

NATURAL CLIMATE PROCESSES

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21 Earth's average surface temperature has been surprisingly steady over millennia. Factors that control the temperature are solar output, Earth orbital characteristics, ocean currents, atmospheric circulations, plate tectonics, volcanoes, clouds, ice caps, gas emissions, particulates, aerosols, and life.

Some of these factors tend to reduce climate change, and are classified as negative feedbacks. Others amplify change, and are positive feedbacks. The steadiness of Earth's surface temperature suggests that negative feedbacks dominate.

Idealized models of climate processes are presented next. These "toy models" are oversimplifications of nature, yet they serve to illustrate fundamental climatic controls and responses.

RADIATIVE EQUILIBRIUM

The Earth is heated by shortwave radiation from the sun, and is cooled by longwave (infrared, IR) radiation to space (Fig. 21.1). Geological records show that the Earth's surface has had nearly constant annual-average temperature (less than 4% change of absolute temperature over the past 100 million years), suggesting a balance between radiation inflow and outflow.

Incoming minus reflected sunlight, times the interception area, gives the total radiative input to Earth:

$$\text{Radiation In} = (1 - A) \cdot S_0 \cdot \pi \cdot R_{\text{Earth}}^2 \quad \bullet (21.1)$$

where $A = 0.30$ is **global albedo** (see Focus Box), $S_0 = 1366 \text{ W} \cdot \text{m}^{-2}$ is the annual average **total solar irradiance (TSI)** over all wavelengths as measured

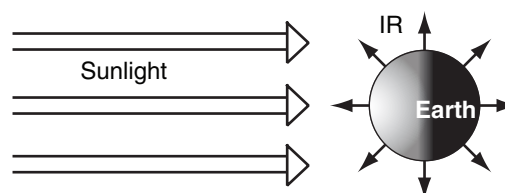
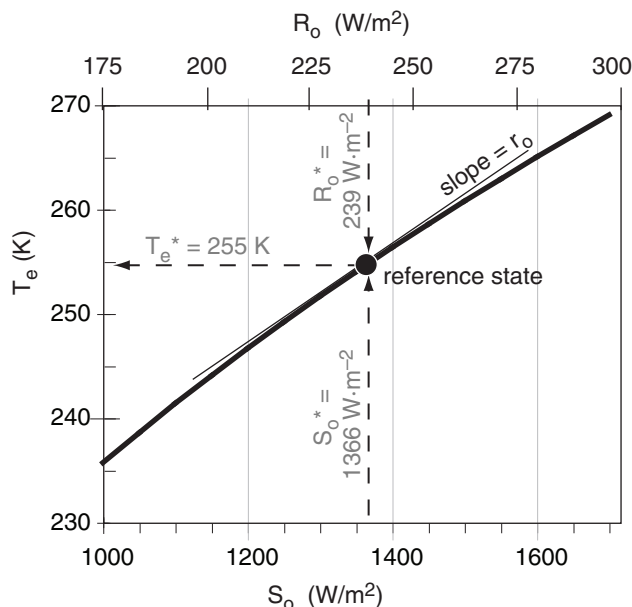


Figure 21.1
Solar input (white arrows) and infrared output (dark arrows) from Earth.

FOCUS • Albedo

The average global reflectivity (portion of incoming sunlight that is reflected back to space) for Earth is called the **albedo**, A . But the exact value of A is difficult to measure, due to natural variability of cloud and snow cover, and due to calibration errors in the satellite radiation sensors.

Trenberth, Fasullo & Kiehl (2009, *Bul. Amer. Meteor. Soc.*) report a range of values of $27.9\% \leq A \leq 34.2\%$. They suggest $A = 29.8\%$. We use $A \approx 30\%$ here.

**Figure 21.2**

Thick curved line shows the variation of Earth effective radiation temperature T_e with solar irradiance S_o , assuming an albedo of $A = 0.30$. Dashed lines show present conditions, which will be used as a reference point or reference state (R_o^* , T_e^*). Thin black straight line shows the slope r_o of the curve at the reference point. $R_o = (1-A) \cdot S_o/4$ is net solar input for each square meter of the Earth's surface (see Focus Box, next page).

Solved Example

Find the radiative equilibrium temperature of the Earth-atmosphere system.

Solution:

Given: $S_o = 1366 \text{ W}\cdot\text{m}^{-2}$ solar irradiance
 $A = 0.30$ global albedo (reflectivity)
 Find: $T_e = ? \text{ K}$ radiative equilibrium temp.

Use eq. (21.3):

$$T_e = \left[\frac{(1-0.3) \cdot (1366 \text{ W}\cdot\text{m}^{-2})}{4 \cdot (5.67 \times 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4})} \right]^{1/4}$$

$$= 254.8 \text{ K} \approx 255 \text{ K} = \underline{-18^\circ\text{C}}$$

Check: Units OK. Physics OK.

Discussion: The effective radiation temperature does not depend on the radius of the planet.

by satellites, and R_{Earth} is the Earth radius. The area of a disk πR^2 rather than the surface area of a sphere $4\pi R^2$ is used because the area of intercepted solar radiation is equivalent to the area of the shadow cast behind the Earth. The TSI has fluctuated about $\pm 0.5 \text{ W}\cdot\text{m}^{-2}$ (the thickness of the dashed line in Fig. 21.2) during the past 33 years due to the average sunspot cycle. See the Radiation chapter for TSI details.

Assume the Earth is a black body in the infrared, and that it emits IR radiation from its whole surface area (continents and oceans). Multiplying the Stefan-Boltzmann law (eq. 2.15), which gives the emissions per unit area, times surface area of the Earth (assumed spherical here) gives:

$$\text{Radiation Out} = 4\pi \cdot R_{\text{Earth}}^2 \cdot \sigma_{\text{SB}} \cdot T_e^4 \quad \bullet(21.2)$$

where $\sigma_{\text{SB}} = 5.67 \times 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$ is the Stefan-Boltzmann constant, and T_e is the **effective radiation emission temperature**.

The real Earth is quite heterogeneous, with oceans, continents, forests, clouds and ice caps of differing temperatures. Each of these regions emits different amounts of radiation. Also, some radiation is emitted from the Earth's surface, some from various heights in the atmosphere, and some from clouds. The effective radiation temperature is defined so that the total emissions from the actual Earth-ocean-atmosphere system equal the theoretical emissions from a uniform Earth of temperature T_e .

For a steady-state climate (i.e., no temperature change with time), outputs must balance the inputs. Thus, equate outgoing and incoming radiation from eqs. (21.1 and 21.2), and then solve for T_e :

$$T_e = \left[\frac{(1-A) \cdot S_o}{4 \cdot \sigma_{\text{SB}}} \right]^{1/4} = \left[\frac{R_o}{\sigma_{\text{SB}}} \right]^{1/4} \quad \bullet(21.3)$$

$$\approx 254.8 \text{ K} \approx -18^\circ\text{C}$$

where $R_o = (1-A) \cdot S_o/4$. Fig. 21.2 shows this reference state and shows how the effective temperature varies with total solar irradiance, assuming constant albedo. This effective temperature is too cold compared to the observed surface temperature of 15°C . Hence, this simple model must be missing some important physics.

This radiation balance is a **negative feedback**. Namely, increases of incoming solar radiation are mostly compensated by increases in outgoing IR, yielding only small temperature changes.

Because the radiative feedback dominates over all others, the Earth's climate is relatively stable, as geological records demonstrate (Fig. 21.3). Fifty million years ago, the average surface temperature

was about 6°C warmer than now. During the past ten thousand years, the climate has oscillated about $\pm 1^\circ\text{C}$ about the recent 15°C average. Although these temperature changes are small (range of ΔT is of order 10°C) relative to the Earth's absolute temperature, they can cause dramatic changes in sea level and glaciation.

Greenhouse Effect

You can increase the sophistication of the previous model by adding a layer of atmosphere (see Fig. 21.4) that is opaque to infrared radiation, but transparent to visible. Sunlight heats the Earth's surface. Radiation from the surface heats the opaque atmosphere. Radiation from the top of the atmosphere is lost to space, while radiation from the bottom of the atmosphere heats the Earth.

The incoming radiation from the sun must balance the outgoing radiation from the atmosphere, in order for the Earth-atmosphere system to remain in radiative equilibrium. Hence

$$T_A = T_e = 255 \text{ K} = -18^\circ\text{C} \quad (21.4)$$

as before, where T_e is the effective emission temperature of the whole Earth-atmosphere system, and T_A is the temperature of the atmosphere. For this case, the two temperatures are equal, because the atmosphere is opaque.

Other energy balances can be made for the atmosphere separately, and for the Earth's surface separately, with output from one system being the input to the other. The opaque atmosphere is assumed to emit as much IR radiation upward to space, as downward toward the Earth. The amount emitted both up and down from the atmosphere is given by the Stefan-Boltzmann law as $\sigma_{SB} T_A^4$. Incoming IR radiation from the Earth's surface is $\sigma_{SB} T_s^4$, where T_s is the Earth-surface temperature. If the atmosphere is at **steady state**, then absorbed IR equals emitted IR:

$$\sigma_{SB} T_s^4 = 2 \cdot \sigma_{SB} T_e^4 \quad (21.5)$$

The sum of atmospheric and solar radiation reaching the Earth's surface is exactly twice that of the solar radiation alone. This extra heating from the atmosphere causes the Earth to warm until it emits exactly twice the radiation as before. This allows the Earth's surface temperature to be estimated by:

$$\begin{aligned} T_s &= 2^{1/4} \cdot T_e \\ &= 303 \text{ K} = 30^\circ\text{C} \end{aligned} \quad \bullet(21.6)$$

Warming of the Earth's surface due to radiation emitted from the atmosphere is known as the **Greenhouse effect**. Water vapor is a greenhouse gas.

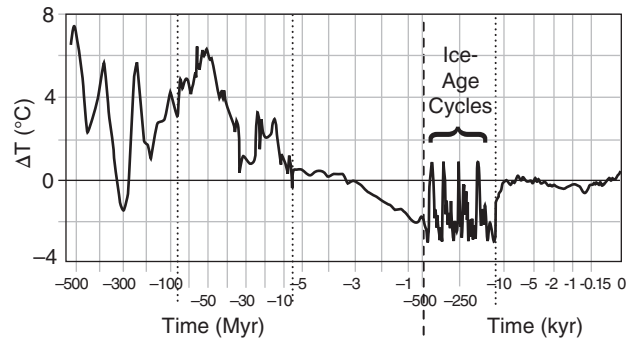


Figure 21.3

Paleotemperature estimate (smoothed) as a difference ΔT from the 1961 to 1990 temperature average. Time is relative to calendar year 2000. Scale breaks at the dotted and dashed lines. Glacial/interglacial oscillations (i.e., ice-age cycles) prior to 500,000 yr ago have been smoothed out.

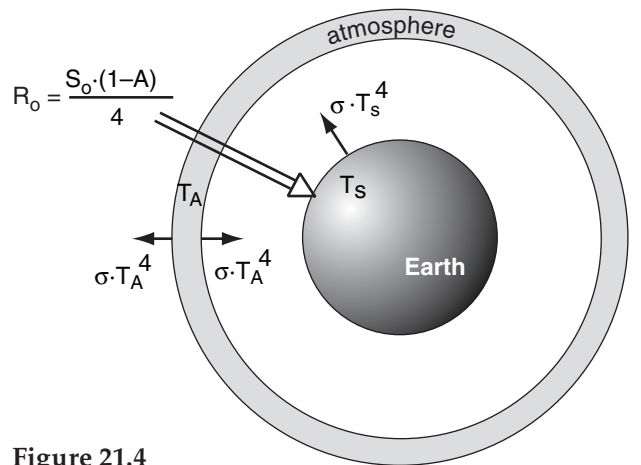


Figure 21.4

Greenhouse effect with an idealized one-layer atmosphere. Distance between Earth and atmosphere is exaggerated. Assume that the radius of the Earth approximately equals the radius at the top of the atmosphere. R_0 is the net solar input, and σ is the Stefan-Boltzmann constant.

FOCUS • Why does $R_0 = (1-A) \cdot S_0 / 4$?

In Figs. 21.4 and 21.5, the **net solar input** R_0 is given as $(1-A) \cdot S_0 / 4$ instead of S_0 . The $(1-A)$ factor is because any sunlight reflected back to space is not available to heat the Earth system. But why also divide by 4? The reason is due to geometry.

Of the radiation streaming outward from the sun, the shadow cast by the Earth indicates how much radiation was intercepted. The shadow is a circle with radius equal to the Earth's radius R_{Earth} , hence the interception area is πR_{Earth}^2 .

But we prefer to study the energy fluxes relative to each square meter of the Earth's surface. The surface area of a sphere is $4\pi R_{Earth}^2$. Namely, the surface area is four times the area intercepted by the sun.

Hence, the average net solar input that we allocate to each square meter of the Earth's surface is:

$$R_0 = (1-A) \cdot S_0 / 4 = (1-0.3) \cdot (1366 \text{ W} \cdot \text{m}^{-2}) / 4 = \mathbf{239 \text{ W} \cdot \text{m}^{-2}}.$$

Solved Example

For the previous solved example, find the Earth surface temperature under a one-layer atmosphere.

Solution

Given: $T_e = 255 \text{ K}$

Find: $T_s = ? \text{ K}$

Use eq. (21.6): $T_s = 1.189 \cdot (255 \text{ K}) = 303 \text{ K} = 30^\circ\text{C}$

Check: Units OK. Physics OK.

Discussion: Although the radiation reaching the Earth's surface doubles, the surface temperature increases by a smaller portion.

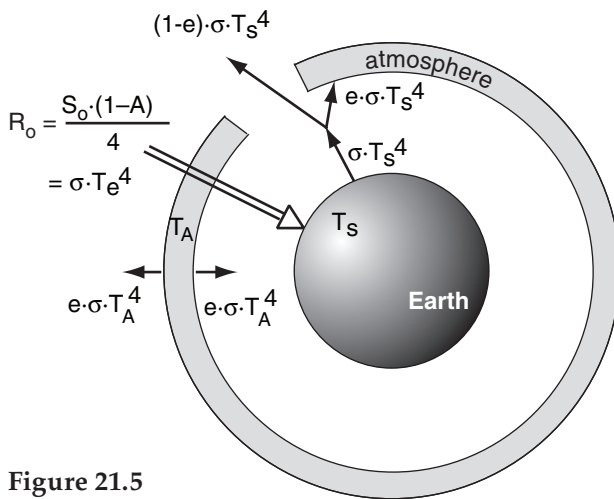


Figure 21.5

Radiation balance in a hypothetical one-layer atmosphere with an IR window.

Solved Example

For the previous solved example, assume $e_\lambda = 0.899$. Find the surface and atmosphere temperatures.

Solution

Given: $T_e = 255 \text{ K}$, $e_\lambda = 0.899$

Find: $T_s = ? \text{ K}$

Use eq. (21.10):

$$T_s = \left(\frac{2}{2 - 0.899} \right)^{1/4} \cdot (254.8 \text{ K}) = 296 \text{ K} = \underline{23^\circ\text{C}}$$

From Eq. (21.7) we see that the effective emission temperature of the atmosphere is:

$$T_A = (0.5)^{1/4} \cdot T_s = 0.84 \cdot (296 \text{ K}) = 249 \text{ K} = \underline{-24^\circ\text{C}}$$

Check: Units OK. Physics OK.

Discussion: This model is more sophisticated than the previous one, and gives a result closer to the observed temperature of 15°C . But the wrong temperature suggests that some important physics is missing.

But 30°C is too warm compared to the observed average surface temperature of 15°C . Hence, we infer that some important physics are still missing.

Atmospheric Window

Between 8 and $14 \mu\text{m}$ wavelengths, the atmosphere is semi-transparent to infrared (IR) radiation (see the Remote Sensing chapter). This is the **atmospheric window**. Suppose this window allows 10.1% of the IR radiation from the Earth to be lost directly to space, without first being absorbed by the atmosphere. Using Kirchhoff's law from the Radiation chapter, this gives an absorptivity = emissivity (e_λ) = 89.9% for this hypothetical atmosphere.

As before, the incoming fluxes must balance the outgoing, for an equilibrium atmosphere. The same holds for the Earth's surface. As before, the flux output from one system is often an input to another.

For a one-layer atmosphere with a window as sketched in Fig. 21.5, the equilibrium energy balance (input = output) of the atmosphere is:

$$e_\lambda \cdot \sigma_{SB} \cdot T_s^4 = 2 \cdot e_\lambda \cdot \sigma_{SB} \cdot T_A^4 \quad (21.7)$$

For the Earth's surface, the corresponding balance is:

$$R_o + e_\lambda \cdot \sigma_{SB} \cdot T_A^4 = \sigma_{SB} \cdot T_s^4 \quad (21.8)$$

Solving these coupled equations for the atmosphere and surface temperatures yields:

$$T_A^4 = \frac{T_e^4}{2 - e_\lambda} \quad \bullet(21.9)$$

$$T_s^4 = \frac{2 \cdot T_e^4}{2 - e_\lambda} \quad \bullet(21.10)$$

Thus:

$$T_s = 296 \text{ K} = 23^\circ\text{C} \quad \text{and} \quad T_A = 249 \text{ K} = -24^\circ\text{C} \quad (21.11)$$

This surface temperature is closer to realistic, but is still slightly too warm.

If **anthropogenic greenhouse gases** carbon dioxide (CO_2), nitrous oxide (N_2O), methane (CH_4), and freon CFC-12 ($\text{C Cl}_2 \text{ F}_2$) accumulate in the atmosphere, they can close the atmospheric window by absorbing and re-emitting more IR radiation emitted from Earth. This could cause surface temperatures to warm toward those of the previous solved example. This is the threat of **global warming**.

Finally, be assured that Fig. 21.5 is just a cartoon designed to illustrate that some IR radiation from Earth can escape to space through a translucent atmosphere. A spatial hole in the atmosphere does not really exist (although a hole of ozone has sometimes existed over the Earth's poles).

Average Energy Budget

Radiation is not the only physical process that transfers energy. Between the Earth and bottom layer of the atmosphere can also be sensible-heat transfer via conduction (see the Heat chapter), and latent-heat transfer via evapotranspiration (see the Moisture chapter). Once this heat and moisture are in the air, they can be transported further into the atmosphere by the mean wind, by deep convection such as thunderstorms, by shallow convection such as thermals, and by smaller turbulent eddies in the atmospheric boundary layer.

One estimate of the annual mean of energy fluxes between all these components, when averaged over the surface area of the globe, is sketched in Fig. 21.6. According to the Stefan-Boltzmann law, the emitted IR radiation from the Earth's surface ($396 \text{ W}\cdot\text{m}^{-2}$) corresponds to an average surface temperature of 16°C , which is close to observed temperature of 15°C .

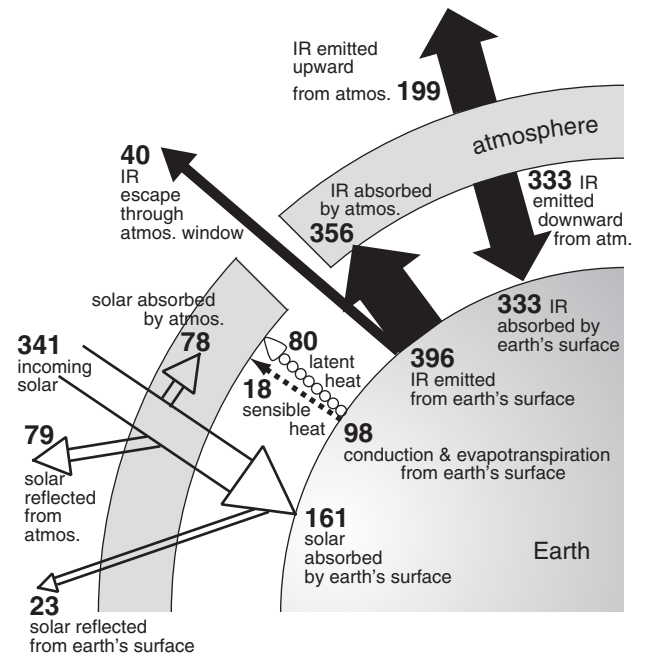


Figure 21.6

Global annual mean energy budget for the Earth system, based on data from 2000-2004. The “atmosphere” component includes the air, clouds, and particles in the air. Numbers are approximate energy fluxes in units of $\text{W}\cdot\text{m}^{-2}$. Errors are of order 3% for fluxes at the top of the atmosphere, and errors are 5 to 10% for surface fluxes. The incoming solar radiation, when averaged over all the Earth's surface, is $S_0/4 = 341 \text{ W}\cdot\text{m}^{-2}$. After subtracting the outgoing (reflected) solar radiation ($341 - 79 - 23 = 239 \text{ W}\cdot\text{m}^{-2}$), the result is the same R_0 that we used before. (Based partially on info from Trenberth, Fasullo and Kiehl, 2009, *Bull. Amer. Meteor. Soc.*)

ASTRONOMICAL INFLUENCES

If the incoming solar radiation (**insolation**) changes, then the radiation budget of the Earth will adjust, possibly altering the climate. Two causes of average-daily-insolation \bar{E} change at the top of the atmosphere are changes in solar output S_0 and changes in Earth-sun distance R (recall eq. 2.21).

Milankovitch Theory

In the 1920s, astrophysicist Milutin Milankovitch proposed that slow fluctuations in the Earth's orbit around the sun can change the sun-Earth distance. Distance changes alter insolation due to the inverse square law (eq. 2.16), thereby contributing to climate change. Ice-age recurrence (inferred from ice and sediment cores) was later found to be correlated with orbital characteristics, supporting his hypothesis. Milankovitch focused on three orbital characteristics: eccentricity, obliquity, and precession.

Eccentricity

The shape of Earth's elliptical orbit around the sun (Fig. 21.7a) slowly fluctuates between nearly circular (**eccentricity** $e \approx 0.0034$) and slightly elliptical ($e \approx 0.058$). For comparison, a perfect circle has $e = 0$. Gravitational pulls by other planets (mostly Jupiter and Saturn) cause these eccentricity fluctuations.

Earth's eccentricity at any time (past or future) can be approximated by a sum of cosine terms:

Solved Example

Confirm that energy flows in Fig. 21.6 are balanced.

Solution

$$\begin{aligned} \text{Solar: } \text{incoming} &= [\text{reflected}] + (\text{absorbed}) \\ 341 &= [79 + 23] + (78 + 161) \\ \mathbf{341} &= \mathbf{341} \text{ W}\cdot\text{m}^{-2} \end{aligned}$$

$$\begin{aligned} \text{Earth's surface: } \text{incoming} &= \text{outgoing} \\ (161 + 333) &= (98 + 396) \\ \mathbf{494} &= \mathbf{494} \text{ W}\cdot\text{m}^{-2} \end{aligned}$$

$$\begin{aligned} \text{Atmosphere: } \text{incoming} &= \text{outgoing} \\ (78 + 18 + 80 + 356) &= (199 + 333) \\ \mathbf{532} &= \mathbf{532} \text{ W}\cdot\text{m}^{-2} \end{aligned}$$

$$\begin{aligned} \text{Earth System: } \text{in from space} &= \text{out to space} \\ 341 &= (23 + 79 + 40 + 199) \\ \mathbf{341} &= \mathbf{341} \text{ W}\cdot\text{m}^{-2} \end{aligned}$$

Check: Yes, all budgets are balanced.

Discussion: The solar budget must balance because the Earth-atmosphere system does not create/emit solar radiation. However, IR radiation is created by the sun, Earth, and atmosphere; hence, the IR fluxes by themselves need not balance.

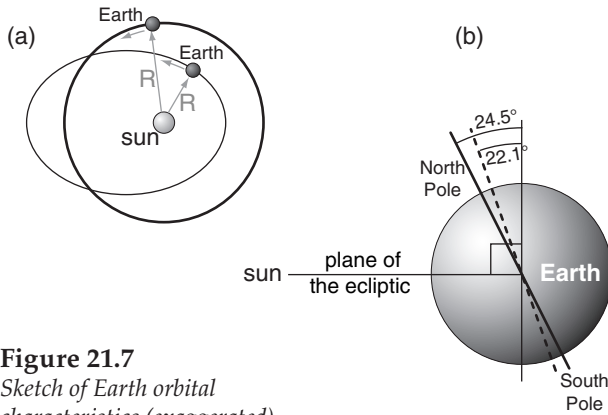


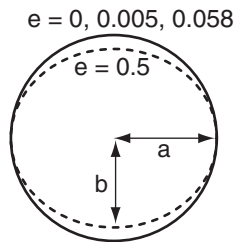
Figure 21.7
Sketch of Earth orbital characteristics (exaggerated).
(a) Eccentricity variations. (b) Obliquity variations.

Solved Example

Plot ellipses for eccentricities of 0, 0.005, 0.058 & 0.50. Hints: the (x, y) coordinates are given by $x = a \cos(t)$, and $y = b \sin(t)$, where t varies from 0 to 2π radians. Assume a semi-major axis of $a = 1$, and get the semi-minor axis from $b = a(1 - e^2)^{1/2}$.

Solution

Plotted at right:



Discussion. The first 3 curves (solid lines) look identical. Thus, for the full range of Earth's eccentricities ($0.0034 \leq e \leq 0.58$), we see that Earth's orbit is nearly circular ($e = 0$).

Solved Example.

Find the Earth-orbit eccentricity 600,000 years ago (i.e., 600,000 years before year 2000).

Solution.

Given: $t = -600,000$ years (negative sign for past).
Find: e (dimensionless).

Use eq. (21.12) with data from Table 21-1:

$$e \approx 0.0275579 + 0.010739 \cos[2\pi(-600000\text{yr}/405091\text{yr} + 170.739^\circ/360^\circ)] + 0.008147 \cos[2\pi(-600000\text{yr}/94932\text{yr} + 109.891^\circ/360^\circ)] + 0.006222 \cos[2\pi(-600000\text{yr}/123945\text{yr} - 60.044^\circ/360^\circ)] + 0.005287 \cos[2\pi(-600000\text{yr}/98857\text{yr} - 86.140^\circ/360^\circ)] + 0.004492 \cos[2\pi(-600000\text{yr}/130781\text{yr} + 100.224^\circ/360^\circ)]$$

$$e \approx 0.0275579 + 0.0107290 + 0.0081106 + 0.0062148 - 0.0019045 - 0.0016384 \approx \mathbf{0.049} \text{ (dimensionless)}$$

Check: $0.0034 \leq e \leq 0.058$, within the expected eccentricity range for Earth. Agrees with Fig. 21.8.

Discussion: An **interglacial** (non-ice-age) period was ending, & a new **glacial** (ice-age) period was starting.

$$e \approx e_0 + \sum_{i=1}^N A_i \cdot \cos \left[C \cdot \left(\frac{t}{P_i} + \frac{\phi_i}{360^\circ} \right) \right] \quad \bullet(21.12)$$

where $e_0 = 0.0275579$, A_i are amplitudes, t is time in years relative to calendar year 2000, P_i are oscillation periods in years, ϕ_i are phase shifts in degrees, and i is an index that counts the 1 to N terms. C should be either 360° or 2π radians, depending on the type of trigonometric argument required by your calculator, spreadsheet, or programming language.

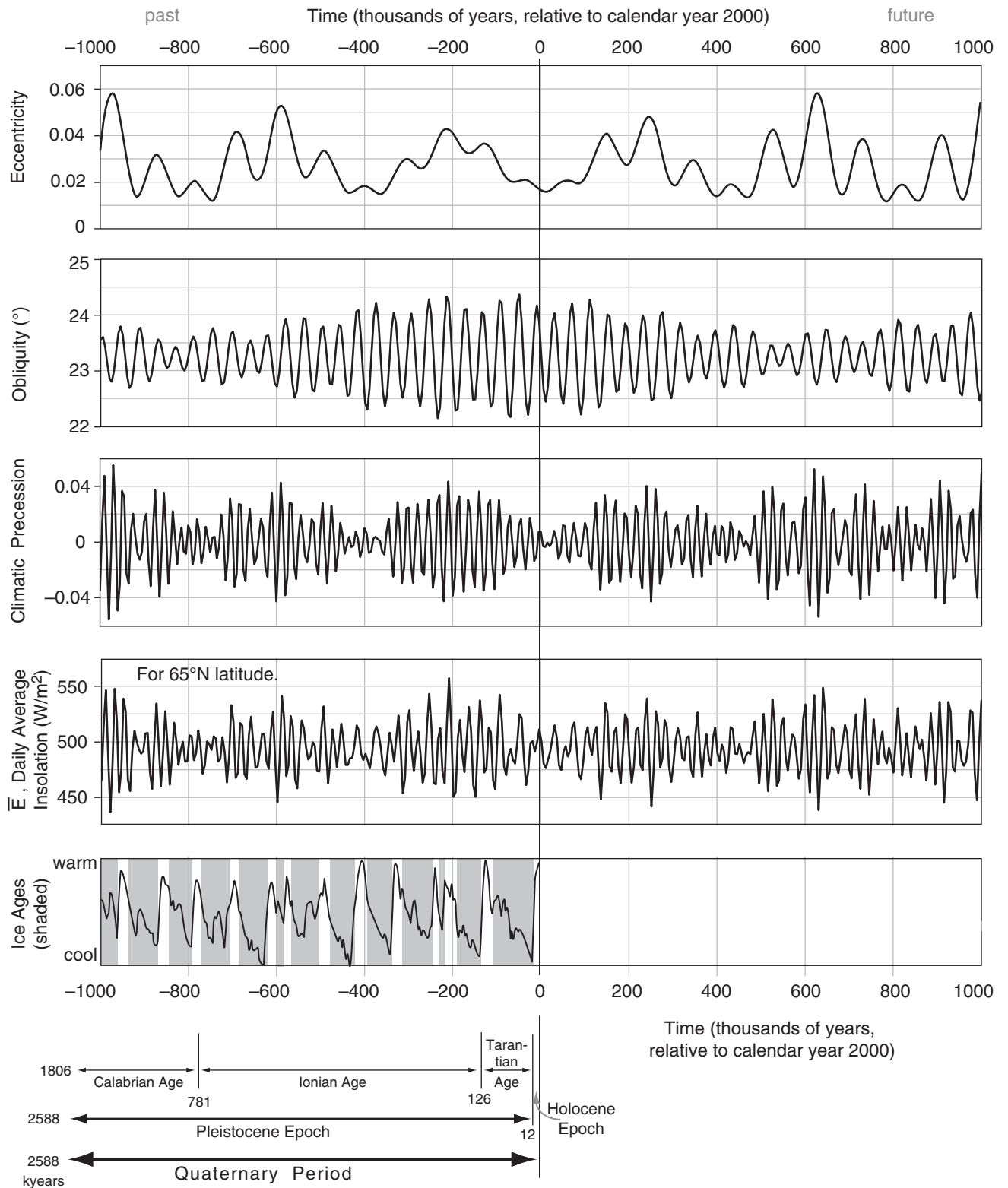
Table 21-1 shows values for these orbital factors for $N = 5$ terms in this series. **[CAUTION:** A more accurate description requires $N = 20$ or more terms. The first 10 terms are listed in Table 21-1b near exercise U9 at the end of this chapter.] I used a spreadsheet to solve these 5 terms for times ranging from 1 million years in the past to 1 million years into the future (see results in Fig. 21.8).

By eye, we see two superimposed oscillations of eccentricity with periods of about 100 and 400 kyr (where “kyr” = kilo years = 1000 years). The first oscillation correlates very well with the roughly 100 kyr period for cool events associated with **ice ages**. The present eccentricity is about 0.0167. Short-term forecast: eccentricity will change little during the next 100,000 years. Variations in e affect Earth-sun distance R via eq. (2.4) in the Radiation chapter.

Table 21-1. Factors in orbital series approximations.

index	A	P (years)	ϕ (degrees)
Eccentricity:			
i= 1	0.010739	405,091.	170.739
2	0.008147	94,932.	109.891
3	0.006222	123,945.	-60.044
4	0.005287	98,857.	-86.140
5	0.004492	130,781.	100.224
Obliquity:			
j= 1	0.582412°	40,978.	86.645
2	0.242559°	39,616.	120.859
3	0.163685°	53,722.	-35.947
4	0.164787°	40,285.	104.689
Climatic Precession:			
k= 1	0.018986	23,682.	44.374
2	0.016354	22,374.	-144.166
3	0.013055	18,953.	154.212
4	0.008849	19,105.	-42.250
5	0.004248	23,123.	90.742

Simplified from Laskar, Robutel, Joutel, Gastineau, Correia & Levrard, 2004: A long-term numerical solution for the insolation quantities of the Earth. “Astronomy & Astrophysics”, 428, 261-285.

**Figure 21.8**

Past and future orbital characteristics in Milankovitch theory, as calculated using only the 4 or 5 highest-order terms (see Table 21-1) in the series approximations of eqs. (21.12 - 21.20). Bottom curve shows relative temperature changes estimated from ice and sediment cores. Ice ages (glacial periods) are shaded grey. Precession is for the summer solstice (i.e., only the $e \sin(\omega)$ term is used).

Solved Example

Estimate Earth's obliquity 90,000 yrs in the future (relative to calendar year 2000).

Solution

Given: $t = 90,000$ yr (positive sign indicates future)

Find: $\varepsilon = ?^\circ$

Use eq. (21.13) with coefficients from Table 21-1.

$$\begin{aligned}\varepsilon &= 23.2545^\circ \\ &+ 0.582412^\circ [2\pi(90000\text{yr}/40978\text{yr} + 86.645^\circ/360^\circ)] \\ &+ 0.242559^\circ [2\pi(90000\text{yr}/39616\text{yr} + 120.859^\circ/360^\circ)] \\ &+ 0.163685^\circ [2\pi(90000\text{yr}/53722\text{yr} - 35.947^\circ/360^\circ)] \\ &+ 0.164787^\circ [2\pi(90000\text{yr}/40285\text{yr} + 104.689^\circ/360^\circ)]\end{aligned}$$

$$\begin{aligned}\varepsilon &= 23.2545^\circ - 0.537349^\circ - 0.189273^\circ - 0.145639^\circ - 0.162777^\circ \\ &= \underline{\underline{22.219^\circ}}\end{aligned}$$

Check: Units OK. Within Earth range $22.1^\circ \leq \varepsilon \leq 24.5^\circ$.

Discussion: Agrees with Fig. 21.8. At this small obliquity, seasonal contrasts are smaller than at present, allowing glaciers to grow due to cooler summers.

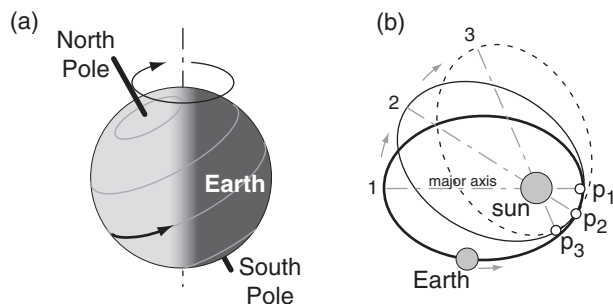


Figure 21.9

Sketch of additional Earth orbital characteristics. (a) Precession of Earth's axis. (b) Precession of the orbit-ellipse major axis (aphelion precession) at successive times 1 - 3. Small white dots p_1, p_2, p_3 show locations of the perihelion (point on Earth's orbit closest to sun). All the orbit ellipses are in the same plane.

Obliquity

The tilt (obliquity) of Earth's axis slowly fluctuates between 22.1° and 24.5° . This tilt angle is measured from a line perpendicular to Earth's orbital plane (the ecliptic) around the sun (Fig. 21.7b).

A series approximation for obliquity ε is:

$$\varepsilon = \varepsilon_0 + \sum_{j=1}^N A_j \cdot \cos \left[C \cdot \left(\frac{t}{P_j} + \frac{\phi_j}{360^\circ} \right) \right] \quad \bullet(21.13)$$

where $\varepsilon_0 = 23.254500^\circ$, and all the other factors have the same meaning as for the eq. (21.12). Table 21-1 shows values for these orbital factors for $N = 4$ terms in this series.

Fig. 21.8 shows that obliquity oscillates with a period of about 41,000 years. The present obliquity is 23.439° and is gradually decreasing.

Obliquity affects insolation via the solar declination angle (eq. 2.5 in the Radiation chapter, where a different symbol Φ_r was used for obliquity). Recall from the Radiation chapter that Earth-axis tilt is responsible for the seasons, so greater obliquity would cause greater contrast between winter and summer. One hypothesis is that ice ages are more likely during times of lesser obliquity, because summers would be cooler, causing less melting of glaciers.

Precession

Various precession processes affect climate.

Axial Precession. Not only does the tilt magnitude (obliquity) of Earth's axis change with time, but so does the tilt direction. This tilt direction rotates in space, tracing one complete circle in 25,680 years (Fig. 21.9a) relative to the fixed stars. It rotates opposite to the direction the Earth spins. This precession is caused by the gravitational pull of the sun and moon on the Earth, because the Earth is an oblate spheroid (has larger diameter at the equator than at the poles). Hence, the Earth behaves somewhat like a spinning toy top.

Aphelion Precession. The direction of the major axis of Earth's elliptical orbit also precesses slowly (with a 174,000 to 304,00 year period) relative to the fixed stars (Fig. 21.9b). The rotation direction is the same as the axial-precession, but the precession speed fluctuates due to the gravitational pull of the planets (mostly Jupiter and Saturn).

Equinox Precession. Summing axial and aphelion precession rates (i.e., adding the inverses of their periods) gives a total precession with fluctuating period of about 22,000 years. This combined precession describes changes to the angle (measured at the sun) between Earth's orbital positions at the perihelion (point of closest approach) and at the moving vernal (Spring) equinox (Fig. 21.10). This

fluctuating angle (ϖ , a math symbol called “variant pi”) is the equinox precession.

For years when angle ϖ is such that the winter solstice is near the perihelion (as it is in this century), then the cooling effect of the N. Hemisphere being tilted away from the sun is slightly moderated by the fact that the Earth is closest to the sun; hence winters and summers are not as extreme as they could be. Conversely, for years with different angle ϖ such that the summer solstice is near the perihelion, then the N. Hemisphere is tilted toward the sun at the same time that the Earth is closest to the sun — hence expect hotter summers and colder winters. However, seasons near the perihelion are shorter duration than seasons near the aphelion, which moderates the extremes for this latter case.

Climatic Precession. ϖ by itself is less important for insolation than the combination with eccentricity e : $e \cdot \sin(\varpi)$ and $e \cdot \cos(\varpi)$. These terms, known as the climatic precession, can be expressed by series approximations, where both terms use the same coefficients from Table 21-1, for $N = 5$.

$$e \cdot \sin(\varpi) \approx \sum_{k=1}^N A_k \cdot \sin \left[C \cdot \left(\frac{t}{P_k} + \frac{\phi_k}{360^\circ} \right) \right] \quad \bullet(21.14)$$

$$e \cdot \cos(\varpi) \approx \sum_{k=1}^N A_k \cdot \cos \left[C \cdot \left(\frac{t}{P_k} + \frac{\phi_k}{360^\circ} \right) \right] \quad (21.15)$$

Other factors are similar to those in eq. (21.12).

A plot of eq. (21.14), labeled “climatic precession”, is shown in Fig. 21.8. It shows high frequency (22,000 year period) waves with amplitude modulated by the eccentricity curve from the top of Fig. 21.8. Climatic precession affects sun-Earth distance, as described next.

Insolation Variations

Eq. (2.21) from the Radiation chapter gives average daily insolation \bar{E} . It is repeated here:

$$\bar{E} = \frac{S_0}{\pi} \cdot \left(\frac{a}{R} \right)^2 \cdot \left[h_o' \cdot \sin(\phi) \cdot \sin(\delta_s) + \cos(\phi) \cdot \cos(\delta_s) \cdot \sin(h_o) \right] \quad \bullet(21.16)$$

where $S_0 = 1366 \text{ W/m}^2$ is the solar irradiance, $a = 149.457 \text{ Gm}$ is the semi-major axis length, R is the sun-Earth distance for any day of the year, h_o' is the hour angle in radians, ϕ is latitude, and δ_s is solar declination angle. Factors in this eq. are given next.

Instead of solving this formula for many latitudes, climatologists often focus on one key latitude: $\phi = 65^\circ\text{N}$. This latitude crosses Alaska, Canada,

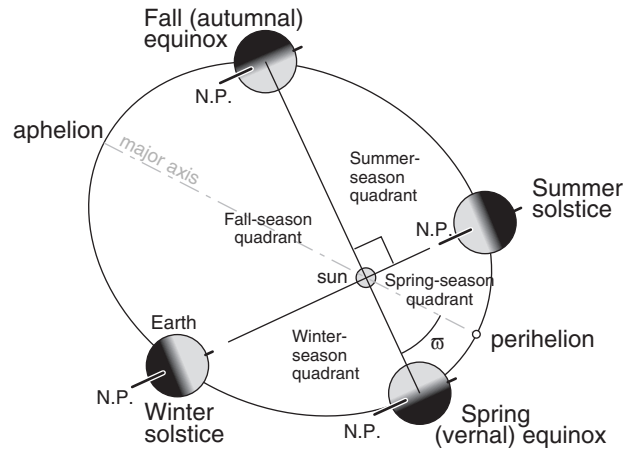


Figure 21.10

Illustration for sometime in the future, showing the angle (ϖ) of the perihelion from the moving vernal equinox. N.P. = North Pole. Seasons are for the N. Hemisphere.

Solved Example

Estimate the climatic precession (sine term) for year 2000.

Solution

Given: $t = 0$

Find: $e \cdot \sin(\varpi) = ?$ (dimensionless)

Use eq. (21.14) with coefficients from Table 21-1.

$$\begin{aligned} e \cdot \sin(\varpi) = & + 0.018986 \cdot [2\pi \cdot (0\text{yr}/23682\text{yr} + 44.374^\circ/360^\circ)] \\ & + 0.016354 \cdot [2\pi \cdot (0\text{yr}/22374\text{yr} - 144.166^\circ/360^\circ)] \\ & + 0.013055 \cdot [2\pi \cdot (0\text{yr}/18953\text{yr} + 154.212^\circ/360^\circ)] \\ & + 0.008849 \cdot [2\pi \cdot (0\text{yr}/19105\text{yr} - 42.250^\circ/360^\circ)] \\ & + 0.004248 \cdot [2\pi \cdot (0\text{yr}/23123\text{yr} + 90.742^\circ/360^\circ)] \end{aligned}$$

$$\begin{aligned} e \cdot \sin(\varpi) = & 0.0132777 - 0.0095743 + 0.0056795 \\ & - 0.0059498 + 0.0042476 = \underline{\underline{0.00768}} \end{aligned}$$

Check: Agrees with curve plotted in Fig. 21.8.

Discussion: The small eccentricity in year 2000 (i.e., almost at present) causes small climatic precession.

Table 21-2. Key values of λ , the angle between the Earth and the moving vernal equinox. Fig. 21.10 shows that solstices and equinoxes are exactly 90° apart.

Date	λ (radians, $^\circ$)	$\sin(\lambda)$	$\cos(\lambda)$
Vernal (Spring) equinox	0, 0	0	1
Summer solstice	$\pi/2$, 90°	1	0
Autumnal (Fall) equinox	π , 180°	0	-1
Winter solstice	$3\pi/2$, 270°	-1	0

Solved Example

For the summer solstice at 65°N latitude, find the average daily insolation for years 2000 and 2200.

Solution

Given: $\lambda = \pi/2$, $\phi = 65^\circ\text{N}$, (a) $t = 0$, (b) $t = 200$ yr
Find: $\bar{E} = ? \text{ W/m}^2$ (Assume $S_0 = 1366 \text{ W/m}^2$)

(a) From previous statements and solved examples:

$$e = 0.0167, \quad \delta_s = \varepsilon = 23.439^\circ, \quad e \cdot \sin(\varpi) = 0.00768$$

Use eq. (21.17): $h_o = \arccos[-\tan(65^\circ) \cdot \tan(23.439^\circ)]$
 $= 158.4^\circ = 2.7645 \text{ radians} = h_o'$

Eq. (21.20): $(a/R) = (1 + 0.00768)/[1 - (0.0167)^2] = 1.00796$
 Plugging all these into eq. (21.16):

$$\bar{E} = [(1366 \text{ W/m}^2)/\pi] \cdot (1.00796)^2 \cdot [2.7645 \cdot \sin(65^\circ) \cdot \sin(23.439^\circ) + \cos(65^\circ) \cdot \cos(23.439^\circ) \cdot \sin(158.4^\circ)]$$

$$\bar{E} = \mathbf{503.32 \text{ W/m}^2} \quad \text{for } t = 0.$$

(b) Eq. (21.12) for $t = 200$: $e \approx 0.0168$

Eq. (21.13): $\varepsilon = 23.2278^\circ$. Eq. (21.14): $e \cdot \sin(\varpi) = 0.00730$

Eq. (21.20): $a/R = 1.00758$. Eq. (21.16): $\bar{E} = \mathbf{499.3 \text{ W/m}^2}$

Check: Units OK. Magnitude \approx that in Fig. 2.11 (Ch2).

Discussion: Milankovitch theory indicates that insolation will decrease slightly during the next 200 yr.

Greenland, Iceland, Scandinavia, and Siberia, and is representative of genesis zones for ice caps.

The hour angle h_o at sunrise and sunset is given by the following set of equations:

$$\alpha = -\tan(\phi) \cdot \tan(\delta_s) \quad (21.17a)$$

$$\beta = \min[1, (\max(-1, \alpha))] \quad (21.17b)$$

$$h_o = \arccos(\beta) \quad (21.17c)$$

Although the hour angle h_o can be in radians or degrees (as suits your calculator or spreadsheet), the hour angle marked with the prime (h_o') must be in radians.

The solar declination angle, used in eq. (21.16), is

$$\delta_s = \arcsin[\sin(\varepsilon) \cdot \sin(\lambda)] \approx \varepsilon \cdot \sin(\lambda) \quad (21.18)$$

where ε is the obliquity from eq. (21.13), and λ is the **true longitude** angle (measured at the sun) between the position of the Earth and the position of the moving vernal equinox. Special cases for λ are listed in Table 21-2.

Recall from the Radiation chapter (eq. 2.4) that the sun-Earth distance R is related to the semi-major axis a by:

$$\frac{a}{R} = \frac{1 + e \cdot \cos(v)}{1 - e^2} \quad (21.19)$$

where v is the true anomaly (position of the Earth from the perihelion), and e is the eccentricity. For the special case of the summer solstice (see the Sun-Earth Distance Focus Box), this becomes:

$$\frac{a}{R} = \frac{1 + [e \cdot \sin(\varpi)]}{1 - e^2} \quad (21.20)$$

where e is from eq. (21.12) and $e \cdot \sin(\varpi)$ is from eq. (21.14)

I used all the information above to solve eq. (21.16) on a spreadsheet for average daily insolation at latitude 65°N during the summer solstice for 1 Myr in the past to 1 Myr in the future. This results in the curve for \bar{E} plotted in Fig. 21.8. This is the climatic signal resulting from Milankovitch theory. Some scientists have found a good correlation between it and the historical temperature curve plotted at the bottom of Fig. 21.8, although the issue is still being debated.

FOCUS • Sun-Earth Distance

Recall from Chapter 2 (eq. 2.4) that the sun-Earth distance R is related to the semi-major axis a by:

$$\frac{a}{R} = \frac{1 + e \cdot \cos(v)}{1 - e^2} \quad (21.19)$$

where v is the true anomaly (position of the Earth from the perihelion), and e is the eccentricity from eq. (21.12). By definition, $v = \lambda - \varpi$ (21.a)

where λ is position of the Earth from the moving vernal equinox, and ϖ is position of the perihelion from the moving vernal equinox. Thus,

$$\frac{a}{R} = \frac{1 + e \cdot \cos(\lambda - \varpi)}{1 - e^2} \quad (21.b)$$

Using trigonometric identities, the cosine term is:

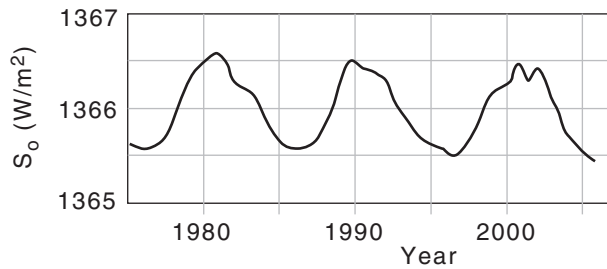
$$\frac{a}{R} = \frac{1 + \cos(\lambda) \cdot [e \cdot \cos(\varpi)] + \sin(\lambda) \cdot [e \cdot \sin(\varpi)]}{1 - e^2} \quad (21.c)$$

The terms in square brackets are the **climatic precisions** that you can find with eqs. (21.14) and (21.15).

To simplify the study of climate change, researchers often consider a special time of year; e.g., the summer solstice ($\lambda = \pi/2$ from Table 21-2). This time of year is important because it is when glaciers may or may not melt, depending on how warm the summer is. At the summer solstice, the previous equation simplifies to:

$$\frac{a}{R} = \frac{1 + [e \cdot \sin(\varpi)]}{1 - e^2} \quad (21.20)$$

Also, during the summer solstice, the solar declination angle from eq. (21.18) simplifies to: $\delta_s = \varepsilon$

**Figure 21.11**

Variation of annual average total solar irradiance S_0 at the top of Earth's atmosphere during 3 recent sunspot cycles.

Solar Output

The average total solar irradiance (TSI) currently reaching the top of Earth's atmosphere is $S_0 = 1366 \text{ W}\cdot\text{m}^{-2}$, but the magnitude fluctuates daily due to solar activity, with extremes of 1362 and $1368 \text{ W}\cdot\text{m}^{-2}$ measured by satellite since 1975. Satellite observations have calibration errors on the order of $\pm 7 \text{ W}\cdot\text{m}^{-2}$.

Sunspot Cycle

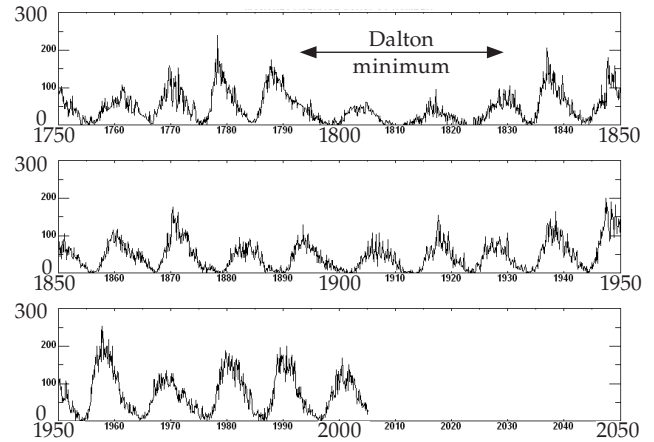
When averaged over each year, the resulting smoothed curve of TSI varies with a noticeable 11-yr cycle (Fig. 21.11), corresponding to the 9.5 to 11-yr sunspot cycle (Fig. 21.12) as observed by telescope. Because the TSI variation is only about 0.1% of its total magnitude, the sunspot cycle is believed to have only a minor (possibly negligible) effect on recent climate change.

Longer-term Variations in Solar Output

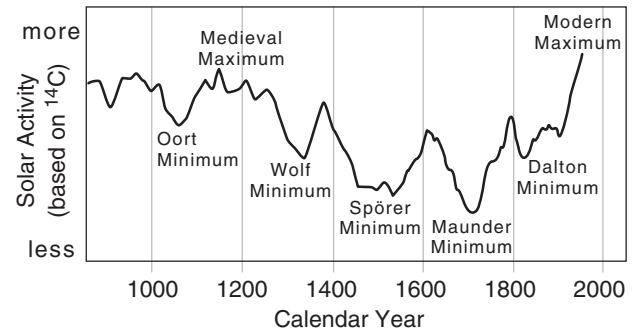
To get solar output information for the centuries before telescopes were invented, scientists use **proxy** measures. Proxies are measurable phenomena that vary with solar activity, such as beryllium-10 concentrations in Greenland ice cores, or carbon-14 concentration in tree rings (**dendrochronology**). Radioactive carbon-14 dating suggests solar activity for the past 1,000 years as plotted in Fig. 21.13. Noticeable is the 200-yr cycle in solar-activity minima.

Going back 11,000 yr in time, Fig. 21.14 shows a highly-smoothed carbon-14 estimate of solar/sunspot activity. There is some concern among scientists that Earth-based factors might have confounded the proxy analysis of tree rings, such as extensive volcanic eruptions that darken the sky and reduce tree growth over many decades.

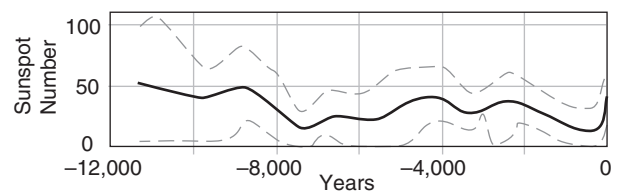
Even further back (3 to 4 Myr ago), the sun was younger and weaker, and emitted only about 70% of what it currently emits. About 5 billion years into the future, the sun will grow into a red-giant star (killing all life on Earth), and later will shrink into a white dwarf star.

**Figure 21.12**

Monthly average sunspot number during 1750 - 2005. NASA.

**Figure 21.13**

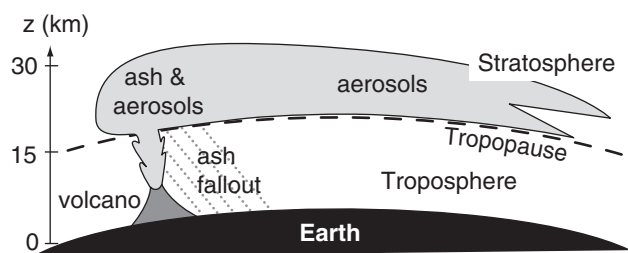
Carbon 14 concentration as a proxy for solar activity. Approximate dates for these minima: **Oort Minimum:** 1040 - 1080, **Wolf Minimum:** 1280 - 1350, **Spörer Minimum:** 1450 - 1550, **Maunder Minimum:** 1645 - 1715, and **Dalton Minimum:** 1790 - 1830. The 11-year sunspot cycle still exists during this time span, but was smoothed out for this plot.

**Figure 21.14**

Smoothed proxy estimate of sunspots. Time is relative to calendar year 2000. Dashed lines show envelope of fluctuations.

**Figure 21.15**

Location of continents about 225 million years ago to form the supercontinent Pangea (also spelled as Pangaea and Pangaea). This supercontinent spanned north-south from pole to pole.

**Figure 21.16**

Eruption of volcanic ash and aerosols into the stratosphere. Average times for particles to fall 1 km are: 36 yr for 0.1 μm diameter particles, 3.2 yr for 0.5 μm , 328 days for 1 μm , 89 days for 2 μm , 14.5 days for 5 μm , and 3.6 days for 10 μm .

TECTONIC INFLUENCES

Continent Movement

As Earth-crust tectonic plates move, continents form, change, merge and break apart. About 225 million years ago the continents were merged into the **Pangea** supercontinent (Fig. 21.15). The interior of such large continents are sufficiently far from the moderating influence of oceans that extreme daily and seasonal temperature variations can occur. Large monsoon circulations can form due to the temperature contrasts between continent and ocean. The more recent (200 million years ago) supercontinents of Gondwanaland and Laurasia would likely have had similar monsoon circulations, and extreme temperature variations in their interiors.

Colliding tectonic plates can cause **mountain building (orogenesis)**. Large mountain ranges can block portions of the global circulation, changing the Rossby waves in the jet stream, and altering cyclogenesis regions. Mountains can partially block the movement of airmasses, changing temperature and precipitation patterns.

Changes in continent location and ocean depths cause changes in ocean circulations. **Thermohaline** circulations are driven by water-density differences caused by temperature and salt-concentration variations. These large, slow circulations connect deep water and surface waters of all oceans, and are important for redistributing the excess heat from the equator toward the poles. Climatic changes in ocean-surface temperatures can alter atmospheric temperature, humidity, winds, cloudiness, and precipitation distribution.

Volcanism

When volcanoes erupt explosively, the eruptive heat and vertical momentum ejects tons of tiny rock particles called **volcanic ash** into the upper troposphere and lower stratosphere (Fig. 21.16). During the few weeks before this ash falls out of the air, it can block some sunlight from reaching the surface.

Gases such as **sulfur dioxide (SO_2)** are also emitted in large volumes (0.1 to 6000 Mtons from the larger eruptions), and are much more important for climate change. SO_2 oxidizes in the atmosphere to form sulfuric acid and sulfate **aerosols**, tiny droplets that fall so slowly they seem suspended in the air.

Those sulfate particles in the stratosphere can remain suspended in the atmosphere for a few years as they are spread around the world by the winds. The sulfates absorb and reflect some of the incoming solar radiation, allowing less to be transmitted

Science Graffito

"The most important fact about Spaceship Earth: an instruction book didn't come with it."

– R. Buckminster Fuller.

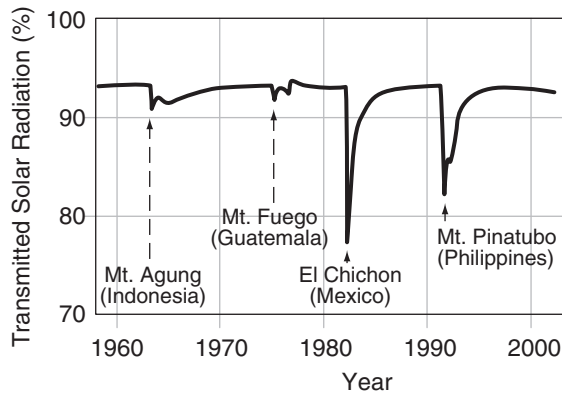


Figure 21.17

Reduction in transmitted shortwave radiation from the sun due to aerosols and particulates erupted into the atmosphere from volcanoes, as observed at Muana Loa.

toward the ground (Fig. 21.17). Also, these aerosols absorb upwelling IR radiation from the Earth and troposphere.

The net result for aerosol particle diameters less than $0.5\ \mu\text{m}$ is that the stratosphere warms while the Earth's surface and lower troposphere cool. This cooling causes short-term (2 - 3 yr) climate change known as **volcanic winter** (analogous to the effects of nuclear winter). The surface cooling can be so great as to ruin vegetation and crops around the world, as was experienced throughout history (Table 21-3). For a brief explosive eruption, the stratospheric effects are greatest initially and gradually diminish over a 3-year period as the aerosols fall out. For violent continuous eruptions, the climatic effects can last decades to millennia.

Volcanoes also erupt hundreds of millions tons of **greenhouse gases** (gases that absorb IR radiation as can alter the global heat budget), including carbon dioxide (CO_2) and water vapor (H_2O). These gases can cause longer term global warming after the aerosol-induced cooling has ended.

After the large eruptions, you can see amazingly beautiful sunsets and sunrises with a bright **red glow** in the sky. Sometimes you can see a **blue moon** at night. These phenomena are due to scattering of sunlight by sulfate aerosols with diameters $\leq 0.5\ \mu\text{m}$ (see the Optics Chapter).

Table 21-3. Some notable volcanic eruptions.

Volcano (Date, Location) • Effect

Siberian Traps, creating floods of basaltic lava (~250 Myr ago, Russia) - Lasted a million years, emitting tremendous volumes of sulfates and greenhouse gases into the atmosphere that caused a mass extinction at the **Permian-Triassic** geological-period boundary. Known as the "**Great Dying**", where 96% of all marine species and 70 percent of terrestrial vertebrate species disappeared.

Major volcanic eruptions likely contributed to other mass extinctions: End-Ordovician (~450 Myr ago) and Late Devonian (~375 Myr ago).

Deccan Traps, creating floods of basaltic lava (~65.5 Myr ago, India) • The Cretaceous–Paleogene extinction event (K–Pg event, formerly known as Cretaceous–Tertiary or **K-T event**) was when the **dinosaurs** disappeared (except for those that evolved into modern-day birds). The sulfate and greenhouse-gas emissions from this massive eruption are believed to be a contributing factor to the K-T event, which was exacerbated by a massive asteroid impact in Mexico.

Yellowstone Caldera (640 kyr ago, USA) • Emissions reduced N. Hem. temperatures by about 5°C

Lake Toba (~90 kyr ago, Indonesia) • Emissions killed most of the humans on Earth, possibly causing a **genetic bottleneck** of reduced genetic diversity. Stratospheric aerosol load ≈ 1000 Mtons. Optical depth $\tau \approx 10$ (see Optics chapter for definition of τ).

Mt. Tambora (1815, Indonesia) • Emissions caused so much cooling in the N. Hemisphere that 1816 is known as the "**Year Without Summer**". Widespread famine in Europe and N. America due to crop failure and livestock deaths. Followed by an exceptionally cold winter. Stratospheric aerosol load ≈ 200 Mtons. Optical depth $\tau \approx 1.3$.

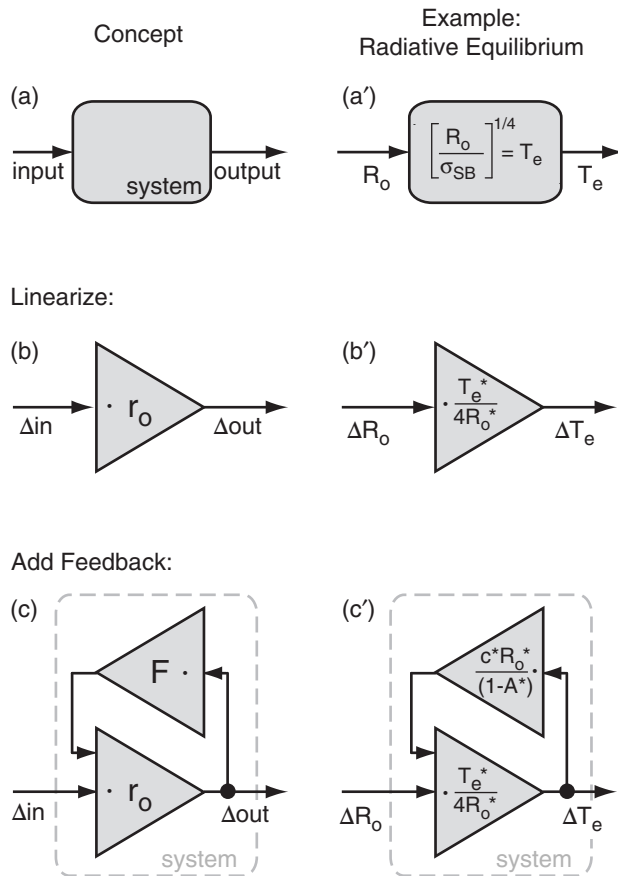
Krakatau (1883, Indonesia) • Violent explosion heard 3,500 km away. Ash reached 80 km altitude. Global temperature decrease $\approx 0.3 - 1.2^\circ\text{C}$. 50 Mtons of aerosols into stratosphere. $\tau \approx 0.55$.

Science Graffito

Darkness

"I had a dream, which was not all a dream.
The bright sun was extinguish'd, and the stars
Did wander darkling in the eternal space,
Rayless, and pathless, and the icy Earth
Swung blind and blackening in the moonless air;
Morn came and went — and came, and
brought no day,
And men forgot their passions in the dread
Of this their desolation; and all hearts
Were chill'd into a selfish prayer for light."

- Lord Byron (1816 –the Year Without Summer.)

**Figure 21.18**

Left column: feedback concepts. Right column: example of ice-albedo feedback within the radiative-equilibrium process, as described by nonlinear eq. (21.3). The reference state is (R_o^*, T_e^*, A^*, c^*) .

Solved Example

Suppose $r_o = 0.1$ and $F = -3$. Find the feedback factor, gain, and compare system responses with and without feedback.

Solution

Given: $F = -3$, no-feedback system response $r_o = 0.1$
Find: $f = ?$, $G = ?$, $r = ?$ = response with feedback

Use eq. (21.24):

$f = (0.1) \cdot (-3) = \underline{-0.3}$ is the feedback factor

Use eq. (21.26):

$G = 1 / [1 - (-0.3)] = \underline{0.77}$ is the gain

Use eq. (21.28):

$r = (0.77) \cdot (0.1) = \underline{0.077}$ is the system response with feedback, compared to $r_o = 0.1$ without.

Check: Units all dimensionless. Magnitudes OK.

Discussion: This system is damped. It has negative feedback. This feedback diminishes the response. For many systems, the output units differ from the input units, in which case r_o , F , and r would have units.

FEEDBACKS**Concept**

Consider a system or physical process with an input and output (Fig. 21.18a). This system might be **linear** (described by a straight line when output is plotted vs. input) or **nonlinear** (described by a curved line). Various values of input give various values of output.

Choose one particular set of input and output values as a **reference state** of the system. Find the slope r_o of the line at this reference point. This slope describes the **response** of the output (Δout) to a small change of input (Δin). Namely,

$$\Delta out = r_o \cdot \Delta in \quad (21.21)$$

as sketched in Fig. 21.18b. The slope is a straight line. Thus, even if the original system is nonlinear, this slope describes a **linearized approximation** to the system response at the reference point.

Feedback (Fig. 21.18c) is where the output signal is added to the input via some feedback process F :

$$\Delta out = r_o \cdot [\Delta in + F \cdot \Delta out] \quad (21.22)$$

A linear approximation is also used for this feedback. The feedback is internal to the system (Fig. 21.18c). Thus the amount of feedback responds to changes in the system. Contrast that to inputs, which are considered **external forcings** that do not respond to changes within the system.

The previous feedback equation can be rewritten as

$$\Delta out = r_o \cdot \Delta in + f \cdot \Delta out \quad \bullet (21.23)$$

where the **feedback factor** f is given by

$$f = r_o \cdot F \quad (21.24)$$

Solving this equation for Δout gives:

$$\Delta out = \frac{1}{(1 - f)} \cdot r_o \cdot \Delta in \quad (21.25)$$

Eq. (21.25) looks similar to eq. (21.21) except for the new factor $1/(1-f)$. In electrical engineering, this factor is called the **gain**, G :

$$G = \frac{1}{(1 - f)} \quad (21.26)$$

[CAUTION: Some climate researchers use a different definition of gain than electrical engineers. See the Focus Box on the next page.]

If we rewrite eq. (21.25) as

$$\Delta_{out} = r \cdot \Delta_{in} \quad (21.27)$$

then we see that the linearized process WITH feedback (eq. 21.27) looks identical to that without (eq. 21.21) but with a different **response**:

$$r = G \cdot r_o \quad (21.28)$$

If $f \geq 1$, then the feedback is so strong that the response increases without limit (i.e., a **runaway** response), and no equilibrium exists. If $0 < f < 1$, then there is **positive feedback** with gain $G > 1$ that leads to an amplified but stable new equilibrium. If $f < 0$, then there is **negative feedback** with gain $G < 1$ that damps the response toward a different stable equilibrium.

For N feedback processes that are independent of each other and additive, the total gain is

$$G = \left[1 - \sum_{i=1}^N f_i \right]^{-1} \quad (21.29)$$

where f_i is the feedback factor for any one process i .

Idealized Example

To illustrate feedback, use the simple no-atmosphere blackbody radiative-equilibrium system of Fig. 21.1. The net solar input per square meter of the Earth's surface is

$$R_o = (1 - A) \cdot S_o / 4 \quad (21.30)$$

(see the Focus box on the 3rd page of this chapter). The IR-radiation output is described by the Stefan-Boltzmann law, resulting in the following energy balance of input = output:

$$R_o = \sigma_{SB} \cdot T_e^4 \quad (21.31)$$

as indicated in Fig. 21.18a'.

The dashed lines in Fig. 21.2 show how an input of $R_o = 239 \text{ W/m}^2$ ($S_o = 1366 \text{ W/m}^2$) results in an output $T_e \approx 255 \text{ K}$. Use this blackbody Earth-equilibrium condition as the **reference state**, and denote the reference values with asterisks (R_o^* , T_e^*). The thick curved line in Fig. 21.2 shows how T_e varies nonlinearly for different inputs of R_o , assuming $A = 0.3$ is constant (i.e., no feedback other than the background outgoing IR).

No Feedback

Focus on this no-feedback example first (Fig. 21.18b'). For small solar-input variations ΔR_o about its reference state, the corresponding temperature

FOCUS • Gain — Different Definitions

Some climate researchers define the **gain** g as

$$g = \frac{\Delta_{out} - \Delta_{out_o}}{\Delta_{out}}$$

where Δ_{out_o} is the background response with NO feedbacks. Thus:

$$g = \frac{G \cdot r_o \cdot \Delta_{in} - r_o \cdot \Delta_{in}}{G \cdot r_o \cdot \Delta_{in}} = 1 - \frac{1}{G}$$

But $G = 1/(1-f)$ from eq. (21.26). Thus

$$g = f$$

Namely, the “gain” used in some climate studies is the feedback factor used in this chapter and in electrical engineering.

Science Graffito

“There are no great people in this world, only great challenges which ordinary people rise to meet.”

Admiral William F. Halsey, Jr.

BEYOND ALGEBRA • No-feedback Response

Start with no feedback, as described by the energy balance of eq. (21.31).

$$R_o = \sigma_{SB} \cdot T_e^4$$

Take the derivative:

$$dR_o = 4 \sigma_{SB} \cdot T_e^3 dT_e$$

Rearrange to find the change of output with input (dT_e/dR_o),

$$\frac{\partial T_e}{\partial R_o} = \frac{1}{4 \cdot \sigma_{SB} \cdot T_e^3}$$

Finally, convert to finite differences:

$$r_o = \frac{\Delta T_e}{\Delta R_o} = \frac{1}{4 \cdot \sigma_{SB} \cdot T_e^3} \quad (21.33)$$

Another form of this answer can be found by multiplying the numerator and denominator by T_e , and then substituting eq. (21.31) in the denominator:

$$r_o = \frac{T_e}{4 \cdot R_o}$$

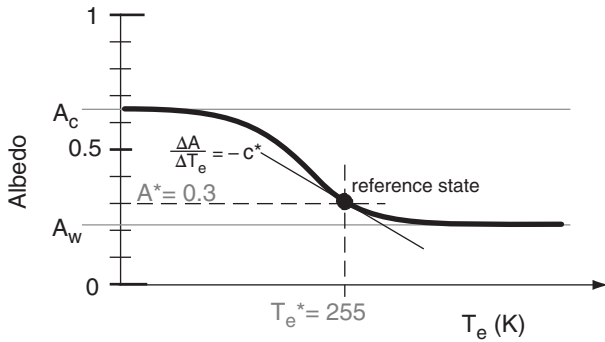


Figure 21.19

Thick line is a sketch of response of Earth's albedo to changes in radiative equilibrium temperature T_e , associated with changes in snow- and ice-cover. The thin line of slope $-c^*$ corresponds to the slope of the thick line at the reference point. Values for A_c and A_w are not accurate — only for illustration.

Solved Example

For ice-albedo radiative-equilibrium with a blackbody-Earth reference state, find the feedback factor and gain. Compare system responses with and without feedback.

Solution

Given: $T_e^* = 255 \text{ K}$, $R_o^* = 239 \text{ W/m}^2$, $A^* = 0.3$, $c^* = 0.01 \text{ K}^{-1}$
Find: $r_o = ? \text{ (K/W}\cdot\text{m}^2\text{)}$, $f = ?$, $G = ?$, $r = ? \text{ (K/W}\cdot\text{m}^2\text{)}$

Use eq. (21.33) for the no-feedback system response:
 $r_o = (255 \text{ K}) / (4 \cdot 239 \text{ W/m}^2) = \underline{0.267 \text{ K/(W}\cdot\text{m}^2)}$

Use eq. (21.36) for the feedback factor:
 $f = (0.01 \text{ K}^{-1}) \cdot (255 \text{ K}) / [4 \cdot (1 - 0.3)] = \underline{0.91 \text{ (dim'less)}}$

Use eq. (21.26): $G = 1 / (1 - 0.91) = \underline{11.1 \text{ (dimensionless)}}$

Use eq. (21.28):
 $r = (11.1) \cdot [0.267 \text{ K/(W}\cdot\text{m}^2)] = \underline{2.96 \text{ K/(W}\cdot\text{m}^2)}$

Check: Response is reasonable. Units OK.

Discussion: This illustrates positive feedback; namely, the response is **amplified** by the feedback. As an example, $\Delta S_o \approx 1 \text{ W/m}^2$ due to sunspot cycle (Fig. 21.11) implies $\Delta R_o = 0.175 \text{ W/m}^2$, giving $\Delta T_e = r \cdot \Delta R_o = 0.52^\circ\text{C}$ variation in equilibrium radiation temperature due to sunspots.

change ΔT_e can be approximated by a straight line tangent to the curve at the reference point (Fig. 21.2), as defined by

$$r_o = \Delta T_e / \Delta R_o \quad (21.32)$$

Based on eq. (21.31) the slope r_o of this straight line is (see Beyond Algebra box on the previous page):

$$r_o = \frac{1}{4 \cdot \sigma_{SB} \cdot T_e^{*3}} = \frac{T_e^*}{4 \cdot R_o^*} = 0.267 \text{ K/(W}\cdot\text{m}^2) \quad (21.33)$$

Ice Albedo Feedback

Next, add **ice-albedo feedback** by allowing the albedo to vary with temperature. Changes in temperature T_e alter the areal extent of highly reflective ice caps and glaciers, thereby changing the albedo (A) between two extremes (Fig. 21.19). A warm limit A_w assumes 0% snow cover, and is based on the reflectivities of the ground, crops, cities, and oceans. A cold-limit A_c represents 100% snow cover, causing a highly reflective Earth. The blackbody Earth state is useful as a reference point ($T_e^* = 255 \text{ K}$, $A^* = 0.3$).

Use the slope (thin line in Fig. 21.19) of the A vs. T_e curve at the reference point as a **linear approximation** of the nonlinear feedback (thick line). Numerical simulations of the global climate suggest this slope is:

$$\frac{\Delta A}{\Delta T} = -c^* \quad (21.34)$$

where $c^* = 0.01 \text{ K}^{-1}$ is the magnitude of the slope at the reference point. The ice-albedo feedback described here is highly oversimplified, for illustrative purposes.

For this idealized ice-albedo radiative-equilibrium case, the feedback equation (21.23) becomes:

$$\Delta T_e = r_o \cdot \Delta R_o + f \cdot \Delta T_e \quad (21.35)$$

where r_o was given in eq. (21.33), and where

$$f = \frac{c^* \cdot T_e^*}{4 \cdot (1 - A^*)} \quad (21.36)$$

(see the Beyond Algebra box on the next page).

The resulting gain is

$$G = \frac{4 \cdot (1 - A^*)}{4 \cdot (1 - A^*) - c^* \cdot T_e^*} \quad (21.37)$$

After multiplying r_o by the gain to give r , we find that the temperature response to solar-input variations (when feedbacks ARE included) is:

$$r = \frac{(1 - A^*) \cdot T_e^*}{R_o^* \cdot [4 \cdot (1 - A^*) - c^* \cdot T_e^*]} \quad (21.38)$$

Fig. 21.20 shows how the total response r with feed-

BEYOND ALGEBRA • Feedback Example

For ice-albedo feedback, infinitesimal variations in solar irradiance dS_o and albedo dA cause infinitesimal changes in equilibrium temperature dT_e :

$$dT_e = \frac{\partial T_e}{\partial S_o} dS_o + \frac{\partial T_e}{\partial A} dA \quad (21.a)$$

To find the factors that go into this equation, start with a simple radiative-equilibrium balance:

$$T_e = \left[\frac{(1-A) \cdot S_o}{4 \cdot \sigma_{SB}} \right]^{1/4} \quad (21.3)$$

and take the derivative $\partial T_e / \partial S_o$:

$$\frac{\partial T_e}{\partial S_o} = \frac{1}{4} \left[\frac{(1-A) \cdot S_o}{4 \cdot \sigma_{SB}} \right]^{-3/4} \cdot \left[\frac{1-A}{4 \cdot \sigma_{SB}} \right]$$

Substituting eq. (21.3) into this gives:

$$\frac{\partial T_e}{\partial S_o} = \frac{1}{4} [T_e]^{-3} \cdot \left[\frac{1-A}{4 \cdot \sigma_{SB}} \right]$$

which we can rewrite as:

$$\frac{\partial T_e}{\partial S_o} = \frac{1}{4} \cdot T_e \cdot \left[\frac{1-A}{4 \cdot \sigma_{SB} \cdot T_e^4} \right]$$

Solve (21.3) for S_o and substituting into the eq. above:

$$\frac{\partial T_e}{\partial S_o} = \frac{1}{4} \cdot T_e \cdot \left[\frac{1}{S_o} \right]$$

You can similarly find $\partial T_e / \partial A$ from eq. (21.3) to be:

$$\frac{\partial T_e}{\partial A} = \frac{-T_e}{4 \cdot (1-A)}$$

When these derivatives are plugged into eq. (21.a) and applied at the reference point (S_o^* , T_e^* , A^*), the result is:

$$dT_e = \frac{T_e^*}{4 \cdot S_o^*} dS_o - \frac{T_e^*}{4 \cdot (1-A^*)} dA$$

Albedo decreases when Earth's temperature increases, due to melting snow and retreating glacier coverage:

$$dA = -c^* \cdot dT_e$$

where c^* is the magnitude of this variation at the reference point. Plugging this into the previous eq. gives:

$$dT_e = \frac{T_e^*}{4 \cdot S_o^*} dS_o + \frac{c^* \cdot T_e^*}{4 \cdot (1-A^*)} dT_e \quad (21.b)$$

(continues in next column)

BEYOND ALGEBRA • Feedback (continued)

Next, take the derivative of eq. (21.30):

$$dR_o = [(1-A)/4] \cdot dS_o$$

Dividing this by eq. (21.30) shows that:

$$\frac{dR_o}{R_o} = \frac{dS_o}{S_o}$$

Plug this into eq. (21.b), and convert from calculus to finite differences:

$$\Delta T_e = \frac{T_e^*}{4 \cdot R_o^*} \cdot \Delta R_o + \frac{c^* \cdot T_e^*}{4 \cdot (1-A^*)} \cdot \Delta T_e$$

Compare this with eq. (21.23) to realize that:

$$r_o = \frac{T_e^*}{4 \cdot R_o^*} \quad (\text{eq. 21.33}) \quad \text{and} \quad f = \frac{c^* \cdot T_e^*}{4 \cdot (1-A^*)}$$

Thus,

$$F = \frac{f}{r_o} = \frac{c^* \cdot R_o^*}{(1-A^*)} \quad \text{and} \quad G = \frac{4 \cdot (1-A^*)}{4 \cdot (1-A^*) - c^* \cdot T_e^*}$$

For the idealized (toy model) example in this section, the reference values are:

$T_e^* = 255 \text{ K}$ = radiative equilibrium temperature

$S_o^* = 1366 \text{ W/m}^2$ = solar irradiance

$R_o^* = 239 \text{ W/m}^2$ = solar input / m^2 Earth's surface

$A^* = 0.3$ (dimensionless) = albedo

$c^* = 0.01 \text{ K}^{-1}$ = albedo response to temperature change

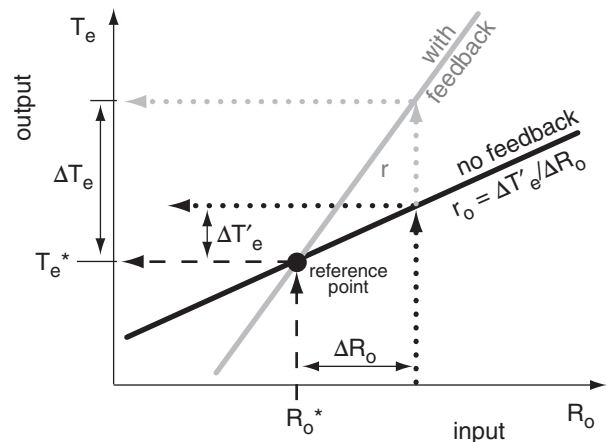


Figure 21.20

Response of system with feedback (grey) and without (black). The prime ' denotes the no-feedback response.

Table 21-4. Feedbacks affecting Earth's climate.

(+) = positive, (–) = negative feedbacks.

F = Feedback parameter ($\text{W}\cdot\text{m}^{-2}/\text{K}$).

f = feedback factor (dimensionless)

G = Gain (dimensionless)

r = system response ($\text{K}/(\text{W}\cdot\text{m}^{-2})$). This is also known as the **sensitivity**, λ .

Processes for gradual climate change:

Feedback	F	f = $r_o \cdot F$	G = $1/(1-f)$	r = $G \cdot r_o$
IR Radiative (–) • blackbody Earth • realistic Earth				$r_o =$ 0.27 0.31
Water-vapor (+)	1.8 ± 0.18	0.48	1.93	0.51
Lapse-rate (–)	-0.84 ± 0.26	-0.22	0.82	0.22
Cloud (+)	0.69 ± 0.38	0.18	1.2	0.33
Surface albedo (+)	0.26 ± 0.08	0.07	1.1	0.29
Ocean CO ₂ (+)				
Biological (–)				

Processes for Abrupt climate change:

- Meridional Overturning Ocean Circulation
- Fast Antarctic & Greenland Ice-sheet Collapse
- Volcanoes
- Methane Release (Hydrate Instability & Permafrost)
- Biogeochemical

Based on IPCC AR4, 2007.

back differs from the r_o without. With feedback, eq. (21.32) becomes:

$$\Delta T_e = r \cdot \Delta R_o \quad (21.39)$$

Table 21-4 lists some feedbacks that can affect Earth's climate. These are briefly discussed next.

Infrared Radiative (IR) Feedback

IR is the strongest feedback — it dominates all others. It was already discussed as the “radiation out” term in eq. (21.2). This important negative feedback allows the Earth to have an equilibrium state. Even a small change in Earth's temperature results in a strong compensating energy loss because IR radiation is to the 4th power of T_e in the Stefan-Boltzmann equation. This strong damping keeps Earth's climate relatively steady.

As we have seen, the reference response to changes in net solar input is $r_o = 0.267 \text{ K}/(\text{W}\cdot\text{m}^{-2})$. Global climate models that include more realistic atmospheric absorption and emission of IR radiation find $r_o = 0.30$ to $0.32 \text{ K}/(\text{W}\cdot\text{m}^{-2})$.

Water-vapor Feedback

Warmer air can hold more water vapor, which absorbs and re-radiates more IR radiation back to the surface and reduces the atmospheric window sketched in Fig. 21.5. The warmer surface warms the air, resulting in a very strong positive feedback (see Table 21-4). The upper troposphere is where this feedback is most effective.

Water vapor is an important natural greenhouse gas. If the climate warms and the upper tropospheric water-vapor content increases to the point where the atmospheric window is mostly closed (net atmospheric emissivity $e \approx 1$), then we return to the negative feedback sketched in Fig. 21.4 and steady Earth temperature of $T_s = 30^\circ\text{C}$ from eq. (21.6).

Lapse-rate Feedback

The simple model of Fig. 21.4 uses the same air temperature T_A to determine IR radiation both up to space and down toward the Earth. However, in the real atmosphere the temperature varies with altitude — decreasing with lapse rate $6.5^\circ\text{C}/\text{km}$ in the troposphere. Thus, colder temperatures at the top of the troposphere emit less upward IR radiation to space than is emitted downward from the warmer bottom of the troposphere. This difference in upward and downward IR radiation is part of our normal climate, causing our normal temperatures.

Global warming tends to reduce the lapse rate. Namely, the temperature difference diminishes between the top and bottom of the troposphere. Rela-

Solved Example

Given the feedbacks listed in Table 21-4, what is the total response.

Solution

Given: Table 21-4. Use realistic Earth for r_o .

Find: $r = ? \text{ K}/(\text{W}\cdot\text{m}^{-2})$.

Assume: total response is based on only those feedback processes for which numbers are given in the table.

Use eq. (21.29): $G = 1 / [1 - \Sigma f]$

$$G = 1 / [1 - (0.48 - 0.22 + 0.18 + 0.07)]$$

$$G = 1 / [1 - 0.51] = 2.04$$

Use eq. (21.28): $r = G \cdot r_o$

$$r = 2.04 \cdot 0.31 = \underline{0.63} \text{ K}/(\text{W}\cdot\text{m}^{-2}).$$

Check: Units OK. Physics OK.

Discussion: For each $\Delta R = 1 \text{ W}/\text{m}^2$ change in solar input, the Earth-system response with all feedbacks is $\Delta T = 0.63 \text{ K}$, which is twice the response (0.31 K) without feedback (other than the background IR feedback). {CAUTION: The total response is NOT equal to the sum of the individual responses r . Instead, use the sum of feedback factors f to get the net feedback.}

tive to each other, IR radiation out to space will increase and IR radiation down toward the Earth will decrease. The net effect is to cool the Earth system. Thus, lapse-rate feedback is negative (Table 21-4).

Water-vapor and lapse-rate feedbacks are closely related, and are often considered together. Their combined effect is a modest positive feedback.

Cloud Feedback

Cloud feedback is likely important, but the details are poorly understood. Relative to clear skies, clouds at all heights cause cooling by reflecting incoming solar radiation. But clouds also cause warming by trapping (absorbing and re-radiating back towards the surface) upwelling IR radiation. The small difference between these large opposing forcings can have very large error, depending on cloud type and altitude. For our present climate, the net effect of clouds is cooling.

Table 21-4 indicates a positive feedback for clouds, based on global climate modeling. Namely, an increase in radiative input to the Earth system would decrease the cloud cover, thus causing less cooling. But much more research must be done to gain more confidence in cloud-feedback processes.

Sometimes **aerosol feedback** is grouped with cloud feedback. Aerosols (microscopic solid or liquid particles in the air) can have similar effects as cloud droplets. However, some aerosols such as sulfates are darker than pure water droplets, and can absorb more solar radiation to cause atmospheric warming. As already discussed, volcanoes can inject sulfate aerosols into the stratosphere. **Phytoplankton** (microscopic plant life) in the ocean can release into the atmospheric boundary layer a chemical called dimethyl sulfide, which can later be oxidized into sulfate aerosols.

Ice–albedo (Surface) Feedback

This positive feedback was discussed earlier in the idealized feedback example, where we found a response of $r = 2.96 \text{ K}/(\text{W}\cdot\text{m}^{-2})$. More realistic estimates from Global Climate Model simulations are $r = 0.27$ to $2.97 \text{ K}/(\text{W}\cdot\text{m}^{-2})$. At our current reference temperature, this feedback mostly affects the climate at high latitudes.

This positive-feedback sensitivity applies only over the limited range of Fig. 21.19 for which albedo can vary with temperature. At the cold extreme of a completely snow covered Earth (**snowball Earth**), the albedo is constant at its cold-limit value A_c . At the warm extreme the Earth has no ice caps or snow cover, and albedo is constant at A_w .

At these two extremes, the negative feedback of IR radiation again dominates, leading to steady

Solved Example

Use Table 21-4 to estimate the combined water-vapor and lapse-rate response, $r_{wv.lr}$.

Solution

Given Table 21-4: $f_{\text{water.vapor}} = 0.48$, $f_{\text{lapse.rate}} = -0.22$

Find: $r_{wv.lr} = ? \text{ K}/(\text{W}\cdot\text{m}^{-2})$

Assume: $r_o = 0.267 \text{ K}/(\text{W}\cdot\text{m}^{-2})$ from eq. (21.33)

For multiple feedbacks, we must use eq. (21.29):

$$G = 1 / [1 - (0.48 - 0.22)] = 1.35$$

Use eq. (21.28):

$$r_{wv.lr} = 1.35 \cdot [0.267 \text{ K}/(\text{W}\cdot\text{m}^{-2})] = \mathbf{0.361 \text{ K}/(\text{W}\cdot\text{m}^{-2})}$$

Check: Units OK. Magnitude OK.

Discussion: This response is midway between water vapor alone ($r = 0.51$) and lapse-rate alone ($r = 0.22$).

FOCUS • Greenhouse Gases

A **greenhouse gas** is an atmospheric gas that absorbs and emits IR radiation. Of particular concern are gases that absorb and emit within the 8 to $14 \mu\text{m}$ wavelength range of the **atmospheric window**. As the concentrations of such gases increase, the atmospheric window can close due to the increased atmospheric emissivity (Fig. 21.5), resulting in global warming.

Greenhouse gases, ranked by importance, are:

- (1) water vapor (H_2O),
- (2) carbon dioxide (CO_2),
- (3) methane (CH_4),
- (4) nitrous oxide (N_2O),
- (5) ozone (O_3),
- (6) halocarbons [e.g., freon CFC-12 ($\text{C Cl}_2 \text{ F}_2$)]

Except for (6), most greenhouse gases have both natural and **anthropogenic** (man-made) sources. CO_2 is of particular concern because it is released by humans burning fossil fuels for energy.

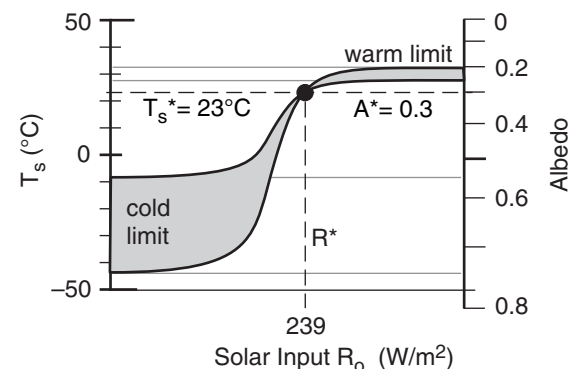


Figure 21.21

Illustration of how Earth surface temperature is bounded once the Earth becomes fully snow covered (cold limit) or fully snow-free (warm limit). Grey shading shows temperature uncertainty due to uncertainty in the limiting albedo values.

FOCUS • Effervescence and CO₂

Carbonated beverages retain their tart flavor when cold. The tartness or sourness comes from carbonic acid (H₂CO₃) due to CO₂ dissolved in water (H₂O). But as the beverage warms, it cannot hold as much dissolved CO₂, forcing some of the CO₂ to **effervesce** (bubble out). Namely, the beverage loses its fizz, causing it to taste “flat”.
The same phenomenon happens to the oceans. If oceans warm, they lose some of their fizz.

FOCUS • Climate Sensitivity

The Intergovernmental Panel on Climate Change (IPCC) states that **climate sensitivity** is “a measure of the climate system response to sustained radiative forcing. It is defined as the equilibrium global average surface warming” (ΔT_s) “following a doubling of CO₂ concentration.”
Based on research by international teams using global climate models, IPCC suggests in their 4th Assessment Report (AR4, 2007) that “this sensitivity is likely to be in the range of” $\Delta T_s =$ “2 to 4.5°C with a best estimate of 3°C” for a doubling of the greenhouse gas CO₂. This temperature rise would be equivalent to a **radiative forcing** (ΔR_0 in Fig. 21.18b’ and c’; abbreviated as **RF** by IPCC) of 3.7 to 4.0 W/m².
But CO₂ has not doubled yet. What are the actual forcings so far. The IPCC is particularly concerned about anthropogenic forcings, because these can be mitigated by people. Based on estimates of anthropogenic changes between years 1750 (the start of the industrial age) and 2005, they find that people have inadvertently altered the net radiative forcing of the Earth system as follows:

Process	ΔR_0 or RF (W/m ²)
CO ₂ increase	1.66
CH ₄ increase	0.48
N ₂ O increase	0.16
Halocarbons	0.34
O ₃ in stratosphere	−0.05
O ₃ in troposphere	0.35
H ₂ O from CH ₄ in stratosphere	0.07
Surface albedo/land use	−0.2
Surface albedo/carbon on snow	0.1
Aerosols/ direct	−0.5
Aerosols/ via cloud albedo	−0.7
Jet contrails	0.01
Total anthropogenic	1.6
	range [0.6 to 2.4]

Compare this to a natural process:
Solar irradiance change **0.12**

The large anthropogenic effect relative to the natural solar variations is what motivates policy makers to take action.

Earth temperatures. For snowball Earth, cold-limit albedo estimates are in the range of $0.55 \leq A_c \leq 0.75$. When used in the radiation balance of eq. (21.3) the cold-limit blackbody temperature is $197 \text{ K} \leq T_e \leq 228 \text{ K}$, yielding a surface temperature ($T_s = 1.161 \cdot T_e$ from eq. 21.10) of $-44^\circ\text{C} \leq T_s \leq -8^\circ\text{C}$.
For the warm limit (assuming that clouds having 37% albedo still cover roughly 70% of the Earth) $0.20 \leq A_w \leq 0.25$, and the equilibrium blackbody temperature is $259 \text{ K} \leq T_e \leq 263 \text{ K}$. This corresponds to a surface-temperature of $28^\circ\text{C} \leq T_s \leq 33^\circ\text{C}$ using eq. (21.10). Thus, runaway global warming or cooling is not possible beyond these bounds (see Fig. 21.21).

Ocean CO₂ Feedback

Warmer seawater can hold less dissolved carbon dioxide (CO₂) than colder water. Hence global warming will cause oceans to release CO₂ into the atmosphere. Greater atmospheric CO₂ concentrations reduce the atmospheric window due to an increase in net atmospheric emissivity e , and thus cause the atmosphere to re-radiate more IR radiation back toward the Earth surface. The net result is positive feedback.

Biological CO₂ Feedback

As plants grow, they consume CO₂ to make hydrocarbons and carbohydrates via photosynthesis. As a result, the carbon is **sequestered** (stored) as the body of the plant. Some of the carbohydrates (e.g., sugars) are consumed by the plant, and are converted back to CO₂ and **transpired** (exhaled by the plant) into the atmosphere.
When plants die and decay, or when they burn in a **wild fire**, they release their remaining carbon as CO₂ back to the atmosphere. If plants are buried under sediment before they decay or burn, their carbon can be fossilized into coal, oil, natural gas (**fossil fuels**). Carbonate rocks can also sequester carbon from the atmosphere, but the carbon can later be released through weathering of the rocks.
The change of carbon from CO₂ to other forms is called the **global carbon cycle**. It is extremely difficult to model the global carbon cycle, which leads to large uncertainty in estimates of climate feedback. Also, anthropogenic effects of fossil-fuel and biomass burning affect the cycle. At present, it is believed that biological CO₂ causes a positive feedback. Namely, global warming will cause more CO₂ to be released than sequestered.

Processes of **abrupt climate change** in Table 21-4 are left to the reader to explore (via an Understanding and Critical Evaluation exercise). But next we introduce the intriguing *gaia* hypothesis.

DAISYWORLD

Does life on Earth merely respond and adapt to changes of climate? James Lovelock suggested that life controls the climate in a way that favors continuance of life (**homeostasis**). This theory is called the **gaia** hypothesis, after the Greek word for “mother Earth”.

To illustrate such “biological homeostasis of the global environment”, Lovelock and Andrew Watson presented a simple hypothetical model called **Daisyworld**. On daisyworld, only two types of plants exist: black daisies and white daisies. These daisies grow and die depending on the temperature. However, as their populations increase, they alter the global albedo due to their color, and thus alter the global radiation temperature (Fig. 21.22).

Physics

A highly simplified and parameterized set of equations were suggested by Lovelock to illustrate biological homeostasis. We will present the equations in an order and with modifications that aid spreadsheet computations.

First, there are two constants. One is the Stefan-Boltzmann constant $\sigma_{SB} = 5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$. The other is the solar irradiance, $S_o = 1366 \text{ W/m}^2$. A solar luminosity parameter, $L \approx 1$, provides a convenient way to experiment with increases or decreases in solar output. From these quantities, a solar forcing ratio is defined as

$$q = \frac{L \cdot S_o}{4 \cdot \sigma_{SB}} \quad (21.40)$$

Parameters are:

$A_W = 0.75$ albedo of white daisies

$A_B = 0.25$ albedo of black daisies

$A_G = 0.5$ albedo of bare ground.

$D = 0.3$ daisy death rate (equal for black and white).

$Tr = 0.6$ horizontal transport parameter (varies between 0 for no transport, to 1 for total mixing in the horizontal). When $Tr = 0$, the temperature in a daisy patch is controlled locally by the albedo of that patch. For $Tr = 1$, the average global albedo determines the local temperature.

The areas of the globe covered by daisies are:

C_W = fractional area covered by white daisies

C_B = fractional area covered by black daisies.

Our initial condition will be no daisies: $C_W = C_B = 0$. Also

$$C_G = 1 - C_B - C_W \quad (21.41)$$

is the fractional area of bare ground.

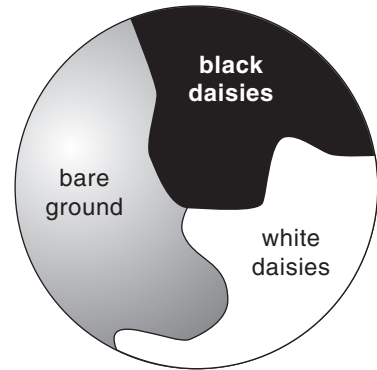


Figure 21.22

Coverage of daisies on a Daisyworld Earth.

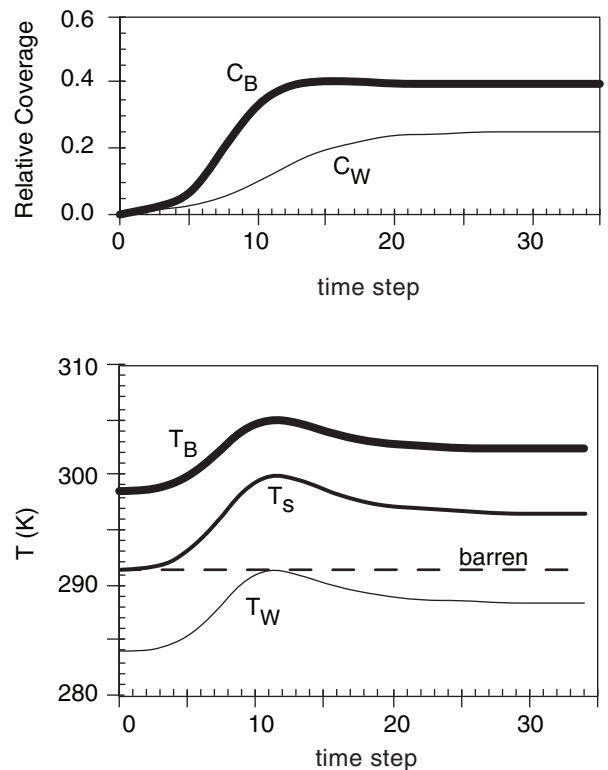


Figure 21.23

Sample Daisyworld forecast as it evolves toward an equilibrium, for $L = 1.2$.

Solved Example (§)

Use the parameter values of the previous page, along with $L = 1.2$, to make a Daisyworld forecast.

Solution

Given: $L=1.2$, $Tr=0.6$, $D=0.3$, $A_W=0.75$, $A_B=0.25$, $A_G=0.5$

Find: $C_B = ?$, $C_W = ?$, $T_s = ?$ K, $T_B = ?$ K, $T_W = ?$ K

First use eq. (21.40):

$$q = \frac{1.5 \cdot (1366 \text{ W} \cdot \text{m}^{-2})}{4 \cdot (5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4})} = 7.24 \times 10^9 \text{ K}^4$$

Initialize with $C_B = C_W = 0$, but seed with $C_s = 0.01$.

Iterate, solving eqs. (21.41) through (21.46) on a spreadsheet:

t	C_B	C_W	C_G	A	T_s (K)	T_B (K)	T_w (K)	β_B	β_W
0	0.00	0.00	1.00	0.50	291.6	298.7	284.1	0.97	0.57
1	0.01	0.01	0.98	0.50	291.6	298.7	284.1	0.97	0.57
2	0.02	0.01	0.97	0.50	291.8	298.8	284.2	0.96	0.58
3	0.03	0.02	0.96	0.50	292.0	299.0	284.4	0.96	0.60
4	0.04	0.02	0.94	0.49	292.5	299.3	284.8	0.95	0.63
5	0.07	0.03	0.90	0.49	293.2	299.9	285.4	0.94	0.67
...	etc.
98	0.39	0.25	0.36	0.46	296.7	302.5	288.5	0.84	0.84

Check: Units OK. Physics OK.

Discussion: Because of the “seed” coverage, values of C_W and C_B of 0.01 actually correspond to no daisies.

At steady state, the growth parameters should be equal: $\beta_B = \beta_W$. This is a good check to see if steady state has been obtained.

The temperature of a barren planet happens to equal the value of T_s from the first iteration (before any daisies). Thus, $T_s \text{ barren} = 291.6$ K.

These answers are plotted in Fig. 21.23.

The planetary albedo (A) is an average of the albedoes (A_i) of the daisies, weighted by their relative coverage (C_i):

$$A = C_G \cdot A_G + C_D \cdot A_D + C_L \cdot A_L \quad (21.42)$$

The effective radiation temperature (Kelvin) is:

$$T_e^4 = q \cdot (1 - A) \quad (21.43)$$

and the surface temperature under a one-layer windowless atmosphere is

$$T_s^4 = 2 \cdot T_e^4 \quad (21.44)$$

Surface temperatures of individual daisy patches are:

$$T_D^4 = (1 - Tr) \cdot q \cdot (A - A_D) + T_s^4 \quad (21.45a)$$

and

$$T_L^4 = (1 - Tr) \cdot q \cdot (A - A_L) + T_s^4 \quad (21.45b)$$

for black and white daisies, respectively.

These daisies grow fastest at an optimum temperature of $T_o = 22.5^\circ\text{C} = 295.5$ K. Growth rate is less for warmer or colder temperature, and growth ceases altogether at temperatures colder than 5°C or warmer than 40°C . The growth rate β is:

$$\beta_D = 1 - b \cdot (T_o - T_D)^2 \quad (21.46a)$$

$$\beta_L = 1 - b \cdot (T_o - T_L)^2 \quad (21.46b)$$

where parameter $b = 0.003265 \text{ K}^{-2}$, and subject to the constraint that $\beta \geq 0$.

This biological homeostasis model is a prognostic model. It forecasts how daisy coverage changes with time as populations of black and white daisies grow and interact. The population equation for daisy coverage is

$$C_{D \text{ new}} = C_D + \Delta t \cdot C_D \cdot (C_G \cdot \beta_D - D) \quad (21.47a)$$

$$C_{L \text{ new}} = C_L + \Delta t \cdot C_L \cdot (C_G \cdot \beta_L - D) \quad (21.47b)$$

which gives the new values after a time step of Δt . We use $\Delta t = 1$. The daisy coverages are constrained such that $C_{B \text{ new}} \geq C_s$ and $C_{W \text{ new}} \geq C_s$, where $C_s = 0.01$ is the seed coverage. Without such seeding of the planet, no daisies could grow.

These new coverages are then used back in eq. (21.41), and the computation process is repeated for many time steps. After sufficient steps, the model approaches a stable equilibrium.

Equilibrium and Homeostasis

For $L = 1.2$ from the solved example, daisy populations reach equilibrium values of $C_B = 0.39$ and $C_W = 0.25$ (Fig. 21.23). The equilibrium surface temperature is $T_s = 296.75$ K, which is warmer than the corresponding temperature of a barren planet of 291.65 K. This latter quantity is found using $A = A_G$ in eqs. (21.43 and 21.44).

By rerunning the model for various values of luminosity, we can discover the equilibrium states that are possible. These are plotted in Fig. 21.24. **Homeostasis** is evident in that the fecund planet maintains a nearly-steady surface temperature conducive to life, for a wide range of luminosities. It does this by producing predominantly black daisies for low luminosity, in order to maximize absorption of the solar energy. Alternately, white daisies dominate for high luminosity to reflect the excess solar energy and maintain temperate conditions.

Although this conceptual model is too simplistic to describe the real Earth, it makes us think about new possibilities.

GCMS

Global climate models (**GCMs**) are special types of numerical weather prediction (NWP) models (see the NWP chapter). A regular grid of nodes (grid points) covers the globe, and climate simulations are made at each node for each time step by numerically solving sets of finite-difference equations that describe the governing equations for heat, momentum, mass, moisture, etc. Some GCMs use spectral rather than grid-point numerics in the horizontal, where a sum of sine waves of various wavelengths are used to approximate the spatial structure of the climate.

Climate feedback processes are explicitly included via the dynamics, thermodynamics, and physics in the model. Other feedback processes must be approximated and parameterized. Many climate models couple atmosphere, ocean, biosphere, cryosphere (ice) and soil forecasts. The net result is a model that includes many but not all of the complex interactions that exist in the real atmosphere.

To simulate the climate, the run duration must be long enough to model years to decades to centuries. For these models to run on present-day computers, sacrifices must be made in horizontal grid resolution to reduce the run time. **Coarse horizontal resolutions** of about 1° to 5° latitude are used on the whole globe, and only a limited number of vertical grid layers are used.

The coarse resolution is a drawback in that much of the important physics (boundary layers, turbu-

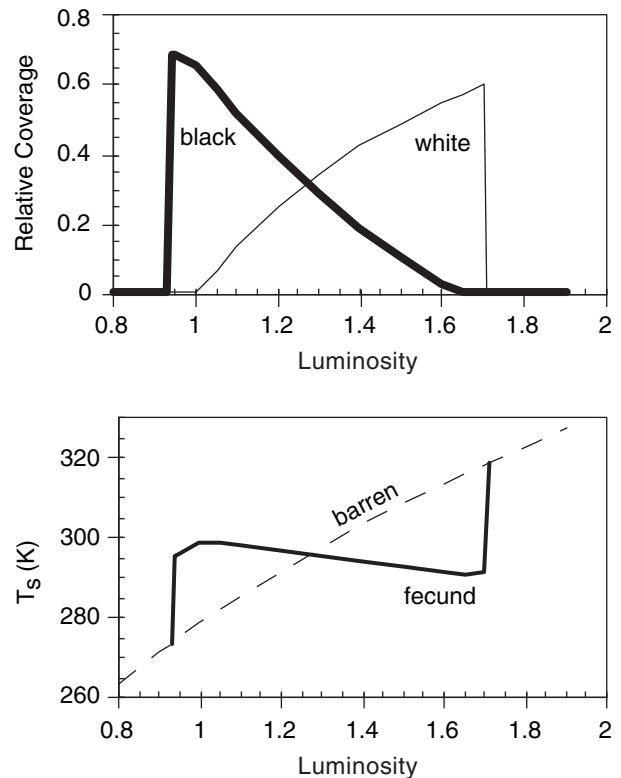


Figure 21.24

Equilibrium conditions in Daisyworld for various solar luminosities. Top: areal coverage of white and black daisies. Bottom, surface temperature for the lushly flowered (fecund) and unflowered (barren) worlds.

Science Graffito

"We tend to predict impacts on people by assuming a static human society. But humans are adapting quickly, and time scales of human change are faster than the climate change. There's no unique perfect temperature at which human societies operate. Run them a little warmer, and they'll be fine; run them a little cooler, they'll be fine. People are like roaches — hard to kill. And our technology is changing rapidly, and we're getting richer, internationally and individually, at an absurd rate. We're rapidly getting less and less sensitive to climatic variations."

— David Keith
(from an interview published in *Discover*, Sep 2005.)

Table 21-6. Köppen climate codes and defining criteria. (Peel et al, 2007)

1 st	2 nd	3 rd	Description	Criteria*
A			Tropical	$T_{cold} \leq 18$
	f m w		Rainforest Monsoon Savannah	$P_{dry} \geq 60$ [not Af] and $[P_{dry} \geq (100 - MAP/25)]$ [not Af] and $[P_{dry} < (100 - MAP/25)]$
B			Arid	$MAP < [10 \cdot P_{threshold}]$
	W S		Desert Steppe	$MAP < [5 \cdot P_{threshold}]$ $MAP \geq [5 \cdot P_{threshold}]$
		h k	Hot Cold	$MAT \geq 18$ $MAT < 18$
C			Temperate	$[T_{hot} \geq 10]$ and $[0 < T_{cold} < 18]$
	s w f		Dry summer Dry winter Without dry season	$[P_{sdry} < 40]$ and $[P_{sdry} < (P_{wwet}/3)]$ $P_{wdry} < [P_{swet}/10]$ [not Cs] and [not Cw]
		a b c	Hot summer Warm summer Cold summer	$T_{hot} > 22$ [not a] and $[N_{monT10} \geq 4]$ [not (a or b)] and $[1 \leq N_{monT10} < 4]$
D			Cold	$[T_{cold} \leq 0]$ and $[T_{hot} > 10]$
	s w f		Dry summer Dry winter Without dry season	$[P_{sdry} < 40]$ and $[P_{sdry} < (P_{wwet}/3)]$ $P_{wdry} < [P_{swet}/10]$ [not Ds] and [not Dw]
		a b c d	Hot summer Warm summer Cold summer Very cold winter	$T_{hot} \geq 22$ [not a] and $[T_{mon10} \geq 4]$ [not (a or b or d)] [not (a or b)] and $T_{cold} < -38$
E			Polar	$T_{hot} < 10$
	T F		Tundra Frost	$T_{hot} > 0$ $T_{hot} \leq 0$

hydrosphere/cryosphere/biosphere/land-surface system. Climate is found by averaging or filtering the hourly or daily weather data over 30 years, with the last year ending in zero (e.g., 1971-2000, or 1981-2010). Mean values of weather, temporal and spatial variability, and extremes are the statistics that are recorded as components of the climate signal.

Köppen Climate Classification

In 1884, Wladimir Köppen used temperature and precipitation to define different types of climate. He drew maps, classifying every point on land according to these climates. The climate at each location controls the dominant type of plants that naturally grow there. The Köppen climate map was modified by Rudolf Geiger in the early 1900s, and was significantly updated in 2007 by Peel, Finlayson and McMahon (*Hydrol. Earth Syst. Sci.*, **11**, 1633-1644) using modern weather data.

A two-or-three-letter code is used to identify each climate type. The criteria used to classify the climate types are in Table 21-6. Fig. 21.25 shows a small portion of the global climate map.

Table 21-6 (continuation). *Footnotes

MAP	= mean annual precipitation (mm/yr)
MAT	= mean annual temperature (°C)
N_{monT10}	= number of months where the temperature is above 10 (°C)
P_{dry}	= precipitation of the driest month (mm/month)
P_{sdry}	= precipitation of the driest month in summer (mm/month)
P_{swet}	= precipitation of the wettest month in summer (mm/month)
P_{wdry}	= precipitation of the driest month in winter (mm/month)
P_{wwet}	= precipitation of the wettest month in winter (mm/month)
T_{hot}	= average temperature of the hottest month (°C)
T_{cold}	= average temperature of the coldest month (°C)
$P_{threshold}$ (mm)	= is given by the following rules: If (70% of MAP occurs in winter) then $P_{threshold} = (2 \cdot MAT)$ Elseif (70% of MAP occurs in summer) then $P_{threshold} = (2 \cdot MAT) + 28$ Else $P_{threshold} = (2 \cdot MAT) + 14$

Summer (winter) is defined as the warmer (cooler) six month period of [October through March] and [April through September].

Table 21-7. Climate oscillations. SST = sea-surface temperature, SLP = sea-level pressure.

Name	Period (yr)	Key Place	Key Variable
El Niño (EN)	2 to 7	Tropical E. Pacific	SST
Southern Oscillation (SO)	2 to 7	Tropical Pacific	SLP
Pacific Decadal Oscillation (PDO)	20 to 30	Extratrop. NE Pacific	SST
North Atlantic Oscillation (NAO)	variable	Extratrop. N. Atlantic	SLP
Arctic Oscillation (AO)	variable	N. half of N. hem.	Pressure
Madden-Julian Oscillation (MJO)	30 to 60 days	Tropical Indian & Pacific	Convective Precipitation

FOCUS • What is an Oscillation?

Oscillations are regular, repeated variations of a measurable quantity such as temperature (T) in space x and/or time t . They can be described by sine waves: $T = A \cdot \sin[C \cdot (t - t_0) / \tau]$ where A is a constant **amplitude** (signal strength), τ is a constant **period** (duration one oscillation), t_0 is a constant **phase shift** (the time delay after $t = 0$ for the signal to first have a positive value), and C is 2π radians or 360° (depending on your calculator).

True oscillations are **deterministic** — the signal T can be predicted exactly for any time in the future. The climate “oscillations” discussed here are not deterministic, cannot be predicted far in the future, and do not have constant amplitude, period, or phase.

NATURAL OSCILLATIONS

The 30-year-average climate is called the **normal climate**. Any shorter-term (e.g., monthly) average weather that differs or varies from the climate norm is called an **anomaly**. Natural climate “oscillations” are recurring, multi-year anomalies with irregular amplitude and period, and hence are not true oscillations (see the Focus Box).

Table 21-7 lists these oscillations, and their major characteristics. Although each oscillation is associated with, or defined by, a location or place on Earth (Fig. 21.26), most of these oscillations affect climate over the whole world. At the defining place(s), a combination of key weather variables is used to define an “**index**”, which is the varying climate signal. Fig. 21.27 shows these oscillation indices.

El Niño - Southern Oscillation (ENSO)

(El Niño; La Niña) occurs when the tropical sea-surface temperature (SST) in the central and eastern Pacific ocean is (warmer; cooler) than the multi-decadal climate average.

The El Niño index plotted in Fig. 21.27a is the Pacific SST anomaly in the tropics (averaged within $\pm 20^\circ$ latitude of the equator) minus the Pacific SST anomaly outside the tropics (averaged over more poleward latitudes). Alternative El Niño indices are also useful.

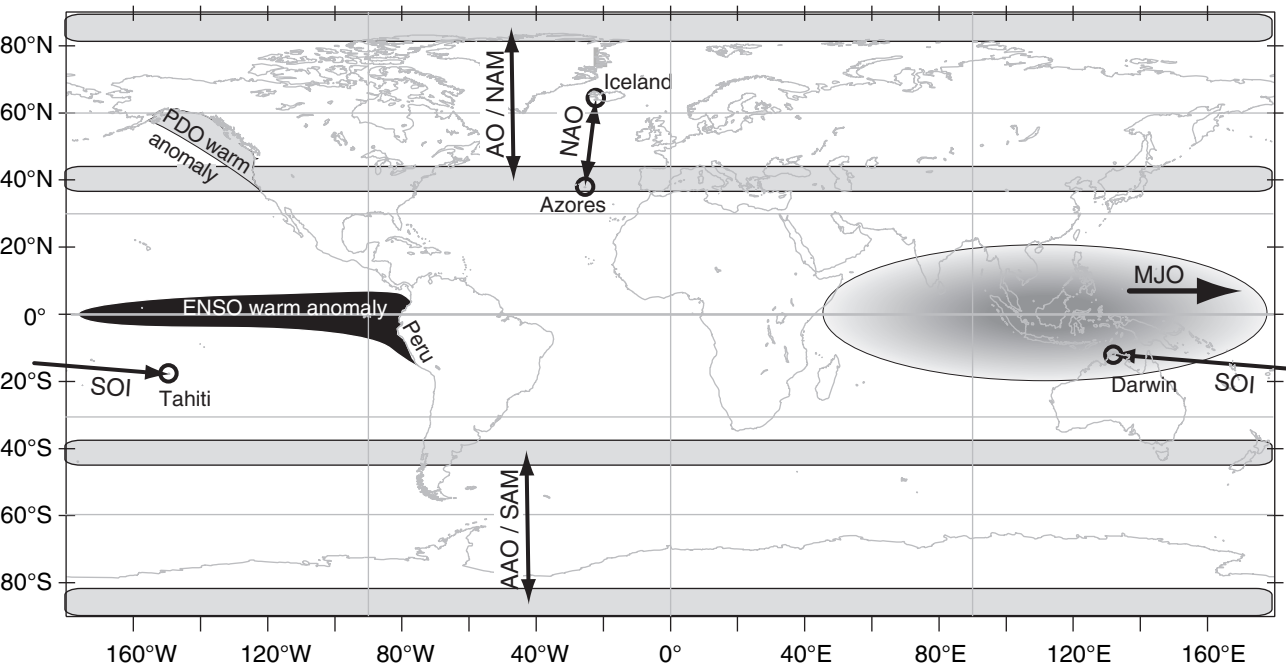
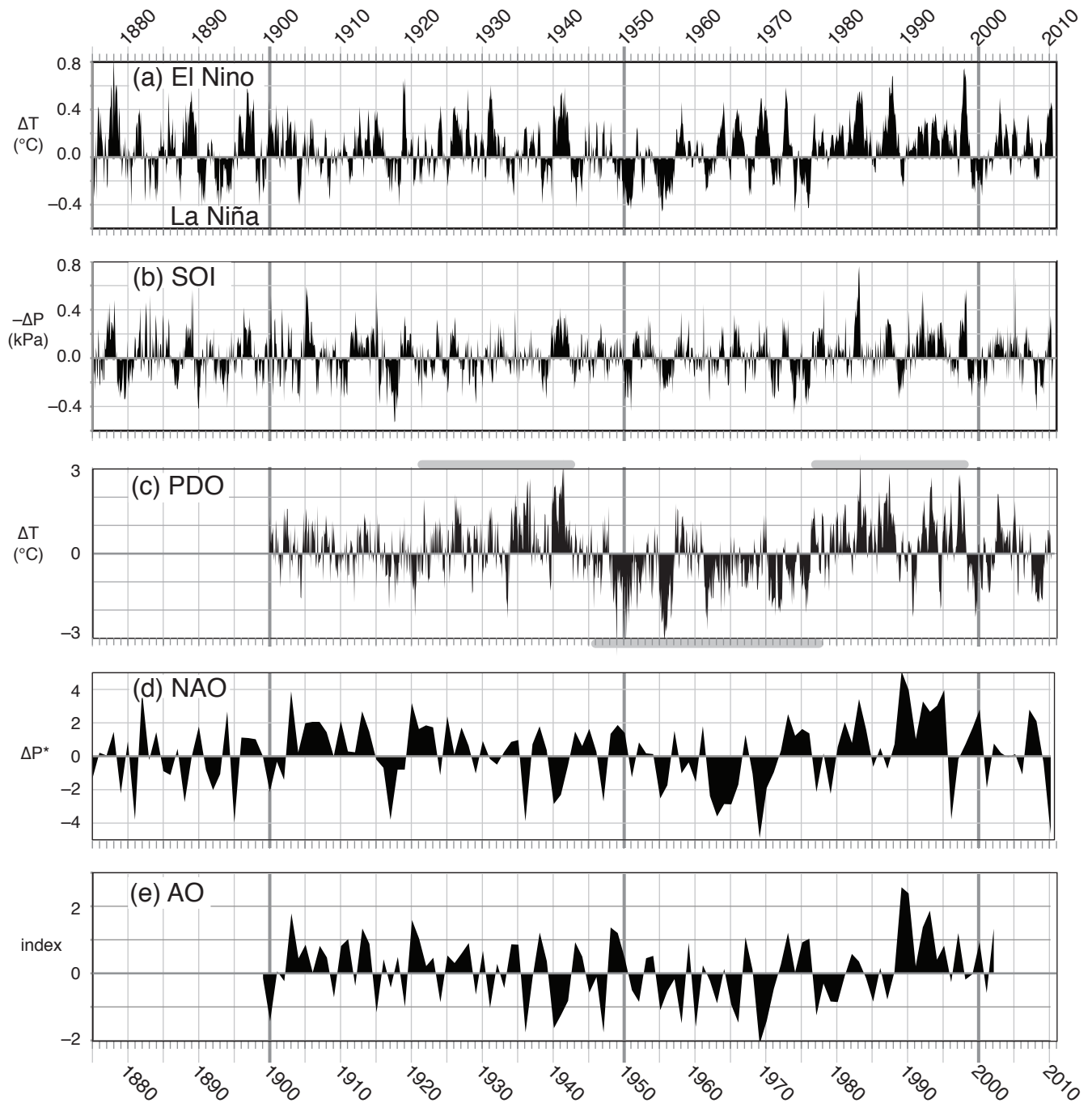


Figure 21.26 Places and key variables that define some climate oscillations. Acronyms are defined in the text.

**Figure 21.27**

Natural oscillations of some climate indices.

(a) **El Niño** index, based on average sea-surface temperature (SST) anomalies ΔT [data courtesy of Todd Mitchell of the Univ. of Washington Joint Inst. for Study of the Atmosphere and Ocean (US JISAO)].

(b) Negative of the **Southern Oscillation index (SOI)**, based on sea-level pressure (SLP) difference: $-\Delta P = P_{\text{Darwin}} - P_{\text{Tahiti}}$ [data courtesy of the Climate Analysis Section of the National Center for Atmospheric Research's (NCAR) Climate and Global Dynamics Division (NCAR/CGD/CAS)].

(c) **Pacific Decadal Oscillation (PDO)** index, based on the leading principal component for monthly Pacific SST variability north of 20° latitude [data courtesy of Nate Mantua and Steven Hare of UW JISAO]. Grey bands show warm and cool phases.

(d) **North Atlantic Oscillation (NAO)** index, based on the difference of normalized SLP: $\Delta P^* = P^*_{\text{Azores}} - P^*_{\text{Iceland}}$ for winter (Dec - Mar) [data courtesy of Jim Hurrell, NCAR/CGD/CAS].

(e) **Arctic Oscillation (AO)**, based on the leading principal component of winter SLP in the N. Hemisphere for winter (Jan-Mar) [courtesy of Todd Mitchell of UW JISAO].

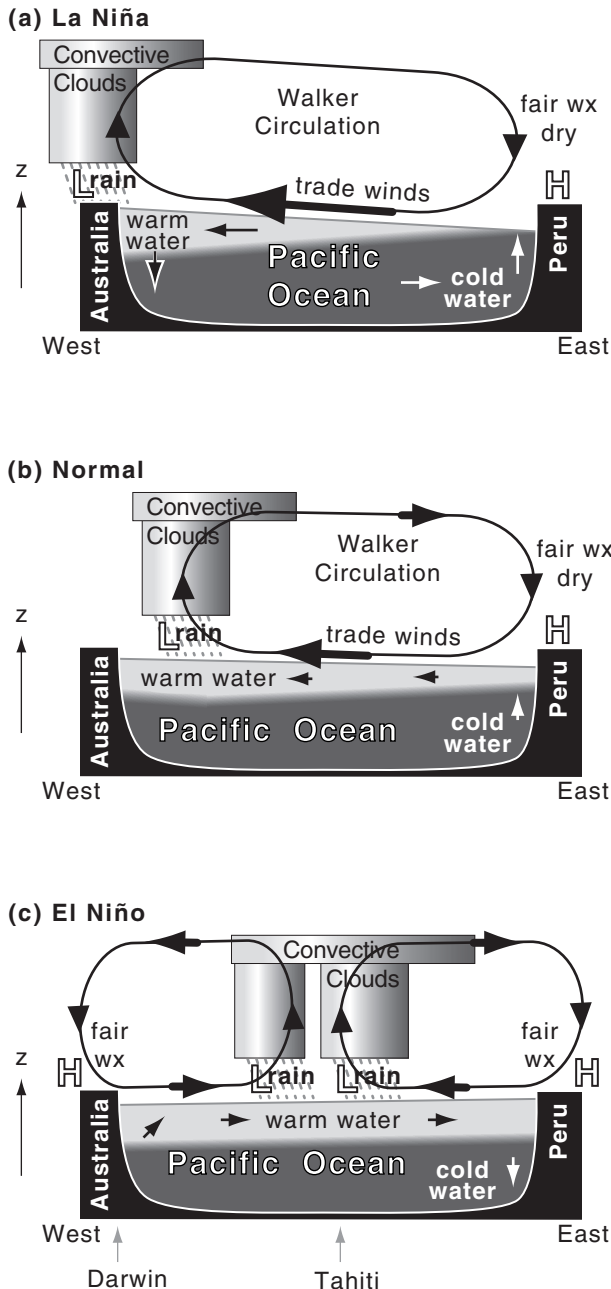


Figure 21.28

Processes that drive ENSO. (a) La Niña conditions, where cold upwelling nutrient-rich water off the coast of Peru enables increases in fish populations. Also, cool equatorial water upwells near the central Pacific. Strong trade winds. (b) Normal conditions, with modest trade winds. (c) El Niño conditions, associated with a warm-water anomaly in the eastern Pacific, and a decline in fish stocks there. H and L indicate centers of high and low atmospheric pressure, respectively.

The **Southern Oscillation** refers to a change in sign of the east-west atmospheric pressure gradient across the tropical Pacific and Indian Oceans. One indicator of this pressure gradient is the sea-level pressure difference between Tahiti (an island in the central equatorial Pacific, Fig. 21.26) and Darwin, Australia (western Pacific). This indicator ($\Delta P = P_{Tahiti} - P_{Darwin}$) is called the **Southern Oscillation index (SOI)**.

The negative of the SOI, plotted in Fig. 21.27b, agrees with the El Niño index (Fig. 21.27a). The reason for this agreement is that El Niño (EN) and the Southern Oscillation (SO) are the ocean and atmospheric signals of the same coupled phenomenon, abbreviated as **ENSO**.

During normal and La Niña conditions, the east-to-west decrease in air pressure drives the easterly trade winds, which drag much of the warm ocean-surface waters to the west side of the Pacific basin (Fig. 21.28a). The trade winds are part of the **Walker Circulation**, which includes cloudy rainy updrafts over the warm SST anomaly in the western tropical Pacific, a return flow aloft, and fair-weather (fair wx) dry downdrafts. Heavy rain can fall over Australia. Other changes are felt around the world, including cool winters over northern North America, with drier warmer winters over the southern USA.

During El Niño, this pressure gradient breaks down, causing the trade winds to fail, and allowing anomalously warm western Pacific waters to return to the eastern Pacific (Fig. 21.28c). Thunderstorms and rain tend to be strongest over the region of warmest SST. As the SST anomaly shifts eastward, droughts (allowing wild fires) occur in Australia, and heavy rains (causing floods and landslides) occur in Peru. Other changes are felt around the world, including warm winters over northern N. America and cool, wet winters over southern N. America.

Pacific Decadal Oscillation (PDO)

The PDO is a 20 to 30 year variation in the extratropical SST near the American coast in the NE Pacific. It has a warm or positive phase (i.e., warm in the key place sketched in Fig. 21.26) and a cool or negative phase. Phase durations are highlighted by the thick grey lines in Fig. 21.27c. The PDO signal is found using a statistical method called **principal component analysis (PCA)**, also called **empirical orthogonal function analysis, EOF**; see the Focus box on the next three pages.)

The PDO affects weather in northwestern North America as far east as the Great Lakes, and affects the salmon production between Alaska and the Pacific Northwest USA. There is some debate among scientists whether the PDO is a statistical artifact associated with the ENSO signal.

FOCUS • PCA and EOF

Principal Component Analysis (PCA) and **Empirical Orthogonal Function (EOF)** analysis are two names for the same statistical analysis procedure. It is a form of data reduction designed to tease out dominant patterns that naturally occur in the data. Contrast that with Fourier analysis, which projects the data onto sine waves regardless of whether the data naturally exhibits wavy patterns or not.

You can use a spreadsheet for most of the calculations, as is illustrated here with a contrived example.

1) The Meteorological Data to be Analyzed

Suppose you have air temperature T measurements from different locations over the North Pacific Ocean. Although PCA works for any number of locations, we will use just 4 locations for simplicity:

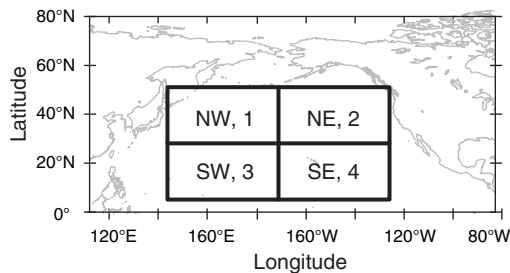


Fig. 21.a. Locations for weather data.

These 4 locations might have place names such as Northwest quadrant, Northeast quadrant, etc., but we will index them by number: $j = 1$ to 4. You can number your locations in any order.

For each location, suppose you have annual average temperature for $N = 10$ years, so your raw input data on a spreadsheet might look this:

Location Index:	$j = 1$	2	3	4
Location:	NW	NE	SW	SE
Year	Air Temperature ($^{\circ}\text{C}$)			
$i = 1$	0	3	8	5
2	4	7	6	8
3	0	6	6	7
4	4	9	5	8
5	0	8	5	6
6	0	9	6	6
7	4	2	10	4
8	0	4	9	4
9	4	6	8	5
10	0	7	5	5
Average:	1.6	6.1	6.8	5.8

(continues)

FOCUS • PCA and EOF (continuation)

These meteorological data are plotted below:

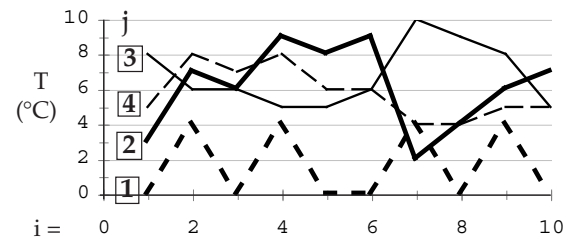


Fig. 21.b. Graphs of raw data at each of 4 locations, where numbers in boxes indicate the j index for each line.

Looking at this data by eye, we see that location 2 has a strong signal. Location 4 varies somewhat similarly to 2 (i.e., has positive correlation); location 3 varies somewhat oppositely (negative correlation). Location 1 looks uncorrelated with the others.

2) Preliminary Statistical Computations

Next, use your spreadsheet to find the average temperature at each location (shown at the bottom of the previous table). Then subtract the average from the individual temperatures at the same locations, to make a table of temperature **anomalies** T' :

$$T'_i = T_i - T_{avg} \quad (21.48)$$

Location	$j = 1$	2	3	4
Year	Air Temperature Anomalies T' ($^{\circ}\text{C}$)			
$i = 1$	-1.6	-3.1	1.2	-0.8
2	2.4	0.9	-0.8	2.2
3	-1.6	-0.1	-0.8	1.2
4	2.4	2.9	-1.8	2.2
5	-1.6	1.9	-1.8	0.2
6	-1.6	2.9	-0.8	0.2
7	2.4	-4.1	3.2	-1.8
8	-1.6	-2.1	2.2	-1.8
9	2.4	-0.1	1.2	-0.8
10	-1.6	0.9	-1.8	-0.8

[Check: The average of each T' column should be 0.]

The covariance between variables A and B is:

$$\text{covar}(A, B) = \frac{1}{N} \sum_{i=1}^N A'_i \cdot B'_i \quad (21.49)$$

where $\text{covar}(A, A)$ is the variance. Use a spreadsheet to compute a table of covariances, where T_1 are the temperatures at location $j=1$, T_2 are from location 2, etc.:

covar(T1,T1)	covar(T1,T2)	covar(T1,T3)	covar(T1,T4)
covar(T1,T2)	covar(T2,T2)	covar(T2,T3)	covar(T2,T4)
covar(T1,T3)	covar(T2,T3)	covar(T3,T3)	covar(T3,T4)
covar(T1,T4)	covar(T2,T4)	covar(T3,T4)	covar(T4,T4)

(continues)

FOCUS • PCA and EOF (continuation)

The numbers in that table are symmetric around the main diagonal, so you need to compute the covariances for only the black cells, and then copy the appropriate covariances into the grey cells.

This table is called a **covariance matrix**. When I did it in a spreadsheet, I got the values below:

<u>3.84</u>	-0.16	0.72	0.72
-0.16	<u>5.29</u>	-3.48	2.22
0.72	-3.48	<u>2.96</u>	-1.74
0.72	2.22	-1.74	<u>1.96</u>

The underlined values along the main diagonal are the variances. The sum of these diagonal elements (known as the **trace of the matrix**) is a measure of the total variance of our original data (= 14.05 °C²).

3) More Statistical Computations

Next, find the **eigenvalues** and **eigenvectors** of the covariance matrix. You cannot do that in most spreadsheets, but other programs can do it for you.

I used a website: http://www.amdt-bruennner.de/mathe/scripts/engl_eigenwert.htm where I copied my covariance matrix into the web page. It gave back the following info, which I pasted into my spreadsheet. Under each eigenvalue is the corresponding eigenvector.

k=	1	2	3	4
Eigenvalues (largest first)				
	8.946	4.118	0.740	0.247
Eigenvectors (columns under evals)				
j=1	-0.049	0.959	-0.144	-0.238
2	0.749	0.031	-0.579	0.323
3	-0.548	0.126	-0.267	0.782
4	0.369	0.251	0.757	0.477

The sum of all eigenvalues = 14.05 °C² = the total variance of the original data. Those eigenvectors with the largest eigenvalues explain the most variance. Eigenvectors are dimensionless.

4) Find the Principal Components

Next, compute the **principal components** (PCs) — one for each eigenvector (*evec*). The number of eigenvectors ($K = 4$ in our case) equals the number of raw variables ($J = 4$ in our case).

Each PC will be a column of N numbers ($i = 1$ to 10 in our case) in our spreadsheet. Compute each of these N numbers in your spreadsheet using:

$$PC(k, i) = \sum_{j=1}^J [evec(k, j) \cdot T(j, i)] \quad (21.50)$$

For each data table (T data; eigenvectors; PCs) used in eq. (21.50), the first index indicates the column and the second indicates the row.

(continues)

FOCUS • PCA and EOF (continuation)

Namely:

- k indicates which PC it is ($k = 1$ for the first PC, $k = 2$ for the second PC, up through $k = 4$ in our case) and also indicates the eigenvector (*evec*).
- i is the observation index ($i = 1$ to 10 yrs in our case), and indicates the corresponding element in any PC.
- j is the data variable index (1 for NW quadrant, etc.), and indicates an element within each eigenvector.

For example, focus on the first PC (i.e., $k = 1$); i.e., the first column in the table below. The first row ($i = 1$) is computed as:

$$PC(1,1) = -0.049 \cdot 0 + 0.749 \cdot 3 - 0.548 \cdot 8 + 0.369 \cdot 5 = -0.29$$

For the second row ($i = 2$), it is:

$$PC(1,2) = -0.049 \cdot 4 + 0.749 \cdot 7 - 0.548 \cdot 6 + 0.369 \cdot 8 = 4.71$$

Each PC element has the same units as the original data (°C) in our case. The resulting PCs are:

k =	1	2	3	4
	PC1	PC2	PC3	PC4
i=1	-0.29	2.36	-0.08	9.61
2	4.71	6.82	-0.17	9.81
3	3.79	2.70	0.23	9.97
4	6.76	6.75	-1.06	9.68
5	5.46	2.38	-1.42	9.35
6	5.66	2.54	-2.26	10.46
7	-2.70	6.16	-1.37	9.42
8	-0.46	2.26	-1.68	10.24
9	1.76	6.29	-2.39	9.63
10	4.34	2.10	-1.60	8.55
evalue:	8.946	4.118	0.740	0.247

Each PC applies to the whole data set, not just to one location. These PCs, plotted below, are like alternative T data:

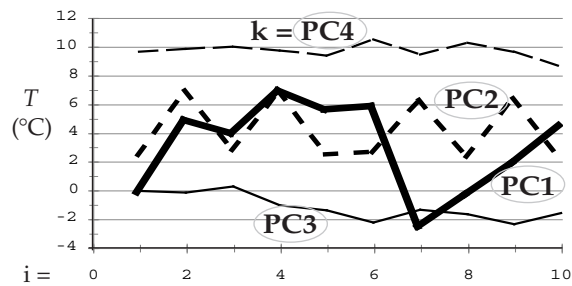


Fig. 21.c. PCs.

- PC1 captures the dominant pattern that is in the raw data for locations 2, 3, and 4. This 1st PC explains $8.946/14.05 = 63.7\%$ of the original total variance.
- PC2 captures the oscillation that is in the raw data for location 1. Since location 1 was uncorrelated with the other locations, it couldn't be captured by PC1, and hence needed its own PC. The 2nd PC explains $4.118/14.05 = 29.3\%$ of the variance.
- PC3 captures a nearly linear trend in the data.
- PC4 captures a nearly constant offset.

(continues)

FOCUS • PCA and EOF (continuation)**5) Synthesis**

Now that we've finished computing the PCs, we can see how well each PC explains the original data. Let T_k be an approximation to the original data, reconstructed using only the k^{th} single principal component. Such reconstruction is known as **synthesis**.

$$T_k(j, i) = \text{evec}(k, j) \cdot PC(k, i) \quad (21.51)$$

using the same (column, row) notation as before.

For $k = 1$ (the dominant PC), the reconstructed T_1 data is:

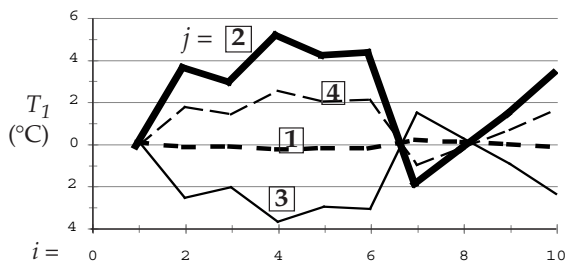


Fig. 21.d. Reconstruction of temperature for each geographic location $j = 1$ to 4 (boxed numbers), but using only the 1st PC.

It is amazing how well just one PC can capture the essence (including positive and negative correlations) of the signal patterns in the raw data.

The spatial pattern associated with PC1 can be represented on a map by the elements of the first eigenvector (arbitrarily scaled by 100 here & rounded):

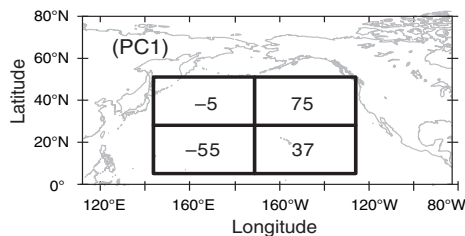


Fig. 21.e. Geographic pattern of influence of PC1.

6) Interpretation:

A dominant pattern in the NE quadrant extends in diminished form to the SE quadrant, with the SW quadrant varying oppositely. But this pattern is not experienced very much in the NW quadrant. Similar maps can be created for the other eigenvectors. When data from more than 4 stations are used, you can contour the eigenvector data when plotted on a map.

Eq. (21.51) can also be used to partially reconstruct the data based on only PC2 (i.e., $k = 2$). As shown in Fig. 21.f, you won't be surprised to find that it almost completely captures the signal in the NW quadrant ($j = 1$), but doesn't explain much in the other regions.

(continues)

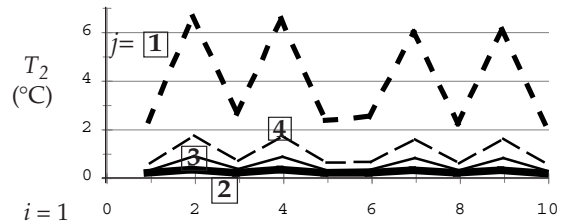
FOCUS • PCA and EOF (continuation)

Fig. 21.f. Similar to Fig. 21.d, but using only PC2.

Plotting the elements of the second eigenvector scaled by 100 on a map at their geographic locations:

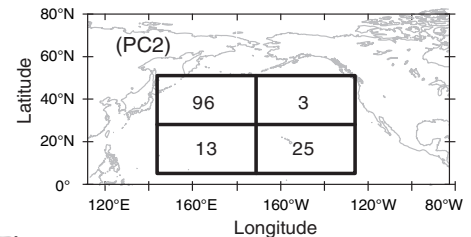


Fig. 21.g. Geographic pattern of influence of PC2.

7) Data Reduction

Suppose that you compute graphs similar to Figs. 21.d & 21.f for the other PCs, and then add all location 1 curves from each PC. Similarly, add all location 2 curves together, etc. The result would have perfectly synthesized the original data as was plotted in Fig. 21.a. Although the last 2 PCs don't explain much of the variance (fluctuations of the raw data), they are important to get the mean temperature values.

But what happens if you synthesize the temperature data with only the first 2 PCs (see plot below). These first 2 PCs explain 93% of the total variance. Indeed, the graph below captures most of the variation in the original signals that were in Fig. 21.a, including positive, negative, and near-zero correlations. This allows data reduction: We can use only the dominant 2 PCs to explain most of the signals, instead of using all 4 columns of raw data.

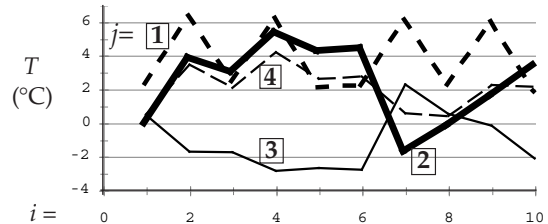


Fig. 21.h. Reconstruction of temperature using only the first 2 PCs.

8) Conclusions

For our small data set, the data reduction might not seem like much, but for larger data sets with hundreds of geographic points, this can be an important way to find dominant patterns in noisy data, and to describe these patterns efficiently. For this reason, PCA/EOF is valued by climate researchers.

North Atlantic Oscillation (NAO)

In the Global Circulation chapter (Fig. 11.31) we identified an Iceland low and Azores (or Bermuda) high on the map of average sea-level pressure (SLP). The NAO is a variation in the strength of the pressure difference between these two pressure centers (Fig. 21.26).

Greater differences (called the positive phase of the NAO) correspond to stronger north-south pressure gradients that drive stronger west winds in the North Atlantic — causing mid-latitude cyclones and associated precipitation to be steered toward N. Europe. Weaker differences (negative phase) cause weaker westerlies that tend to drive the extratropical cyclones toward southern Europe.

An NAO index is defined as $\Delta P^* = P^*_{Azores} - P^*_{Iceland}$, where $P^* = (P - P_{clim})/\sigma_P$ is a normalized pressure that measures how many standard-deviations (σ_P) the pressure is from the mean (P_{clim}). The winter average (Dec - Mar) is often most relevant to winter storms in Europe, as plotted in Fig. 21.27d.

Arctic Oscillation (AO)

The AO is also known as the **Northern Annular Mode (NAM)**. It is based on the leading PCA mode for sea-level pressure in the N. Hemisphere, and varies over periods of weeks to decades. Fig. 21.27e shows the winter (Jan-Mar) averages.

The AO acts over the whole tropospheric depth, and is associated with the **polar vortex** (a belt of mid-troposphere through lower-stratosphere westerly winds circling the pole over the polar front). It compares the average pressure anomaly over the whole Arctic with pressure anomaly averaged around the 37 to 45°N latitude band. Thus, the AO pattern is zonal, like a bulls-eye centered on the N. Pole (Fig. 21.26). You can think of the NAO as a regional portion of the AO.

The positive, high-index, warm phase corresponds to greater pressure difference between mid-latitudes and the North Pole. This is correlated with faster westerly winds, warmer wetter conditions in N. Europe, warmer drier winters in S. Europe, and warmer winters in N. America. The negative, low-index, cool phase has weaker pressure gradient and winds, and opposite precipitation and temperature patterns.

Madden-Julian Oscillation (MJO)

The MJO is 45-day variation (± 15 days) between enhanced and reduced convective (CB) precipitation in the tropics. It is the result of a region of enhanced convection (sketched as the grey shaded oval in Fig. 21.26) that propagates toward the east at roughly 5 to 10 m/s.

The region of enhanced tropical convection consists of thunderstorms, rain, high anvil cloud tops (cold, as seen by satellites), low SLP anomaly, convergent U winds in the boundary layer, and divergent U winds aloft. MJO can enhance tropical cyclone and monsoonal-rain activity. Outside of this region is reduced convection, clearer skies, subsiding air, less or no precipitation, divergent U winds in the boundary layer, convergent U winds aloft, and reduced tropical-cyclone activity.

The convective regions are strongest where they form over the Indian Ocean and as they move eastward over the western and central Pacific Ocean. By the time they reach the eastern Pacific and the Atlantic Oceans, they are usually diminished, although they can still have observable effects.

For example, when the thunderstorm region approaches Hawaii, the moist storm outflow can merge with a southern branch of the jet stream to feed abundant moisture toward the Pacific Northwest USA and SW Canada. This moist river of air is known as the **Pineapple Express**, and can cause heavy rains, flooding, and landslides due to orographic uplift of the moist air over the coastal mountains (see the Extratropical Cyclone chapter).

A useful analysis method to detect the MJO and other propagating phenomena is to plot key variables on a **Hovmöller diagram** (see Focus box). Eastward propagating weather features would tilt down to the right in such a diagram. Westward moving ones would tilt down to the left. The slope of the tilt indicates the zonal propagation speed.

Other Oscillations

Other climate oscillations have been discovered:

- **Antarctic Oscillation (AAO)**, also known as the **Southern Annular Mode (SAM)** in S. Hem. SLP (sketched in Fig. 21.26). 2 - 7 yr variation.
- **Arctic Dipole Anomaly (ADA)**, 2 - 7 yr variation with high SLP over northern Canada and low SLP over Eurasia.
- **Atlantic Multi-decadal Oscillation (AMO)**, 30 - 40 yr variations in SST over the N. Atlantic
- **Indian Ocean Dipole (IOD)**. 7.5 yr oscillation between warm east and warm west SST anomaly in the Indian Ocean.
- **Interdecadal Pacific Oscillation (IDPO or IPO)** for SST in the N. and S. Hemisphere extratropics. 15-30 yr periods.
- **Pacific North American Pattern (PNA)** in SLP and in 50 kPa geopotential heights.
- **Quasi-biennial Oscillation (QBO)** in stratospheric equatorial zonal winds
- **Quasi-decadal Oscillation (QDO)** in tropical SST, with 8-12 yr periods.

Other climate variations may also exist.

SUMMARY

Climate is weather averaged over 30 yr. Earth's climate is controlled by both external and internal processes. External processes include the balance between incoming solar radiation and outgoing infrared (IR) radiation. Internal processes include changes to the composition or structure of the atmosphere that alter how energy is distributed within the Earth-ocean-atmosphere-cryosphere-biosphere system.

Externally, changes in Earth's orbit around the sun as predicted by Milankovitch can alter the amount of sunlight reaching Earth, and changes in the tilt of Earth's axis can alter the severity of the seasons. Solar output can also vary. Changes in Earth's average albedo (via snow cover, cloud cover, or coverage of vegetation) can modulate the amount of IR radiation lost from the Earth system.

Internally, normal chemicals (water vapor, carbon dioxide, etc.) in the atmosphere absorb IR radiation, causing the air to become warmer and re-radiate a large amount of IR radiation back toward the Earth's surface. This greenhouse effect does not have perfect efficiency, because some wavelength bands of IR radiation can escape directly to space through the so-called "atmospheric window." Anthropogenic and natural changes to these greenhouse gases can alter the greenhouse efficiency and shift the climate equilibrium — an effect called "climate change."

Natural internal processes such as volcanic eruptions can also change the climate via emission of sulfates into the stratosphere. Movement of tectonic plates can shift the locations of continents and oceans relative to the equator, altering global-scale and continental-scale circulations and climate.

Some processes in the atmosphere cause negative feedbacks that tend to stabilize the climate, while other positive-feedback processes tend to amplify climate variations. Many of these feedbacks apply to external, internal, natural, and anthropogenic forcings. Overall, our climate is remarkable steady.

Short-term (few years to few decades) variations in Earth's climate are also observed. El Niño is one example out of many such "oscillations" in the climate signal.

Researchers have classified the current climate (Köppen). They also utilize tools including global climate models (GCMs), principal component analysis (PCA), Hovmöller diagrams, and conceptual models (Daisyworld) to study climate variability.

FOCUS • Hovmöller Diagram

A Hovmöller diagram is a plot of a meteorological variable as a function of date and longitude. Usually date is along the ordinate — increasing downward. Longitude is along the abscissa. The meteorological variable is contoured with grey or color fill. The latitude or latitude band of these meteorological observations is fixed, and is specified in the figure legend. These diagrams are useful for detecting the zonal movement of weather features over time.

For example, suppose that on 1 January you flew an airplane around the world along the equator (0° latitude), and remotely measured the sea surface temperature (SST). After subtracting the average SST, you would be left with a temperature anomaly T' at each point around the equator. Suppose the first graph below shows what you measured.

Later, on 15 January, make the same flight, and plot your measurements (second graph below). Repeat the flights for many different dates.

Instead of plotting each circumnavigation on a separate graph, we can write all the T' numbers on the same diagram of time vs. longitude. Then analyze the result as you learned in the Map Analysis chapter, by drawing isotherms of T' . The result is a Hovmöller diagram (third diagram below). You can use any meteorological variable, not just T' .

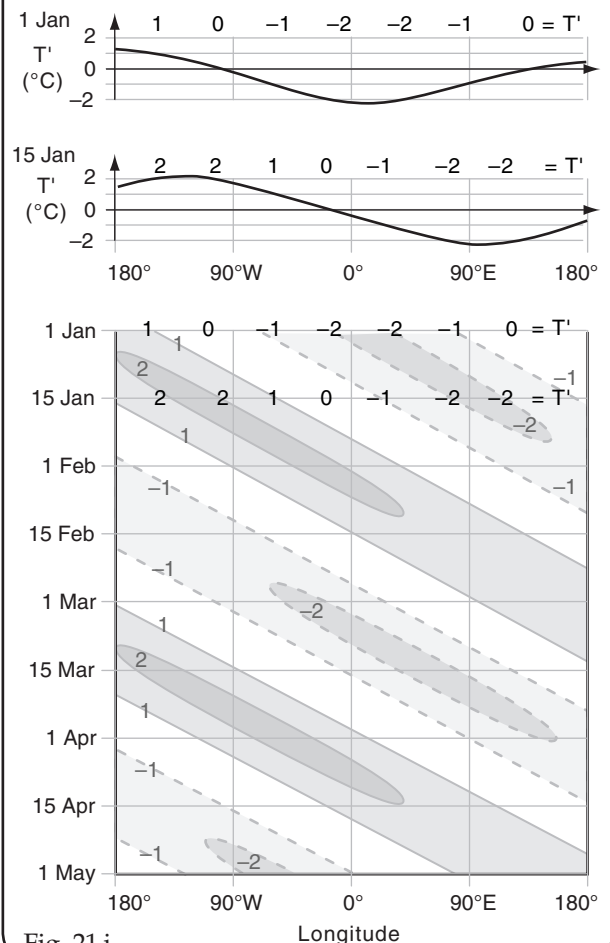


Fig. 21.i

Threads

Many of the climate-change issues relate to radiation (Chapter 2) within the setting of our existing atmospheric structure (Chapter 1). The radiative input alters the heat budget (Chapter 3), which controls water evaporation (Chapter 4), clouds (Chapter 6) and precipitation (Chapter 7).

Climate is partly governed by, and in turn alters, the global circulation (Chapter 11), which can affect the frequency, intensity, and track of cyclones (Chapters 13 and 16) and thunderstorms (Chapters 14 and 15).

Much of the concern about climate change is related to pollutants (Chapter 19) emitted into the atmosphere by people in the boundary layer (Chapter 18). Venting of greenhouse gases out of the boundary layer, and longevity of these tracers in the upper atmosphere depends, in part, on the vertical stability of the atmosphere (Chapter 5).

Global climate models (GCMs) are forms of numerical weather prediction models (Chapter 20).

ON DOING SCI. • Ethics & Data Abuse

In the race to reach scientific conclusions, and with the natural human desire to be successful, scientists sometimes inappropriately manipulate their data. The following are considered to be unethical in science:

“• **Massaging** – performing extensive transformations... to make inconclusive data appear to be conclusive;

• **Extrapolating** – developing curves [or proposing theories] based on too few data points, or predicting future trends based on unsupported assumptions about the degree of variability measured;

• **Smoothing** [or **trimming**] – discarding data points too far removed from expected or mean values;

• **Slanting** [or **cooking**] – deliberately emphasizing and selecting certain trends in the data, ignoring or discarding others which do not fit the desired or preconceived pattern;

• **Fudging** [or **forging**] – creating data points to augment incomplete data sets or observations; and

• **Manufacturing** – creating entire data sets de novo, without benefit of experimentation or observation.”

[from C. J. Sindermann, 1982: *Winning the Games Scientists Play*. Plenum. 290 pp.; and Sigma Xi, 1984: *Honor in Science*. Sigma Xi Pubs., 41 pp.]

EXERCISES

Numerical Problems

N1. Find the radiation input to the Earth system for an albedo of:

- a. 0.1 b. 0.15 c. 0.2 d. 0.25 e. 0.35 f. 0.4
g. 0.45 h. 0.5 i. 0.55 j. 0.6 k. 0.65 m. 0.7

N2. Find the Earth effective radiation temperature for the albedoes in the previous exercise.

N3. For a global albedo of 0.3, find the radiation input to the Earth system for total solar irradiance values (W/m^2) of:

- a. 1350 b. 1352 c. 1354 d. 1356 e. 1358
f. 1360 g. 1362 h. 1364 i. 1365 j. 1367
k. 1368 m. 1370 n. 1372 o. 1374

N4. For a global albedo of 0.3, find the Earth effective radiation temperature for solar constant values of the previous exercise.

N5. Find the net radiation out from the Earth system, for an Earth effective radiation emission temperature ($^{\circ}\text{C}$) of

- a. -19 b. -17 c. -15 d. -13 e. -11
f. -9 g. -7 h. -5 i. -3 j. -1
k. 1 m. 3 n. 5 o. 10 p. 15

N6. Calculate the effective radiation temperature for the following planets and dwarf planets.

Planet	Albedo	Planet	Albedo
a. Mercury	0.12	b. Venus	0.75
c. Mars	0.25	d. Jupiter	0.34
e. Saturn	0.34	f. Uranus	0.30
g. Neptune	0.29	h. Pluto	0.40

N7. For an idealized one-layer atmosphere that is opaque to all IR radiation, find the surface temperature for:

- (i) exercise N2 (ii) exercise N4 (iii) exercise N6

N8. For an atmosphere with a window in the IR, find Earth's surface temperature under a one-layer atmosphere with emissivity:

- a. 0.70 b. 0.73 c. 0.76 d. 0.80 e. 0.83
f. 0.86 g. 0.88 h. 0.91 i. 0.93 j. 0.95
k. 0.97 m. 0.99

Assume an effective emission temperature of 255 K.

N9. Use Fig. 21.6 to answer these questions:

- a. What fraction of energy absorbed at the Earth's surface comes from solar radiation?

b. What fraction of energy leaving the Earth's surface consists of (latent + sensible) heat?

c. What is the net flux of IR radiation between the Earth's surface and the atmosphere, and how does this compare to the flux of (latent + sensible) heat?

d. What is the total amount of upwelling energy (solar, IR, sensible, latent) that reaches the bottom of the idealized atmosphere in that figure?

e. Why is the 79 W/m^2 of "solar reflected from atmos." not included in the energy budget of the atmosphere as one of the outgoing fluxes (Hint: See the Solved Example after that figure)?

f. Why is the (sensible + latent) heat flux upward from the top of the atmosphere not shown?

N10(S). On a very large graph, carefully plot two ellipses. One has eccentricity of 0.0 (i.e., is a circle), and the other has an eccentricity of:

- a. 0.05 b. 0.1 c. 0.15 d. 0.2 e. 0.25 f. 0.3
g. 0.35 h. 0.4 i. 0.6 j. 0.7 k. 0.8 m. 0.9

N11(S). Using the data from Table 21-1, find the Earth-orbit eccentricity for the following time:

- a. 400 kyr ago b. 1.1 Myr ago c. 1.2 Myr ago
d. 1.4 Myr ago e. 1.6 Myr ago f. 2 Myr ago
g. 625 kyr in the future h. 1.1 Myr in future
i. 1.3 Myr in future j. 2 Myr in future

N12(S). Same as for N11, but find the obliquity ($^\circ$).

N13(S). Same as for N11, but find the climatic precession at the summer solstice (i.e., the $e \sin(\varpi)$ term).

N14(S). Same as for N11 (and N13), but find the ratio of semi-major axis to sun-Earth distance: a/R .

N15(S). Same as N11 (and using your results from N12 - N14), but find the average daily insolation at 65°N during the summer solstice.

N16. Find the sun-Earth distance R (Gm) given an eccentricity of 0.05 and a climatic precession value of:

- a. -0.05 b. -0.04 c. -0.03 d. -0.02 e. -0.01
f. 0.01 g. 0.02 h. 0.03 i. 0.04 j. 0.05

N17. Use the definition of optical depth from the Beer's Law section of the Radiation chapter to estimate the optical depth for background "clean" air, which transmits about 94% of incident solar radiation. Also, use data from Fig. 21.17 on recent volcanic eruptions, and compare the "clean" optical depth to the optical depth for:

- a. Mt. Agung (Indonesia)
b. Mt. Fuego (Guatemala)
c. El Chichon (Mexico)
d. Mt. Pinatubo (Philippines)

N18. Find the feedback factor, gain, and system response, given a no-feedback response of $r_o = 0.2$, and a feedback value (F) of:

- a. -8 b. -6 c. -4 d. -2 e. 1 f. 2 g. 3
h. 4 i. 5 j. 6 k. 7 m. 8 n. 9 o. 10

N19. Based on the answer to N18, is the feedback negative, stable positive, or run-away positive?

N20. For a no-feedback reference state for Earth, find the no-feedback climate response r_o for a blackbody Earth equilibrium radiative temperature of T_e^* (K):

- a. 200 b. 210 c. 220 d. 230 e. 240 f. 250
g. 260 h. 270 i. 280 j. 290 k. 300 m. 310

N21. Given a reference Earth blackbody radiative temperature of $T_e^* = 255 \text{ K}$ and a reference albedo of $A^* = 0.3$. Find the feedback factor, gain, and total system response if ice-albedo feedback behaves such that the change of Earth albedo (associated with ice coverage changes) with surface temperature ($\Delta A/\Delta T = -c$), where $c \text{ (K}^{-1}\text{)}$ has a value of:

- a. 0.002 b. 0.005 c. 0.008 d. 0.012 e. 0.015
f. 0.02 g. 0.03 h. 0.05 i. 0.10 j. 0.2

N22. Suppose that doubling of CO_2 in the global atmosphere had the same effect as increasing the incoming solar radiation by $\Delta R_o = 4 \text{ W/m}^2$ (due to the increased absorption of solar radiation, as explained in the Focus box on Climate Sensitivity). Given Table 21-4, how much would Earth's equilibrium temperature change (i.e., find ΔT_e in $^\circ\text{C}$) for a reference response with no feedbacks, and for the following feedback process:

- a. water-vapor only b. lapse-rate only
c. cloud only d. surface-albedo only
e. all of a - d combined

N23(S). Program the Daisyworld equations into your spreadsheet. Calculate the surface temperature (K), coverage of white daisies, and coverage of black daisies, for luminosity of:

- a. 0.9 b. 1.0 c. 1.1 d. 1.2 e. 1.3
f. 1.4 g. 1.5 h. 1.6 i. 1.7 j. 1.8

N24. Describe the climate at

- a. Oregon coast
b. Olympic Mtns (NW corner of Washington)
c. central Vancouver Island (British Columbia)
d. at the USA-Canada border labels in Fig. 21.25
e. north-central California
f. north-central Idaho
g. western Nevada
h. extreme SE California
i. central part of Alberta (portion shown in map)
j. south central Idaho

Understanding & Critical Evaluation

U1. If the atmosphere can be idealized as having two layers, each opaque to IR, extend the results from the Greenhouse section to find an equation for surface temperature as a function of effective radiation temperature.

U2. After a major (hypothetical) volcanic eruption, assume that the ash in the atmosphere quickly spreads evenly around the globe. This dirty air absorbs 25% of any sunlight that shines through the atmosphere. For simplicity, assume that IR absorption/emission is not affected by the ash, and that the albedo of the Earth's surface has not changed by ash deposits. Using a simplified climate model with a one-layer atmosphere with no atmospheric window, what is the new temperature of the Earth's surface (averaged over the whole globe)?

U3. For the last 2 million years the global climate is believed to have been 1.5 to 3 degrees colder than now. For the 100 million years before that, the climate ranged from 2 to 5 degrees warmer than now. If our climate is presently warming, does that mean we are approaching or diverging from "normal"?

U4. Explain how the "atmospheric window" in Fig. 21.5 relates to Figs. 8.3 and 8.4 in the Remote Sensing chapter.

U5. Given a single layer atmosphere with no window. Suppose that initially the Earth-atmosphere system is in equilibrium. Then, suddenly, the sun becomes hotter and the insolation doubles. Describe how the fluxes and energy budgets of Fig. 21.6 would change while the Earth and atmosphere adjust toward their new equilibrium. Also, which of the fluxes would change the quickest; and which the slowest?

U6. In music there is a concept of "**beat frequency**". Namely, when two sounds are produced that have nearly the same tone (frequency), you can hear a third tone that depends on the difference between the two frequencies. Look-up the definition of "beat frequency", and use that info to explain why the Climatic Precession curve in Fig. 21.8 looks the way it does.

U7. a. Given the max and min values of climatic precession plotted in Fig. 21.8, what are the max and min of values of (a/R) ; namely, the ratio of semi-major axis to sun-Earth distance?

b. Simplify the equation (21.18) for the solar declination angle for the special case of the summer solstice.

c. Given the max and min of obliquities plotted in Fig. 21.8, what are the max and min of solar declination angles ($^\circ$) for the summer solstice?

d. Combine the info in Figs. 21.11 - 21.13 to estimate the max and min values of solar irradiance (W/m^2) during the past 1000 years.

e. Given your answers to (a) - (d) above, which has the largest effect on the value of daily-average insolation at 65°N during the summer equinox: climatic precession, obliquity, eccentricity, or solar irradiance?

U8. In Fig. 21.8, the Daily Average Insolation graph is based on a combination of the previous three graphs in that figure. Looking at the nature of the signals in all four top graphs in Fig. 21.8, which of the top 3 graphs best explains the Insolation, and which of the top 3 least explain the Insolation? Why?

U9(\$). Table 21-1 shows only the first $N = 4$ or 5 terms for Earth's orbital factors. Use the first $N = 10$ terms (in Table 21-1b, on the next page) to more accurately re-calculate the first 3 graphs in Fig. 21.8. [The original paper by Laskar et al has over 20 terms for each orbital series.]

U10. Use the orbital data for Earth to answer the following. How many years in the future is the first time when the perihelion will coincide with the summer solstice? This is important because it means that summers will be hotter and winters colder in the N. Hemisphere, because the N. Hemisphere will be tilted toward the sun at the same date that the Earth is closest to the sun.

U11 Show that the right-hand approximation in eq. (21.18) for the solar declination is the same as eq. (2.5). [Hints: Consider that in summer, Earth is 90° further along its orbit than it was in Spring. Use trig identities that relate sines to cosines. Consider the definitions of various orbital angles, as summarized in the Focus box on Sun-Earth Distance.]

U12. Explain why Pangea's climate would be different than our present climate. [Hint: Consider global circulations and monsoonal circulations, as discussed in the Global Circulation chapter.]

U13. a. Use the definitions of optical depth and visual range from the Beer's Law section of the Radiation chapter to find a relationship for visibility (km) as a function of optical depth τ .

b. Use your answer from (a) with the data from Table 21-3 to estimate the atmospheric visibilities associated with the following volcanic eruptions:

(i) Lake Toba (ii) Mt. Tambora (iii) Krakatau

Table 21-1b. Factors in orbital series approximations.			
index	A	P (years)	ϕ (degrees)
Eccentricity:			
i= 1	0.010739	405,091.	170.739
2	0.008147	94,932.	109.891
3	0.006222	123,945.	-60.044
4	0.005287	98,857.	-86.140
5	0.004492	130,781.	100.224
6	0.002967	2,373,298.	-168.784
7	0.002818	977,600.	57.718
8	0.002050	105,150.	49.546
9	0.001971	486,248.	148.744
10	0.001797	688,038.	137.155
Obliquity:			
j= 1	0.582412°	40,978.	86.645
2	0.242559°	39,616.	120.859
3	0.163685°	53,722.	-35.947
4	0.164787°	40,285.	104.689
5	0.095382°	41,697.	-112.872
6	0.094379°	41,152.	60.778
7	0.087136°	9,572,151.	39.928
8	0.064348°	29,842.	-15.13
9	0.072451°	28,886.	-155.175
10	0.080146°	40,810.	-70.983
Climatic Precession:			
k= 1	0.018986	23,682.	44.374
2	0.016354	22,374.	-144.166
3	0.013055	18,953.	154.212
4	0.008849	19,105.	-42.250
5	0.004248	23,123.	90.742
6	0.002742	19,177.	61.600
7	0.002386	19,025.	74.660
8	0.001796	22,570.	-145.502
9	0.001908	19,260.	119.333
10	0.001496	16,465.	141.244
<i>Simplified from Laskar, Robutel, Joutel, Gastineau, Correia & Levrard, 2004: A long-term numerical solution for the insolation quantities of the Earth. "Astronomy & Astrophysics", 428, 261-285.</i>			

U14. For every type of climate feedback, explain why runaway climate change is not possible. [Or if it is possible, show how runaway climate changes applies for only a limited change of Earth temperatures before a new stable climate equilibrium is reached. Can you recall any Hollywood movies that showed runaway climate change, with or without reaching a stable equilibrium?]

U15. For the feedback interconnections shown in Fig. 21.18, the feedback outcomes are only: negative (damped), positive (amplified), or runaway (increasing without limit). Can you think of a different type of interconnection or process that could cause the system response to oscillate? Explain.

U16. Ice-albedo processes were used for an idealized illustration of feedback. Try to develop your own feedback concepts that account for the following (and ignores all other feedbacks except the fundamental radiative feedback). Suppose there is a CO₂ feedback process whereby warmer Earth temperatures cause the oceans to release more CO₂ into the air. But more CO₂ in the air acts to partially close the IR window sketched in Fig. 21.5.

a. What info or what relationships do you need to be able to figure out the feedback? [Just list what info you need here — do not actually try to find these relationships.]

b. If you are good at calculus and if assigned by your instructor, then try to create a governing equation similar to that used for ice-albedo feedback, then take the derivatives as was done in the Beyond Algebra Feedback Example box. There is no single correct answer — many different approximations and solutions might be reasonable. Justify your methods.

U17. For feedbacks in the Earth climate system, use error-propagation methods as discussed in Appendix A, to propagate the errors listed in the feedback (F) column of Table 21-4 into the other columns for feedback factor (f), gain (G), & system response (r).

U18. Do outside readings to explain how the following abrupt climate-change processes work (Table 21-4). Also, explain if feedback is positive or negative.

- meridional overturning ocean circulation
- fast Antarctic & Greenland ice-sheet collapse
- volcanoes
- methane release due to hydrate instability and permafrost)
- biogeochemical

U19.(§) For a daisyworld

- Redo the calculation shown in this chapter.
- What happens to the temperature over black and white daisies if the transport parameter changes to 0 or 1?
- What happens if you start with initial conditions of 100% coverage by white or black daisies?
- What happens if you take a time step increment of 2, 4, or 8?
- What parameter values prevent homeostasis from occurring (i.e., eliminate the nearly constant temperature conditions on a fecund daisyworld?
- Is it possible for homeostasis to occur on Earth, given the large fraction of area covered by oceans?

U20.(§). In Fig. 21.24 for Daisyworld, the daisies are able to control the climate only over a limited range of luminosities. What changes to the growth

Table 21-8. Names of Köppen climate classes.

Exercise	Code	Name
a	Af	equatorial climate
b	Am	monsoon climate
c	Aw	tropical savanna climate
d	BWh	warm desert climate
e	BWk	cold desert climate
f	BSH	warm semi-arid climate (warm steppe clim.)
g	BSk	cold semi-arid climate (cold steppe climate)
h	Csa	warm mediterranean climate
i	Csb	temperate mediterranean climate
j	Cwa	warm humid subtropical climate
k	Cwb	temperate humid subtropical climate
l	Cwc	cool humid subtropical climate
m	Cfa	warm oceanic/marine; or humid subtropical
n	Cfb	temperate oceanic/marine climate
o	Cfc	cool oceanic/marine climate
p	Dsa	warm continental (mediterranean continental)
q	Dsb	temperate continental (mediter. continental)
r	Dsc	cool continental climate
s	Dsd	cold continental climate
t	Dwa	warm humid continental climate
u	Dwb	temperate humid continental climate
v	Dwc	cool continental clim.; cool subarctic climate
w	Dwd	cold continental clim.; cold subarctic climate
x	Dfa	warm humid continental climate
y	Dfb	temperate humid continental climate
z	Dfc	cool continental clim.; cool subarctic climate
aa	Dfd	cold continental clim.; cold subarctic climate
ab	ET	tundra climate
ac	EF	ice cap climate

characteristics of the daisies would allow control of the climate over a wider range of luminosities (and thus would allow greater chance of survival of an extreme luminosity change event)? Check your theory by modifying the growth equations accordingly, and performing the calculation on a spreadsheet or other program.

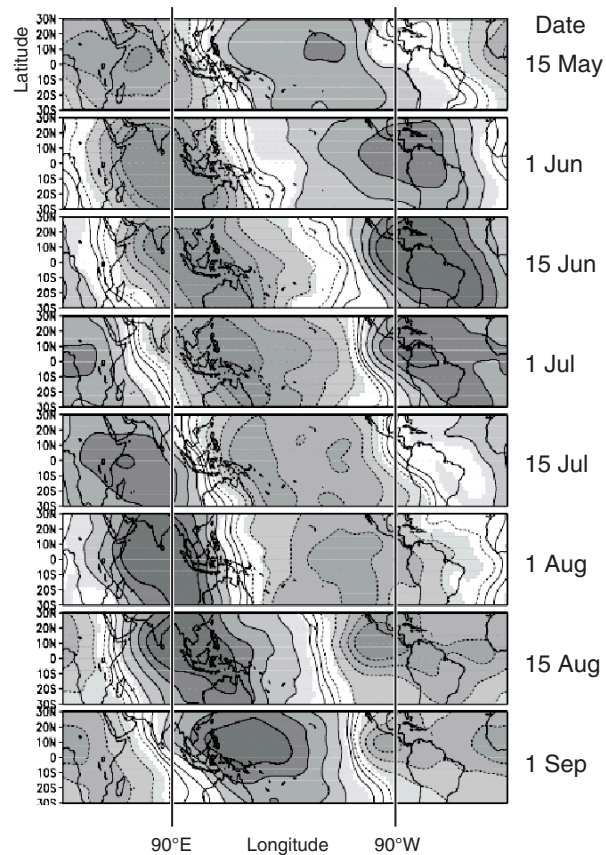
U21. Where in the world (but NOT in any region plotted in Fig. 21.25) would you expect to find the climate in Table 21-8 above. (Hint, consider info in the Global Circulation chapter, and also use the Köppen climate definitions in Table 21-6.)

U22(\$-difficult). Find the covariance matrix, the eigenvectors and eigenvalues (using some other resource such as a web-based eigenvector calculator), leading 2 principal components (PCs), interpret the results, and reconstruct (synthesize) the original data as approximated using the dominant 2 PCs, given time series in Table 21-9. Assume that the locations for these time series correspond to the geographic locations shown in Fig. 21.a of the PCA Focus box.

U23. Given the Hovmöller diagram in Fig. 21.29, determine the zonal speed and direction that features move.

Table 21-9. Time series for PCA exercise.

Location Index:	j = 1	2	3	4
Location:	NW	NE	SW	SE
Year	Air Temperature (°C)			
i = 1	9	1	1	8
2	7	2	1	8
3	5	3	2	7
4	7	4	5	6
5	9	5	9	5
6	7	6	9	5
7	5	5	5	5
8	7	4	5	6
9	9	3	4	7
10	7	2	1	8

**Figure 21.29**

Hovmöller Diagram showing 20 kPa winds. (Shading is arbitrary for this exercise.) Modified from data courtesy of the Climate Prediction Center, National Weather Service, National Oceanic and Atmospheric Admin.

Web-Enhanced Questions

W1. Search the web for graphs comparing different satellite measurements of the solar irradiance (i.e., the solar constant). Discuss the relative magnitudes of the variation of solar irradiance with time vs. the errors associated with measuring solar irradiance. [Hint: Recall info about accuracy and precision from Appendix A.]

W2. Search the web for maps showing the ice cover on Earth, and how ice cover has changed during the past decades. Discuss these changes in relation to ice-albedo feedback.

W3. Use info from the web to discuss the difference between **Bond albedo** and **geometric albedo**. Which one of these albedos is most appropriate to use for studying the radiation budget of other planets?

W4. Search the web for animations or graphs showing/predicting the changes in eccentricity, obliquity, and precession of the Earth over time. Discuss the simplifications made in these diagrams/animations to communicate this info to the general public.

W5. Search the web for more accurate Earth orbital parameters, to improve and expand the data that were in Tables 21-1 and 21-1b. Save and print this table.

W6. Search the web for accurate graphs of ice ages. Discuss what ice-age details are missing or poorly resolved in the ice-age graph in Fig. 21.8.

W7. Search the web for graphs of solar activity or sunspot number, going as far back in time as you can find. Discuss the regular and irregular components of the sunspot cycle.

W8. Search the web for animations or a sequence of still-frames showing how the continents moved to create Pangea, and how they continued to move into their current configuration. Discuss how monsoonal circulations change and alter the continental climates as the continents move.

W9. Search the web for photos of one recent volcanic eruptions that show: (a) ash fallout close to the volcano, and (b) the plume of aerosols injected into the stratosphere, showing their track around the Earth. Discuss the relative amount of global impact due to the one volcanic eruption.

W10. Search the web for photos of beautiful sunsets that were enhanced by volcanic aerosols in the stratosphere. Read ahead in the Optics chapter to see why you can see red skies and blue moons after some volcanic eruptions.

W11. Search the web for info about other major volcanic eruptions that were not listed in Table 21-3. Discuss the impacts of these eruptions on species.

W12. Search the web for the most recent Assessment Report from the Intergovernmental Panel on Climate Change (IPCC), and identify new information and understanding that updates information in this Chapter.

W13. Search the web for additional climate feedback processes that were not listed in Table 21-4, and discuss their relative importance to the ones already listed.

W14. Search the web for data on actual amounts of global warming during the past two decades, and compare to the predictions presented in older IPCC reports. How accurate are climate predictions?

W15. Search the web for animations of Daisyworld evolution. Enjoy.

W16. Lovelock, the inventor of Daisyworld, wrote a more recent book called "The Revenge of Gaia." If you don't have time to read the book, search the web for a summary of its key points, and argue either for or against his thesis.

W17. Search the web (particularly IPCC Assessment Reports) that summarize the attributes of the various global climate models (GCMs) that were used for the most recent IPCC climate report. Summarize the commonalities and differences between these various GCMs.

W18. Search the web for color maps showing Köppen climate classification for the whole world. Focus on where you live, and discuss how the climate classification does or does not agree with the climate you observed in your area.

W19. Search the web for color diagrams illustrating the processes involved in these climate oscillations:
a. El Niño or ENSO, including the relationship between trade winds, upwelling water, and SST
b. PDO c. NAO d. AO e. MJO

W20. What is meant by an "anomaly map" (for example, of sea surface temperature (SST))? Find ex-

amples of such maps or animations on the web for ENSO, and compare to the actual SST map. How are the SST anomalies associated with rainfall shifts, cloud coverage changes, etc.?

W21. Search the web for maps showing the geographic distribution of dominant principal components for any one of the climate oscillations.

W22. Search the web for a Hovmöller diagram for climate oscillations such as MJO in the most recent 6 or 12 months, and analyze it to explain the direction and movement of features.

W23. Use the web to find what is meant by **teleconnections**.

W24. Search the web to find the worldwide weather consequences a recent major El Niño event.

W25. Use the web to learn about the US government agency NCDC. How might it be useful to you? What similar agencies exist in other countries?

W26. Search the web graphs showing observed changes in many of the greenhouse gases.

Synthesis Questions

S1. What if emitted IR radiation were constant — not a function of temperature. How would Earth's climate be different, if at all?

S2. Create an energy balance to determine T_s and T_e for an atmosphere with:

- 2 opaque atmospheric layers (no window)
- 3 opaque atmospheric layers (no window)
- extrapolate your theory so it works with a large number of windows

S3. What if the Earth did not rotate, the atmosphere did not move, and there was no conduction from the warm side to the cold side of the Earth. Create an energy balance (and justify your assumptions) for:

- a single-layer atmosphere with no atmospheric window.
- a single-layer atmosphere with an atmospheric window.

S4. What if Earth had an internal heat source that created as much excess heat as the insolation at the top of the atmosphere. Describe the resulting energy balances of all relevant systems, and find the final equilibrium temperatures of the Earth's surface and the atmosphere (assume single layer with no window), and discuss feedbacks.

S5. Estimate the variation in insolation if Earth's orbit were to have eccentricity of 0.8.

S6. Suppose that transparent windows in the atmosphere opened over the north and south pole, similar to the ozone hole. Given that these spatial windows are over only the colder parts of the Earth's surface, calculate the resulting energy balances for all relevant systems, and compute the average Earth surface and atmosphere temperatures. State all assumptions.

S7. What if solar radiation from the sun ionized the sunny side of the atmosphere resulting in complete closure of the IR atmospheric window there. But assume that the shady side remains un-ionized, and a window can exist. Calculate the resulting energy budgets, and the equilibrium temperatures of the Earth and the atmosphere. State all assumptions.

S8. Re-derive the Daisyworld equations, but for a situation where daisies always cover the whole Earth. The relative coverages of black and white daisies can vary, but there is never any bare ground. How do the results differ, both in equilibrium and sensitivity to disturbances, from the Daisyworld described in this chapter?

S9. If runaway climate change caused the Earth to reach a new equilibrium, how difficult would it be to return to our present equilibrium? Would there also be a positive feedback to return to our present climate?

S10. Would global warming alter:

- the global circulation?
- thunderstorm severity?
- hurricane severity?

Explain how.

S11. Discuss the scientific accuracy of recent Hollywood movies where the theme was weather or climate disasters.

S12. Repeat the principal component analysis from the focus box, but when you calculate the table of principal components (PC), use the temperature anomaly T' in eq. (21.50) instead of T . Then finish all the remaining calculations using these new PCs.

What is the sum of elements in each PC (i.e., the sum of each column in the PC table)? What is the relationship between your plotted PCs and Fig. 21.c? When you synthesize the signal using only PC1, how does your result compare to Fig. 21.d?

Outcome: you are synthesizing T' and not T . Most researchers use PCA to focus on the anomaly.