocky Mountains (Fig. 13.9). At the surface, a long stationary front spans from Montana through Texas, and extends northeastward towards Pennsylvania as a dying cold front (Fig. 13.10a). [See the focus box two pages before, for locations of the states.]

Divergence in the upper-tropospheric jet stream between the trough and a ridge east of Texas causes a broad region of upward motion over the southern plains. This **upper-level disturbance** forms a cloud shield and precipitation. As upward motion continues, a surface low (i.e., a cyclone) develops on the stationary front in west Texas in early morning, and moves into east Texas by afternoon (Fig. 13.10a).

As the cyclone tracks northeast toward Illinois (Fig. 13.10b) during 22 to 23 Feb, it intensifies and forms well-developed cold and warm fronts. A squall line of intense thunderstorms with hail and weak tornadoes is triggered in the Southern Plains

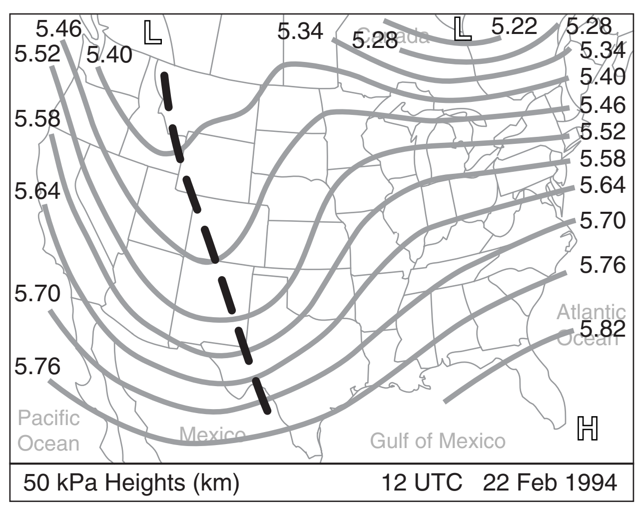


Figure 13.9
Geopotential heights (grey lines) in the mid-troposphere prior to cyclone development. Black dashed line shows the trough axis of low heights (corresponding to low pressure on a constant altitude surface).

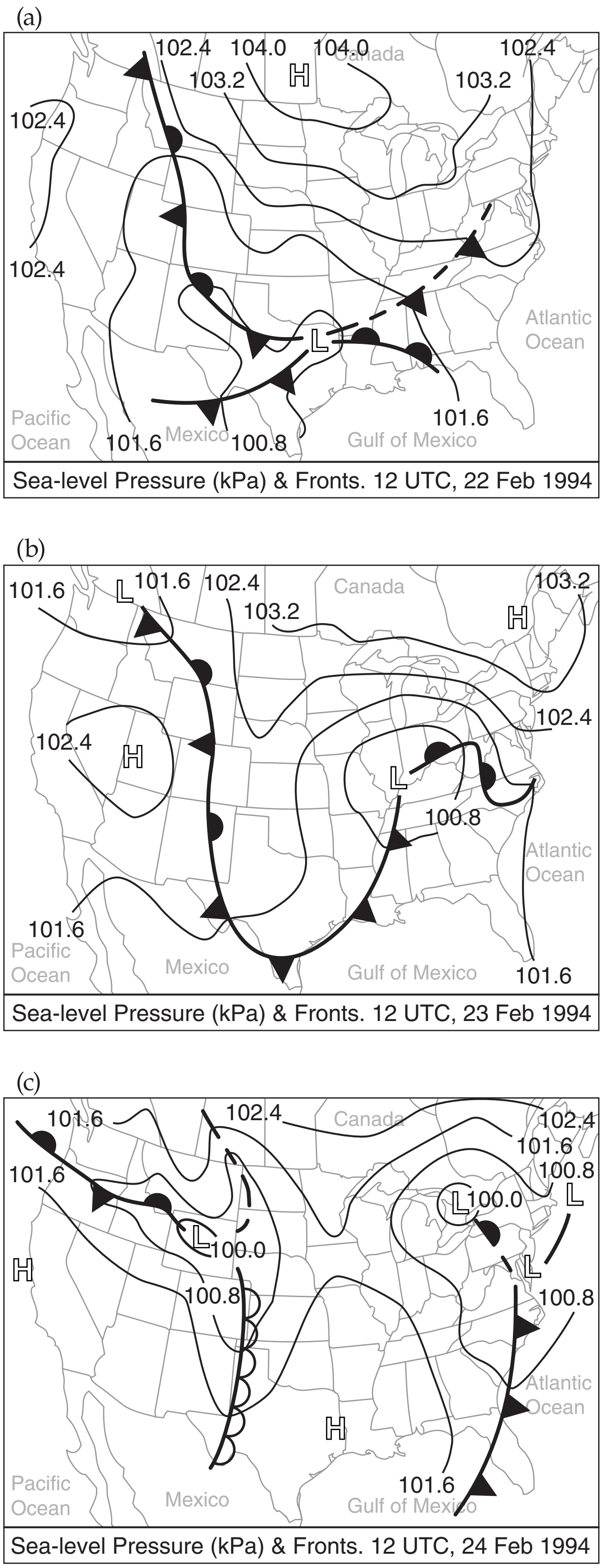


Figure 13.10
Surface weather maps, showing evolution of a case-study mid-latitude cyclone. 𝕃 = low-pressure center, ℍ = high-pressure center. Thin black lines are isobars. Thick lines are fronts.

along and ahead of the cold front. North of the low, heavy snow falls in the upper plains and midwest.

The main low quickly tracks further northeast (Fig. 13.10c), dropping heavy snow across the Ohio valley. The cyclone occludes and begins to weaken while producing heavy snow and freezing rain over the northeast USA.

Meanwhile, a new secondary cyclone forms off the Carolina coast (Fig. 13.10c, and location "g" in Fig. 13.11), caused in part by support from the upper-level flow. This storm rapidly intensifies over the warm Gulf-stream. During 24 Feb 1994 the old cyclone merges/shifts into the new low (Fig. 13.11 locations f –> h). The combined cyclone deepens rapidly while continuing to move into Canada. It causes blizzard conditions from Washington DC to Prince Edward Island, Canada, in the next three days. A third low forms (not drawn) in Alberta, Canada, east of the Rockies. It quickly sweeps across the northern plains bringing strong winds, which whip up the previously fallen snow creating whiteout conditions and deep snow drifts. Such fast-moving lows are called **Alberta Clippers**.

In summary, the original cyclone is born, completes its full life cycle, and dies within just a few days while causing misery across a large portion of N. America east of the Rockies (Fig. 13.11). Many more weather maps showing characteristics of this cyclone are given in this chapter.

Storm Data

• **20 Feb 94 –** Heavy rains (15 cm) cause flooding in south Texas.

• **21 Feb –** Thunderstorms and heavy rain continue in north Texas and southern Oklahoma, with 2 cm diameter hail, an F0 tornado (see Fujita scale in the Thunderstorm chapters), and straight-line winds gusting to 25 m/s. Heavy snow begins in the Colorado Rockies.

• **22 Feb –** Thunderstorms continue in Texas and Oklahoma, with more hail (most 2 cm diameter, but some 5 cm). Lightning starts some fires, and heavy rains cause flash floods that wash out highways and close bridges.

Snow ends in the mountains of Colorado, leaving up to 38 cm of new snow on the ground. Elsewhere, snow and heavy snow fall all day, leaving 20 cm accumulation in Kansas, and 25 cm in Nebraska, causing schools to close early. Heavy snow starts in South Dakota, Iowa, N. Missouri, and Illinois.

Snow mixed with freezing rain begins in southern Missouri, leaving an ice glaze 3.5 cm thick. Snow starts in Ohio, dropping 15 cm, followed by a little freezing rain and gusts to 12 m/s.

• **23 Feb** – Thunderstorms continue in Texas with gusts up to 22 m/s, rolling a mobile home. An F1 tornado damages barns, mobile homes, and an aircraft hangar. Hail 4.5 cm in diameter falls in Texas.

Winds of 38 m/s are reported in ski areas near and east of the continental divide in the Colorado Rockies.

Heavy snow continues in Nebraska (35 cm), South Dakota (30 cm), Kansas (26 cm), Missouri (25 cm), and Illinois (23 cm at O'Hare airport in Chicago, with strong winds and drifting snow). Heavy snow starts in North Dakota.

Thundersnow in Iowa accumulates at a rate of 9 cm/h near Des Moines and Waterloo. A band of convective snow showers leaves up to 28 cm in a 3 h period as it sweeps across N.E. Iowa. Winds of 17 m/s reduce visibilities to 0.1 km, cause drifts up to 1.5 m, resulting in power outages and many closed roads. A dry airmass advects into extreme S.E. Iowa.

Thunderstorm straight-line winds and an F0 tornado damage numerous buildings in Alabama and Ohio. Heavy rains cause flash floods in Tennessee, which wash out bridges and cars, and require evacuation of 5 families.

Heavy wet snow spreads through more of the midwest, starting in SE. Minnesota (23 cm) and S. Wisconsin (40 cm, with 18 m/s winds, whiteout conditions in blowing snow, numerous accidents, schools closed). In Michigan, 33 cm of new snow with winds gusting to 22 m/s cause near blizzard conditions (schools closed in 12 counties).

Heavy snow and freezing rain spreads into New England, including Maine, New Hampshire, Vermont, Massachusetts, New York, and New Jersey.

• **24 Feb** – Mountain downslope winds increase in Colorado, with gusts up to 48 m/s at the National Center for Atmospheric Research in Boulder. A roof blows off a community college in Lakewood, CO, causing evacuation of 1500 people.

Blizzard conditions with heavy snow occurs in North Dakota (23 cm new snow, with gusts to 20 m/s). Near blizzard conditions are in South Dakota as snowfall ends there.

Michigan reports 34 cm total snow, with drifts up to 1 m. A 25-car pileup involves a fully-loaded gasoline tanker on interstate 96, resulting in one death. Heavy snow ends in Wisconsin.

Heavy rains and flash floods end in Tennessee.

Heavy snow and ice storms continue in New England, enhanced by the new cyclone moving up the East Coast. Many states are hit: Maine (38 cm snow in NW., but more freezing rain in SE.), New Hampshire (28 cm), Vermont (45 cm snow, with sleet and freezing rain mixed in S.), Massachusetts (20 cm snow, with sleet and freezing rain along E. coast), and New York (55 cm snow, with ice storms near coast causing downed power lines).

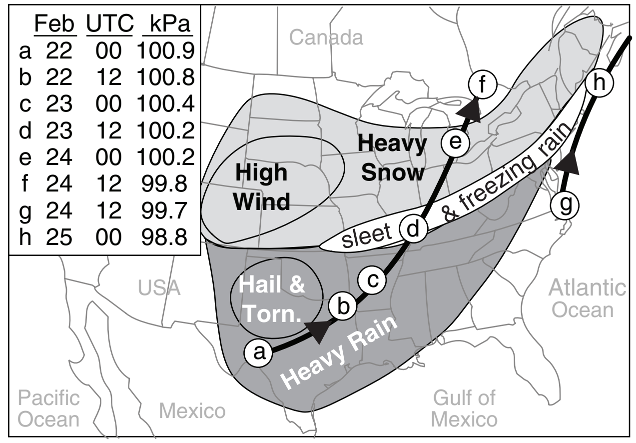


Figure 13.11
Track (thick black line) of the case-study cyclone (a-f), its weather, and central-pressure evolution during Feb 1994. The old cyclone at f merges into a new cyclone at g, which moves to h.

• **25 Feb** – Strong winds cause a blizzard in N. Illinois, with an additional 15 cm new snow, gusts to 17 m/s, whiteout conditions, dangerous wind chill, and schools closed. Heavy snow in Indiana leaves 25 cm in places.

The "Alberta Clipper" cyclone sweeps across the Plains States, causing a ground blizzard (gusts to 22 m/s with 10 cm new snow), dangerous wind chills, and a convective snow shower of 85 dBZ observed by a WSR-88D radar. Blowing of old snow causes drifts up to 3 m. Strong gusts occur in Kansas and North Dakota. Snow starts again in Wisconsin, with 25 cm new snow, with additional lake-effect snow near Great Lakes. Heavy snow falls in Ohio.

Heavy snow changes to sleet and freezing rain, and ends in New Hampshire, Vermont, New York.

• **26 Feb** – Blizzard ends in N. Illinois with peak gusts 30 m/s, closing 4 interstate highways and leaving hundreds of stranded cars. Similar conditions in N. Indiana, with blowing and drifting snow, whiteout conditions, gusts to 25 m/s, and drifts to 1 m, leave 1400 stranded motorists. A snow emergency declared in 8 counties. In N. Ohio is 15 cm of new snow, gusts to 20 m/s, and convective snowfall rates of 5 cm/h. Heavy snow ends in S. New York, leaving up to 23 cm of new snow.

Next, we will study the characteristics of the cyclone at 12 UTC on 23 Feb 1994 — a time near its peak intensity. A stack of charts at different isobaric surfaces are presented, each valid at the same time. Taken together, these weather maps give us an understanding of the 3-D structure of the storm.

FOCUS • Isosurfaces & Their Utility

Lows and other synoptic features are five-dimensional beasts (3-D spatial structure + 1-D time evolution + 1-D multiple variables). To accurately analyze and forecast the weather, you should try to form in your mind a multi-dimensional picture of the weather. Although some computer-graphics packages can display 5-dimensional data, most of the time we are stuck with flat 2-D weather maps.

Thus, to examine the inner workings of cyclones, look at 2-D atmospheric slices (Fig. c) in the horizontal, vertical, or along special curved surfaces called iso-surfaces. In this way you can study a two-dimensional dissection of these beasts, which often reveals insights into their higher-dimensional physics.

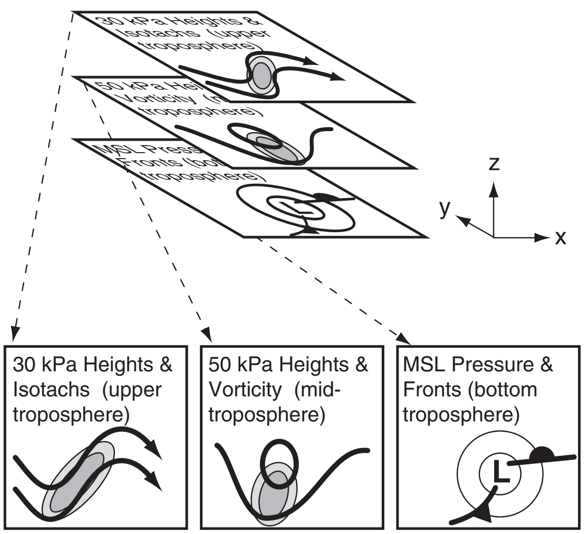


Figure c
A set of weather maps for different altitudes helps you gain a 3-D perspective of the weather. MSL = mean sea level.

Different iso-surfaces have different advantages, as discussed in this Focus Box. See Table 1-6 of Chapter 1 for a comprehensive list of iso-surface names.

Height

The only constant height surface used regularly is the surface at **mean sea level** (**MSL**). Weather maps analyzed on this surface are somewhat fictitious, because sea-level is underground virtually everywhere except at the oceans. (A few land-surface locations are below sea level, such as Death Valley and the Salton Sea USA, or the Dead Sea in Israel and Jordan). Nevertheless, this MSL surface is used to represent the weather where people live. It also represents the bottom boundary on the atmosphere.

A frequently plotted map is MSL pressure with surface fronts.

(continues in next column)

FOCUS • Isosurfaces *(continuation)*

Pressure

Isobaric (equal pressure) surfaces must be identified by their pressure. Commonly-used isobaric charts include the: 95 kPa surface, 85 kPa surface, 70 kPa surface, 50 kPa surface, 30 kPa surface, and 20 kPa surface. Because atmospheric pressure monotonically decreases with height, we know that the 20 kPa surface is at higher altitude than the 50 kPa surface, and so forth.

In mountainous regions such as the Rocky mountains, the 95 kPa and 85 kPa surfaces do not exist because they would be below ground. These surfaces are not usually plotted in that region (unless data is extrapolated to a hypothetical isobaric surface below ground).

On an isobaric chart, we can plot various weather elements (anything except pressure). For example, we can plot temperatures observed on the 50 kPa surface. We could draw lines of equal temperature (isotherms) along this isobaric surface. The meteorological jargon for such an analysis is a chart of "50 kPa isotherms".

An analysis of heights-above-sea-level along a 50 kPa isobaric surface is called a chart of "50 kPa heights". Similarly, we could look at "85 kPa isohumes", "20 kPa isotachs", "50 kPa absolute vorticity", or any other combination of our choosing.

Although confusing at first, sometimes two different weather maps are plotted on the same chart. This is done to illustrate physical relationships between both fields of data. An example is a chart of 50 kPa heights and isotherms. Such a chart tells us about temperature advection by the wind.

Sometimes the two superimposed charts are not on the same atmospheric surface, such as sea-level pressure (a constant height chart) and 95 kPa isotherms (a constant pressure chart). By using perhaps solid lines for isobars and dashed lines for isotherms, the result is hopefully not too cluttered.

Thickness

Often a map of height-thickness between pressures of 100 kPa and 50 kPa is analyzed. Greater thickness corresponds to warmer temperature, according to the hypsometric equation. In meteorological jargon, this map would be called the "100-50 kPa thickness" chart.

Horizontal temperature gradients cause horizontal thickness gradients. The **thermal wind** flows parallel to thickness lines, with low thickness (cold air) to the left of the thermal wind vector in the Northern Hemisphere. Recall that the thermal wind indicates how the geostrophic wind changes with height. Thermal-wind advection of geostrophic vorticity is important for cyclogenesis, as will be explained later in this chapter.

(continues on next page)

FOCUS • Isosurfaces (continued)

Potential Temperature

An **isentropic** surface connects points of equal potential temperature. Such a surface curves up and down. For example, the global average surface temperature decreases from equator to pole, and the average potential temperature increases with altitude in the troposphere. These two effects cause isentropic surfaces to slope downward toward the equator, on average. Any snapshot of an isentropic surface can differ from the average, particularly near fronts.

Isentropic surfaces are identified by their temperature, usually in Kelvin. An example is the "300 K isentropic surface", which is close to sea level near the equator, but rises to 4 to 8 km near the poles. The "350 K isentropic surface" is at a higher altitude, mostly in the stratosphere. Again, we can plot other weather elements on such a surface, such as 300 K heights or isohumes.

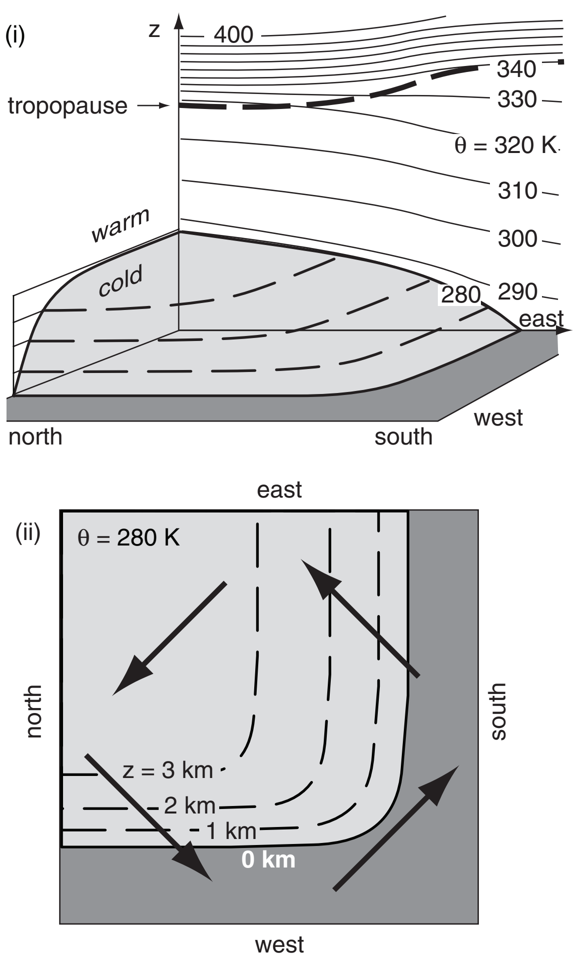


Figure d
Isentropic surface. (i) Three-dimensional sketch of the 280 K isentropic surface, corresponding to Fig. 5.20. (ii) Vertical view of the same scene. This is an isentropic chart. Dashed lines are height contours of the 280 K isentropic surface, and vectors are wind directions along that surface.
(continues in next column)

FOCUS • Isosurfaces (continued)

Isentropic surfaces cannot cross other isentropic surfaces. They can intersect the ground.

Isentropic surfaces are used because air parcels follow them when blown by the wind, under adiabatic conditions (see Figs. 5.20 and 12.13). Diabatic processes such as radiation, turbulence, and condensation can warm or cool the parcel, and cause it to change (jump) to a different isentropic surface.

Fig. d sketches an idealized isentropic surface corresponding to the 280 K isentrope of Fig. 5.20. Fig. d(i) is a 3-D representation, where there is a cold dome of air in the north-east. This could be the cold air behind a cold front. The 280 K surface slopes upward to the north-east. The dashed lines on this figure represent height contours of the isentropic surface above mean sea level.

A vertical view of the same scene is shown in Fig. d(ii). This is an **isentropic chart**. Height contours are shown again as dashed lines. Other variables could also be shown on the isentropic surface, such as temperature, humidity, wind, vorticity, etc.

This figure shows wind vectors on the isentropic surface, for the idealized case of circulation around a low-center in the N. Hemisphere. Because air parcels tend to follow isentropic surfaces as they advect, the wind vector in the upper-right portion of Fig. d(ii) implies that air parcels are rising as they blow toward the higher height contours. From such rising, we would expect adiabatic cooling and clouds to form in that region.

At the lower left side of this figure, air is blowing down the isentropic surface toward lower altitudes, in this hypothetical circulation. Descending air parcels warm adiabatically, and become cloud free.

Potential Vorticity (PVU)

The 1.5 PVU isentropic-potential-vorticity surface can be plotted as a marker for the tropopause. It often is found at high altitude (15 to 18 km) near the equator, and lower altitude (6 km) near the poles. These surfaces can fold back on themselves, such as near fronts. Vertical cross sections of PVU can show intrusions of stratospheric air into the troposphere near fronts – the tropopause folding that was discussed in Fig. 12.34. Another example of such a fold is shown in Fig. 13.19a later in this chapter for the synoptic-storm case study.

Surface

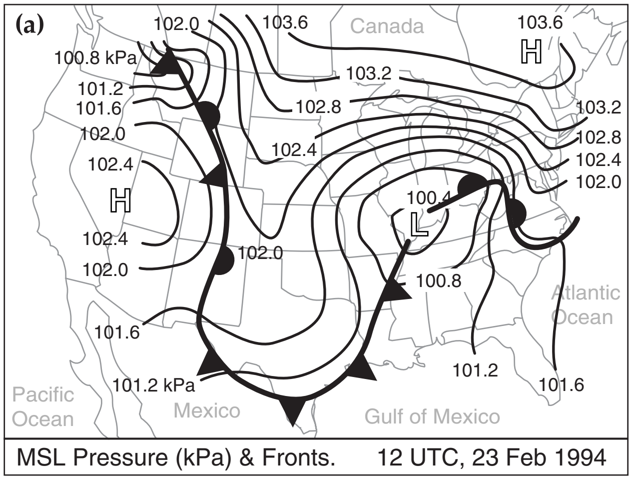
The "**surface weather map**" shows weather at the elevation of the Earth's surface. Namely, it follows the terrain up and down, and is not necessarily at mean sea level.

Surface Charts

Although the cyclone central pressure at southern Illinois is not very low in an absolute sense (Fig. 13.12a), it is quite low relative to the neighboring intense highs. The strong pressure gradient between high and low is sufficient to drive very strong winds (Fig. 13.12e; and storm data given earlier).

Quite a large surface temperature change exists across the frontal zone (Fig. 13.12b). Cold air (θ ≤ 0°C) is advecting from the north behind the low, while warm air (θ ≈ 20°C) is advecting from the south ahead of the low.

The cold air (see the 0°C isentrope) is dammed-up against the Rockies, causing the western part of the front to become stationary. The Appalachian mountains also slow the northward march of the warm front, resulting in some frontal kinks east of the low center. Humid air (high dew-point temperatures) are in the southeast (Fig. 13.12c), with drier air north.



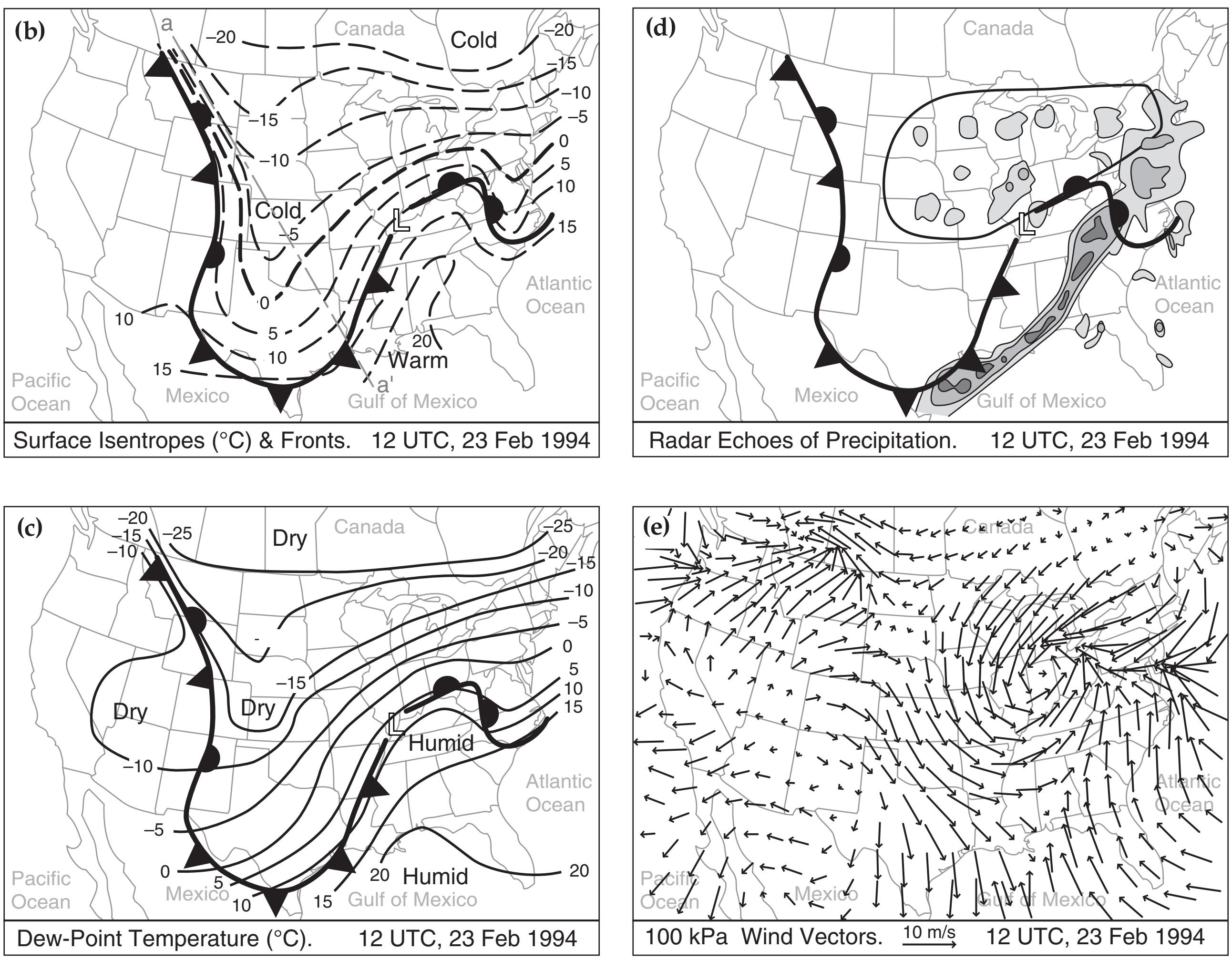


Figure 13.12

Surface weather maps. (a) Mean sea level pressure (P, kPa), & surface fronts. (b) Surface isentropes (θ, °C) & fronts. (c) Humidity (Td, °C) & fronts. (d): Radar echoes of precipitation & fronts. Levels: 1 - light precip. (light gray); 3 - heavy precip. (medium gray); 5 - intense precip. (dark). Black line in (d) encircles snow. (e) Near-surface (100 kPa) wind vectors. [Analysis by Jon Martin.]

85 kPa Charts

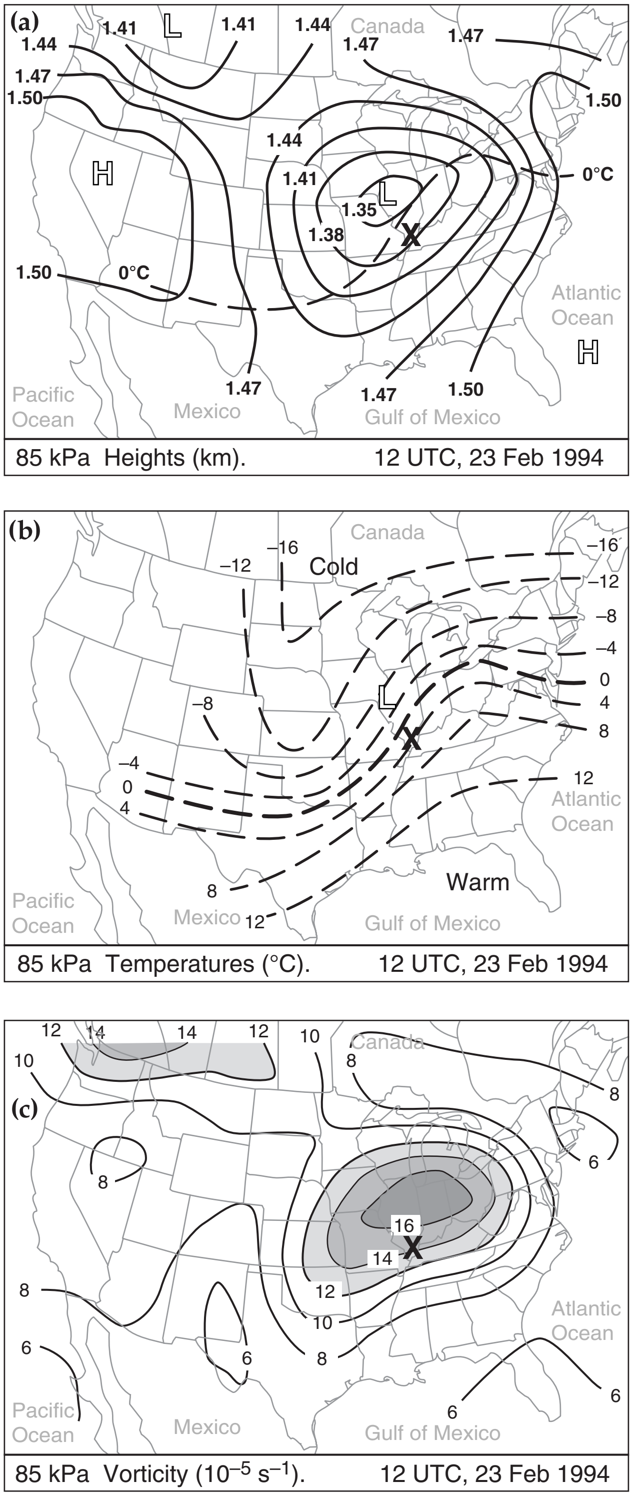


Figure 13.13
85 kPa weather maps. (a) Geopotential height (km). (b) Temperature (°C). (c) Absolute vorticity (10^{-5} s^{-1}). X marks the surface low. [Courtesy of Jon Martin.]

70 kPa Charts

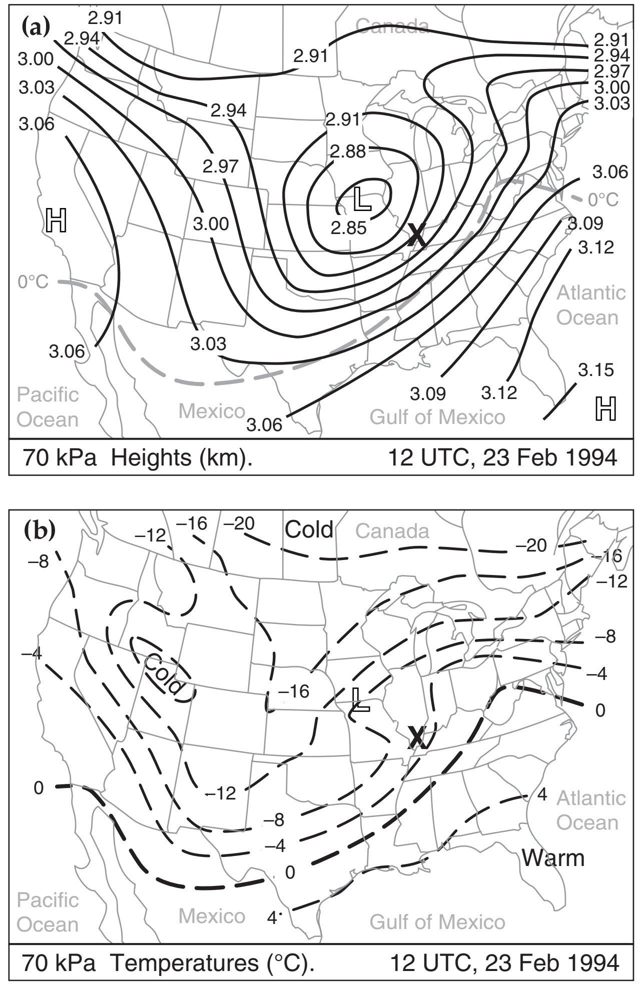


Figure 13.14
70 kPa weather maps. (a) Geopotential heights (km). (b) Temperature (°C). X marks surface low. [Courtesy of Jon Martin.]

100 - 50 kPa Thickness

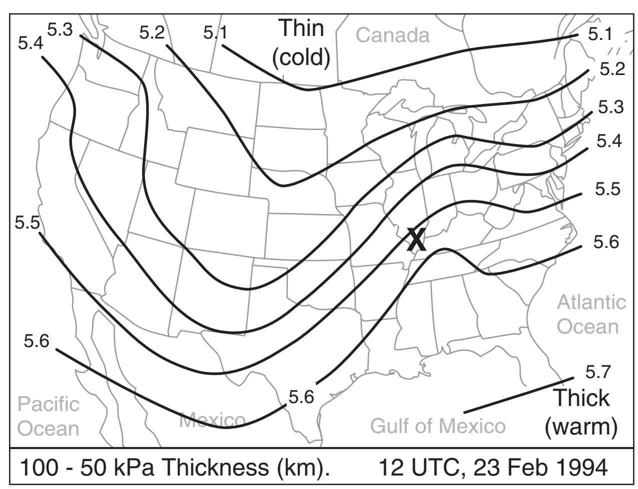


Figure 13.15
100–50 kPa thickness (km), from a 24 h forecast of the NWS NGM, valid at 12 UTC on 23 Feb 94. X marks surface low.

50 kPa Charts

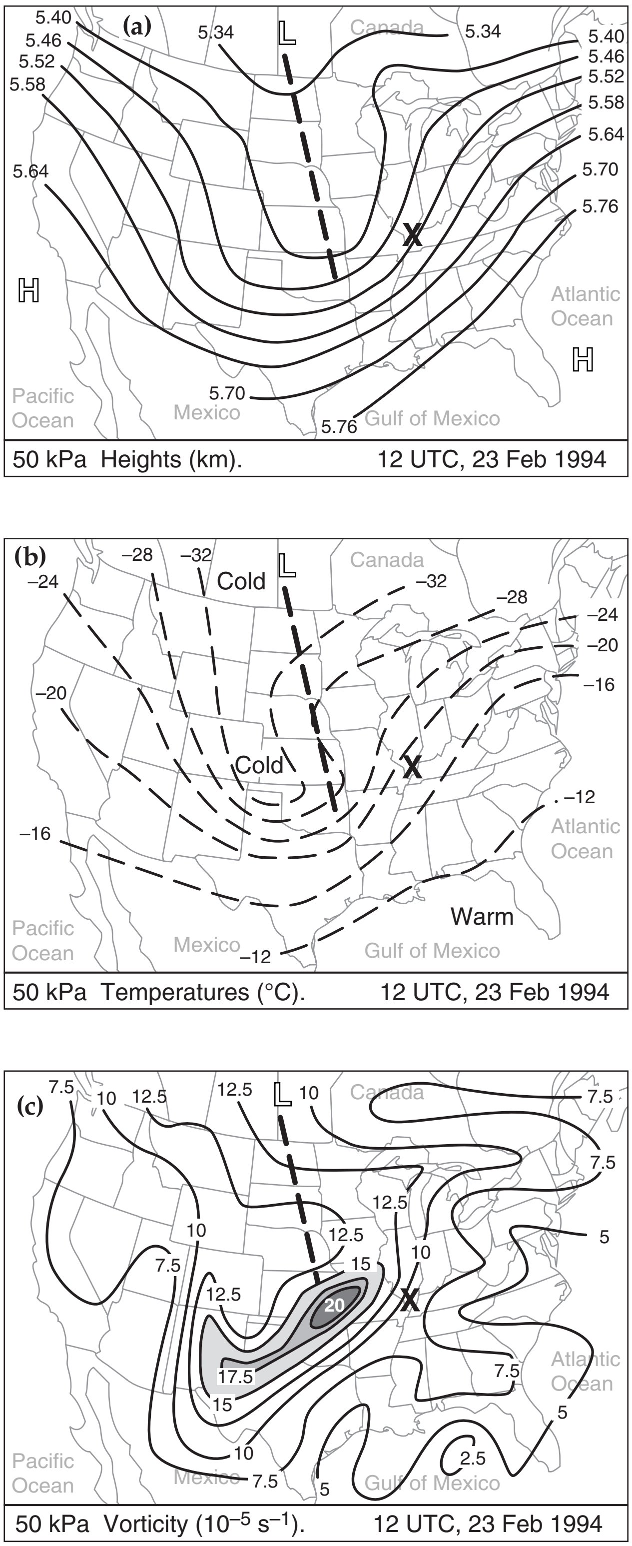


Figure 13.16
50 kPa weather maps. (a) Geopotential heights (km). (b) Temperature (°C). (c) Absolute vorticity (10^{-5} s^{-1}). Thick dashed line is trough axis. X marks the surface low. [courtesy of Jon Martin.]

20 kPa Charts

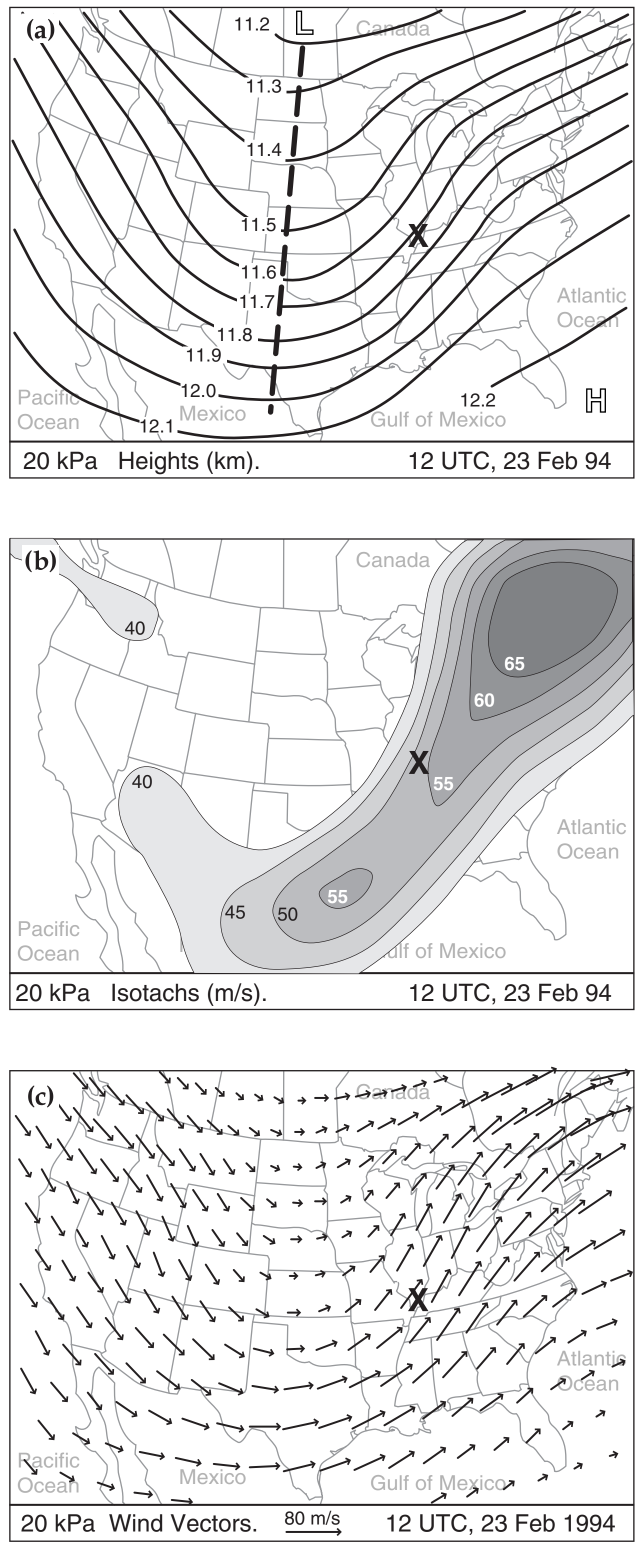


Figure 13.17
20 kPa weather maps. (a) Geopotential heights (km). (b) Isotachs (m/s). (c) Wind vectors. X marks the surface low. [courtesy of Jon Martin.]

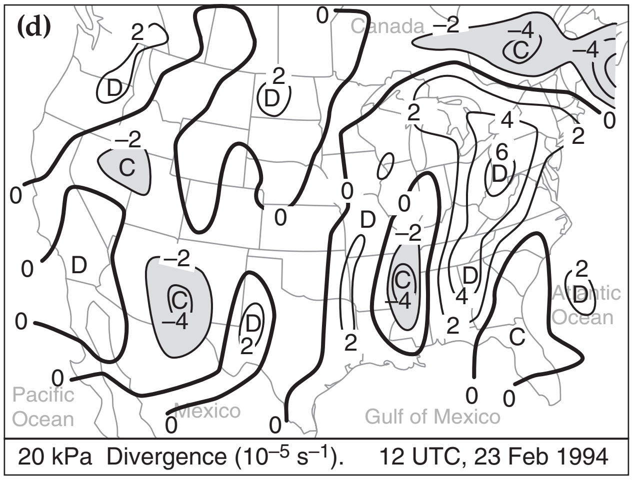
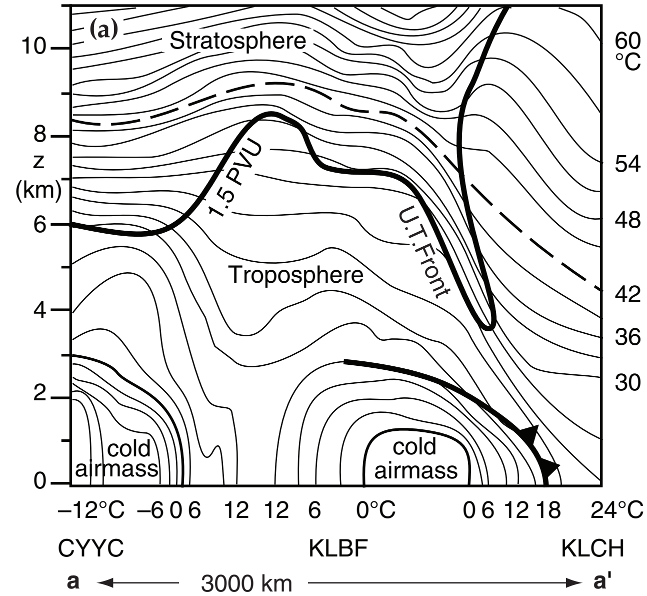


Figure 13.17 *(continued)*
(d) 20 kPa Horizontal divergence (10^{-5} s^{-1}) at the top of the troposphere. D = divergence, C = convergence.

Isentropic Charts



20 kPa Hemispheric Chart

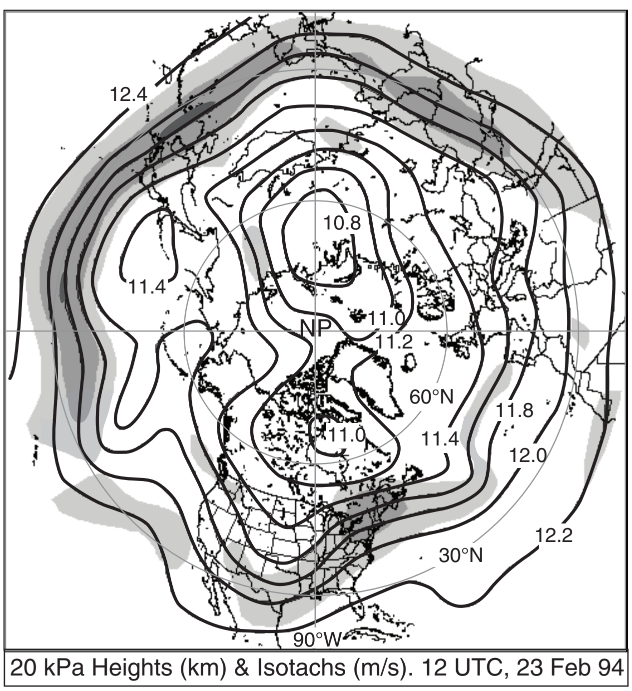


Figure 13.18
20 kPa Northern hemispheric chart centered on the North Pole (NP). Solid lines show geopotential heights, and shading shows isotachs. Light shading has winds ≥ 40 m/s; medium shading has winds ≥ 60 m/s; and dark shading has winds ≥ 80 m/s. Shading highlights the jet stream. [Adapted from NCEP Reanalysis Plotter output, courtesy of Christopher Godfrey, the University of Oklahoma School of Meteorology. Data is from the National Centers for Environmental Prediction/National Center for Atmospheric Research 40-Year Reanalysis Project.]

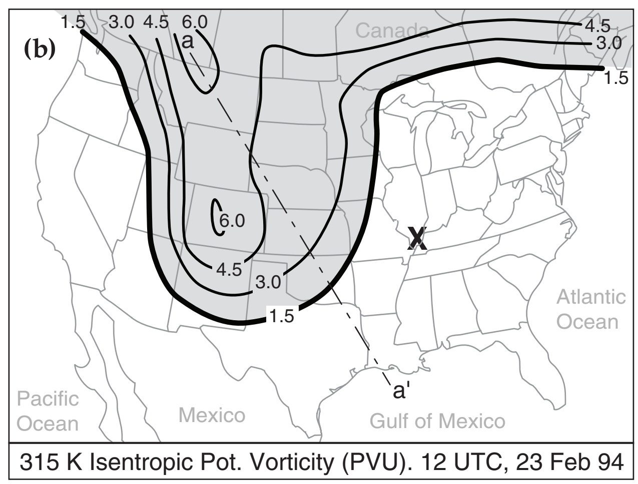


Figure 13.19
(a) Vertical cross section from northwest (CYYC = Calgary, Alberta) through the central USA (KLBF = North Platte, Nebraska) to the southeast (KLCH = Lake Charles, Louisiana). Thin lines are isentropes (θ, °C), labeled at right and bottom. Thick solid line in top half of Fig. is the 1.5 potential vorticity unit (PVU) contour, which indicates the tropopause. Cold-frontal symbols indicate the surface front and frontal inversion top. UT Front indicates upper-tropospheric front. z is height above sea level. Valid 00 UTC 24 Feb 94. [This is 12 h later than the other figs.] (b) Isentropic potential vorticity on the θ = 315 K surface. This surface corresponds to the 42°C dashed line in (a). Shading indicates where the 315 K surface is in the stratosphere. Dash-dot line in (b) shows the cross-section slice in (a). Units are potential vorticity units (PVU), where 1 PVU = 10^{-6} K·m^2·s^{-1}·kg^{-1}.

North of the low in the cold air (Fig. 13.12d) is a broad area of light to moderate snow that the radar cannot detect. Imbedded are regions of moderate to heavy snow that the radar can see. Closer to the low, and just east of it where temperatures are near 0°C, sleet and freezing rain are falling.

Well in advance of the cold front in the southeast USA is a squall line of intense thunderstorms. These storms produce heavy rain, hail, and some small tornadoes. The squall line is feeding on latent heat, because of the high humidities near the Gulf of Mexico. The wind field (Fig. 13.12e) shows a strong wind shift near the squall line, suggesting the main dynamic activity is not at the cold front but at the squall line.

85 kPa Charts

Pressure level 85 kPa corresponds to about 1.4 km above sea level. At this height, the low center is over western Illinois (Fig. 13.13a). This is northwest of the surface low. Because winds at this pressure level follow the height contours, expect northerly winds in the Central Plains (Nebraska and Kansas), and southerly winds to the east of the low center.

From Texas to Pennsylvania is a strong baroclinic zone, which is evident by the close spacing between isotherms (Fig. 13.13b). Strong cold-air advection exists in Oklahoma, Texas, Arkansas, and Missouri. This is marked by the height contours crossing the isotherms, with winds advancing the cold air. Warm-air advection occurs east of the low center. Cyclonic rotation around the low creates a vorticity maximum (**vort max**; shaded in Fig. 13.13c) over the midwest.

70 kPa Charts

Pressure level 70 kPa is roughly 3 km above sea level. At this height, the low is centered over northern Missouri (Fig. 13.14a) — significantly northwest of the surface low. The tightly packed height contours from Arkansas through West Virginia suggest a strong southwesterly wind in that region.

Cold-air advection is occurring in Oklahoma, Texas, and New Mexico (Fig. 13.14b). Subsidence and clearing skies are expected in regions of cold-air advection aloft. Warm-air advection is associated with ascending air, clouds, and precipitation, such as over and east of the Great Lakes.

100–50 kPa Thickness Chart

The cold air pushing into the central plains of the USA from Canada has small 100–50 kPa thickness (Fig. 13.15), as expected from the hypsometric equation. A thermal wind parallel to the thickness lines would create a jet stream that dips south toward Texas and then NE. toward Ohio, as is indeed observed.

50 kPa Charts

With nearly equal amount of air mass above and below it, the 50 kPa pressure level is near the mass-weighted middle of the atmosphere. It is roughly 5.5 km above sea level at mid-latitudes.

Instead of a closed low center as in the charts for lower altitudes, the 50 kPa chart shows a trough of low pressure (Fig. 13.16a) through the central plains. This trough axis (heavy dashed line in the figure) is much further west of the surface low. The trough is a portion of the Rossby wave, such as discussed in the Global Circulation chapter.

The region of cold air (Fig. 13.16b) is west of the trough. Hence, you can expect self-development to occur (explained later in this chapter), with strengthening of the wave amplitude.

Fig. 13.16c shows absolute vorticity on the 50 kPa pressure surface. The vorticity maximum (**vort max**) is often at, or just behind, the low center. Winds advect the vort max (**positive vorticity advection**), thereby supporting propagation of the low. This vorticity map is from a 24 h forecast of the U.S. National Weather Service (NWS) Nested Grid Model (NGM), valid at 12 UTC on 23 Feb 94.

20 kPa Charts

The 20 kPa pressure level is roughly at 11.5 km above sea level. In Fig. 13.17a, the trough over central North America is quite evident.

The wind-speed chart (Fig. 13.17b) shows a jet stream from the desert southwest USA through Texas and then northeast over the Great Lakes toward Nova Scotia, Canada. A **jet streak** (relative maximum of wind speed) is over the northeast portion of the figure. Horizontal divergence aloft (Fig. 13.17d) favors cyclone development at the surface near Lake Ontario, near the left entrance region to this jet streak. Such development indeed occurred, as indicated by the storm track of Figs. 13.10 & 13.11.

Winds are nearly geostrophic, as suggested by the fact that the wind vectors in Fig. 13.17c are roughly parallel to the height contours of Fig. 13.17a.

A hemispheric chart at the 20 kPa level is plotted in Fig. 13.18. Superimposed are heights and isotachs. From this N. Hemispheric perspective, you can see that the trough over the USA is part of a global Rossby-wave pattern between about 30° and 50°N latitude around the North Pole. The trough axis at roughly 100°W longitude over the USA is one of about 7 jet-stream troughs in the hemisphere.

Some high-latitude countries use 30 kPa charts instead of 20 kPa, because of the lower tropopause.

Isentropic Charts

Fig. 13.19a shows a **vertical cross section** from northwest to southeast, across North America (along section line **a – a′** in Figs. 13.12b and 13.19b). Plotted are isentropes (equal potential temperature), and a single contour of isentropic potential vorticity.

The cold air mass is evident in Kansas (to the right of KLBF in this plot). The surface front is just northwest of Lake Charles, Louisiana (KLCH), as indicated by the tight packing of the isentropes near the surface.

The 1.5 PVU line marks the tropopause. Above this line, the isentropes are more closely spaced, indicating the greater static stability in the stratosphere.

Between North Platte and Lake Charles is a **tropopause fold**, as indicated by the 1.5 PVU line dipping down from the stratosphere. An upper-level front is evident in this region, based on the close spacing between isentropes.

Fig. 13.19b is a plot of isentropic potential vorticity on the θ = 315 K (= 42°C) isentropic surface. This surface corresponds to the dashed line in Fig. 13.19a. Shading shows where this surface is in the stratosphere, by the values of isentropic potential vorticity that are greater than 1.5 PVU there. The troposphere is shallower near the poles than near the equator. Thus, the 315 K isentropic surface is usually in the stratosphere near the pole (as suggested by the widespread shading at high latitudes in Fig. 13.19b), and in the troposphere near the equator.

In the next sections, we see how dynamics can be used to explain cyclone formation and evolution.

FOCUS • Multi-field Charts

Most of the weather maps presented in the previous case study contained plots of only one field, such as the wind field or height field. Because many fields are related to each other or work together, meteorologists often plot multiple fields on the same chart.

To help you discriminate between the different fields, they are usually plotted differently. One might use solid lines and the other dashed. Or one might be contoured and the other shaded (see Fig. e or Fig. 13.7a). Look for a legend or caption that describes which lines go with which fields, and gives units.

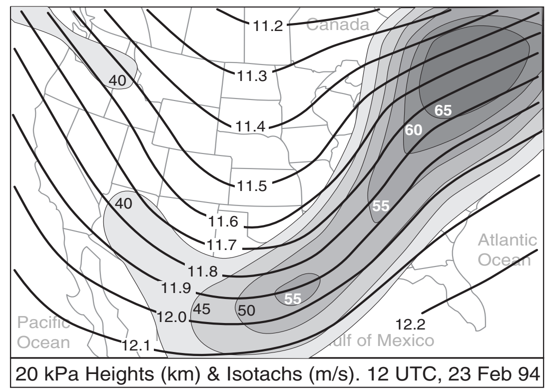


Fig. e.
Geopotential heights (lines) and wind speed (shaded).

LEE CYCLOGENESIS

Waves in the upper-air (jet-stream) flow can create mid-latitude cyclones at the surface, as sketched in Figs. 13.6 and 13.7. Rossby showed in 1939 that evolution of planetary waves (i.e., **Rossby waves**) can be described by conservation of absolute vorticity, as was discussed in the Global Circulation chapter. One way to create Rossby waves is by topographic modification of the flow, as is described next.

Stationary Planetary Waves

When air flows across a mountain range such as the Rocky Mountains (Fig. 13.20a), planetary-scale Rossby waves that meander north and south are triggered in the atmosphere (Fig. 13.20b). These Rossby waves are stationary relative to the ground. Otherwise, they behave like the barotropic and baroclinic

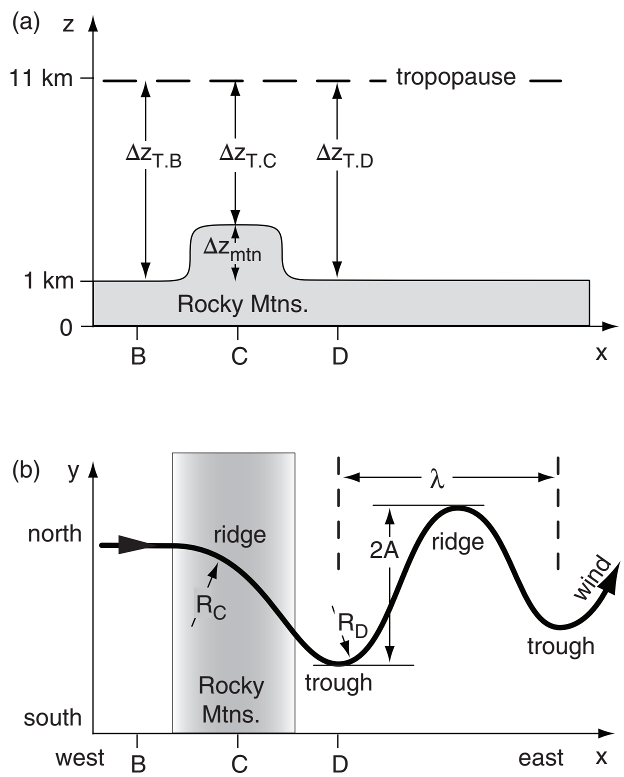


Figure 13.20
Cyclogenesis to the lee of the mountains. (a) Vertical cross section. (b) Map of jet-stream flow. "Ridge" and "trough" refer to the wind, not the topography.

FOCUS • Synoptics

Synoptic meteorology is the study and analysis of weather maps, often with the aim to forecast the weather on horizontal scales of 400 to 4000 km. **Synoptic weather maps** give a snapshot of atmospheric conditions over a large area, as created from weather observations made simultaneously.

Typical weather phenomena at these **synoptic scales** include cyclones (Lows), anticyclones (Highs), and airmasses. Fronts are also included in synoptics because of their length, even though frontal zones are so narrow that they can also be classified as mesoscale. See Table 10-6 and Fig. 10.24 in the Dynamics chapter for a list of different atmospheric scales.

The material in this chapter and in the previous one fall solidly in the field of **synoptics**. People who specialize in synoptic meteorology are called **synopticians**.

The word "synoptics" is from the Greek "synoptikos", and literally means "seeing everything together". It is the big picture.

Solved Example

For wind blowing 16.2 m/s over a 1 km-high mountain range at 45°N, find the wavelength and amplitude of the lee planetary wave. Assume the tropospheric depth is 10 km upstream of the mountains.

Solution

Given: ϕ = 45°N, M = 16.2 m/s, Δz_{mtn} = 1 km, Δz_T = 10 km.
Find: λ = ? km, A = ? km

First, find β at 45°N, using eq. (13.2):
$\beta = (2.294\text{x}10^{-11}\ \text{m}^{-1}\cdot\text{s}^{-1}) \cdot \cos(45°)$
$= 1.62\text{x}10^{-11}\ \text{m}^{-1}\cdot\text{s}^{-1}$

Next, use eq. (13.1):

$$\lambda \approx 2\cdot\pi\cdot\left[\frac{16.2\text{m}\cdot\text{s}^{-1}}{1.62\times10^{-11}\text{m}^{-1}\cdot\text{s}^{-1}}\right]^{1/2} = \underline{\textbf{6283 km}}$$

Use eq. (13.4):
A = [(1 km) / (10 km)] · (6371 km) = **637 km**

Check: Units OK. Physics OK.
Discussion: The circumference of the 45° meridian is $2\cdot\pi\cdot R_{Earth}\cdot\cos(45°)$ = 28,306 km. Thus, about 4.5 wavelengths fit round the Earth – it is indeed a planetary-scale wave. Also, the north-south meander of the wave spans $2\cdot A \approx 11°$ latitude.

waves in the global circulation, discussed in the Global Circulation chapter. The north-south (meridional) component of wave-amplitude often decays to the east of the mountains, as boundary-layer turbulence and deep convection cause damping.

At the parts of the wave furthest from the equator, the air is turning anticyclonically (clockwise in the Northern Hemisphere). Hence, over the mountains (location C in Fig. 13.20b), one would generally expect to find a predominance of highs.

Just east of the mountain range, a trough forms (location D in Fig. 13.20b). This location is to the **lee** (downwind) of the mountains. East of that trough, upper-air divergence can favor **lee cyclogenesis** at the surface.

The dominant wavelength λ of these upper-atmosphere waves is approximately

$$\lambda \approx 2\cdot\pi\cdot\left[\frac{M}{\beta}\right]^{1/2} \qquad (13.1)$$

where M is the average wind speed. As you have seen in an earlier chapter, the rate β of increase of the Coriolis parameter f_c with distance north is:

$$\beta = \frac{\Delta f_c}{\Delta y} = \frac{2\cdot\Omega}{R_{earth}}\cdot\cos\phi \qquad \bullet(13.2)$$

where $2\cdot\Omega = 1.458\text{x}10^{-4}\ \text{s}^{-1}$ is twice the angular rotation rate of the Earth. β is on the order of 1.5 to $2\text{x}10^{-11}\ \text{m}^{-1}\cdot\text{s}^{-1}$ at mid latitudes.

The north-south amplitude A depends on the height of the mountains Δz_{mtn} relative to the upstream depth of the troposphere Δz_T :

$$A \approx \frac{f_c}{\beta}\cdot\frac{\Delta z_{mtn}}{\Delta z_T} \qquad (13.3)$$

The ratio $f_c/\beta = R_{Earth}\cdot\tan(\phi)$, where R_{Earth} = 6371 km is the average Earth radius, and ϕ is latitude. At mid-latitudes where $\tan(\phi) \approx 1$, the amplitude is

$$A \approx \frac{\Delta z_{mtn}}{\Delta z_T}\cdot R_{earth} \qquad (13.4)$$

The north-south distance Δy between crest and trough is $\Delta y = 2\cdot A$.

Thus, wavelength depends on wind speed, but not mountain height. Wave amplitude depends on mountain height, but not wind speed.

Conservation of Potential Vorticity

Another way to examine lee-side creation of Rossby waves is by their vorticity. Neglecting shear and assuming constant wind-speed M for simplicity, eq. (11.25) for potential vorticity ζ_p becomes:

$$\zeta_p = \frac{(M/R) + f_c}{\Delta z} = \text{constant} \qquad \bullet(13.5)$$

which states that potential vorticity is conserved.

Suppose air at point B (Fig. 13.20b), west of the Rockies, blows directly from west to east, and initially has no curvature ($M/R = 0$). Thus, at location B eq. (13.5) reduces to

$$\zeta_p = \frac{f_{c.B}}{\Delta z_{T.B}} \qquad (13.6)$$

where $\Delta z_{T.B}$ is the average depth of troposphere at point B. This initial condition sets the constant value of the potential vorticity for this example.

Let Δz_{mtn} be the relative mountain height above the surrounding land (Fig. 13.20a). As the air blows over the mountain range, the troposphere becomes thinner as it is squeezed between mountain top and the tropopause: $\Delta z_{T.C} = \Delta z_{T.B} - \Delta z_{mtn}$. But the latitude of the air hasn't changed much yet, so $f_{c.C} \approx f_{c.B}$. The previous two equations combine (because $\zeta_{p.B} = \zeta_{p.C}$) to give the anticyclonic radius of curvature over the west portion of the mountains (at location C):

$$R_C = \frac{-M}{f_{c.B} \cdot (\Delta z_{mtn} / \Delta z_{T.B})} \qquad (13.7)$$

Namely, in eq. (13.5), when Δz became smaller while f_c was constant, M/R had to also become smaller to keep the ratio constant. But since M/R was initially zero, the new M/R had to become negative. Negative R means anticyclonic curvature.

As the air moves south, the Coriolis parameter decreases, thereby requiring the radius of anticyclonic curvature to increase (i.e., to become less curved). Eventually, the flow is straight, blowing toward the southeast near the east side of the Rocky Mountains. As the air overshoots south, and as the air column stretches as it leaves the mountains, the curvature must become positive (cyclonic) in order to keep potential vorticity constant. The radius of curvature R_D is similar in magnitude to R_C, but of opposite sign. This is the region where a lee trough would form in the jet stream flow. Cyclogenesis is favored just east of the lee trough.

Solved Example

Suppose a noncurving wind blows from the west at 20 m/s (assume constant), at latitude 40°N. Initially, the tropospheric depth is 10 km. Then, the air flows over the Rocky Mountains, where you can assume a relative terrain depth of 1 km above the neighboring plains. Find the potential vorticity, and the radius of curvature over the mountains.

Solution

Given: ϕ = 40°N, M = 20 m/s, $R_{initial} = \infty$,
Δz_T = 10 km, Δz_{mtn} = 1 km.
Find: ζ_p = ? m$^{-1}\cdot$s^{-1}, R_C = ? km

Assume: shear can be neglected.

First, find the Coriolis parameter using eq. (10.16):
$f_c = (1.458\text{x}10^{-4}\ \text{s}^{-1})\cdot\sin(40°) = 9.38\text{x}10^{-5}\ \text{s}^{-1}$

Use eq. (13.6):

$$\zeta_p = \frac{9.38\times10^{-5}\,\text{s}^{-1}}{10\text{km}} = \underline{\mathbf{9.38x10}}^{-9}\ \text{m}^{-1}\ \text{s}^{-1}$$

Use eq.(13.7) to find anticyclonic radius of curvature:

$$R_C = \frac{-(20\text{m/s})}{(9.38\times10^{-5}\text{s}^{-1})\cdot(1\text{km}/10\text{km})} = \underline{\mathbf{-2132\ km}}$$

Check: Units OK. Physics OK.
Discussion: We expect $R_D \approx -R_C$, namely about 2000 km. Thus, mountains trigger long-wavelength ridges and troughs. Cyclones are favored east of the Rockies, just east of the first lee trough.

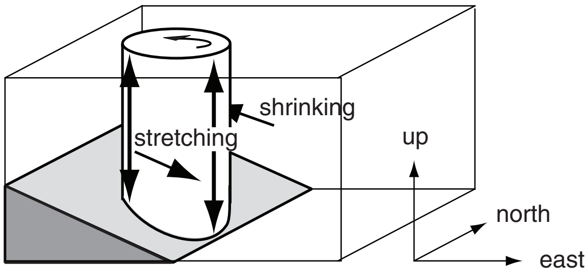


Figure 13.21
Cyclone on the eastern slope of a mountain in N. Hemisphere.

Solved Example
An extratropical cyclone of radius 500 km has potential vorticity of $2\text{x}10^{-8}$ m^{-1}·s^{-1}. It sits on a mountain slope of 1/1000. Find the increase of relative vorticity along the south side of the cyclone in N. Hemisphere.

Solution
Given: R = 500 km, $\zeta_p = 2\text{x}10^{-8}$ m^{-1}·s^{-1}, $\alpha = 0.001$
Find: $\Delta\zeta_r$ = ? s^{-1} .
Assumption: Neglect changes of latitude.

Use eq. (13.8):
$\Delta\zeta_r = 2 \cdot (500{,}000 \text{ m}) \cdot (0.001) \cdot (2\text{x}10^{-8} \text{ m}^{-1}\cdot\text{s}^{-1})$
$= \underline{\mathbf{2x10}}^{-5}$ s^{-1} .

Check: Units OK. Physics OK.
Discussion: This intensification on the cyclone fringe is easily as large as the average relative vorticity within center of the cyclone, resulting in southward cyclone movement toward the region of increasing relative vorticity.

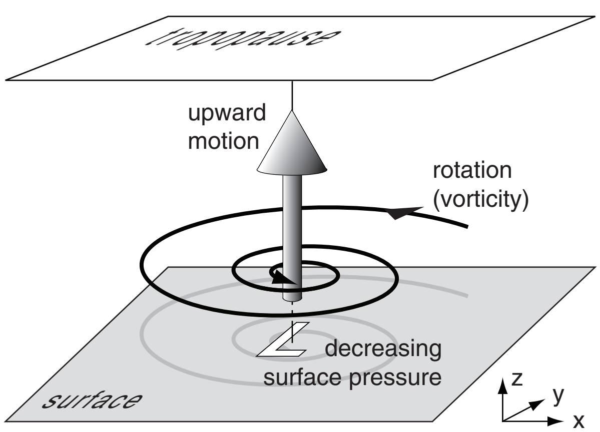


Figure 13.22
Attributes of cyclogenesis. Updrafts remove air molecules from near the ground, which lowers surface pressure. The pressure gradient drives winds, which rotate due to Coriolis force.

Equatorward Propagation Along the Lee Side

Picture a cyclone that already exists at the lee side (i.e., east) of the mountain range (Fig. 13.21). The bottom of this cyclone rests on a sloping surface (slope $\alpha = \Delta z/\Delta x$). Circulation around the low will cause downslope flow on the equatorward side, and upslope on the poleward side.

Rotation about the low will cause vertical columns of air along the equatorward side of the cyclone to stretch. Those on the poleward side shrink. For any potential vorticity ζ_p of the cyclone, stretching intensifies the relative vorticity ζ_r on the equatorward side, according to:

$$\Delta\zeta_r = 2 \cdot R \cdot \alpha \cdot \zeta_p \qquad (13.8)$$

A similar change of opposite sign occurs on the poleward side. This causes the low to propagate equatorward along the eastern side of the mountains, toward the region of greater relative vorticity.

Another way to examine the same process is by the adiabatic warming that occurs at the bottom of the column (on the equatorward side) as it descends down the mountain slope. No warming occurs at the top of the column, because air is not flowing downward there.

This differential heating reduces the overall static stability of the troposphere; namely, $\Delta\theta/\Delta z$ decreases. A decrease of static stability must be accompanied by an increase of relative vorticity, in order to maintain constant isentropic potential vorticity ζ_{IPV} (eq. 11.26). Thus, relative vorticity intensifies on the equatorward side of the cyclone, and decreases on the poleward side. The low tends to propagate equatorward along the eastern side of the mountains, toward the region of greater relative vorticity.

CYCLONE SPIN-UP

Even in the absence of lee effects, baroclinic instability (see the Global Circulation chapter) can cause cyclogenesis. Cyclogenesis is associated with upward motion, decreasing surface pressure, and increasing vorticity. You can gain insight into cyclogenesis by studying all three characteristics, even though they are intimately related (Fig. 13.22). Let us start with vorticity.

Vorticity-increase at low altitudes is one measure of increasing cyclone strength. The equation that forecasts change of vorticity with time is called the **vorticity tendency equation**.

Vorticity Tendency

A forecast equation for relative vorticity ζ_r is:

•(13.9)

$$\frac{\Delta\zeta_r}{\Delta t} = -U\frac{\Delta\zeta_r}{\Delta x} - V\frac{\Delta\zeta_r}{\Delta y} - V\frac{\Delta f_c}{\Delta y} + f_c\frac{\Delta W}{\Delta z}$$

spin-up · *horiz. advection* · *beta* · *stretching*

$$-W\frac{\Delta\zeta_r}{\Delta z} + \zeta_r\frac{\Delta W}{\Delta z} + \frac{\Delta U}{\Delta z}\cdot\frac{\Delta W}{\Delta y} - \frac{\Delta V}{\Delta z}\cdot\frac{\Delta W}{\Delta x} - C_D\frac{M}{z_i}\zeta_r$$

vert.advect. · *stretching* · *tilting* · *turb. drag*

The first term gives the **spin-up** rate of a cyclone, because it indicates the increase of vorticity. This term is also called the **vorticity tendency** term, because it shows how vorticity tends to change with time. A positive tendency corresponds to a strengthening cyclone.

Fig. 13.23 shows a physical interpretation of the other terms. Advection (Fig. 13.23a) changes vorticity by blowing-in air with different vorticity from some other location. **Positive vorticity advection** (**PVA**) occurs when greater vorticity is blown in.

The opposite is **negative vorticity advection** (**NVA**). Advection can work in both horizontal directions and in the vertical direction, although just one horizontal direction is sketched in Fig. 13.23a.

The **beta effect** is named because $\Delta f_c/\Delta y = \beta$ = positive in the N. Hemisphere (see eq. 13.2). If wind moves air northward (i.e., V = positive) to where f_c is larger, then relative vorticity ζ_r becomes smaller (as indicated by the negative sign in front of the beta term) to conserve potential vorticity (Fig. 13.23b).

Stretching has already been discussed in the Global Circulation chapter. As shown in Fig. 13.23c, a short vortex tube will stretch if the top rises faster than the base. This difference in rise is given by $\Delta W/\Delta z$. The two stretching terms account for the Earth's rotation and the relative vorticity.

When such a tube stretches in the vertical, mass conservation requires horizontal convergence (C), which shrinks the tube radius. Because of this convergence/divergence effect, the stretching term is also known as the **divergence term**. Horizontal convergence spins up cyclonic vorticity, and divergence spins up anticyclonic vorticity.

The **tilting term** shows how horizontal vorticity can be tilted into vertical vorticity. For example, Fig. 13.23d shows vorticity associated with the vertical shear of horizontal wind near the ground. The vortex tubes are shown between the U wind vectors. If these tubes were then tilted by a vertical velocity that increases toward the north, then the horizontal vortex tubes would become more vertical, thereby contributing to the vertical vorticity.

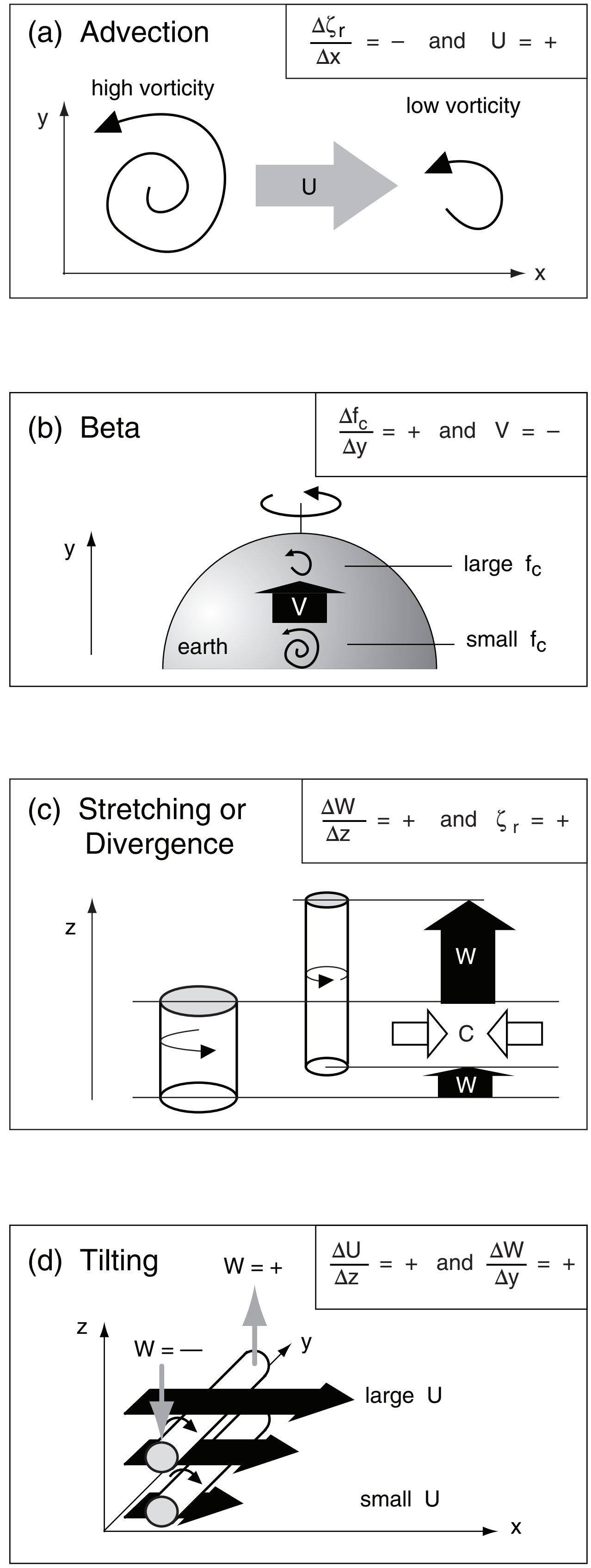


Figure 13.23
Physical interpretation of terms in the vertical vorticity equation (continued next column).

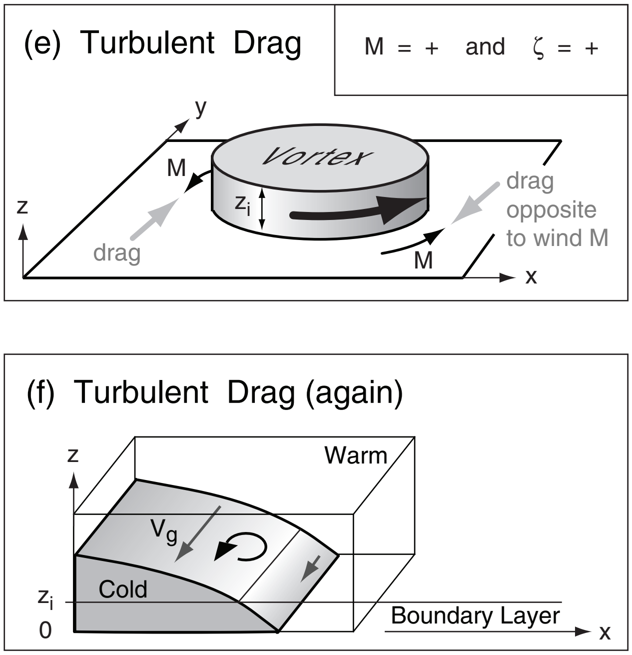


Figure 13.23 *(continuation)*
Physical interpretation of terms in the vertical vorticity equation

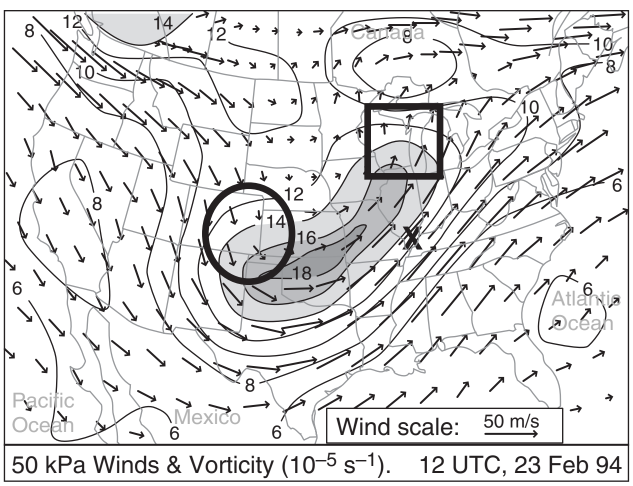


Figure 13.24
Absolute vorticity (shaded) and winds (vectors) at 50 kPa, highlighting regions of positive (square) and negative (oval) vorticity advection for the case-study storm.

Finally, the **turbulent-drag** term has two effects. One effect is to reduce rotation regardless of direction, which causes **spin-down** (Fig. 13.23e). This occurs in the turbulent boundary layer (of depth z_i) due to frictional drag against the ground. In the absence of continued spin-up processes, the cyclone will spin-down and die.

The other turbulent-drag effect causes spin up, although it is not apparent from eq. (13.9). Recall from Fig. 12.17 (geostrophic adjustment) that a geostrophic wind forms along sloping isentropic surfaces. If such a surface intersects the boundary layer, then turbulent drag will slow the winds near the ground, but not above the boundary layer (Fig. 13.23f). The resulting shear produces cyclonic relative vorticity.

The net spin-up or spin-down depends on the sum of all these terms.

You can identify the action of some of these terms by looking at weather maps.

Fig. 13.24 shows the wind vectors and absolute vorticity on the 50 kPa isobaric surface (roughly in the middle of the troposphere) for the case-study cyclone. **Positive vorticity advection** (PVA) occurs where wind vectors are crossing the vorticity contours from high toward low vorticity, such as highlighted by the dark box in Fig. 13.24. Namely, higher vorticity air is blowing into regions that contained lower vorticity. This region favors cyclone spin up.

Negative vorticity advection (NVA) is where the wind crosses the vorticity contours from low to high vorticity values (dark oval in Fig. 13.24). By using the absolute vorticity instead of relative vorticity, Fig. 13.24 combines the advection and beta terms.

Fig. 13.25 shows vertical velocity in the middle of the atmosphere. Since vertical velocity is near zero at the ground, regions of positive vertical velocity at 50 kPa must correspond to stretching in the bottom half of the atmosphere. Thus, the updraft regions in the figure favor cyclone spin-up (i.e., cyclogenesis).

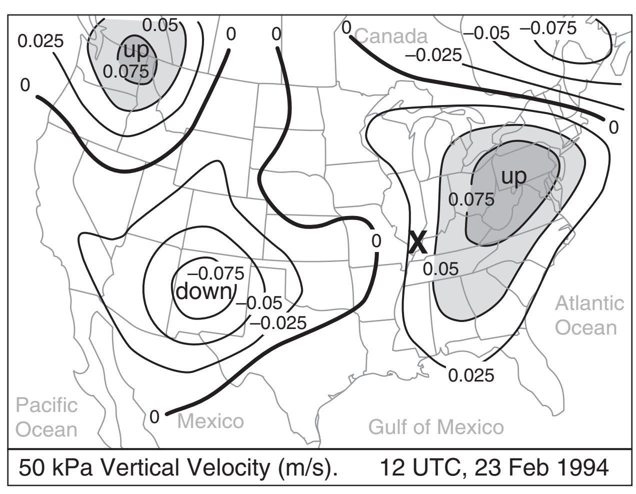
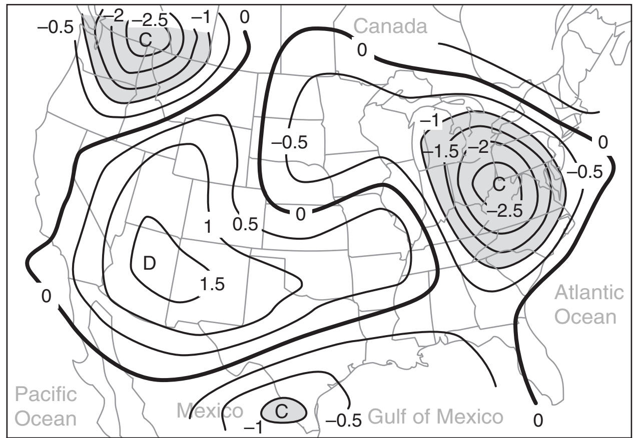


Figure 13.25 (at right)
Vertical velocity (m/s) for the case-study storm. At this altitude, a value of w = 0.075 m/s corresponds to ω = –0.5 Pa/s.

Figure 13.26 (right)
Horizontal divergence ($D = \Delta U/\Delta x + \Delta V/\Delta y$) for the case-study storm. C = horizontal convergence (= – D).



In the bottom half of the troposphere, regions of stretching must correspond to regions of convergence of air, due to mass continuity. Fig. 13.26 shows the **divergence** field at 85 kPa. Negative divergence corresponds to convergence. The regions of low-altitude convergence (shaded) favor cyclone spin-up.

Low-altitude spin-down due to turbulent drag occurs wherever there is rotation. Thus, the positive vorticity regions in Fig. 13.13c (shaded) are where spin-down is occurring for the case-study cyclone. The tilting term will be discussed in the Thunderstorm chapters.

Quasi-Geostrophic Approximation

Above the boundary layer (and away from fronts, jets, and thunderstorms) the terms in the second line of the vorticity equation are smaller than those in the first line, and can be neglected. Also, for synoptic scale, extratropical weather systems, the winds are almost geostrophic (**quasi-geostrophic**).

These weather phenomena are simpler to analyze than thunderstorms and hurricanes, and can be well approximated by a set of equations (quasi-geostrophic vorticity and omega equations) that are less complicated than the full set of **primitive equations** of motion (Newton's second law, the first law of thermodynamics, continuity, and ideal gas law).

As a result of the simplifications above, the vorticity forecast equation simplifies to the following **quasi-geostrophic vorticity equation**:

$$\underbrace{\frac{\Delta\zeta_g}{\Delta t}}_{\text{spin-up}} = \underbrace{-U_g\frac{\Delta\zeta_g}{\Delta x} - V_g\frac{\Delta\zeta_g}{\Delta y}}_{\text{horizontal advection}} \underbrace{-V_g\frac{\Delta f_c}{\Delta y}}_{\text{beta}} \underbrace{+f_c\frac{\Delta W}{\Delta z}}_{\text{stretching}} \qquad \bullet(13.10)$$

where the relative geostrophic vorticity ζ_g is defined similar to the relative vorticity of eq. (11.20), except using geostrophic winds U_g and V_g:

$$\zeta_g = \frac{\Delta V_g}{\Delta x} - \frac{\Delta U_g}{\Delta y} \qquad \bullet(13.11)$$

For solid body rotation, eq. (11.22) becomes:

$$\zeta_g = \frac{2 \cdot G}{R} \qquad \bullet(13.12)$$

where G is the geostrophic wind speed and R is the radius of curvature.

Solved Example

Suppose an initial flow field has no geostrophic relative vorticity, but there is a straight north to south geostrophic wind blowing at 10 m/s at latitude 45°. Also, the top of a 1 km thick column of air rises at 0.01 m/s, while its base rises at 0.008 m/s. Find the rate of geostrophic-vorticity spin-up.

Solution

Given: $V = -10$ m/s, $\phi = 45°$, $W_{top} = 0.01$ m/s, $W_{bottom} = 0.008$ m/s, $\Delta z = 1$ km.
Find: $\Delta\zeta_g/\Delta_t = ?$ s^{-2}

First, get the Coriolis parameter using eq. (10.16):
$f_c = (1.458\text{x}10^{-4}\ \text{s}^{-1})\cdot\sin(45°) = 0.000103\ \text{s}^{-1}$

Next, use eq. (13.2):

$$\beta = \frac{\Delta f_c}{\Delta y} = \frac{1.458\times10^{-4}\text{s}^{-1}}{6.357\times10^{6}\text{m}}\cdot\cos 45°$$
$$= 1.62\text{x}10^{-11}\ \text{m}^{-1}\cdot\text{s}^{-1}$$

Use the definition of a gradient (see Appendix A):

$$\frac{\Delta W}{\Delta z} = \frac{W_{top} - W_{bottom}}{z_{top} - z_{bottom}} = \frac{(0.01-0.008)\text{m/s}}{(1000-0)\text{m}}$$
$$= 2\text{x}10^{-6}\ \text{s}^{-1}$$

Finally, use eq. (13.10). We have no information about advection, so assume it is zero. The remaining terms give:

$$\underbrace{\frac{\Delta\zeta_g}{\Delta t}}_{\text{spin-up}} = \underbrace{-(-10\text{m/s})\cdot(1.62\times10^{-11}\text{m}^{-1}\cdot\text{s}^{-1})}_{\text{beta}}$$
$$\underbrace{+(0.000103\text{s}^{-1})\cdot(2\times10^{-6}\ \text{s}^{-1})}_{\text{stretching}}$$
$$=(1.62\text{x}10^{-10} + 2.06\text{x}10^{-10})\ \text{s}^{-2} = \underline{\mathbf{3.68x10}}^{-10}\ \text{s}^{-2}$$

Check: Units OK. Physics OK.
Discussion: Even without any initial geostrophic vorticity, the rotation of the Earth can spin-up the flow if the wind blows appropriately.

BEYOND ALGEBRA • The Laplacian

A Laplacian operator ∇^2 can be defined as

$$\nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$$

where A represents any variable. Sometimes we are concerned only with the horizontal (H) portion:

$$\nabla_H^2(A) = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}$$

What does it mean? If $\partial A/\partial x$ represents the slope of a line when A is plotted vs. x on a graph, then $\partial^2 A/\partial x^2 = \partial[\ \partial A/\partial x\ ]/\partial x$ is the change of slope; namely, the curvature.

How is it used? Recall from the Dynamics chapter that the geostrophic wind is defined as

$$U_g = -\frac{1}{f_c}\frac{\partial \Phi}{\partial y} \qquad V_g = \frac{1}{f_c}\frac{\partial \Phi}{\partial x}$$

where Φ is the geopotential ($\Phi = |g|\cdot z$). Plugging these into eq. (13.11) gives the geostrophic vorticity:

$$\zeta_g = \frac{\partial}{\partial x}\left(\frac{1}{f_c}\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{f_c}\frac{\partial \Phi}{\partial y}\right)$$

or

$$\boxed{\zeta_g = \frac{1}{f_c}\nabla_H^2(\Phi)} \qquad (13.11b)$$

This illustrates the value of the Laplacian — as a way to more concisely describe the physics.

For example, a low-pressure center corresponds to a low-height center on an isobaric sfc. That isobaric surface is concave up, which corresponds to positive curvature. Namely, the Laplacian of $|g|\cdot z$ is positive, hence, ζ_g is positive. Thus, a low has positive vorticity.

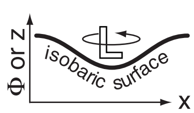


Figure f.

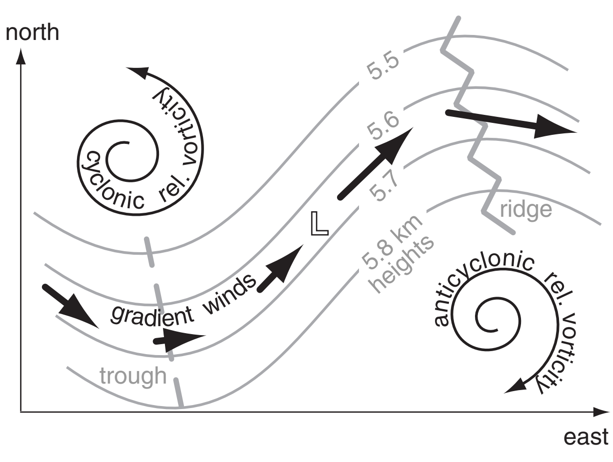


Figure 13.27
An idealized 50 kPa chart with equally-spaced height contours, as introduced by J. Bjerknes in 1937. The location of the surface low L is indicated.

The prefix "quasi-" is used for the following reasons. If the winds were perfectly geostrophic or gradient, then they would be parallel to the isobars. Such winds never cross the isobars, and could not cause convergence into the low. With no convergence there would be no vertical velocity.

However, we know from observations that vertical motions do exist and are important for causing clouds and precipitation in cyclones. Thus, the last term in the quasi-geostrophic vorticity equation includes W, a wind that is not geostrophic. When such an **ageostrophic** vertical velocity is included in an equation that otherwise is totally geostrophic, the equation is said to be **quasi-geostrophic**, meaning partially geostrophic. The quasi-geostrophic approximation will also be used later in this chapter to estimate vertical velocity in cyclones.

Within a quasi-geostrophic system, the vorticity and temperature fields are closely coupled, due to the dual constraints of geostrophic and hydrostatic balance. This implies close coupling between the wind and mass fields, as was discussed in the Global Circulation and Fronts chapters in the sections on geostrophic adjustment. While such close coupling is not observed for every weather system, it is a reasonable approximation for synoptic-scale, extratropical systems.

Application to Idealized Weather Patterns

An idealized weather pattern ("toy model") is shown in Fig. 13.27. Every feature in the figure is on the 50 kPa isobaric surface (i.e., in the mid troposphere), except the 𝕃 which indicates the location of the surface low center. All three components of the geostrophic vorticity equation can be studied.

Geostrophic and gradient winds are parallel to the height contours. The trough axis is a region of cyclonic (counterclockwise) curvature of the wind, which yields a large positive value of geostrophic vorticity. At the ridge is negative (clockwise) relative vorticity. Thus, the **advection** term is positive over the 𝕃 center and contributes to spin-up of the cyclone because the wind is blowing higher positive vorticity into the area of the surface low.

For any fixed pressure gradient, the gradient winds are slower than geostrophic when curving cyclonically ("slow around lows"), and faster than geostrophic for anticyclonic curvature, as sketched with the thick-line wind arrows in Fig. 13.27. Examine the 50 kPa flow immediately above the surface low. Air is departing faster than entering. This imbalance (divergence) draws air up from below. Hence, W increases from near zero at the ground to some positive updraft speed at 50 kPa. This **stretching** helps to spin-up the cyclone.

The **beta** term, however, contributes to spin-down because air from lower latitudes (with smaller Coriolis parameter) is blowing toward the location of the surface cyclone. This effect is small when the wave amplitude is small. The sum of all three terms in the quasigeostrophic vorticity equation is often positive, providing a net spin-up and intensification of the cyclone.

In real cyclones, contours are often more closely spaced in troughs, causing relative maxima in jet stream winds called jet streaks. Vertical motions associated with horizontal divergence in jet streaks are discussed later in this chapter. These motions violate the assumption that air mass is conserved along an "isobaric channel". Rossby also pointed out in 1940 that the gradient wind balance is not valid for varying motions. Thus, the "toy" model of Fig. 13.27 has weaknesses that limit its applicability.

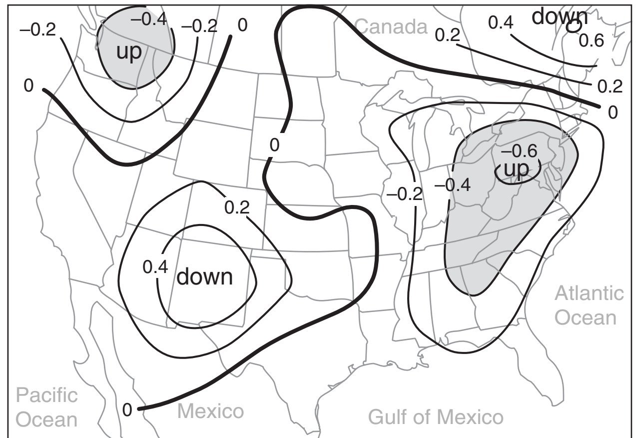


Figure 13.28
Vertical velocity (omega) in pressure coordinates, for the case-study cyclone. Negative omega corresponds to updrafts.

UPWARD MOTION

Upward vertical motion is another measure of cyclone vigor and cloud formation. For the case-study cyclone, Fig. 13.25 shows upward motion near the middle of the troposphere (at 50 kPa).

In height z coordinates, the vertical velocity is defined as $W = \Delta z/\Delta t$, where t is time. An analogous vertical velocity, **omega** (ω), can be defined in pressure coordinates:

$$\omega = \frac{\Delta P}{\Delta t} \qquad \bullet(13.13)$$

where units of omega are Pa/s.

Because pressure decreases with height, ω is negative for updrafts, while W is positive. Using the hydrostatic equation, the relationship between ω and W is:

$$\omega = -\rho \cdot |g| \cdot W \qquad \bullet(13.14)$$

where ρ is air density and $|g| = 9.8 \text{ m·s}^{-2}$ is gravitational acceleration magnitude. For example, at P = 50 kPa where standard atmospheric density is $\rho \approx 0.69 \text{ kg·m}^{-3}$, an updraft of W = 1 m/s gives $\omega = -6.8$ Pa/s. Either W or ω can be used to represent vertical motion.

Most numerical weather prediction models (see the NWP chapter) use ω. Fig. 13.28 shows upward motion (ω) near the middle of the troposphere (at 50 kPa).

We will use three approaches to investigate vertical motion: continuity equation, omega equation, and Q-vectors. The continuity approach shows how jet-stream winds diverging horizontally at the top of the troposphere cause vertical motion in the mid

Solved Example

At an elevation of 5 km MSL, suppose (a) a thunderstorm has an updraft velocity of 40 m/s, and (b) the subsidence velocity in the middle of an anticyclone is –0.01 m/s. Find the corresponding omega values.

Solution

Given: (a) W = 40 m/s. (b) W = – 0.01 m/s. z = 5 km.
Find: ω = ? kPa/s for (a) and (b).

To estimate air density, use the standard atmosphere table from Chapter 1: $\rho = 0.7361 \text{ kg/m}^3$ at z = 5 km.

Next, use eq. (13.14) to solve for the omega values:

(a) $\omega = -(0.7361 \text{ kg/m}^3)\cdot(9.8 \text{ m/s}^2)\cdot(40 \text{ m/s})$
$= -288.55 \text{ (kg·m}^{-1}\text{·s}^{-2})\text{/s} = -288.55$ Pa/s
= **-0.29 kPa/s**

(b) $\omega = -(0.7361 \text{ kg/m}^3)\cdot(9.8 \text{ m/s}^2)\cdot(-0.01 \text{ m/s})$
$= 0.0721 \text{ (kg·m}^{-1}\text{·s}^{-2})\text{/s} = 0.0721$ Pa/s
= **$7.21\text{x}10^{-5}$ kPa/s**

Check: Units OK. Sign OK.
Discussion: CAUTION. Remember that the sign of omega is opposite that of vertical velocity, because as height increases in the atmosphere, the pressure decreases. As a quick rule of thumb, near the surface where air density is greater, omega (in kPa/s) has magnitude of roughly a hundredth of W (in m/s), with opposite sign.

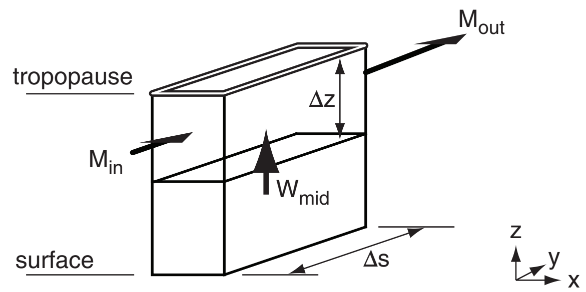


Figure 13.29
A column of tropospheric air, showing that mid-level vertical motion W is related to upper level divergence $M_{out} - M_{in}$, where M is wind speed.

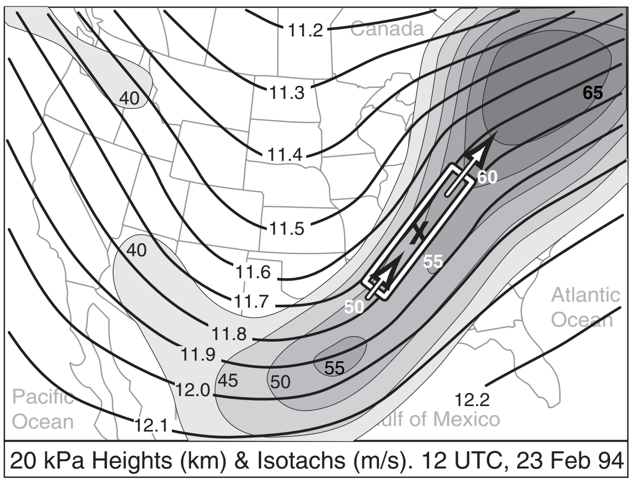


Figure 13.30
Over the surface cyclone (X) is a region (box) with faster jet-stream outflow than inflow (arrows). Isotachs are shaded.

> **Solved Example**
> Jet-stream inflow winds are 50 m/s, while outflow winds are 75 m/s a distance of 1000 km further downstream. What updraft is induced below this 5 km thick divergence region? Assume air density is 0.5 kg/m^3.
>
> **Solution**
> Given: M_{in} = 50 m/s, M_{out} = 75 m/s, Δs = 1000 km, Δz = 5 km.
> Find: W_{mid} = ? m/s
>
> Use eq. (13.17):
> $W_{mid} = [M_{out} - M_{in}]\cdot(\Delta z/\Delta s)$
> = [75 - 50 m/s]·[(5km)/(1000km)] = **0.125 m/s**
>
> **Check**: Units OK. Physics unreasonable, because the incompressible continuity equation assumes constant density — a bad assumption over a 5 km thick layer.
> **Discussion**: Although this seems like a small number, over an hour this updraft velocity would lift air about 450 m. Given enough hours, the rising air might reach its lifting condensation level, thereby creating a cloud or enabling a thunderstorm.

troposphere. The omega equation uses quasi-geostrophic dynamics in the bottom half of the troposphere to diagnose vertical motion from the vorticity and thermal wind. Q-vectors consider ageostrophic motions that help maintain quasi-geostrophic flow. All approaches give complementary insight.

Continuity Effects

Vertical motion in a cyclone is often driven by changes of horizontal wind speed of the upper-tropospheric jet stream. Mass continuity requires that horizontal divergence be compensated by vertical convergence, and vice versa. Namely, air leaving a region horizontally must be balanced by replacement air entering the same region vertically, so as not to leave a vacuum (Fig. 13.29). The jet stream is near the base of the stratosphere, where strong static stability aloft impedes vertical motion above the jet. Thus, most of the compensating vertical motion occurs in the troposphere, beneath the level of the jet.

For the column of air sketched in Fig. 13.29, the incompressible continuity equation becomes

$$W_{mid} = D \cdot \Delta z \tag{13.15}$$

or

$$W_{mid} = \left[\frac{\Delta U}{\Delta x} + \frac{\Delta V}{\Delta y} \right] \cdot \Delta z \tag{13.16}$$

or

$$W_{mid} = \frac{\Delta M}{\Delta s} \cdot \Delta z \tag{13.17}$$

where $\Delta M = M_{out} - M_{in}$ is the change of horizontal wind speed, Δs is horizontal distance between inflow and outflow locations, $\Delta M/\Delta s$ is the upper level **horizontal divergence** ($D = \Delta U/\Delta x + \Delta V/\Delta y$, see Fig. 13.17d), Δz is the thickness of the upper portion of the troposphere (between the 50 kPa surface and the tropopause), and W_{mid} is the vertical velocity across the 50 kPa surface. For simplicity, assume that all of horizontal divergence aloft can be represented by the winds in one direction, as sketched.

Fig. 13.30 shows this scenario for the case-study storm. Geostrophic winds are often nearly parallel to the height contours (solid black lines in Fig. 13.30). Thus, for the region outlined with the white box drawn parallel to the contour lines, the main inflow and outflow are at the ends of the box (arrows). The isotachs (shaded) tell us that the inflow (≈ 50 m/s) is slower than outflow (≈60 m/s).

Two mechanisms that cause jet-stream divergence are examined next: curvature of the jet stream, and jet streaks. The first is a large-scale feature in approximate geostrophic balance. The second is a smaller-scale feature that causes an ageostrophic (non-geostrophic) divergent flow.

Jet-Stream Curvature

Recall that air blows slower around troughs than around ridges, due to centrifugal force (see the Dynamics chapter). Consider a column of air situated east of an upper-level trough. The gradient wind speed horizontally entering the column is less than that leaving (Fig. 13.31). This results in a wind speed difference of ΔM across a distance $\Delta s = d$.

Assume the jet stream oscillates north-south as a sine wave. If the north-south distance between crest and trough is Δy [= 2 · (amplitude of wave)], and the east-west wavelength is λ, then the distance d between crest and trough is:

$$\Delta s = d = \left[(\lambda/2)^2 + \Delta y^2\right]^{1/2} \qquad (13.18)$$

Gradient wind speeds around anticyclones and cyclones were discussed in the Dynamics chapter. The speed difference between outflow (anticyclonic) and inflow (cyclonic) is

$$\Delta M = 0.5 \cdot f_c \cdot R \cdot \left[2 - \sqrt{1 - \frac{4 \cdot G}{f_c \cdot R}} - \sqrt{1 + \frac{4 \cdot G}{f_c \cdot R}}\right] \qquad (13.19)$$

where G is geostrophic wind speed. The radius of curvature R at crests and troughs of sine waves is:

$$R = \frac{1}{2\pi^2} \cdot \frac{\lambda^2}{\Delta y} \qquad (13.20)$$

The effect of upper-level divergence due to jet-stream curvature can be found by combining the equations above:

$$W_{mid} = \frac{\frac{f_c \cdot \Delta z \cdot \lambda^2}{4\pi^2 \cdot \Delta y}\left[2 - \sqrt{1 - \frac{8\pi^2 G \cdot \Delta y}{f_c \cdot \lambda^2}} - \sqrt{1 + \frac{8\pi^2 G \cdot \Delta y}{f_c \cdot \lambda^2}}\right]}{[(\lambda/2)^2 + \Delta y^2]^{1/2}} \qquad (13.21)$$

Thus, knowing the wavelength, amplitude, and geostrophic wind speed, it is possible to estimate the vertical motion.

Jet-stream curvature is one of the factors causing cyclogenesis for the case-study storm (Fig. 13.30). Near the trough over Texas, jet stream winds are 40 - 55 m/s. Near the ridge over the eastern Great Lakes, jet-stream winds are 60 - 68 m/s. For this case, the trough-to-crest distance is about Δs = 2200 km.

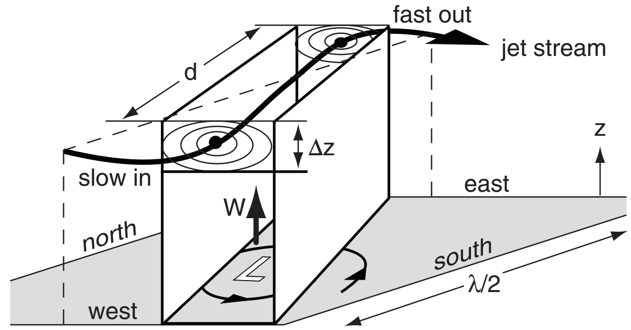


Figure 13.31
Factors affecting total mass within a column of air in the N. Hemisphere. The column is between the two vertical planes that are drawn.

Solved Example

Given a jet stream with thickness 5 km, air density 0.364 kg·m^{-3}, wavelength 6000 km, amplitude 500 km, and average geostrophic wind speed 30 m/s. Find the radius of curvature at the crest, distance d along the flow, inflow vs. outflow wind-speed difference, and mid-level vertical velocity.

Solution

Given: ρ_{top} = 0.364 kg·m^{-3}, G = 30 m/s,
Δz = 5 km, λ = 6000 km,
Δy = 2 · (500 km) = 1000 km.
Find: R = ? km, d = ? km, ΔM= ? m/s, W_{mid} = ? m/s
Assume: f_c = 0.0001 s^{-1}

Use eq. (13.20):

$$R = \frac{1}{2\pi^2} \cdot \frac{(6000\text{km})^2}{(1000\text{km})} = \mathbf{\underline{1824\ km}}$$

From Chapter 10 use:

$$\frac{G}{f_c \cdot R} = Ro_c = \frac{30\text{m/s}}{(0.0001\text{s}^{-1}) \cdot (1824\text{km})} = 0.164$$

Use eq. (13.19):

$$\Delta M = \frac{10\text{m/s}}{2(0.164)} \cdot \left[2 - \sqrt{1 - 4(0.164)} - \sqrt{1 + 4(0.164)}\right]$$

$$= \mathbf{\underline{11.6\ m/s}}$$

Use eq. (13.18):
d = [(3000km)^2 + (1000km)^2]$^{1/2}$ = **3162 km**

Use eq. (13.17):

$$W_{mid} = \left(\frac{11.6\text{m/s}}{3162\text{km}}\right) \cdot 5\text{km} = \mathbf{\underline{0.018\ m/s}}$$

Check: Units OK. Physics OK.
Discussion: A vertical velocity of a couple cm/s averaged over the whole cyclone is typical, although local updrafts in clouds can be faster.

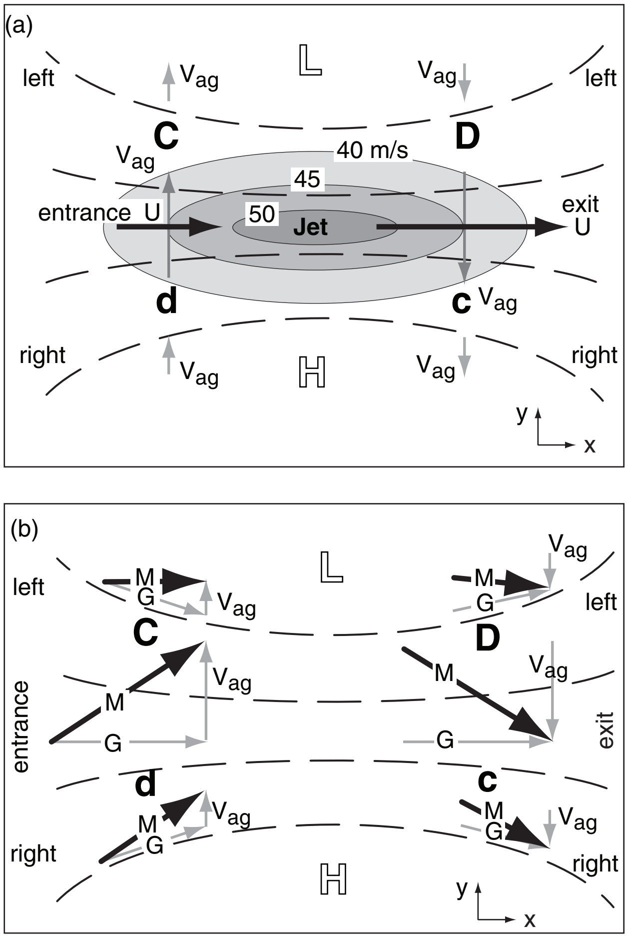


Figure 13.32
Horizontal divergence (D = strong, d = weak) and convergence (C = strong, c = weak) near a jet streak. Shown are isobars or height contours (dashed lines), and ageostrophic components V_{ag} *of wind (thin grey arrows) . (a) Isotachs (shaded contours), jet stream wind and jet axis (heavy arrows), (b) Geostrophic wind G is parallel to the isobars (also shown with grey arrows). The vector sum of the gray vectors is the total wind speed M, shown by the heavy black vectors.*

Solved Example
A west wind of 60 m/s in the center of a jet streak decreases to 40 m/s in the jet exit region 500 km to the east. Find the exit ageostrophic wind component.

Solution
Given: ΔU = 40–60 m/s = –20 m/s, across Δx = 500 km.
Find: V_{ag} = ? m/s

Use eq. (13.25), and assume $f_c = 10^{-4}$ s^{-1}.
The average wind is U = (60 + 40 m/s)/2 = 50 m/s.
V_{ag} = [(50 m/s) / (10^{-4} s^{-1})] · [(–20 m/s) / (5x10^5 m)]
= **<u>–20 m/s</u>**

Check: Units OK. Sign OK. Physics OK.
Discussion: Negative sign means V_{ag} is north wind.

Jet Streaks

A jet streak is a relative maximum of wind speed within the jet stream. Picture a jet streak as sketched in Fig. 13.32a, blowing from west to east. This forms in the upper-tropospheric jet stream in regions where isobars or height contours are tightly packed (i.e., closely spaced). Wind speed reaches a maximum at the center of the streak.

Wind cannot only advect temperature or humidity, but it can also advect itself. As a result, the wind maximum advects itself eastward. An outcome is that the jet maximum finds itself in a region where the isobars are further apart. Namely, the actual wind speed in the exit region of the streak is faster than the theoretical geostrophic wind. This is a situation that causes an ageostrophic (i.e., not in geostrophic balance) wind to the south.

To quantify this effect, start with the full equations of motion from the Dynamics chapter (eqs. 10.51a). Consider only the equation for U wind (assuming a jet aligned west to east), assume approximate steady state (no time change), and neglect boundary-layer drag. The result is:

$$0 = -U\frac{\Delta U}{\Delta x} + f_c\left(V - V_g\right) \qquad (13.22)$$

Define an **ageostrophic wind** V_{ag} as the difference between the actual wind and the geostrophic wind:

$$V_{ag} = V - V_g \qquad \bullet(13.23)$$

Similarly:

$$U_{ag} = U - U_g \qquad \bullet(13.24)$$

With this definition, the equation of motion becomes

$$V_{ag} = \frac{U}{f_c}\cdot\frac{\Delta U}{\Delta x} \qquad \bullet(13.25)$$

For a north-south aligned jet axis, the corresponding ageostrophic wind would be

$$U_{ag} = -\frac{V}{f_c}\cdot\frac{\Delta V}{\Delta y} \qquad \bullet(13.26)$$

In the **exit region** of a west-east aligned jet, U is positive but $\Delta U/\Delta x$ is negative (wind decreases eastward). Thus, eq. (13.25) says that a negative V_{ag}

ageostrophic wind develops (i.e., blows from north to south). In the **entrance region** the U wind increases in the x direction, causing a positive V_{ag} (i.e., blows from south to north, as sketched in Fig. 13.32a).

The magnitude of ageostrophic wind is largest near the jet axis, it decreases in magnitude further off to the side of the jet. As a result, horizontal divergence of the ageostrophic wind exists in the **left exit region** of the jet, and convergence in the right exit region, regardless of the jet orientation. A similar but opposite convergence-divergence pair occurs in the entrance region. These four regions are called the **four quadrants** of the jet streak (Fig. 13.35).

The total wind speed M is the vector sum of the geostrophic components G parallel to the isobars, and the ageostrophic components V_{ag} as indicated in Fig. 13.32b. Focusing on the total wind speed (thick black vectors in Fig. 13.32 b), one can see strong convergence in the left entrance region (i.e., the vectors come towards each other at a large angle). Similarly, large divergence exists in the left exit region.

In the right entrance and exit regions, however, the total wind vectors are almost parallel to the wind vectors along the center axis of the jet. Thus, divergence and convergence are weaker there.

This ageostrophic behavior can also be seen in the case-study event (Fig. 13.33). This figure overlays wind vectors, isotachs, and geopotential height contours near the top of the troposphere (at 20 kPa). The broad area of shading shows the jet stream. Embedded within it are two relative speed maxima (one over Texas, and the other over New England) that we identify as jet streaks. Recall that if winds are geostrophic (or gradient), then they should flow parallel to the height contours.

In Fig. 13.33 the square highlights the exit region of the Texas jet streak, showing wind vectors that cross the height contours toward the right. Namely, inertia has caused these winds to be faster than geostrophic (**supergeostrophic**), therefore Coriolis force is stronger than pressure-gradient force, causing the winds to be to the right of geostrophic. The oval highlights the entrance region of the second jet, where winds cross the height contours to the left. Inertia results in slower-than-geostrophic winds (**subgeostrophic**), causing the Coriolis force to be too weak to counteract pressure-gradient force.

Fig. 13.34 shows the vertical and horizontal circulations induced by the ageostrophic motion near the exit region of the jet streak. Such a flow is called a **secondary circulation**. The vertical components of this circulation, which are caused by the convergence and divergence regions aloft, also induce a pair of horizontal divergence and convergence at the surface.

FOCUS • Ageostrophic right-hand rule

If the geostrophic winds are accelerating, use your right hand to curl your fingers from vertical toward the direction of acceleration (the acceleration vector). Your thumb points in the direction of the ageostrophic wind.

This right-hand rule also works for deceleration, for which case the direction of acceleration is opposite to the wind direction.

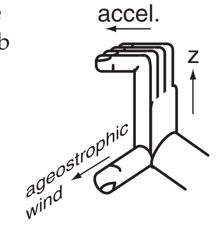


Figure g.

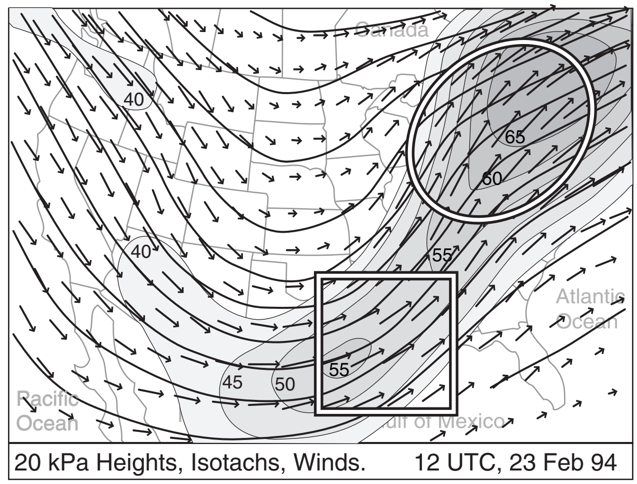


Figure 13.33
Superposition of the 20 kPa charts for geopotential heights (medium-thickness black curved lines), isotachs in m/s (shading), and winds (vectors). The scale for winds and the values for the height contours are identical to those in Figs. 13.17. Regions of relatively darker shading indicate the jet streaks. White/black square outlines the exit region from a small jet streak over Texas, and white/black oval outlines the entrance region for a larger jet streak over the northeastern USA.

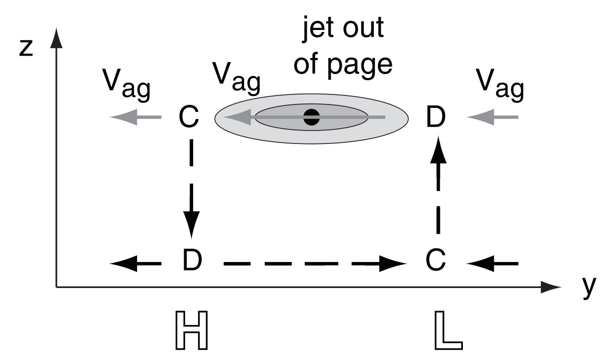


Figure 13.34
Vertical cross-section through the exit region of a jet (shaded), where the jet points out of the page. The V_{ag} ageostrophic winds (thin grey lines) are driven by jet-streak dynamics, while the dashed circulation is driven by mass continuity. Regions of horizontal convergence C and divergence D are indicated. This circulation creates or strengthens low (𝕃) and high (ℍ) pressure centers near the surface.

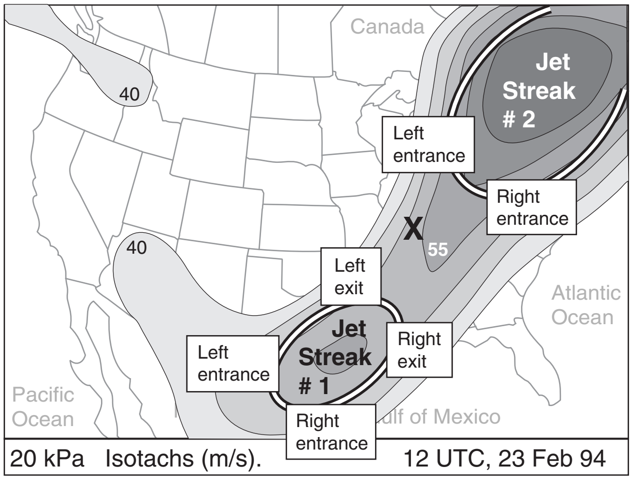


Figure 13.35
Entrance and exit regions of jet streaks (highlighted with ovals). X marks the surface low center.

Solved Example
A jet streak of width 1000 km has a core wind speed of 50 m/s that decreases eastward to 25 m/s across a distance of 1000 km. If the jet region is 5 km thick, find the mid-level vertical velocity. Assume $f_c = 10^{-4}$ s^{-1}.

Solution
Given: $U = 50$ m/s, $\Delta U = 25 - 50 = -25$ m/s,
$\Delta x = \Delta y = 10^6$ m, $\Delta z = 5000$ m.
Find: W_{mid} = ? m/s

Use eq. (13.27):

$$W_{mid} = \frac{(50\text{m/s})\cdot(25\text{m/s})}{(10^{-4}\text{s}^{-1})\cdot(10^6\text{m})}\cdot\frac{(5000\text{m})}{(10^6\text{m})} = \underline{\mathbf{0.0625\ m/s}}$$

Check: Units OK. Physics OK.
Discussion: This is a larger average vertical velocity over a much smaller horizontal area than for the previous solved example. Such a vertical velocity can significantly intensify a cyclone.

FOCUS • Sutcliffe Development Theorem

To help forecast cyclogenesis, Sutcliffe devised

$$D_{top} - D_{bottom} = -\frac{1}{f_c}\left[U_{TH}\frac{\Delta\zeta_{gc}}{\Delta x} + V_{TH}\frac{\Delta\zeta_{gc}}{\Delta y}\right]$$

where **divergence** is $D = \Delta U/\Delta x + \Delta V/\Delta y$, column geostrophic vorticity is $\zeta_{gc} = \zeta_{g\ top} + \zeta_{g\ bottom} + f_c$, and (U_{TH}, V_{TH}) are the thermal-wind components.

This says that if the vorticity in an air column is positively advected by the thermal wind, then this must be associated with greater air divergence at the column top than bottom. When combined with eq. (13.15), this conclusion for upward motion is nearly identical to that from the Trenberth omega eq. (13.29).

The sense (direction) of this circulation below the **exit region** of the jet is **indirect** (opposite to the sense of rotation of a Hadley cell). The sense below the **entrance region** is **direct**.

Regions of divergence aloft remove mass from the column of air, thereby lowering sea-level pressure. Such sea-level pressure drop and associated upward motion cause cyclogenesis. Hence, <u>cyclones are favored below the left exit region and right entrance region of the jet streak</u>.

Looking again at the case-study storm, Fig. 13.35 shows the entrance and exit regions of the two dominant jet streaks in this image (for now, ignore the smaller jet streak over the Pacific Northwest). Thus, you can expect divergence aloft at the left exit and right entrance regions. These are locations that would favor cyclogenesis near the ground. Indeed, a new cyclone formed over the Carolinas (under the right entrance region of jet streak # 2), as was described in the case-study overview. Convergence aloft, favoring cyclolysis (cyclone death), is at the left entrance and right exit regions.

To find the mid-level vertical velocity below the left-exit or right-entrance quadrant, let $\Delta s = \Delta y$ be the north-south half-width of a predominantly west-to-east jet streak. Assume the ageostrophic wind V_{ag} smoothly approaches zero within a distance Δy to the side of the jet. Using eq. (13.25) in eq. (13.16) with V_{ag} in place of M, the upward motion is

$$W_{mid} = \left|\frac{U\cdot\Delta U}{f_c\cdot\Delta x}\right|\cdot\frac{\Delta z}{\Delta y} \qquad (13.27)$$

Omega Equation

The **omega equation** is the name of a diagnostic equation used to find vertical motion in pressure units (omega; ω). We will use a form of this equation developed by K. Trenberth, based on quasi-geostrophic dynamics and thermodynamics.

The full omega equation is one of the nastier-looking equations in meteorology (see the Beyond Algebra box). To simplify it, focus on one part of the full equation, apply it to the bottom half of the troposphere (the layer between 100 to 50 kPa isobaric surfaces), and convert the result from ω to W.

The resulting approximate omega equation is:

$$W_{mid} \cong \frac{-2\cdot\Delta z}{f_c}\left[U_{TH}\frac{\overline{\Delta\zeta_g}}{\Delta x} + V_{TH}\frac{\overline{\Delta\zeta_g}}{\Delta y} + V_{TH}\frac{\beta}{2}\right] \qquad \bullet(13.28)$$

where W_{mid} is the vertical velocity in the mid-troposphere (at $P = 50$ kPa), Δz is the 100 to 50 kPa thickness, U_{TH} and V_{TH} are the thermal-wind components for the 100 to 50 kPa layer, f_c is Coriolis parameter, β is the change of Coriolis parameter with y (see eq. 13.2), ζ_g is the geostrophic vorticity, and the overbar represents an average over the whole depth of the layer. An equivalent form is:

$$W_{mid} \cong \frac{-2 \cdot \Delta z}{f_c}\left[M_{TH} \; \overline{\frac{\Delta\left(\zeta_g + (f_c / 2)\right)}{\Delta s}} \right] \qquad \bullet(13.29)$$

where s is distance along the thermal wind direction, and M_{TH} is the thermal-wind speed.

Regardless of the form, the terms in square brackets represent the advection of vorticity by the thermal wind, where vorticity consists of the geostrophic relative vorticity plus a part of the vorticity due to the Earth's rotation. The geostrophic vorticity at the 85 kPa or the 70 kPa isobaric surface is often used to approximate the average geostrophic vorticity over the whole 100 to 50 kPa layer.

A physical interpretation of the omega equation is that greater upward velocity occurs where there is greater advection of cyclonic (positive) geostrophic vorticity by the thermal wind. Greater upward velocity favors clouds and heavier precipitation. Also, by moving air upward from the surface, it reduces the pressure under it, causing the surface low to move toward that location and deepen.

Weather maps can be used to determine the location and magnitude of the maximum upward motion. The idealized map of Fig. 13.36a shows the height (z) contours of the 50 kPa isobaric surface, along with the trough axis. Also shown is the location of the surface low and fronts.

At the surface, the greatest vorticity is often near the low center. At 50 kPa, it is often near the trough axis. At 70 kPa, the vorticity maximum (**vort max**) is usually between those two locations. In Fig. 13.36a, the darker shading corresponds to regions of greater cyclonic vorticity at 70 kPa.

Fig. 13.36b shows the thickness (Δz) of the layer of air between the 100 and 50 kPa isobaric surfaces. Thickness lines are often nearly parallel to surface fronts, with the tightest packing on the cold side of the fronts. Recall that thermal wind is parallel to the thickness lines, with cold air to the left, and with the greatest velocity where the thickness lines are most tightly packed. Thermal wind direction is represented by the arrows in Fig. 13.36b, with longer arrows denoting stronger speed.

Advection is greatest where the area between crossing isopleths is smallest (the Focus Box on the next page explains why). This rule also works for advection by the thermal wind. The dotted lines represent the isopleths that drive the thermal wind.

Solved Example

The 100 to 50 kPa thickness is 5 km and $f_c = 10^{-4}$ s^{-1}. A west to east thermal wind of 20 m/s blows through a region where avg. cyclonic vorticity decreases by 10^{-4} s^{-1} toward the east across a distance of 500 km. Use the omega eq. to find mid-tropospheric upward speed.

Solution

Given: $U_{TH} = 20$ m/s, $V_{TH} = 0$, $\Delta z = 5$ km,
$\Delta\zeta = -10^{-4}$ s^{-1}, $\Delta x = 500$ km, $f_c = 10^{-4}$ s^{-1}.
Find: W_{mid} = ? m/s

Use eq. (13.28):

$$W_{mid} \cong \frac{-2 \cdot (5000\text{m})}{(10^{-4}\text{s}^{-1})}\left[(20\text{m/s})\frac{(-10^{-4}\text{s}^{-1})}{(5\times 10^{5}\text{m})} + 0 + 0 \right] = \underline{\mathbf{0.4\ m/s}}$$

Check: Units OK. Physics OK.
Discussion: At this speed, an air parcel would take 7.6 h to travel from the ground to the tropopause.

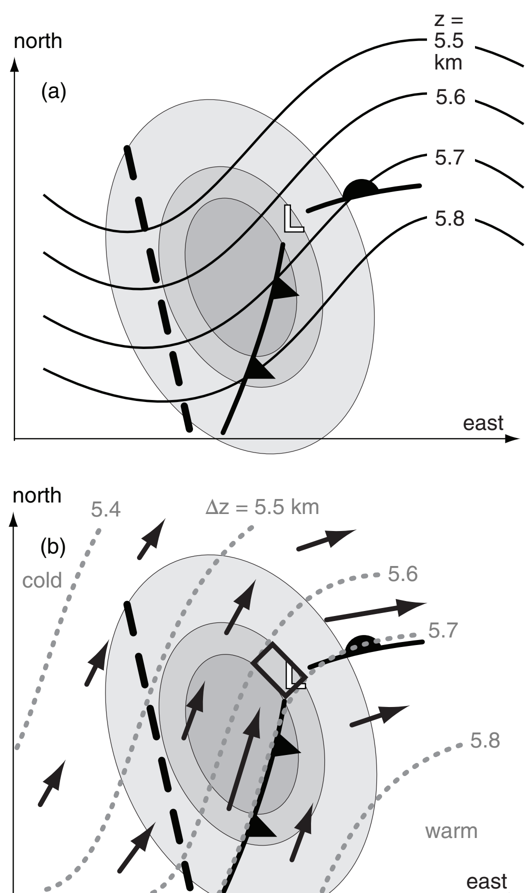


Figure 13.36
(a) Weather at three different pressure heights: (1) 50 kPa heights (solid lines) and trough axis (thick dashed line); (2) surface low pressure center (L) and fronts; (3) 70 kPa vorticity (shaded).
(b) Trough axis, surface low and fronts, and vorticity shading are identical to Fig. (a). Added are: 100 to 50 kPa thickness (dotted lines), thermal wind vectors (arrows), and region of maximum positive vorticity advection by the thermal wind (rectangular box). It is within this box that the omega equation gives the greatest updraft speed, which support cyclogenesis.

BEYOND ALGEBRA • The Omega Eq.

Full Omega Equation

The omega equation describes vertical motion in pressure coordinates. One form of the quasi-geostrophic omega equation is:

$$\left\{\nabla_p^2 + \frac{f_o^{\,2}}{\sigma}\frac{\partial^2}{\partial p^2}\right\}\omega = \frac{-f_o}{\sigma}\cdot\frac{\partial}{\partial p}\left[-\vec{V}_g \bullet \vec{\nabla}_p\left(\zeta_g + f_c\right)\right]$$

$$-\frac{\Re}{\sigma\cdot p}\cdot\nabla_p^2\left[-\vec{V}_g \bullet \vec{\nabla}_p T\right]$$

where f_o is a reference Coriolis parameter f_c at the center of a beta plane, σ is a measure of static stability, V_g is a vector geostrophic wind, ℜ is the ideal gas law constant, p is pressure, T is temperature, ζ_g is geostrophic vorticity, and • means vector dot product.

$\vec{\nabla}_p(\,) = \partial(\,)/\partial x|_p + \partial(\,)/\partial y|_p$ is the **del operator**,

which gives quasi-horizontal derivatives along an isobaric surface. Another operator is the **Laplacian**:

$$\nabla^2{}_p(\,) = \partial^2(\,)/\partial x^2\Big|_p + \partial^2(\,)/\partial y^2\Big|_p \quad .$$

Although the omega equation looks particularly complicated and is often shown to frighten unsuspecting people, it turns out to be virtually useless. The result of this equation is a small difference between very large terms on the RHS that often nearly cancel each other, and which can have large error.

Trenberth Omega Equation

Trenberth developed a more useful form that avoids the small difference between large terms:

$$\left\{\nabla_p^2 + \frac{f_o^{\,2}}{\sigma}\frac{\partial^2}{\partial p^2}\right\}\omega = \frac{2 f_o}{\sigma}\cdot\left[\frac{\partial \vec{V}_g}{\partial p} \bullet \vec{\nabla}_p\left(\zeta_g + (f_c/2)\right)\right]$$

For the omega subsection of this chapter, we focus on the vertical (pressure) derivative on the LHS, and ignore the Laplacian. This leaves:

$$\frac{f_o^{\,2}}{\sigma}\frac{\partial^2\omega}{\partial p^2} = \frac{2 f_o}{\sigma}\cdot\left[\frac{\partial \vec{V}_g}{\partial p} \bullet \vec{\nabla}_p\left(\zeta_g + (f_c/2)\right)\right]$$

Upon integrating over pressure from p = 100 to 50 kPa:

$$\frac{\partial\omega}{\partial p} = \frac{-2}{f_o}\cdot\left[\vec{V}_{TH} \bullet \overline{\vec{\nabla}_p\left(\zeta_g + (f_c/2)\right)}\right]$$

where the definition of thermal wind V_{TH} is used, along with the mean value theorem for the last term.

The hydrostatic eq. is used to convert the LHS: $\partial\omega/\partial p = \partial W/\partial z$. The whole eq. is then integrated over height, with $W = W_{mid}$ at $z = \Delta z$ (= 100 - 50 kPa thickness) and W = 0 at z = 0.

This gives W_{mid} =

$$\frac{-2\cdot\Delta z}{f_c}\left[U_{TH}\,\frac{\overline{\Delta\left(\zeta_g + (f_c/2)\right)}}{\Delta x} + V_{TH}\,\frac{\overline{\Delta\left(\zeta_g + (f_c/2)\right)}}{\Delta y}\right]$$

But f_c varies with y, not x. The result is eq. (13.28).

FOCUS • Max Advection on Wx Maps

One trick to locating the region of maximum advection is to find the region of smallest area between crossing isopleths on a weather (wx) map, where one set of isopleths must define a wind.

For example, consider temperature advection by the geostrophic wind. Temperature advection will occur only if the winds blow across the isotherms at some nonzero angle. Stronger temperature gradient with stronger wind component perpendicular to that gradient gives stronger temperature advection.

But stronger geostrophic winds are found where the isobars are closer together. Stronger temperature gradients are found where the isotherms are closer together. In order for the winds to cross the isotherms, the isobars must cross the isotherms. Thus, the greatest temperature advection is where the tightest isobar packing crosses the tightest isotherm packing. At such locations, the area bounded between neighboring isotherms and isobars is smallest.

This is illustrated in the surface weather map below, where the smallest area is shaded to mark the maximum temperature advection. There is a jet of strong geostrophic winds (tight isobar spacing) running from northwest to southeast. There is also a front with strong temperature gradient (tight isotherm spacing) from northeast to southwest. However, the place where the jet and temperature gradient together are strongest is the shaded area.

Each of the odd-shaped tiles (**solenoids**) between crossing isobars and isotherms represents the same amount of temperature advection. But larger tiles imply that temperature advection is spread over larger areas. Thus, greatest temperature flux (temperature advection per unit area) is at the smallest tiles.

This approach works for other variables too. If isopleths of vorticity and height contours are plotted on an upper-air chart, then the smallest area between crossing isopleths indicates the region of maximum **vorticity advection** by the geostrophic wind. For vorticity advection by the **thermal wind**, plot isoplets of vorticity vs. **thickness contours**.

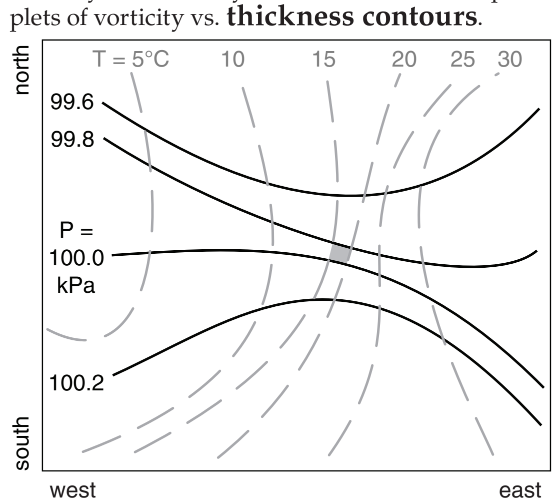
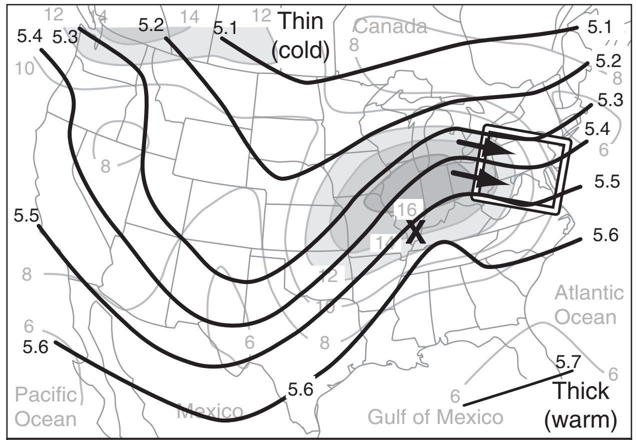


Fig. h. *Solid lines are isobars. Grey dashed lines are isotherms. Greatest temperature advection is at shaded tile.*

Figure 13.37 (right)
Superposition of the vorticity chart (grey lines and shading) at 85 kPa with the chart for thickness (thick black lines) between the 100 and 50 kPa isobaric surfaces, for the case-study storm. The thermal wind (arrows) blows parallel to the thickness lines with cold air to its left. The white box highlights a region of positive vorticity advection (PVA) by the thermal wind, where updrafts, cyclogenesis, and bad weather would be expected.



85 kPa Vorticity & 100-50 kPa Thickness. 12 UTC, 23 Feb 94

In Fig. 13.36 the thin black lines around the shaded areas are isopleths of vorticity. The solenoid at the smallest area between these crossing isopleths indicates the greatest vorticity advection by the thermal wind, and is outlined by a rectangular box. For this particular example, the greatest updraft would be expected within this box.

Be careful when you identify the smallest area. In Fig. 13.36b, another area equally as small exists further south-south-west from the low center. However, the cyclonic vorticity is being advected away from this region rather than toward it. Hence, this is a region of negative vorticity advection by the thermal wind, which would imply downward vertical velocity and cyclolysis or anticyclogenesis.

To apply these concepts to the case-study storm, Fig. 13.37 superimposes the 85 kPa vorticity chart with the 100 - 50 kPa thickness chart. The white box highlights a region of small solenoids, with the thermal wind blowing from high towards low vorticity. Hence, the white box outlines an area of **positive vorticity advection** (**PVA**) by the thermal wind, so anticipate substantial updrafts in that region. Such updrafts would create bad weather (clouds and precipitation), and would encourage cyclogenesis in the region outlined by the white box.

Near the surface low center (marked by the X in Fig. 13.37) is weak negative vorticity advection. This implies downdrafts, which contribute to cyclolysis. This agrees with the actual cyclone evolution, which began weakening at this time, while a new cyclone formed near the Carolinas and moved northward along the USA East Coast.

The Trenberth **omega equation** is heavily used in weather forecasting to help diagnose synoptic-scale regions of updraft and the associated cyclogenesis, cloudiness and precipitation. However, in the derivation of the omega equation (which we did not cover in this book), we neglected components that describe the role of ageostrophic motions in helping to maintain geostrophic balance. The Focus box on the Geostrophic Paradox describes the difficulties of maintaining geostrophic balance in some situations — motivation for Hoskin's Q-vector approach described next.

FOCUS • The Geostrophic Paradox

Consider the entrance region a jet streak. Suppose that the thickness contours are initially zonal, with cold air to the north and warm to the south (Fig. i(a)). As entrance winds (black arrows in Fig. i(a) converge, warm and cold air are advected closer to each other. This causes the thickness contours to move closer together (Fig. i(b), in turn suggesting tighter packing of the height contours and faster geostrophic winds at location "X" via the thermal wind equation. But the geostrophic wind in Fig. i(a) is advecting slower wind speeds to location "X".

Paradox: advection of the geostrophic wind by the geostrophic wind seems to undo geostrophic balance at "X".

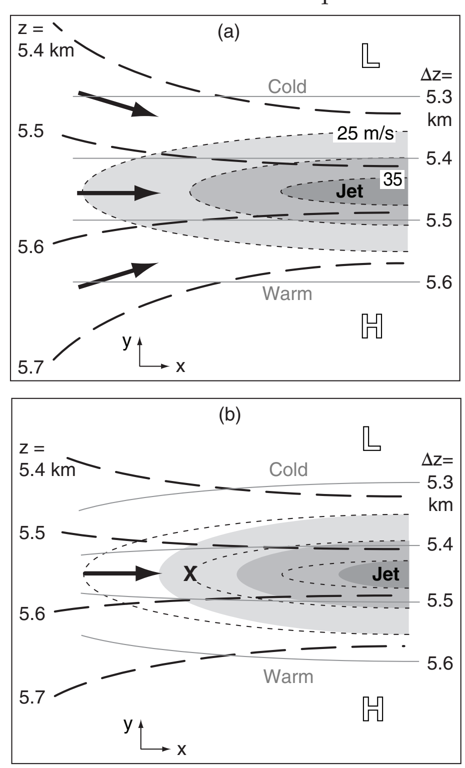


Fig. i. *Entrance region of jet streak on a 50 kPa isobaric surface. z is height (black dashed lines), Δz is thickness (thin grey lines), shaded areas are wind speeds, with initial isotachs as dotted black lines. L & H are low and high heights. (a) Initially. (b) Later.*

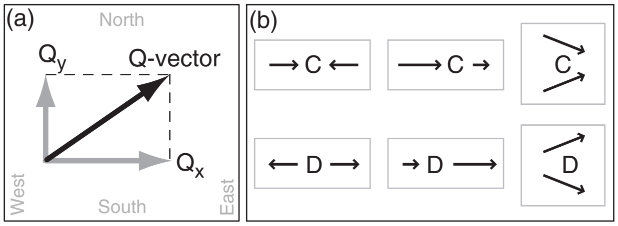


Figure 13.38
(a) Components of a Q-vector. (b) How to recognize patterns of vector convergence (C) and divergence (D) on weather maps.

Solved Example

Given the weather map at right showing the temperature and geostrophic wind fields over the NE USA. Find the Q-vector at the "X" in S.E. Pennsylvania. Side of each grid square is 100 km, and corresponds to G = 5 m/s for the wind vectors.

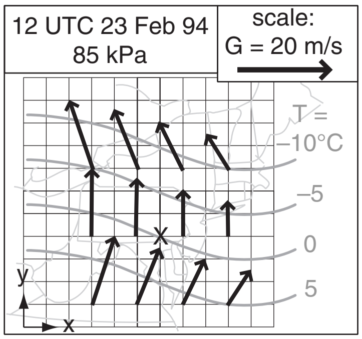


Figure j

Solution

Given: P = 85 kPa, G (m/s) & T (°C) fields on map.
Find: Q_x & Q_y = ? $m^2 \cdot s^{-1} \cdot kg^{-1}$

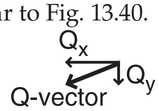
First, <u>estimate</u> U_g, V_g, and T gradients from the map.
$\Delta T/\Delta x = -5°C/600km$, $\Delta T/\Delta y = -5°C/200km$,
$\Delta U_g/\Delta x = 0$, $\Delta V_g/\Delta x = (-2.5m/s)/200km$
$\Delta U_g/\Delta y = (-5m/s)/300km$, $\Delta V_g/\Delta y = 0$,
$\Re/P = 0.287/85 = 0.003376\ m^3 \cdot kg^{-1} \cdot K^{-1}$

Use eq. (13.30): $Q_x = -(0.003376\ m^3 \cdot kg^{-1} \cdot K^{-1}) \cdot$
$[((0)\cdot(-8.3) + (-12.5)\cdot(-25)] \cdot 10^{-12}\ K \cdot m^{-1} \cdot s^{-1}$
$Q_x = \underline{\mathbf{-1.06x10^{-12}}}\ m^2 \cdot s^{-1} \cdot kg^{-1}$

Use eq. (13.31): $Q_y = -(0.003376\ m^3 \cdot kg^{-1} \cdot K^{-1}) \cdot$
$[((-16.7)\cdot(-8.3) + (0)\cdot(-25)] \cdot 10^{-12}\ K \cdot m^{-1} \cdot s^{-1}$
$Q_y = \underline{\mathbf{-0.47x10^{-12}}}\ m^2 \cdot s^{-1} \cdot kg^{-1}$

Use eq. (13.32) to find Q-vector magnitude:
$|Q| = [(-1.06)^2 + (-0.47)^2]^{1/2} \cdot 10^{-12}\ K \cdot m^{-1} \cdot s^{-1}$
$|Q| = \underline{\mathbf{1.16x10^{-12}}}\ m^2 \cdot s^{-1} \cdot kg^{-1}$

Check: Units OK. Physics OK. Similar to Fig. 13.40.
Discussion: The corresponding Q-vector is shown at right; namely, it is pointing from the NNE because both Q_x and Q_y are negative. There was obviously a lot of computations needed to get this one Q-vector. Luckily, computers can quickly compute Q-vectors for many points in a grid, as shown in Fig. 13.40. Normally, you don't need to worry about the units of the Q-vector. Instead, just focus on Q-vector convergence zones such as computers can plot (Fig. 13.41), because these zones are where the bad weather is.



Q-Vectors

Q-vectors allow an alternative method for diagnosing vertical velocity that does not neglect as many terms.

Defining Q-vectors

Define a horizontal Q-vector (units $m^2 \cdot s^{-1} \cdot kg^{-1}$) with x and y components as follows:

$$Q_x = -\frac{\Re}{P} \cdot \left[\left(\frac{\Delta U_g}{\Delta x} \cdot \frac{\Delta T}{\Delta x} \right) + \left(\frac{\Delta V_g}{\Delta x} \cdot \frac{\Delta T}{\Delta y} \right) \right] \quad (13.30)$$

$$Q_y = -\frac{\Re}{P} \cdot \left[\left(\frac{\Delta U_g}{\Delta y} \cdot \frac{\Delta T}{\Delta x} \right) + \left(\frac{\Delta V_g}{\Delta y} \cdot \frac{\Delta T}{\Delta y} \right) \right] \quad (13.31)$$

where $\Re = 0.287\ kPa \cdot K^{-1} \cdot m^3 \cdot kg^{-1}$ is the gas constant, P is pressure, (U_g, V_g) are the horizontal components of geostrophic wind, T is temperature, and (x, y) are eastward and northward horizontal distances. On a weather map, the Q_x and Q_y components at any location are used to draw the Q-vector at that location, as sketched in Fig. 13.38a. Q-vector magnitude is

$$|Q| = \left(Q_x^{\ 2} + Q_y^{\ 2} \right)^{1/2} \quad (13.32)$$

Estimating Q-vectors

Eqs. (13.30 - 13.32) seem non-intuitive in their existing Cartesian form. Instead, there is an easy way to estimate Q-vector direction and magnitude using weather maps. First, look at direction.

Suppose you fly along an isotherm (Fig. 13.39) in the direction of the thermal wind (in the direction that keeps cold air to your left). Draw an arrow describing the geostrophic wind vector that you observe at the start of your flight, and draw a second arrow showing the geostrophic wind vector at the end of your flight. Next, draw the vector difference, which points from the head of the initial vector to the head of the final vector. The Q-vector direction points 90° to the right (clockwise) from the geostrophic difference vector.

The magnitude is

$$|Q| = \frac{\Re}{P} \left| \frac{\Delta T}{\Delta n} \cdot \frac{\Delta \vec{V}_g}{\Delta s} \right| \quad (13.33)$$

where Δn is perpendicular distance between neighboring isotherms, and where the temperature difference between those isotherms is ΔT. Stronger baroclinic zones (namely, more tightly packed isotherms) have larger temperature gradient $\Delta T/\Delta n$. Also, Δs

is distance of your flight along one isotherm, and ΔV_g is the magnitude of the geostrophic difference vector from the previous paragraph. Thus, greater change of geostrophic wind in stronger baroclinic zones has a larger Q-vector. Furthermore, Q-vector magnitude increases with the decreasing pressure P found at increasing altitude.

Using Q-vectors / Forecasting Tips

Different locations usually have different Q-vectors, as sketched in Fig. 13.40 for the case-study storm. Interpret Q-vectors on a synoptic weather map as follows:

- Updrafts occur where Q-vectors converge (Fig. 13.41 gives an example for the case-study storm).
- Subsidence (downward motion) occurs where Q-vectors diverge.
- Frontogenesis occurs where Q-vectors cross isentropes (lines of constant potential temperature) from cold toward warm.
- Updrafts in the TROWAL region ahead of a warm occluded front occur during cyclolysis where the along-isentrope component of Q-vectors converge.

Using the tricks for visually recognizing patterns of vectors on weather maps (Fig. 13.38b), you can identify by eye regions of convergence and divergence in Fig. 13.40. Or you can let the computer analyze the Q-vectors directly to plot Q-vector convergence and divergence (Fig. 13.41). Although Figs. 13.40 and 13.41 are analysis maps of current weather, you can instead look at Q-vector forecast maps as produced automatically by numerical weather prediction models (see the NWP chapter) to help you forecast regions of updraft, clouds, and precipitation.

Remember that Q-vector convergence indicates regions of likely synoptic-scale upward motion and associated clouds and precipitation. Looking at Fig. 13.41, see a moderate convergence region running from the western Gulf of Mexico up through eastern Louisiana and southern Mississippi. It continues as a weak convergence region across Alabama and Georgia, and then becomes a strong convergence region over West Virginia, Virginia and Maryland. A moderate convergence region extend northwest toward Wisconsin.

This interpretation agrees with the general locations of radar echoes of precipitation plotted in Fig. 13.12d. Note that the frontal locations, as analyzed in Fig. 13.12d do not correspond to the precipitation regions. This demonstrates the utility of Q-vectors — even when the updrafts and precipitation are not exactly along a front, you can use Q-vectors to anticipate the bad-weather regions.

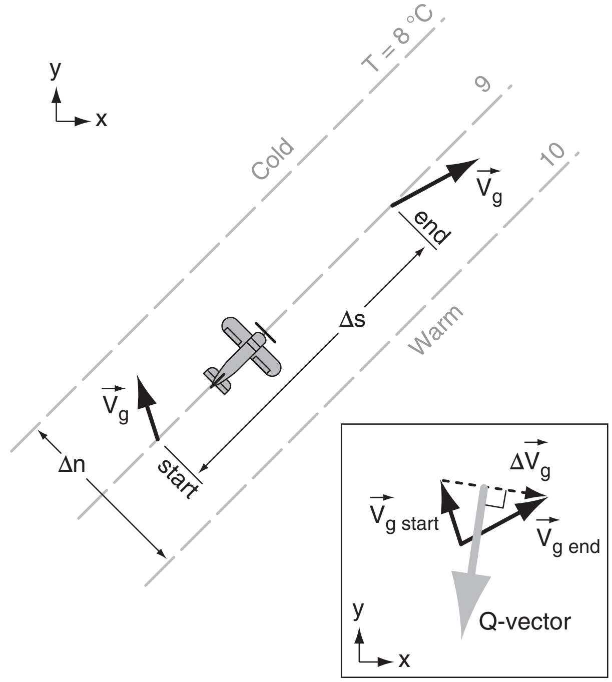


Figure 13.39
Illustration of natural coordinates for Q-vectors. Dashed grey lines are isotherms. Aircraft flies along the isotherms with cold air to its left. Black arrows are geostrophic wind vectors. Grey arrow indicates Q-vector direction (but not magnitude).

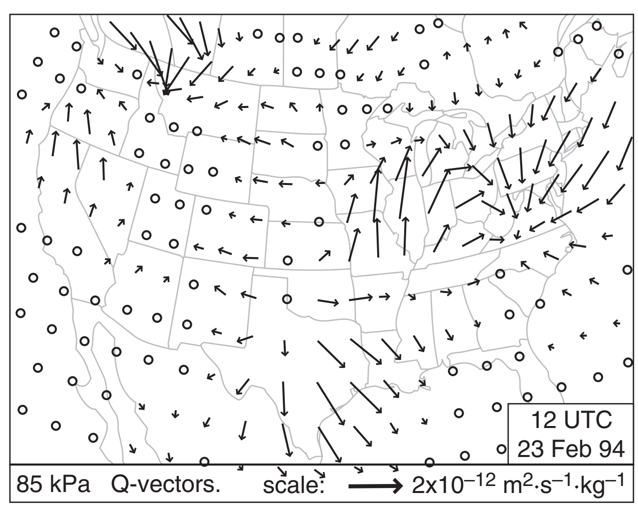


Figure 13.40
*Weather map of Q-vectors. (**o** means small magnitude.)*

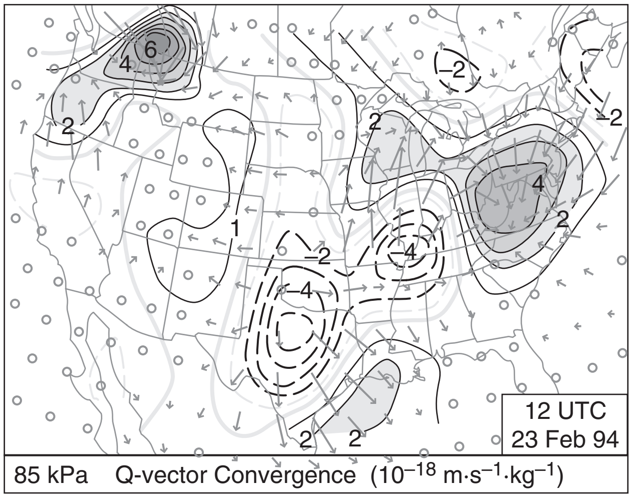


Figure 13.41
Convergence of Q-vectors (shaded). Divergence (dashed lines).

Solved Example

Discuss the nature of circulations and anticipated frontal and cyclone evolution, given the Q-vector divergence region of southern Illinois and convergence in Maryland & W. Virginia, using Fig. 13.41.

Solution

Given: Q-vector convergence fields.
Discuss: circulations, frontal & cyclone evolution

Discussion: From Fig. 13.10b, recall that the low-center is over southern Illinois, right at the location of maximum divergence of Q-vectors in Fig. 13.41. This suggests that: (1) The cyclone is entering the cyclolysis phase of its evolution (synoptic-scale subsidence that opposes any remaining convective updrafts from earlier in the cyclones evolution) as it is steered northeastward toward the Great Lakes by the jet stream (Fig. 13.17b). (2) The cyclone will likely shift toward the more favorable updraft region over Maryland. This shift indeed happened, as discussed in the case-study overview earlier in this Chapter, and as plotted in Figs. 13.10c & 13.11.

The absence of Q-vectors crossing the fronts in western Tennessee and Kentucky suggest no frontogenesis there.

Between Maryland and Illinois, we would anticipate a mid-tropospheric ageostrophic wind from the east-northeast. This would connect the updraft region over western Maryland with the downdraft region over southern Illinois. This circulation would move air from the warm-sector of the cyclone to over the low center, helping to feed warm humid air into the cloud shield over and north of the low.

Also, along the Texas Gulf coast, note that the Q-vectors in Fig. 13.40 are crossing the cold front (Fig. 13.12b) from cold toward warm air. Using the third bullet on the previous page, you can anticipate frontogenesis in this region.

Resolving the Geostrophic Paradox

What about the ageostrophic circulations that were missing from the Trenberth omega equation? Fig. 13.41 suggests updrafts at the Q-vector convergence region over the western Gulf of Mexico, and subsidence at the divergence region of central Texas. Due to mass continuity, expect an ageostrophic circulation of mid-tropospheric winds from the southeast toward the northwest over the Texas Gulf coast, which connects the up- and down-draft portions of the circulation. This ageostrophic wind moves warm pre-frontal air up over the cold front in a **direct circulation** (i.e., a circulation where warm air rises and cold air sinks).

But if you use the 85 kPa height chart of Fig. 13.13a to anticipate geostrophic winds over central Texas, you would expect light winds at 85 kPa from the northwest. These opposing geostrophic and ageostrophic winds agree nicely with the warm-air convergence (creating thunderstorms) for the cold katafront sketch in Fig. 12.16a.

Similarly, over West Virginia and Maryland, Fig. 13.41 shows convergence of Q-vectors at low altitudes, suggesting rising air in that region. This updraft adds air mass to the top of the air column, increasing air pressure in the jet streak right entrance region, and tightening the pressure gradient across the jet entrance. This drives faster geostrophic winds that counteract the advection of slower geostrophic winds in the entrance region. Namely, the ageostrophic winds as diagnosed using Q-vectors help prevent the Geostrophic Paradox (Focus Box).

BEYOND ALGEBRA • Q-vector Omega Eq.

By considering the added influence of ageostrophic winds, the Q-vector omega equation is:

$$\left\{\nabla_p^2 + \frac{f_o{}^2}{\sigma}\frac{\partial^2}{\partial p^2}\right\}\omega = \frac{-2}{\sigma}\cdot\left[\frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y}\right] + \frac{f_o\cdot\beta}{\sigma}\frac{\partial V_g}{\partial p} - \frac{R/C_p}{\sigma\cdot P}\nabla^2(\Delta Q_H)$$

The left side looks identical to the original omega equation (see a previous Beyond Algebra box for an explanation of most symbols). The first term on the right is the convergence of the Q vectors. The second term is small enough to be negligible for synoptic-scale systems. The last term contributes to updrafts if there is a local maximum of sensible heating ΔQ_H.

SEA-LEVEL-PRESSURE TENDENCY

Sea-level pressure is another measure of cyclone intensity. When pressure drops in an intensifying low center, the low is said to **deepen**. In other words, the low becomes lower. This corresponds to **falling** geopotential heights. When sea-level pressure increases, the low **fills** and heights **rise** (i.e., the low weakens).

The pressure change with time is called the **pressure tendency**, and is closely related to the **height tendency** of isobaric surfaces (Fig. 13.42). The amount of air mass in the column of atmosphere above the low center determines the surface pressure in a hydrostatic environment.

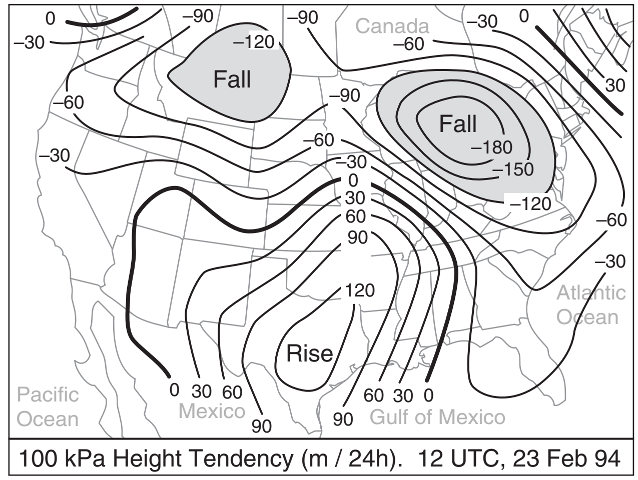


Figure 13.42
Change of geopotential height with time near the surface, for the case-study storm. Shaded regions indicates where heights (and surface pressures) are decreasing (falling); namely, regions of cyclogenesis. Height rises favor anticyclogenesis.

Mass Budget

We will approach this subject heuristically. Picture a small cylinder filled with air, as shown in Fig. 13.43a. Near the middle of the cylinder is a massless, frictionless piston. Above and below the piston, the air pressures are identical, and the air densities are identical (i.e., ignore gravity for now).

Suppose that some of the air is withdrawn from the top of the cylinder, as shown in Fig. 13.43b. Pressure in the top of the cylinder will decrease, which will cause the piston to rise until the bottom pressure decreases to equal the top pressure. At that point, the densities above and below the piston are also identical. Similarly, air mass could have been added to the top of the cylinder, causing the pressure to increase and piston to move down.

Thus, you can use the vertical motion of the piston as a surrogate measure of net mass flow and pressure change. Namely, upward motion indicates a decrease in total air mass in the cylinder (summing both above and below the piston), and is associated with pressure decrease in the cylinder. Conversely, downward motion indicates increases in both mass and pressure.

That rule seems simple. However, picture what would happen if air were withdrawn or added to the bottom of the cylinder. Upward piston motion would correspond to an increase in mass and pressure, not a decrease as before. Thus, you can use vertical piston movement as a surrogate measure of pressure change only if you change the sign of the result depending on whether mass is added at the top or bottom of the column.

Extend this reasoning to the atmosphere, where the cylinder of the previous example will now be visualized as a column of air from the ground to the top of the atmosphere. A complication is that atmospheric density decreases with height. Recall that

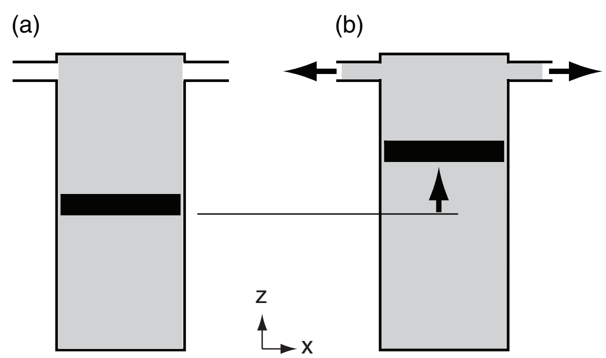


Figure 13.43
(a) Column of air (grey shading) in a cylinder, with a piston (black) in the middle. (b) Piston location changes after some air is withdrawn from the top. Assume that no air leaks past the piston.

Solved Example

If the surface pressure is 100 kPa, how much air mass is in the whole air column above a 1 meter squared surface area?

Solution

Given: $A = 1$ m^2, $P_s = 100$ kPa
Find: $m = ?$ kg

Rearrange eq. (13.34) to solve for m:
$m = P_s \cdot A / |g| = [(100 \text{ kPa}) \cdot (1 \text{ m}^2)] / (9.8 \text{ m/s}^2)$

But from Appendix A: 1 Pa = 1 kg·m^{-1}·s^{-2} , thus:
$m = [(10^5 \text{ kg·m}^{-1}\text{·s}^{-2}) \cdot (1 \text{ m}^2)] / (9.8 \text{ m/s}^2)$
$= \mathbf{10.2 \times 10^3 \text{ kg}} = 10.2$ Mg

Check: Units OK. Physics OK.
Discussion: This calculation assumed that gravitational acceleration is approximately constant over the depth of the atmosphere.

Eq. 13.34 can be used for the pressure at any height in the atmosphere, but only if m represents the mass of air above that height. For example, if the tropopause is at pressure 25 kPa, then the mass of air above the tropopause is one quarter of the previous answer; namely, 2.55 Mg over each square meter.

Subtracting this value from the previous answer shows that of the total 10.2 Mg of mass in the atmosphere above a square meter, most of the air (7.65 Mg) is within the troposphere.

Solved Example

Divergence of air at the top of the troposphere removes air molecules from the top of a tropospheric column, causing a 0.1 m/s updraft at height 8 km above ground level (AGL). No other processes add or remove air mass. What is the corresponding surface pressure tendency?

Solution

Given: $z = 8$ km, $W_{surrogate}(8 \text{ km}) = 0.1$ m/s
Find: $\Delta P_s/\Delta t = ?$ kPa/s

For air density at $z = 8$ km, assume a standard atmosphere. Use $\rho = 0.5252$ kg/m^3.

Use eq. (13.37) with negative sign because the forcing is at the top of the troposphere:
$\Delta P_s/\Delta t = -(9.8 \text{ m/s}^2) \cdot (0.5252 \text{ kg/m}^3) \cdot (0.01 \text{ m/s})$
$= -0.0515 \text{ kg·m}^{-1}\text{·s}^{-3} = -0.0515$ Pa/s
$= \mathbf{-5.15 \times 10^{-5} \text{ kPa/s}}$

Check: Units OK. Physics reasonable.
Discussion: The corresponding hourly pressure tendency is $\Delta P_s/\Delta t = -0.185$ kPa/h. If this rapid deepening of the cyclone were to continue for 24 h, we would classify this explosive cyclogenesis as a **cyclone bomb**.

pressure is force per unit area, and that force is mass times acceleration. Recall from Chapter 1 that sea-level pressure P_s is a measure of the total **mass** m **of air in a column** above the surface

$$P_s = \frac{|g|}{A} \cdot m \qquad \bullet(13.34)$$

where $|g| = 9.8$ m·s^{-2} is gravitational acceleration magnitude, and A is the horizontal cross-section area under the column.

Changes in sea-level pressure with time t are caused by changes in total air mass above the surface:

$$\frac{\Delta P_s}{\Delta t} = \frac{|g|}{A} \cdot \frac{\Delta m}{\Delta t} \qquad \bullet(13.35)$$

Recall from the definition of density that

$$m = \rho \cdot Volume = \rho \cdot A \cdot z \qquad (13.36)$$

where z is the height of a hypothetical volume containing constant-density air. Thus, $\Delta m/\Delta t$ causes $\Delta z/\Delta t$, where $\Delta z/\Delta t$ is vertical velocity w. Define the change of height of the hypothetical volume with time as a surrogate velocity $W_{surrogate}$ that is analogous to the piston movement.

Thus, if the mass flow in or out of the column occurs at a height z where the air density $\rho(z)$ is known, then the equations above can be combined to give the pressure tendency:

$$\frac{\Delta P_s}{\Delta t} = \pm |g| \cdot \rho(z) \cdot W_{surrogate}(z) \qquad (13.37)$$

The proper sign for the right-hand-side of the equation must be chosen depending on the cause of the surrogate vertical motion (i.e., use the negative sign if it is driven at the top of the troposphere, and positive sign if driven at the bottom). More than one mechanism can add or subtract mass to a column of air, so generalize the right-hand-side of eq. (13.37) to be a sum of terms.

Four mechanisms will be included here in a simplified model for pressure tendency:

- Upper-level divergence
- Boundary-layer pumping
- Advection
- Diabatic heating

Upper-level **Divergence** has just been described in the previous section as a mass-removal process. Namely, sea-level pressure can drop if the jet stream or a jet streak removes mass aloft. That resulting W_{mid} will be used as one of the forcing terms in the net pressure-tendency equation, to be given later in this section.

Boundary-layer pumping was discussed in the Dynamics chapter. Sometimes lows deepen so rapidly, that the accelerating boundary-layer winds blow directly from high to low pressure as an ageostrophic non-equilibrium flow. Later, if the low persists for a sufficiently-long duration, the winds can turn cyclonically and behave as a classical equilibrium boundary-layer flow, with a vertical velocity as was given in the Dynamics chapter. This W_{BL} will be used later in the net pressure-tendency equation.

Advection describes the movement of cyclones by the steering-level winds. Namely, surface pressure can drop at a fixed point on the ground if the horizontal wind blows in an air column having less total mass than the column of air that is blowing out. Analogous to advection terms in other equations earlier in this book (e.g., heat, moisture, and momentum budget eqs.), the advection term will be approximated as $-M_c \cdot (\Delta P_s / \Delta s)$, where M_c is the speed of movement of the whole column of air along path s, and $\Delta P_s / \Delta s$ is the horizontal surface pressure gradient along that path.

Diabatic (i.e., not adiabatic) **heating** due to condensation is discussed in detail in the next section. Other diabatic effects such as direct radiative heating can also affect the mass budget, but are not discussed here.

Diabatic Heating due to Condensation

When water vapor condenses in clouds, latent heat is released. This heat warms the air in a column, causing it to expand (Fig. 13.44). According to the hypsometric equation, horizontal pressure gradients develop that push the air out of the top of the column. Hence, mass diverges from the column, which lowers the sea-level pressure (see the Hydrostatic Thermal Circulation section of the Global Circulation chapter).

In an overly simplified point-of-view, the heated air expands out of the top of the column and overflows. The amount of overflow out of the top in this simple view equals the amount of mass diverged from the sides in a more realistic view. None-theless, vertical velocity W at the top of the column is a surrogate measure of the mass loss.

If all of the condensation falls as precipitation, then the latent-heating rate is related to the rainfall rate RR:

$$\frac{\Delta T_v}{\Delta t} = \frac{a}{\Delta z} \cdot \frac{L_v}{C_p} \cdot \frac{\rho_{liq}}{\rho_{air}} \cdot RR \qquad (13.38)$$

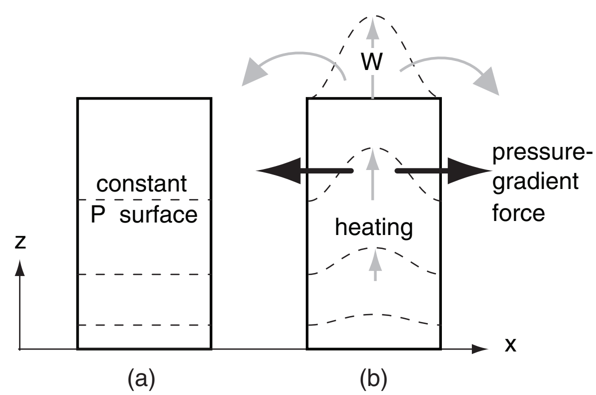


Figure 13.44
(a) Column of air before heating. (b) Column of air after heating, where the air expands and overflows out of the column. P = pressure; W = vertical velocity.

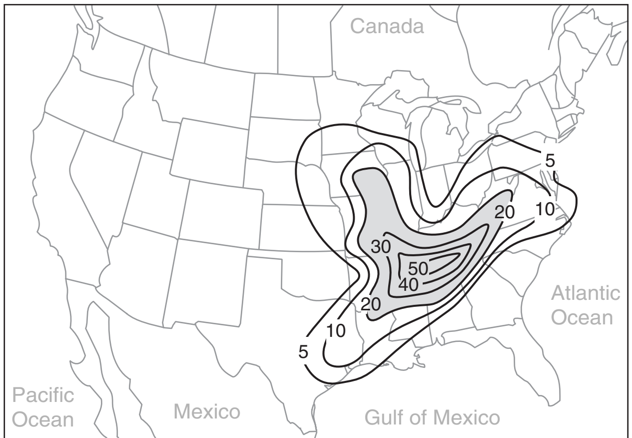


Figure 13.45
Precipitation (liquid equivalent) measured with rain gauges.

Solved Example
For the maximum contoured precipitation rate for the case-study storm (in Fig. 13.45), find the diabatic heating contribution to sea-level pressure tendency.

Solution
Given: RR = 50 mm/24 h
Find: $\Delta P_s/\Delta t$ = ? kPa/h

First, convert RR from 24 h to 1 hr:
RR = 2.1 mm/h

Use eq. (13.40):
$\Delta P_s/\Delta t$ = –(0.082 kPa/mm$_{rain}$)·(2.1 mm/h)
= **–0.17 kPa/h**

Check: Units OK. Physics OK. Magnitude OK.
Discussion: This deepening rate corresponds to 4.1 kPa/day — not large enough to be classified as a cyclone "bomb".

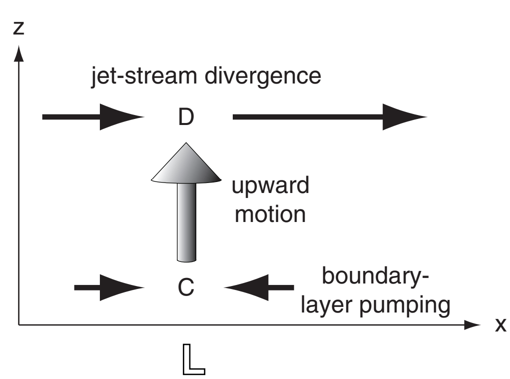


Figure 13.46
Sketch of coupling between convergence (C) in the boundary layer and divergence (D) in the upper atmosphere. Arrows represent winds. L is location of low-pressure center at surface.

where T_v is the virtual temperature in the column, t is time, $a = 10^{-6}$ km/mm, L_v is the latent heat of vaporization, C_p is the specific heat at constant pressure for air, L_v/C_p = 2500 K·kg$_{air}$ /kg$_{liq}$, ρ_{liq} is the density of liquid water (1000 kg/m^3), ρ_{air} is the density of air, Δz is the column depth in km, and where RR often has units of mm/h.

The net effect on sea-level pressure can be found by combining the hypsometric equation and the equation above:

$$\frac{\Delta P_s}{\Delta t} = -\frac{|g|}{\overline{T_v}}\cdot\frac{L_v}{C_p}\cdot\rho_{liq}\cdot RR \qquad \bullet(13.39)$$

where $|g|$ = 9.8 m·s^{-2} is the magnitude of gravitational acceleration, RR is rainfall rate (often converted to units of mm/h), and $\overline{T_v}$ is the average virtual temperature in the column (≈ 300 K). Because RR is the net precipitation reaching the ground, the equation above represents the net or average effect of latent heating over the whole cyclone depth.

Although this equation depends on absolute temperature (which can vary), you can approximate the equation as

$$\frac{\Delta P_s}{\Delta t} \approx -b\cdot RR \qquad \bullet(13.40)$$

where $b = \frac{|g|}{\overline{T_v}}\cdot\frac{L_v}{C_p}\cdot\rho_{liq} \approx$ 0.082 kPa/mm$_{rain}$.

You can estimate rainfall rate with weather radar, or you can measure it with rain gauges. Fig. 13.45 shows measured precipitation liquid-equivalent depth (after melting any snow) for the case-study storm.

Net Pressure Tendency

Combining all of the mechanisms just described using the idealized geometry of Fig. 13.46 yields the following form for the net **sea-level pressure tendency equation**:

$$\frac{\Delta P_s}{\Delta t} = -M_c\frac{\Delta P_s}{\Delta s} + |g|\cdot\rho_{BL}\cdot W_{BL} - |g|\cdot\rho_{mid}\cdot W_{mid} - b\cdot RR \qquad \bullet(13.41)$$

pressure tendency | *horizontal advection* | *boundary-layer pumping (in)* | *upper-level horiz. divergence (out)* | *latent heating*

where (ρ_{BL}, ρ_{mid}) and (W_{BL}, W_{mid}) are the average air densities and vertical velocities at the top of the boundary layer and midpoint of the column, respectively. The horizontal translation speed of the column is M_c measured positive in the direction of movement, along horizontal path s. RR is rainfall rate at the surface, $|g|$ = 9.8 m·s^{-2} is gravitational acceleration magnitude, and $b \approx 0.082$ kPa/mm$_{rain}$. A negative value of $\Delta P_s/\Delta t$ corresponds to pressure drop and cyclone intensification.

Imagine a scenario where baroclinic instability in the jet stream causes a Rossby wave to move over some location on the Earth that initially lacks any strong highs or lows. If the upper-level trough is west of this location and ridge east, then divergence aloft can begin to suck air out of the column (time *A* in Fig. 13.47). Similarly, the presence of a jet streak at the right location could initiate upper-level divergence.

As sea-level pressure falls, pressure gradient force begins to spin-up the circulation around the low. Initially the circulation is weak, resulting in weak boundary-layer pumping. At this stage the upper level divergence is continuing to withdraw air from the column faster than boundary-layer pumping fills it, allowing the low to intensify (time *B*).

The rising air in the cyclone causes clouds to form, with the resulting precipitation. Latent heating warms the air, which contributes to the loss of mass and decrease of sea-level pressure.

As the low matures, it wraps the cold and warm airmasses further around the low center, thereby modifying the temperature gradients and thermal winds. The low center begins to lag behind the region of maximum divergence aloft. The surface low and associated air masses also interact with and modify the jet-stream aloft.

After the time of maximum divergence (time *C* in Fig. 13.47), withdrawal of air aloft decreases while input of air from the boundary layer increases, eventually reaching the point (time *D*) where input and output match. The cyclone has reached its lowest pressure at this point.

Later as the low occludes, it lags so far behind the upper-air wave that jet-stream divergence ceases over the column. Meanwhile the still-spinning surface low continues to pump boundary-layer air into the column. With the upper-level trough east of the surface low, convergence aloft also pumps air into the column from above. Thus, input of mass is now occurring at both the top and bottom (time *E*), causing the sea-level pressure to rise. The cyclone rapidly fills and spins down as pressure gradients weaken.

Finally at (time *F*), the low has filled completely, and circulations disappear. During its lifetime, the low has done its job of moving warm air poleward and cold air equatorward. Analogous to Le Chatelier's principle, it has partially undone the baroclinic instability that produced it in the first place.

Solved Example

Suppose upper level divergence has an effective vertical velocity of 0.03 m/s over a cyclone, while boundary-layer pumping causes a vertical velocity of 0.01 m/s. The rainfall rate is 5 mm/h. Sea-level pressure increases 1 kPa across a distance of 200 km as you move west of the cyclone, and there is a steering wind from the west at 20 m/s (so *s* is positive toward the east). Find the sea-level pressure tendency.

Solution

Given: W_{mid} = 0.03 m/s , W_{BL} = 0.01 m/s,
$\Delta P/\Delta s$ = –1 kPa/200 km, RR = 5 mm/h
Find: $\Delta P_s/\Delta t$ = ? kPa/h
Assume: ρ_{BL} = 1.112 kg·m^{-3} , ρ_{mid} = 0.5 kg·m^{-3}

Use eq. (13.41): $\Delta P_s/\Delta t$ =

$$-\left(20\frac{\text{m}}{\text{s}}\right)\left(\frac{-1\text{kPa}}{2\times10^5\text{m}}\right)+\left(9.8\frac{\text{m}}{\text{s}^2}\right)\left(1.112\frac{\text{kg}}{\text{m}^3}\right)\left(0.01\frac{\text{m}}{\text{s}}\right)$$
$$-\left(9.8\frac{\text{m}}{\text{s}^2}\right)\left(0.5\frac{\text{kg}}{\text{m}^3}\right)\left(0.03\frac{\text{m}}{\text{s}}\right)-\left(0.084\frac{\text{kPa}}{\text{mm}}\right)\left(5\frac{\text{mm}}{\text{h}}\right)$$

$\Delta P_s/\Delta t$ = 0.0001 +0.00011 –0.00015 –0.00012 kPa/s
advect +B.L. – jet diverg. – diabatic

= – 0.00006 kPa/s = **– 0.22 kPa/h**

Check: Units OK. Physics OK. Magnitude OK.
Discussion: The cyclone would continue to deepen, given this scenario. Advection is quite important. Don't forget to convert from Pa to kPa in the boundary-layer (BL) and jet-divergence terms. Also convert the rainfall term to kPa/s from kPa/h.

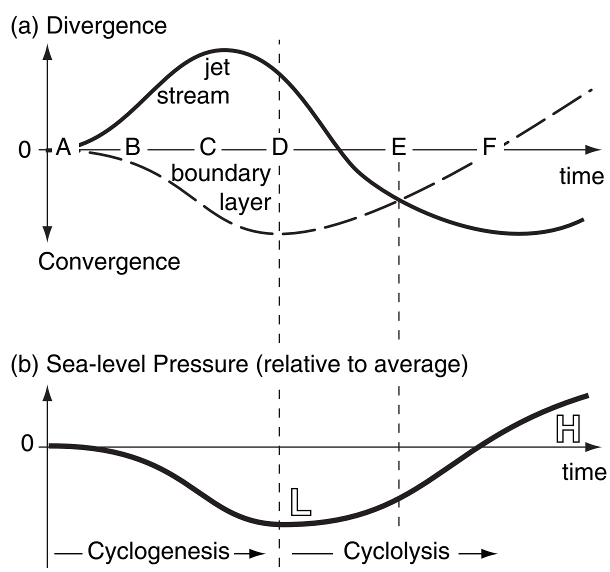


Figure 13.47
(a) Divergence contributions from the jet stream and boundary layer, and (b) their modification of sea-level pressure during cyclone evolution.

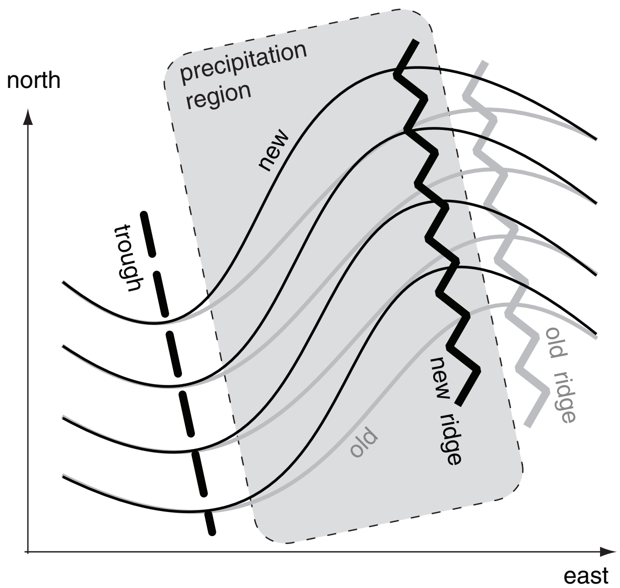


Figure 13.48
50 kPa chart showing cloud and precipitation region in the upper troposphere, causing latent heating and a westward shift of the ridge axis.

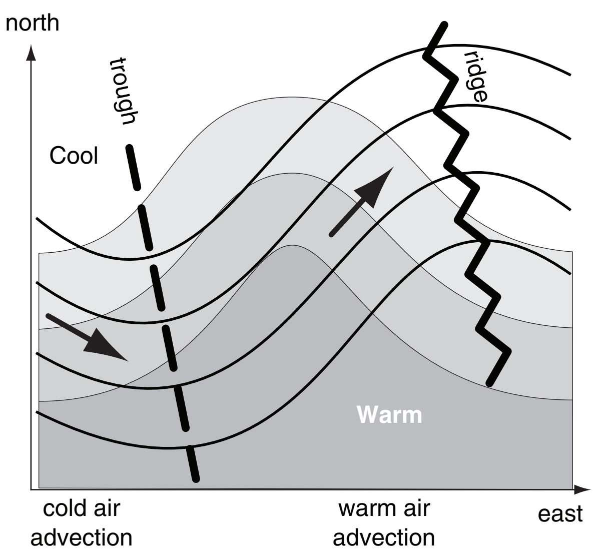


Figure 13.49
50 kPa chart showing a temperature field (shaded) that is 1/4 wavelength west of the wave in the height contours (solid lines).

SELF DEVELOPMENT OF CYCLONES

Up to this point, cyclogenesis has been treated as a response to various external imposed forcings. However, some positive feedbacks allow the cyclone to enhance its own intensification. This is often called **self development**.

Condensation

As discussed in the quasi-geostrophic vorticity subsection, divergence of the upper-level winds east of the Rossby-wave trough (Fig. 13.48) causes a broad region of upward motion there. Rising air forms clouds and possibly precipitation if sufficient moisture is present. Such a cloud region is sometimes called an **upper-level disturbance** by broadcast meteorologists, because the bad weather is not yet associated with a strong surface low.

Latent heating of the air due to condensation enhances buoyancy and increases upward motion. The resulting **stretching** enhances spin-up of the vorticity, and the upward motion withdraws some of the air away from the surface, leaving lower pressure. Namely, a surface low forms.

Diabatic heating also increases the average temperature of the air column, which pushes the 50 kPa pressure surface upward (i.e., increasing its height), according to the hypsometric relationship. This builds or strengthens a ridge in the upper-level Rossby wave west of the initial ridge axis.

The result is a shortening of the wavelength between trough and ridge in the 50 kPa flow, causing tighter turning of the winds and greater vorticity (Fig. 13.48). Vorticity advection also increases.

As the surface low strengthens due to these factors (i.e., divergence aloft, vorticity advection, precipitation, etc.), more precipitation and latent heating can occur. This positive feedback shifts the upper-level ridge further west, which enhances the vorticity and the vorticity advection. The net result is rapid strengthening of the surface cyclone.

Temperature Advection

Cyclone intensification can also occur when warm air exists slightly west from the Rossby-wave ridge axis, as sketched in Fig. 13.49. For this situation, warm air advects into the region just west of the upper-level ridge, causing ridge heights to increase. Also cold air advects under the upper-level trough, causing heights to fall there.

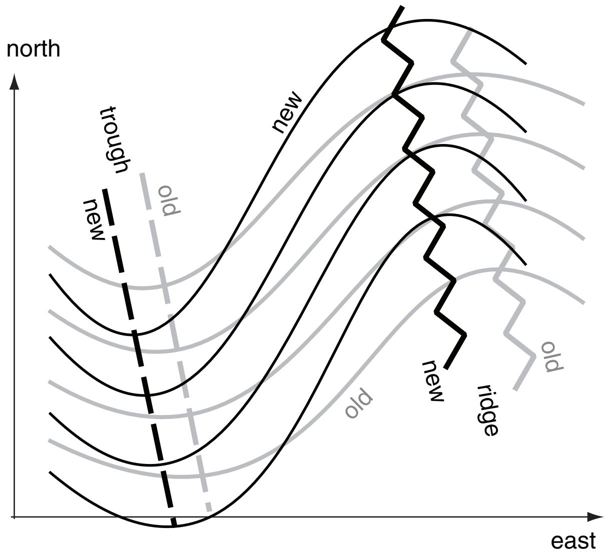


Figure 13.50
50 kPa chart showing the westward shift and intensification of north-south wave amplitude caused by differential temperature advection.

The net result is intensification of the Rossby-wave amplitude (Fig. 13.50) by deepening the trough and strengthening the ridge. Stronger wave amplitude can cause stronger surface lows due to enhanced upper-level divergence.

Propagation of Cyclones

For a train of cyclones and anticyclones along the mid-latitude baroclinic zone (Fig. 13.51a), a Q-vector analysis (Fig. 13.51b) suggests convergence of Q-vectors east of the low, and divergence of Q-vectors west of the low (Fig. 13.51c). But convergence regions imply updrafts with the associated clouds and surface-pressure decrease — conditions associated with cyclogenesis. Thus, the cyclone (𝕃) in Fig. 13.51c would tend to move toward the updraft region indicated by the Q-vector convergence.

The net result is that cyclones tend to propagate in the direction of the thermal wind.

Creation of Baroclinic Zones

Cyclones and anticyclones tend to create or strengthen baroclinic zones such as fronts. This works as follows.

Consider a train of lows and highs in a region with uniform temperature gradient, as shown in Fig. 13.52a. The rotation around the lows and highs tend to distort the center isotherms into a wave, by moving cold air equatorward on the west side of the lows and moving warm air poleward on the east side.

In addition, convergence into lows pulls the isotherms closer together, while divergence around

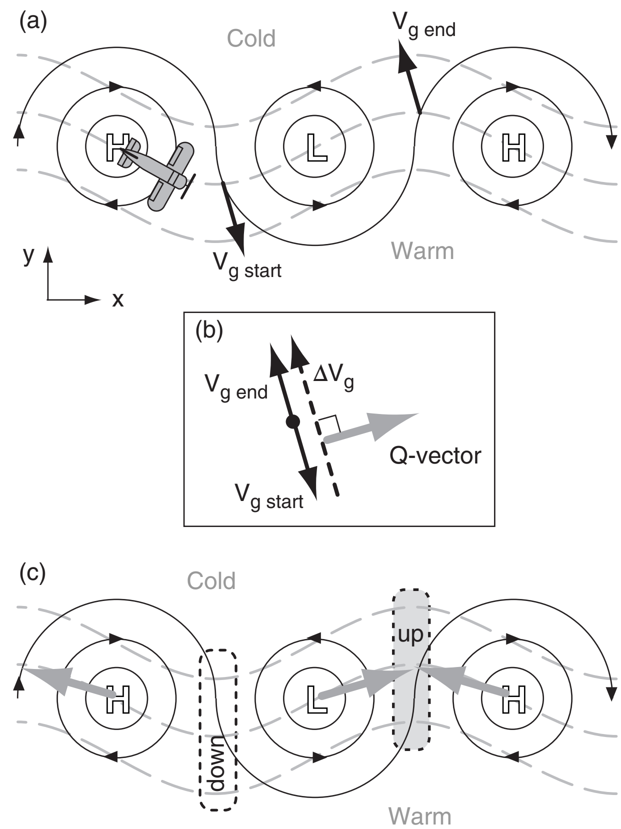


Figure 13.51
Using Q-vectors to estimate cyclone propagation. Grey dashed lines are isotherms (or thickness contours). Thin black lines are isobars (or height contours). (a) Airplane is flying in the direction of the thermal wind. Black arrows are geostrophic wind vectors encountered by the airplane on either side of the cyclone (𝕃). (b) Vector difference (black dashed line) between starting and ending geostrophic-wind vectors (drawn displaced to the right a bit so you can see it) for the portion of aircraft flight across the low. Q-vector (grey thick arrow) is 90° to the right of the dashed vector. (c) Q-vector from (b) is copied back to the cyclone. Similar analyses can be done to find the Q-vectors for the anticyclones (ℍ). Convergence of Q-vectors (grey shaded box with dotted outline) indicates region of updraft. Divergence of Q-vectors (white box with dotted outline) indicates region of downdraft. (for N. Hemisphere.)

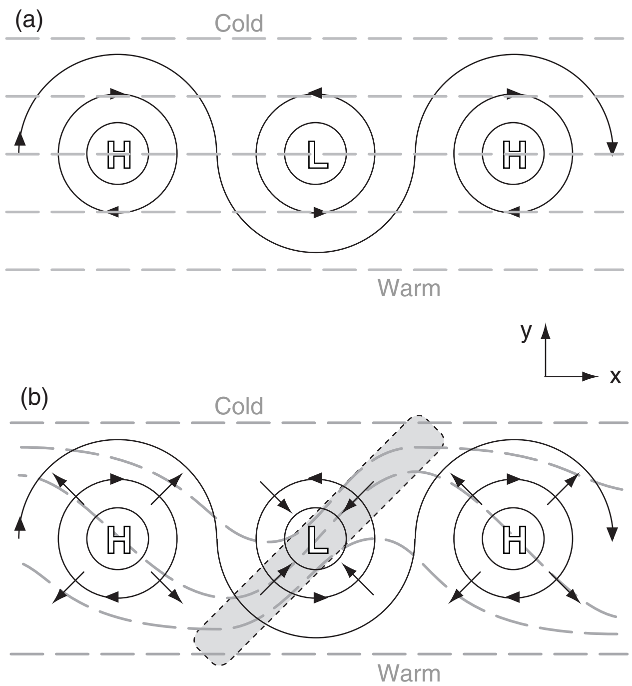


Figure 13.52
Rotational and divergent wind components (thin black arrows) and isotherms (dashed grey lines) in the lower troposphere. (a) Initial train of highs (H) and lows (L) in a uniform temperature gradient in the N. Hemisphere. (b) Later evolution of the isotherms into frontal zones (shaded rectangle is a baroclinic zone).

> **Science Graffito & On Doing Science • Truth vs. Uncertainty**
>
> "There are no certainties in science. Scientists do not assert that they know what is immutably 'true' — they are committed to searching for new, temporary theories to explain, as best they can, phenomena." Science is "a process for proposing and refining theoretical explanations about the world that are subject to further testing and refinement." – U.S. Supreme Court.
>
> [*CAUTION: The definition, role, and activities of science cannot be defined or constrained by a legal court or religious inquisition. Instead, science is a philosophy that has gradually developed and has been refined by scientists. The "On Doing Science" boxes in this book can help you to refine your own philosophy of science.*]

highs tends to push isotherms further apart. The combination of rotation and convergence/divergence tends to pack the isotherms into frontal zones near lows, and spread isotherms into somewhat homogeneous airmasses at highs (Fig. 13.52b).

Much of the first part of this chapter showed how cyclones can develop over existing baroclinic regions. Here we find that cyclones can help create those baroclinic zones — resulting in a positive feedback where cyclones modify their environment to support further cyclogenesis. Thus, cyclogenesis and frontogenesis often occur simultaneously.

Propagation of Cold Fronts

Recall from Fig. 13.8 that the circulation around a cyclone can include a deformation and diffluence region of cold air behind the cold front. If the diffluent winds in this baroclinic zone are roughly geostrophic, then you can use Q-vectors to analyze the ageostrophic behavior near the front.

Fig. 13.53a on the next page is zoomed into the diffluence region, and shows the isotherms and geostrophic wind vectors. By flying an imaginary airplane along the isotherms and noting the change in geostrophic wind vector, estimate the Q-vectors at the front as drawn in Fig. 13.53b. Further from the front, the Q-vectors are near zero.

Thus, Q-vector convergence is along the leading edge of the cold front (Fig. 13.53c), where warm air is indeed rising over the front. Behind the cold front is Q-vector divergence, associated with downward air motion of cool dry air from higher in the troposphere (Fig. 13.8).

SUMMARY

Cyclones are regions of low pressure, cyclonic vorticity, and upward motion. These lows can cause clouds, precipitation and strong winds. Cyclogenesis is the birth or growth of cyclones, while cyclolysis is their decline and death.

Mountain ranges can disturb the planetary flow, causing north-south waves in the otherwise west-to-east jet stream. The troughs of such waves contain cyclonic curvature, and usually exist just east of the mountains. This tends to enhance cyclogenesis in the lee of mountain ranges, such as in Colorado and Alberta just east of the Rocky Mountains.

Vorticity is one measure of cyclone intensity. Vorticity changes with terrain height, advection, stretching, tilting, turbulent drag, and with latitude.

Upward motion is another measure of cyclone intensity. It is often driven by jet streaks, and by curvature in the jet stream. Greater updrafts are possible in environments with weaker static stability, and with closely-packed and tightly-curved isobars on sea-level weather maps. Advection of geostrophic vorticity by the thermal wind also contributes to upward motion, as described by the omega equation and Sutcliffe's development equation. Upward motion can also be found in regions where Q-vectors converge.

Divergence aloft tends to reduce the sea-level pressure, and lows can deepen due to latent heating associated with precipitation. This is partially counteracted by inflow of air due to boundary-layer pumping, which is relentless in trying to cause cyclolysis. Once a surface low exists, it can enhance itself via temperature advection and latent-heat release in clouds. Cyclogenesis and frontogenesis often occur simultaneously.

A variety of weather maps, such as for the 23 February 1994 case-study shown in this chapter, can be used to study extratropical cyclones and other synoptic weather features.

Threads

Almost every chapter preceding this one contributed to the understanding of extratropical cyclones. We utilized concepts of temperature, density, pressure, and the standard atmosphere from Chapter 1. Cyclones form as nature's way to redistribute heat caused by the radiative imbalance (Chapter 2) between the equator and poles. This radiative imbalance causes baroclinicity primarily in the boundary layer (Chapter 18) via the heat budget (Chapter 3).

The bad weather of cyclones includes clouds (Chapters 4 - 6), precipitation (Chapter 7), and strong winds (Chapter 10). Condensation — as detected in precipitation areas by weather radar (Chapter 8) — releases latent heat which can strengthen cyclones. Cyclones are triggered by waves in the jet stream of the global circulation (Chapter 11), and help to transport heat, momentum, and moisture at mid-latitudes. The convergence at cyclones draws together different airmasses (Chapter 12), thereby forming fronts and enhancing baroclinicity.

Cyclones can be predicted numerically by solving the primitive equations (Chapter 20). Thunderstorms and tornadoes (Chapters 14 and 15) are triggered by the frontal weather near cyclones. Extratropical cyclones are **cold core systems**, which differ from warm core tropical cyclones (i.e., hurricanes, Chapter 16). Polluted air (Chapter 19) converges near cyclones and fronts, but pollutants can be washed out by rain. Changes in climate (Chapter 21) might alter the tracks or intensities of cyclones.

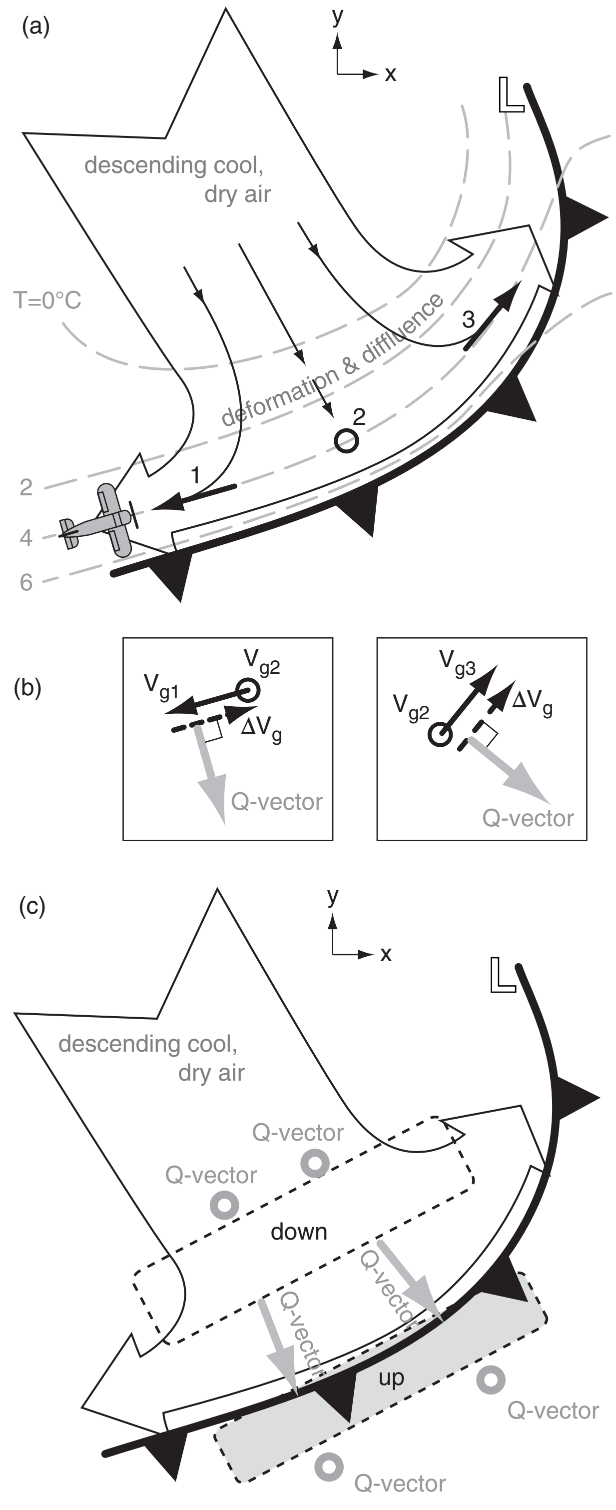


Figure 13.53
Using Q-vectors to locate regions of upward and downward motion due to diffluence of air behind a cold front. (a) Thin black lines are wind direction. Dashed grey lines are isotherms, along which an imaginary airplane flies. Thick black arrows show geostrophic wind vectors, with "o" representing zero wind. (b) Estimation of Q-vectors between points 1 and 2, and also between points 2 and 3. (c) The Q-vectors from (b) plotted in the baroclinic zone, with near-zero Q-vectors (grey "o") elsewhere. Q-vector convergence in grey shaded region suggests updrafts.

FOCUS • Landfalling Pacific Cyclones

Mid-latitude cyclones that form over the warm ocean waters east of Japan often intensify while approaching the dateline (180° longitude). But by the time they reach the eastern North Pacific ocean they are often entering the cyclolysis phase of their evolution. Thus, most landfalling cyclones that reach the Pacific Northwest coast of N. America are already occluded and are spinning down.

The cyclone labeled L_1 in Fig. k has just started to occlude. Satellite images of these systems show a characteristic tilted-"T" ( ) shaped cloud structure (grey shaded in Fig. k), with the cold front, occluded front, and a short stub of a warm front.

As the cyclone translates further eastward, its translation speed often slows and the low center turns northward toward the cold waters in the Gulf of Alaska — a cyclone graveyard where lows go to die. In the late occluded phase, satellite images show a characteristic "cinnamon roll" cloud structure, such as sketched with grey shading for cyclone L_2.

For cyclone L_2, when the cold front progresses over the complex mountainous terrain of the Pacific Northwest (British Columbia, Washington, Oregon), the front becomes much more disorganized and difficult to recognize in satellite and surface weather observations (as indicated with the dashed line over British Columbia).

The remaining portion of L_2's cold front still over the Pacific often continues to progress toward the southeast as a "headless" front (seemingly detached from its parent cyclone L_2).

Sometimes there is a strong "**pre-frontal jet**" of fast low-altitude winds just ahead of the cold front, as shown by the black arrows in Fig. k. If the source

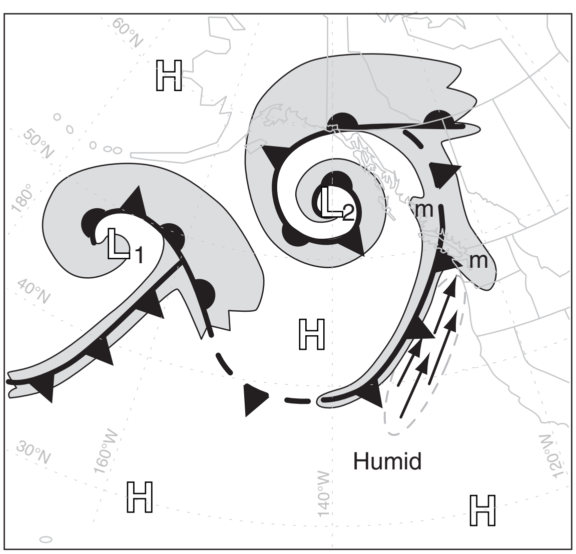


Figure k
Sketch of occluding mid-latitude cyclones approaching the Pacific Northwest coast of N. America.

(continues in next column)

FOCUS • Pacific Cyclones *(continued)*

region of this jet is in the humid sub-tropical air, then copious amounts of moisture can be advected toward the coast. If the source of this jet is near Hawaii, then the conveyor belt of moist air streaming toward N. America is nicknamed the "**Pineapple Express**".

When this humid air hits the coast, the air is forced to rise over the mountains. As the rising air cools adiabatically, clouds and orographic precipitation form over the mountains (indicated by "m" in Fig. k). Sometimes the cold front stalls (stops advancing) while the pre-frontal jet continues to pump moisture toward the mountains. This situation causes extremely heavy precipitation and flooding.

Fig. l shows an expanded view of the Vancouver, Seattle, Victoria region (corresponding to the lower-right "m" from Fig. k). Low-altitude winds split around the Olympic Mountains, only to converge (thick dashed line) in a region of heavy rain or snow called the **Olympic Mountain Convergence Zone** (also known as the **Puget Sound Convergence Zone** in the USA). The windward slopes of mountain ranges often receive heavy orographic precipitation (dotted ovals), while in between is often a **rain shadow** of clearer skies and less precipitation.

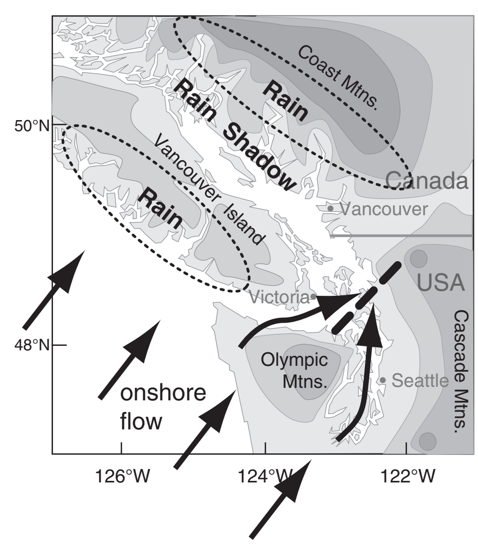


Figure l
Zoomed view of the Pacific Northwest. Pacific Ocean, sounds, and straits are white, while higher terrain is shaded darker. Arrows represent low-altitude wind.

EXERCISES

Numerical Problems

N1. For latitude 50°N, find the approximate wavelength (km) of upper-atmosphere (Rossby) waves triggered by mountains, given an average wind speed (m/s) of:
a. 20 b. 25 c. 30 d. 35 e. 40 f. 45 g. 50
h. 55 i. 60 j. 65 k. 70 l. 75 m. 80 n. 85

N2. Find the rate of increase of the β parameter (i.e., the rate of change of Coriolis parameter with distance north) in units of $m^{-1}s^{-1}$ at the following latitude (°N):
a. 40 b. 45 c. 50 d. 55 e. 60 f. 65 g. 70
h. 80 i. 35 j. 30 k. 25 l. 20 m. 15 n. 10

N3. For latitude 40°N and troposphere of depth 11 km, find the north-south amplitude (km) of upper-atmosphere (Rossby) waves triggered by mountains, given an average mountain-range height (km) of:
a. 0.4 b. 0.6 c. 0.8 d. 1.0 e. 1.2 f. 1.4 g. 1.6
h. 1.8 i. 2.0 j. 2.2 k. 2.4 l. 2.6 m. 2.8 n. 3.0

N4. How good is the approximation of eq. (13.4) to eq. (13.3) at the following latitude (°N)?
a. 40 b. 45 c. 50 d. 55 e. 60 f. 65 g. 70
h. 80 i. 35 j. 30 k. 25 l. 20 m. 15 n. 10

N5. For a troposphere of depth 12 km at latitude 43°N, find the potential vorticity in units of $m^{-1}s^{-1}$, given the following:
[wind speed (m/s) , radius of curvature (km)]
a. 50 , 500
b. 50 , 1000
c. 50 , 1500
d. 50 , 2000
e. 50 , –500
f. 50 , – 1000
g. 50 , –1500
h. 50 , -2000
i. 75 , 500
j. 75 , 1000
k. 75 , 1500
l. 75 , 2000
m. 75 , –1000
n. 75 , – 2000

N6. For air at 55°N with initially no curvature, find the potential vorticity in units of $m^{-1}s^{-1}$ for a troposphere of depth (km):
a. 7.0 b. 7.5 c. 8.0 d. 8.5 e. 9.0 f. 9.5 g. 10.0
h. 10.5 i. 11 j. 11.5 k. 12 l. 12.5 m. 13 n. 13.5

N7. When air at latitude 60°N flows over a mountain range of height 2 km within a troposphere of depth 12 km, find the radius of curvature (km) at location "C" in Fig. 13.20 given an average wind speed (m/s) of: a. 20 b. 25 c. 30 d. 35 e. 40 f. 45 g. 50
h. 55 i. 60 j. 65 k. 70 l. 75 m. 80 n. 85

N8. Regarding equatorward propagation of cyclones on the eastern slope of mountains, if a cyclone of radius 1000 km and potential vorticity of $3x10^{-8}$ $m^{-1}s^{-1}$ is over a slope as given below ($\Delta z/\Delta x$), find the change in relative vorticity (s^{-1}) between the north and south sides of the cyclone.
a. 1/500 b. 1/750 c. 1/1000 d. 1/1250
e. 1/1500 f. 1/1750 g. 1/2000 h. 1/2250
i. 1/2500 j. 1/2750 k. 1/3000 l. 1/3250

N9. Recall from the Dynamics chapter that incompressible mass continuity implies that $\Delta W/\Delta z = -D$, where D is horizontal divergence. Find the Coriolis contribution to the stretching term (s^{-2}) in the relative-vorticity tendency equation, given the average divergence in Fig. 13.26 for the following USA state:
a. AZ b. IL c. OK d. OH e. VA f. CO g. WA
h. WY i. PA j. WI k. central TX l. NY m. IN

N10. Find the spin-up rate (s^{-2}) of quasi-geostrophic vorticity, assuming that the following is the only non-zero characteristic:
a. geostrophic wind of 30 m/s from north within region where geostrophic vorticity increases toward the north by $6x10^{-5}$ s^{-1} over 500 km distance.
b. a geostrophic wind of 50 m/s from west within region where geostrophic vorticity increases toward the east by $8x10^{-5}$ s^{-1} over 1000 km distance.
c. A location at 40°N with geostrophic wind from the north of 25 m/s.
d. A location at 50°N with geostrophic wind from the south of 45 m/s.
e. A location at 35°N with vertical velocity increasing 0.5 m/s with each 1 km increase in height.
f. A location at 55°N with vertical velocity decreasing 0.2 m/s with each 2 km increase in height.

N11. Find the value of geostrophic vorticity (s^{-1}), given the following changes of (U_g, V_g) in m/s with 500 km distance toward the (north, east):
a. (0, 5) b. (0, 10) c. (0, –8) d. (0, –20)
e. (7, 0) f. (15, 0) g. (–12, 0) h. (-25, 0)
i. (5, 10) j. (20, 10) k. (–10, 15) l. (–15, –12)

N12. Find the value of geostrophic vorticity (s^{-1}), given a geostrophic wind speed of 35 m/s with the following radius of curvature (km). Assume the air rotates similar to a solid-body.
a. 450 b. –580 c. 690 d. –750 e. 825 f. –988

g. 1300 h. –1400 i. 2430 j. –2643 k. 2810
l. –2900 m. 3014 n. –3333

N13. What is the value of omega (Pa/s) following a vertically-moving air parcel, if during 1 minute its pressure change (kPa) is:
a. –2 b. –4 c. –6 d. –8 e. –10 f. –12 g. –14
h. –16 i. –18 j. –20 k. 0.00005 l. 0.0004
m. 0.003 n. 0.02

N14. At an altitude where the ambient pressure is 85 kPa, convert the following vertical velocities (m/s) into omega (Pa/s):
a. 2 b. 5 c. 10 d. 20 e. 30 f. 40 g. 50
h. –0.2 i. –0.5 j. –1.0 k. –3 l. –5 m. – 0.03

N15. Using Fig. 13.17d, find the most extreme value horizontal divergence (10^{-5} s^{-1}) at 20 kPa over the following USA state:
a. MI b. NM c. NV d. WV e. TN f. GA
g. ND h. WA i. KY j. PA k. NY l. AR

N16. Find the vertical velocity (m/s) at altitude 9 km in an 11 km thick troposphere, if the divergence (10^{-5} s^{-1}) given below occurs within a 2 km thick layer within the top of the troposphere.
a. 0.2 b. 1 c. 1.5 d. 2 e. 3 f. 4 g. 5 h. 6
i. –0.3 j.–0.7 k. –1.8 l. –2.2 m. –3.5 n. –5

N17. Jet-stream inflow is 30 m/s in a 4 km thick layer near the top of the troposphere. Jet-stream outflow (m/s) given below occurs 800 km downwind within the same layer. Find the vertical velocity (m/s) at the bottom of that layer
a. 35 b. 40 c. 45 d. 50 e. 55 f. 60 g. 65
h. 70 i. 30 j. 25 k. 20 l. 15 m. 10 n. 5

N18. Find the diagonal distance (km) from trough to crest in a jet stream for a wave of 750 km amplitude with wavelength (km) of:
a. 1000 b. 1300 c. 1600 d. 2000 e. 2200
f. 2500 g. 2700 h. 3000 i. 3100 j. 3300
k. 3800 l. 4100 m. 4200 n. 4500

N19. Given the data from the previous exercise, find the radius (km) of curvature near the crests of a sinusoidal wave in the jet stream.

N20. Find the gradient-wind speed difference (m/s) between the jet-stream speed moving through the anticyclonic crest of a Rossby wave in the N. Hemisphere and the jet-stream speed moving through the trough. Use data from the previous 2 exercises, and assume a geostrophic wind speed of 75 m/s for a wave centered on latitude 40°N.

N21. Given the data from the previous 3 exercises. Assuming that the gradient-wind speed difference calculated in the previous exercise is valid over a layer between altitudes 8 km and 12 km, where the tropopause is at 12 km, find the vertical velocity (m/s) at 8 km altitude.

N22. Suppose that a west wind enters a region at the first speed (m/s) given below, and leaves 500 km downwind at the second speed (m/s). Find the north-south component of ageostrophic wind (m/s) in this region. Location is 55°N.
a. (40, 50) b. (30, 60) c. (80, 40) d. (70, 50)
e. (40, 80) f. (60, 30) g. (50, 40) h. (70, 30)
i. (30, 80) j. (40, 70) k. (30, 70) l. (60, 20)

N23. Use the ageostrophic right-hand rule to find the ageostrophic wind direction for the data of the previous problem.

N24. Using the data from N22, find the updraft speed (m/s) into a 4 km thick layer at the top of the troposphere, assuming the half-width of the jet streak is 200 km.

N25. The 100 to 50 kPa thickness is 5.5 km and the Coriolis parameter is 10^{-4} s^{-1}. A west-to-east thermal wind of 25 m/s flows through a region where the average cyclonic vorticity decreases by 2×10^{-4} s^{-1} toward the east across a distance (km) given below. Use the omega equation to find the mid-tropospheric upward speed (m/s).
a. 200 b. 300 c. 400 d. 500 e. 600
f. 700 g. 800 h. 900 i. 1000 j. 1100
k. 1200 l. 1300 m. 1400 n. 1400

N26. On the 70 kPa isobaric surface, $\Delta U_g/\Delta x$ = 4m/s/500 km and $\Delta T/\Delta x$ = ___°C/500 km, where the temperature change is given below. All other gradients are zero. Find the Q-vector components Q_x, Q_y, and the magnitude and direction of Q.
a. 1 b. 1.5 c. 2 d. 2.5 e. 3 f. 3.5 g. 4
h. 4.5 i. 5 j. 5.5 k. 6 l. 6.5 m. 7 n. 7.5

N27. Find Q-vector magnitude on the 85 kPa isobaric surface if the magnitude of the horizontal temperature gradient is 5°C/200 km, and the magnitude of the geostrophic-wind difference-vector component (m/s) along an isotherm is __ /200 km, where __ is:
a. 1 b. 1.5 c. 2 d. 2.5 e. 3 f. 3.5 g. 4
h. 4.5 i. 5 j. 5.5 k. 6 l. 6.5 m. 7 n. 7.5

N28. Find the mass of air over 1 m^2 of the Earth's surface if the surface pressure (kPa) is:
a. 104 b. 102 c. 100 d. 98 e. 96 f. 94 g. 92
h. 90 i. 88 j. 86 k. 84 l. 82 m. 80 n. 78

N29. Assume the Earth's surface is at sea level. Find the vertical velocity (m/s) at height 3 km above ground if the change of surface pressure (kPa) during 1 hour is:
a. –0.5 b. –0.4 c. –0.3 d. –0.2 e. –0.1 f. 0.1
g. 0.2 h. 0.4 i. 0.6 j. 0.8 k. 1.0 l. 1.2 m. 1.4

N30. Given the rainfall (mm) accumulated over a day. If the condensation that caused this precipitation occurred within a cloud layer of thickness 6 km, then find the virtual-temperature warming rate (°C/day) of that layer due to latent heat release.
a. 1 b. 50 c. 2 d. 45 e. 4 f. 40 g. 5
h. 35 i. 7 j. 30 k. 10 l. 25 m. 15 n. 20

N31. For the data in the previous exercise, find the rate of decrease of surface pressure (kPa) per hour, assuming an average air temperature of 5°C.

Understanding & Critical Evaluation

U1. Compare the similarities and differences between cyclone structure in the Northern and Southern Hemisphere?

U2. In the Cyclogenesis & Cyclolysis section is a list of conditions that favor rapid cyclogenesis. For any 3 of those bullets, explain why they are valid based on the dynamical processes that were described in the last half of the chapter.

U3. Knowing the acceleration rate for a parcel of air at sea level (given Newton's second law and assuming that you know the horizontal pressure gradient), speculate on if it is possible for air to accelerate so slowly compared to the deepening of a cyclone bomb that the wind speed is not in geostrophic equilibrium. Discuss using order-of-magnitude numbers.

U4. Make a photocopy of Fig. 13.3. For each one of the figure panels on this copy, infer the centerline position of the jet stream and draw it on those diagrams.

U5. Create a 6-panel figure similar to Fig. 13.3, but for cyclone evolution in the Southern Hemisphere.

U6. Justify the comment that cyclone evolution obeys Le Chatelier's Principle.

U7. Refer back to the figure in the Global Circulation chapter that sketches the position of mountain ranges in the world. Use that information to hypothesize favored locations for lee cyclogenesis in the world, and test your hypothesis against the data in Fig. 13.5.

U8. Why are there no extratropical cyclone tracks from east to west in Fig. 13.5?

U9. Contrast the climatology of cyclone formation and tracks in the Northern vs. Southern Hemisphere (using the info in Fig. 13.5), and explain why there is a difference in behaviors based on the dynamical principles in the last half of the chapter.

U10. Justify why the tank illustration in Fig. 13.6 is a good analogy to atmospheric flow between cyclones and anticyclones.

U11. Fig. 13.7 shows the axis of low pressure tilting westward with increasing height. Explain why this tilt is expected. (Hints: On which side of the cyclone do you expect the warm air and the cold air? The hypsometric equation in Chapter 1 tells us how fast pressure decreases with height in air of different temperatures.)

U12. Redraw Fig. 13.7 a & b for the Southern Hemisphere, by extending your knowledge of how cyclones work in the Northern Hemisphere.

U13. Regarding stacking and tilting of low pressure with altitude, make a photocopy of Fig. 13.3, and on this copy for Figs. b and e draw the likely position of the trough axis near the top of the troposphere. Justify your hypothesis.

U14. Fig. 13.8 shows a warm-air conveyor bring air from the tropics. Assuming this air has high humidity, explain how this conveyor helps to strengthen the cyclone. Using dynamical principles from the last half of the chapter to support your explanation.

U15. Fig. 13.10 has coarse temporal resolution when it shows the evolution of the case-study cyclone. Based on your knowledge of cyclone evolution, draw two weather maps similar to Fig. 13.10, but for the times centered between the maps in the book. Namely, draw weather maps for 00 UTC 23 Feb 94 and 00 UTC 24 Feb 94.

U16. Speculate on why the dividing line between snow and rain in Fig. 13.11 is not parallel to the cyclone track.

U17. From the set of case-study maps in Figs. 13.12 through 13.19, if you had to pick 3 maps to give you the best 5-D mental picture of the cyclones, which 3 would you pick? Justify your answer.

U18. Of the following isosurfaces (height, pressure, thickness, and potential temperature), which one seems the most peculiar to you? What questions would you want to ask to help you learn more about that one isosurface?

U19. Starting with a photocopy of Fig. 13.12a, trace onto it the 70 kPa height map (in one color, such as blue) and the 50 kPa height map (in a different color, such as red). Analyze the tilt with height of the axis of low pressure, and explain why such tilt does or does not agree with the state of cyclone evolution at that time.

U20. Compare and contrast the 85 kPa temperature map of Fig. 13.13b with the thickness map of Fig. 13.15 for the case-study storm. Why or why not would you expect them to be similar?

U21. Compare the wind vectors of Fig. 13.17c with the heights in Fig. 13.17a. Use your understanding of wind dynamics to explain the relationship between the two maps.

U22. In the vertical cross section of Fig. 13.19a, why are the isentropes packed more closely together in the stratosphere than in the troposphere? Also, why is the stratosphere missing from the right side of that figure? [Hints: Consider the standard atmosphere temperature profile from Chapter 1. Consider the Global Circulation chapter.

U23. Why are the surface temperatures in Fig. 13.19a much warmer than the temperatures shown along a – a′ in Fig. 13.12b?

U24. In Fig. 13.20, how would lee cyclogenesis be affected if the tropopause perfectly following the terrain elevation? Explain.

U25. In Fig. 13.20, speculate on why the particular set of Rossby waves discussed in that section are known as "stationary" waves.

U26. What is the wavelength of stationary planetary waves near the equator?

U27. Suppose straight west-to-east flow at latitude 60° and initial troposphere depth 11 km encounters a semi-infinite plateau of terrain height 2 km above sea level. Describe mathematically the nature of the flow over the plateau, assuming the tropopause height doesn't change. Would the triggering of cyclones, and the wavelength of planetary waves be different? Why? Can you relate your answer to weather over the Tibetan Plateau?

U28. If cyclones often form to the lee of mountain ranges, do cyclones often weaken near the upwind side of mountain ranges? Why? How can you verify your answer?

U29. Suppose that instead of the Rocky Mountains, there was a trench in the land as wide as the Rockies are wide, and as deep as the Rockies are high. How would the weather and climate be different, if at all?

U30. In the winter, the tropopause is often lower and the wind speeds faster than during summer. How would the triggering of stationary planetary waves be different between winter and summer, if at all?

U31. Is there poleward propagation of cyclones at the upwind slopes of mountain ranges? Describe.

U32. Fig. 13.22 highlights 3 important attributes of cyclones that are discussed in greater detail in the last half of the chapter. Speculate on why we study these attributes separately, even though the caption to that figure discusses how all 3 attributes are related.

U33. Can the vorticity, vertical velocity, and sea-level pressure tendency equations be used to study or predict cyclolysis? Describe.

U34. The tilting effect of the vorticity equation has two terms, but only one term was sketched in Fig. 13.23d. Sketch the other term.

U35. Except for the last term in the full vorticity tendency equation, all the other terms can evaluate to be positive or negative (i.e., gain or loss of relative vorticity). What is it about the mathematics of the turbulent drag term in that equation that always make it a loss of relative vorticity?

U36. Based on what you learned from Fig. 13.24, what tips would you teach to others to help them easily find regions of PVA and NVA.

U37. In Figs. 13.23, no diagram was shown for vertical advection. Draw a sketch to illustrate this process.

U38. Suppose that the only two terms in the vorticity tendency equation were the spin-up term and the turbulent drag term. If there was some initial nonzero vorticity, describe how the vorticity would evolve with time.

U39. Devise a tilting term for vertical vorticity that is tilted into horizontal vorticity. Could this process happen in nature?

U40. a. Summarize the quasi-geostrophic approximation.
b. What are the limitations of the quasi-geostrophic vorticity equation?

U41. The quasi-geostrophic vorticity equation includes a term related to vorticity advection by the geostrophic wind. The omega equation has a term related to vorticity advection by the thermal wind. Contrast these terms, and how they provide information about cyclogenesis.

U42. Use the case-study weather maps to calculate as many terms as reasonably possible in the vorticity-tendency equation, on the 50 kPa isobaric surface at the location of the "X".

U43. Fig. 13.27 examines flow east of a trough. Draw a similar sketch, except east of a ridge, and indicate the physical effects causing vorticity change. Also, indicate if the result is spin-up or spin-down.

U44. How does the radius of curvature of a planetary wave vary with latitude, everything else being equal?

U45. Why does jet stream curvature contribute to surface cyclogenesis east of the jet trough axis rather than west of the trough axis?

U46. Recall the limitation on the max wind speed and pressure gradient near high-pressure centers, but not near lows. How does that affect the possible strength of the jet-stream curvature effect for cyclogenesis?

U47. Using the case-study weather map (Fig. 13.30) for 20 kPa heights and isotachs, calculate the jet streak and curvature contributions to mid-tropospheric vertical velocity and to sea-level pressure tendency over the Ohio Valley, valid at 12 UTC on 23 Feb 94.

U48. Make a photo copy of Fig. 13.18. Using the jet-stream curvature and jet-streak information in that figure, draw on your copy the locations in the Northern Hemisphere where cyclogenesis is favored.

U49. In Fig. 13.32b, why are some divergence and convergence regions indicated as being stronger than others? Discuss.

U50. Consider a steady, straight jet stream from west to east. Instead of a jet streak of higher wind speed imbedded in the jet stream, suppose the jet streak has lower wind speed imbedded in the jet stream. For the right and left entrance and exit regions to this "slow" jet streak, describe which ones would support cyclogenesis at the surface.

U51. Does the "Ageostrophic right-hand rule" work in the Southern Hemisphere too? Justify your answer based on dynamical principles.

U52. For a jet-streak axis that is aligned from SW to NE, how could you use eqs. (13.25 & 13.26) for this situation?

U53. Fig. 13.34 suggests low-altitude convergence of air toward cyclones (lows), rising motion, and high-altitude divergence. Is this sketch supported by the case-study data from Figs. 13.17d, 13.25, and 13.26?

U54. For Fig. 13.34, what causes the portion of the secondary circulation near the ground? (Remember that Newton's law states that forces are needed to drive winds.)

U55. a. For the set of four maps of horizontal structure of a cold front (in the Airmasses & Fronts chapter), use the "area between crossing isopleths" technique to locate the region of maximum temperature advection.
b. Same, but for the warm-front maps.

U56. Discuss how Fig. 13.37 relates to the omega equation, and how the figure and equation can be used to locate regions that favor cyclogenesis.

U57. Recall that the thermal wind is really the change of geostrophic wind with height, which is proportional to the horizontal temperature gradient. Rewrite the omega equation in terms of:
a. the vertical gradient of geostrophic wind
b. the horizontal temperature gradient.

U58. Show that eq. (13.29) is identical to (13.28). What assumptions are needed to convert between the two equations, if any?

U59. What if there was no baroclinicity (no horizontal temperature gradient). How would that affect the omega eq.?

U60. What role does inertia play in the "geostrophic paradox"?

U61. Suppose that the geostrophic wind vectors are parallel to the height contours in Fig. 13.13a for the case-study storm. Use that information along with the isotherms in Fig. 13.13b to estimate the direction of the Q-vectors at the center of the following USA state:

a. KY b. WV c. WI d. KS e. AR

U62. Use the Q-vector approach to forecast where cyclogenesis might occur in the Pacific Northwest USA (in the upper left quadrant of Figs. 13.40 and 13.41).

U63. What are the limitations on the heuristic arguments of Fig. 13.43, where a vertical velocity was used as a surrogate for terms in the mass budget?

U64. Create a relationship between surface pressure tendency and dBZ values of reflectivity observed by weather radar.

U65. Given that Doppler radar can measure both precipitation rate and radial components of velocity, what quantitative information can be used from it to compute intensification of cyclones?

U66. Is it possible for the latent heating term in the net pressure tendency equation to increase, rather than decrease, the central pressure of a cyclone? Describe the conditions necessary for this to happen.

U67. In Fig. 13.47, describe the dynamics that makes the time of maximum convergence in the boundary layer occur <u>after</u> the time of maximum divergence in the jet stream, during cyclone evolution.

U68. Sketch a figure similar to Fig. 13.49, except for the temperature wave located 1/4 wavelength to the right instead of to the left of the height field. How would that alter the flow (sketch your result similar to Fig. 13.50)? Speculate on how likely it is that the temperature wave is shifted this way.

U69. Would anticyclones have any self development processes analogous to those for cyclones? Describe.

U70. Explain why the Q-vector analysis of Fig. 13.51 indicates the propagation of cyclones is toward the east. Also explain why this relates to self-propagation rather than relating to cyclogenesis driven by the jet-stream flow.

U71. Use Fig. 13.52 to explain why fronts are associated with cyclones and not anticyclones. The same figure can be used to explain why airmasses are associated with anticyclones. Discuss.

U72. Can cyclones form if global baroclinicity is absent (e.g., no air-temperature gradient between the equator and the poles)? Why?

U73. Explain how the up- and down-couplet of air motion in Fig. 13.53c (as diagnosed using Q-vectors), works in a way to strengthen and propagate the cold front.

U74. Use a Q-vector analysis to speculate on the dynamics needed to cause <u>warm</u> fronts to strengthen and propagate.

U75. For the case-study cyclone in this chapter, combine all the information from the vorticity tendency, vertical motion, and surface-pressure tendency to synthesize a coherent description of the dynamics of this storm.

Web-Enhanced Questions

W1. For an exciting storm system that affected you (or a storm assigned by your instructor), develop a case study of weather maps from the web and discuss cyclone evolution based on the physical processes reviewed in this chapter.

W2. Search the web for sources of storm damage data ("Storm Data").

W3. For an intense cyclone, download a set weather maps and upper air maps for a snapshot of the weather. Use this to develop a 5-D image in your mind of the storm, and its dynamics.

W4. Search the web for maps showing typical or historical tracks of extra-tropical cyclones. What are the names of some of the favored cyclone formation sites or tracks (such as the Alberta Clipper)?

W5. Track an intense cyclone as it approaches the west coast of the N. America from the Pacific. Download a sequence of maps showing how the storm changes as it passes over the various mountain ranges in the western part of the continent.

W6. For an exciting storm that caused significant damage, download data from the web to create a summary map of its track and damage, similar to Fig. 13.11.

W7. Search the web for case studies of cyclone bombs, or explosive cyclogenesis. Discuss the evolution of these storms.

W8. From web maps of the jet stream (upper air charts at 20, 25, or 30 kPa), measure the wavelength of the various waves. Also measure the wave amplitude. How do they vary with latitude? With season?

W9. From web maps of the jet stream (upper air charts at 20, 25, or 30 kPa), estimate the vorticity from the radius of curvature and the wind speed.

W10. Download upper-air maps of temperature and height contours for a level such as 85 or 70 kPa. Superimpose these two fields if they are not already on the same map. Use the technique described in the "Advection" focus box to locate the region of maximum temperature advection.

W11. Same as the previous problem, but for a level such as 50 kPa where vorticity and height maps are available to locate the region of maximum vorticity advection. Locate both the regions of max positive and negative vorticity advection by the geostrophic wind. Which region would most likely favor cyclone development?

W12. Download a map of 70 kPa vorticity, and of 100 - 50 kPa thickness contours. From the thickness, determine the thermal wind near a cyclone. Use the result to find the region of max vorticity advection by the thermal wind, which according to the omega equation would favor upward motion and strengthening of cyclones.

W13. Search the web for an isentropic weather map for the current weather. If you cannot find one, then search the web for a map from a previous case study, or from a figure in a paper about isentropic analysis. From this map, discuss the elevation variation of the isentropic surface, state how high it is within the troposphere, and discuss any other info such as if winds are likely to blow up or down the isentropic surface in various regions.

W14. Locate a jet streak on a current weather map, or a past map if necessary, and label the right and left entrance and exit regions. Indicate where on the map you would expect cyclones to develop or strengthen. Given the change of wind velocity at different locations near the entrance or exit of the jet streak, plug those numbers into the formula to estimate upward motion associated with jet streak divergence.

W15. Study the spacing of height contours at a jet stream level (20, 25, or 30 kPa) as the jet stream flows around neighboring ridges and troughs, using a map you download from the web. Does the spacing between contours remain relatively constant around both the ridge and trough? If not, given the changes in spacing, and given what you know about gradient wind speeds (e.g., slow around lows), discuss the variation of actual wind speed around the ridge and trough.

W16. Search the web for radar observations in a cyclone having heavy precipitation. From the dBZ values, estimate the rainfall rate (see an earlier chapter for the relationship). Then, from the rainfall rate, estimate the latent heating contribution to the surface pressure tendency. Also, see if self-development of the cyclone occurs due to latent heating.

W17. Search the web for the site of a major weather map provider, government weather service, university forecasting project, military forecast service, numerical forecast center (NCEP, ECMWF, CMC), or research organization that produces lots of weather maps. In addition to traditional maps on various isosurfaces (e.g., height, pressure, thickness, potential temperature, etc.), what other weird, interesting, and useful maps can you find?

W18. Find a web site that produces maps of Q-vectors or Q-vector divergence. Print this map and a normal surface weather map with fronts, and discuss how you would anticipate the cyclone to evolve based on a Q-vector analysis.

Synthesis Questions

S1. Consider Fig. 13.6. What if frictional drag is zero at the bottom of the atmosphere (in the boundary layer), how would cyclone evolution be different, if at all?

S2. What if the case-study cyclone of Fig. 13.11 occurred in April rather than February, what broad aspects of the storm data would change, if at all?

S3. Suppose no major north-south mountain ranges exist in N. America. How would the weather and climate be different, if at all?

S4. Suppose a major mountain range exists east-west across the center of N. America, and no north-south ranges. How would weather and climate be different, if at all?

S5. What if average temperature decreased toward the east over N. America, rather than toward the north. How would this change in the direction of baroclinicity change the weather and climate, if at all?

S6. What if the Earth rotated twice as fast. How would the triggering of cyclones and the nature of stationary planetary waves be different, if at all?

S7. Most numerical weather prediction (NWP) models use the primitive equations (the conservation equations for heat, moisture, and momentum, along with the continuity eq. and ideal gas law) to make their forecasts (as described in a later chapter).

a. Is it possible to use the quasi-geostrophic vorticity and omega equations to make a forecast instead?

b. How would you get the wind components and temperature from such a forecast?

c. Is such a forecast system simpler or more complicated to solve than the primitive equations?

S8. Suppose the jet stream is at its current altitude, and meanders north and south, and creates horizontal divergence and convergence as it does now. But what if the static stability in the stratosphere equaled that in the troposphere. What change would occur to jet-stream forcing of surface pressure systems, if any? Discuss.

S9. Suppose that the troposphere were as strongly stable as the stratosphere. How would the weather and climate be different, if at all?

S10. Suppose that extratropical cyclones did not exist. How would the weather and climate be different, if at all?

S11. Suppose that Earth was a larger planet that had a very hot core and surface, similar to Jupiter. If the heat from the solid Earth to the atmosphere was the dominant source of heat that balanced net longwave radiative cooling from the atmosphere, how would the weather and climate on Earth be different, if at all?

S12. Re-read this chapter and extract all the forecasting tips to create your own concise synoptic-weather forecast guide.