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CONSTANTS & CONVERSION FACTORS

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UNIVERSAL CONSTANTS

- [from US National Institute of Standards and Technology (NIST), based on 2006 CODATA]
- $c_0 = 299,792,458$ m/s = speed of light in a vacuum
- $c_1 = 3.741 \ 771 \ 18 \ x10^8 \ W \cdot m^{-2} \cdot \mu m^{\overline{4}} = \text{first radiation}$ constant (in Planck's law)
- $c_{1B} = 1.191\ 042\ 759\ x10^8\ W\ m^{-2}\ \mu m^4\ sr^{-1} = first\ radiation constant for spectral radiance$
- $c_2 = 1.4387752 \times 10^4 \mu m \cdot K =$ second radiation constant (in Planck's law)
- $G = 6.674 \ 28 \ \text{x}10^{-11} \text{ m}^3 \cdot \text{s}^{-2} \cdot \text{kg}^{-1} = \text{Newtonian gravi-tational constant}$
- $h = 6.626\ 068\ 96\ x10^{-34}\ J \cdot s = Planck \ constant$
- $k_B = 1.380\,650\,4\,\mathrm{x10^{-23}}$ J·K⁻¹·molecule⁻¹ = Boltzmann constant
- $N_A = 6.022 \ 141 \ 79 \ \text{x} 10^{23} \ \text{mol}^{-1} = \text{Avogadro constant}$
- $\sigma_{SB} = 5.670 \, 400 \, \text{x} 10^{-8} \, \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-4} = \text{Stefan-Boltzmann}$ constant
- $T = -273.15^{\circ}$ C = 0 K = absolute zero (not considered a true universal constant)

MATH CONSTANTS

[from CRC Handbook of Chemistry and Physics] $e = 2.718\ 281\ 828\ 459 =$ base of natural logarithms $1/e = 0.367\ 879\ 441 =$ e-folding ratio $\pi = 3.141\ 592\ 653\ 589\ 793\ 238\ 462\ 643 =$ pi sqrt(2) = 1.414\ 213\ 562\ 373\ 095

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available at http://www.eos.ubc.ca/books/Practical_Meteorology/

a = 149.60 Gm = semi-major axis of Earth orbit A = 0.306 = Bond albedo (NASA 2009)

A = 0.367 = visual geometric albedo (NASA 2009)

 1° latitude = 111 km = 60 nautical miles (nm) [Cau-

tion: This relationship does NOT hold for de-

EARTH CHARACTERISTICS

grees longitude.]

- b = 149.090 Gm = semi-minor axis of Earth orbit
- *d* = 149.59787 Gm = average sun-Earth distance = 1 Astronomical Unit (AU)
- *d_{aphelion}* = 152.10 Gm = furthest sun-Earth distance, which occurs about 4 July (NASA 2009)
- *d*_{perihelion} = 147.09 Gm = closest sun-Earth distance, which occurs about 3 January (NASA 2009)
- $d_r = 173 = 22$ June = approx. day of summer solstice
- e = 0.0167 = eccentricity of Earth orbit around sun
- g = -9.798 m·s⁻² = average gravitational acceleration on Earth at sea level (negative = down-
- ward). (from NASA)
- $$\begin{split} |g| &= g_0 \cdot [1 + A \cdot \sin^2(\phi) B \cdot \sin^2(2\phi)] C \cdot H \\ &= \text{variation of gravitational-acceleration mag$$
 $nitude with latitude ϕ and altitude H(m) above mean sea level. $g_0 = 9.780 318 4 m \cdot s^{-2}$, $A = 0.005 3024$, $B = 0.000 0059$, $C = 3.086 \times 10^{-6} \text{ s}^{-2}$. \end{split}$

 $M = 5.9736 \times 10^{24}$ kg = mass of Earth (from NASA)

$$P_{earth} = 365.254\ 63\ days = Earth orbital period$$

- $P_{moon} = 27.32$ days = lunar orbital period
- $P_{sidereal}$ = 23.934 469 6 h = sidereal day = period for one revolution of the Earth about its axis, relative to fixed stars
- $R_{earth} = 6371.0$ km = volumetric average Earth radius (from NASA)
 - = 6378.1 km = Earth radius at equator
 - = 6356.8 km = Earth radius at poles
- $S = 1366 \text{ W} \cdot \text{m}^{-2} = \text{ solar irradiance (solar constant)}$ at top of atmosphere
 - ≈ 1.125 K ·m ·s⁻¹ = kinematic solar constant (based on mean sea-level density)
- T_e = 254.3 K = effective radiation emission black-body temperature of the Earth system (NASA)
- $\Phi_r = 23.45^\circ = 0.409$ radians = tilt of Earth axis = obliquity relative to the orbital plane
- $\Omega = 0.729 \ 211 \ 6 \ x10^{-4} \ s^{-1} = rotation frequency of Earth$

 $2 \cdot \Omega = 1.458 \ 423 \ \text{x10}^{-4} \ \text{s}^{-1} = \text{Coriolis factor}$

 $2 \cdot \Omega / R_{earth} = 2.294 \ 285 \ \text{x10}^{-11} \ \text{m}^{-1} \cdot \text{s}^{-1} = \text{beta factor}$

AIR AND WATER CHARACTERISTICS

- $a = 0.0337 \text{ (mm/day)} \cdot (W/m^2)^{-1} = \text{water depth evap-}$ oration per latent heat flux
- $B = 3 \times 10^9$ V/km = breakdown potential for dry air C_{vd} = 717 J·kg⁻¹·K⁻¹ = specific heat for dry air at
- constant volume $C_{nd} = 1003 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} = \text{specific heat for dry air at}$
- constant pressure at -23°C
- $C_{nd} = 1004 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} = \text{specific heat for dry air at}$ constant pressure at 0°C
- C_{pd} = 1005 J·kg⁻¹·K⁻¹ = specific heat for dry air at constant pressure at 27°C
- $C_{pv} = 1850 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} = \text{specific heat for water va-}$ por at constant pressure at 0°C
- $C_{pv} = 1875 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} = \text{specific heat for water vapor}$ at constant pressure at 15°C
- $C_{lia} = 4217.6 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} = \text{specific heat of liquid wa-}$ ter at 0°C
- $C_{ice} = 2106 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} = \text{specific heat of ice at } 0^{\circ}\text{C}$ $D = 2.11 \times 10^{-5} \text{ m}^2 \cdot \text{s}^{-1} = \text{molecular diffusivity of wa-}$ ter vapor in air in standard conditions
- $e_o = 0.611$ kPa = reference vapor pressure at 0°C
- k = 0.0253 W·m⁻¹·K⁻¹ = molecular conductivity of air at sea level in standard conditions
- $L_d = 2.834 \times 10^6$ J ·kg⁻¹ = latent heat of deposition at 0°C
- $L_f = 3.34 \text{ x}10^5 \text{ J} \cdot \text{kg}^{-1} = \text{latent heat of fusion at } 0^{\circ}\text{C}$
- $L_{v} = 2.501 \text{ x}10^{6} \text{ J} \cdot \text{kg}^{-1} = \text{latent heat of vaporization}$ at 0°C
- $n = 3.3 \times 10^{28}$ molecules $\cdot m^{-3}$ for liquid water at 0°C $n_{air} \approx 1.000 \ 277 = \text{index of refraction for air}$
- $n_{water} \approx 1.336$ = index of refraction for liquid water
- $n_{ice} \approx 1.312 = index of refraction for ice$
- P_{STP} = 101.325 kPa = standard sea-level pressure (STP = Standard Temperature & Pressure)
- $\Re_d = 0.287\ 053\ \text{kPa} \cdot \text{K}^{-1} \cdot \text{m}^3 \cdot \text{kg}^{-1} = C_{pd} C_{vd}$ = 287.053 J·K⁻¹·kg⁻¹ = gas constant for dry air \Re_v = 461.5 J·K⁻¹·kg⁻¹ = water-vapor gas constant
- $= 4.61 \times 10^{-4} \text{ kPa} \cdot \text{K}^{-1} \cdot \text{m}^{3} \cdot \text{g}^{-1}$
- $Ri_c = 0.25 =$ critical Richardson number
- $s_0 = 343.15 \text{ m/s} = \text{sound speed in standard, calm air}$ $T_{STP} = 15^{\circ}\text{C} = \text{standard sea-level temperature}$

 $\begin{aligned} \varepsilon &= 0.622 \text{ g}_{\text{water}} \cdot g_{\text{air}}^{-1} = \Re_d / \Re_v = \text{gas-constant ratio} \\ \gamma &= 0.0004 (g_{\text{water}} \cdot g_{\text{air}}^{-1}) \cdot \text{K}^{-1} = C_p / L_v \\ &= 0.4 (g_{\text{water}} \cdot \text{k}g_{\text{air}}^{-1}) \cdot \text{K}^{-1} = \text{psychrometric constant} \\ \Gamma_d &= 9.75 \text{ K} \cdot \text{km}^{-1} = |g| / C_p = \text{dry adiabatic lapse rate} \\ \rho_{STP} &= 1.225 \text{ kg} \cdot \text{m}^{-3} = \text{standard sea-level air density} \end{aligned}$ $\rho_{avg} = 0.689 \text{ kg} \cdot \text{m}^{-3} = \text{air density averaged over the}$

troposphere (over z = 0 to 11 km)

- ρ_{lig} = 999.84 kg·m⁻³ = density of liquid water at 0°C
 - = 1000.0 kg·m⁻³ = density of liquid water at 4° C
 - = 998.21 kg·m⁻³ = density of liquid water at 20°C
- = 992.22 kg·m⁻³ = density of liquid water at 40°C
- = 983.20 kg·m⁻³ = density of liquid water at 60° C
- = 971.82 kg·m⁻³ = density of liquid water at 80° C
- = 958.40 kg·m⁻³ = density of liquid water at 100°C $\rho_{sea-water} = 1025 \text{ kg} \cdot \text{m}^{-3} = \text{avg. density of sea water,}$
- (sea water contains 34.482 g of salt ions per kg of water, on average)
- $\rho_{ice} = 916.8 \text{ kg} \cdot \text{m}^{-3} = \text{density of ice at } 0^{\circ}\text{C}$
- $\sigma = 0.076 \text{ N} \cdot \text{m}^{-1} = \text{surface tension of pure water } 0^{\circ}\text{C}$

CONVERSION FACTORS & COMBINED PARAMETERS

- $C_{vd} / C_{vd} = k = 1.400$ (dimensionless)
- = specific heat ratio $C_{md}/|q| = 102.52 \text{ m} \cdot \text{K}^{-1}$

$$C_{pd} / L_v = 0.0004 (g_{water} \cdot g_{air}^{-1}) \cdot K^{-1} = \gamma$$

= 0.4 (g_{water} \cdot kg_{air}^{-1}) \cdot K^{-1}

- (gwater Kgair) = psychrometric constant
- C_{vd} / \Re_d = 3.50 (dimensionless)
- $C_{vd} / C_{pd} = 1/k = 0.714$ (dimensionless)
- $|g|/C_{vd}$ = Γ_d = 9.8 K ·km⁻¹ = dry adiabatic lapse rate
- $|g|/\Re_d = 0.0342 \text{ K} \cdot \text{m}^{-1} = 1/(\text{hypsometric constant})$
- $L_v / C_{pd} = 2.5 \text{ K} / (g_{water} \cdot kg_{air}^{-1})$
- L_v / \Re_v = 5423 K = Clausius-Clapeyron parameter for vaporization
- $\Re_d / C_{pd} = 0.28571$ (dimensionless) = potential-temperature constant
- $\Re_d / \Re_v = \varepsilon = 0.622 \text{ g}_{\text{water}} \cdot g_{\text{air}}^{-1} = \text{gas-constant ratio}$
- $\Re_d / |g| = 29.29 \text{ m} \cdot \text{K}^{-1} = \text{hypsometric constant}$
- $\rho_{air} \cdot C_{vd air} = 1231 \text{ (W·m}^{-2}) / \text{(K·m·s}^{-1}) \text{ at sea level}$
 - = 12.31 mb \cdot K⁻¹ at sea level = 1.231 kPa $\cdot K^{-1}$ at sea level
 - = 12.0 kg \cdot m⁻² \cdot s⁻² at sea level
- $\rho_{air} \cdot |g|$ = 0.12 $\overline{mb} \cdot m^{-1}$ at sea level = 0.012 kPa \cdot m⁻¹ at sea level
- $\rho_{air} \cdot L_v = 3013.5 \text{ (W·m}^{-2}) / [(g_{water} \cdot kg_{air}^{-1}) \cdot (m \cdot s^{-1})] \text{ at}$ sea level
- $\rho_{lig} \cdot C_{lig} = 4.295 \text{ x}10^6 \text{ (W·m}^{-2}) / (\text{K·m} \cdot \text{s}^{-1})$
- 1 megaton nuclear explosion $\approx 4 \times 10^{15} \text{ J}$

 2π radians = 360°

 $(1-\varepsilon)/\varepsilon = 0.61 = virtual$ -temperature constant