2 SOLAR & INFRARED RADIATION

Contents

Solar energy powers the atmosphere. This energy warms the air and drives the air motion you feel as winds. The seasonal distribution of this energy depends on the orbital characteristics of the Earth around the sun.

The Earth's rotation about its axis causes a daily cycle of sunrise, increasing solar radiation until solar noon, then decreasing solar radiation, and finally sunset. Some of this solar radiation is absorbed at the Earth's surface, and provides the energy for photosynthesis and life.

Downward **infrared** (IR) radiation from the atmosphere to the Earth is usually slightly less than upward IR radiation from the Earth, causing net cooling at the Earth's surface both day and night. The combination of daytime solar heating and continuous IR cooling yields a **diurnal** (daily) cycle of **net radiation**.

ORBITAL FACTORS

Planetary Orbits

Johannes Kepler, the 17th century astronomer, discovered that planets in the solar system have elliptical orbits around the sun. For most planets in the solar system, the eccentricity (deviation from circular) is relatively small, meaning the orbits are nearly circular. For circular orbits, he also found that the time period Y of each orbit is related to the distance R of the planet from the sun by:

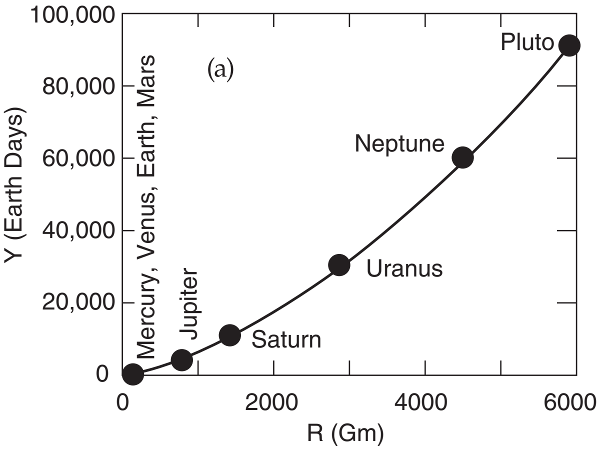
$$Y = a_1 \cdot R^{3/2} \tag{2.1}$$

Parameter $a_1 \approx 0.1996 \text{ d}\cdot(\text{Gm})^{-3/2}$, where d is Earth days, and Gm is gigameters = 10^6 km.

Figs. 2.1a & b show the orbital periods vs. distances for the planets in our solar system. These figures show the duration of a year for each planet, which affect the seasons experienced on the planet.

Orbit of the Earth

The Earth and the moon rotate with a **sidereal** (relative to the stars) period of 27.32 days around their common center of gravity, called the Earth-moon **barycenter**. (Relative to the moving Earth,



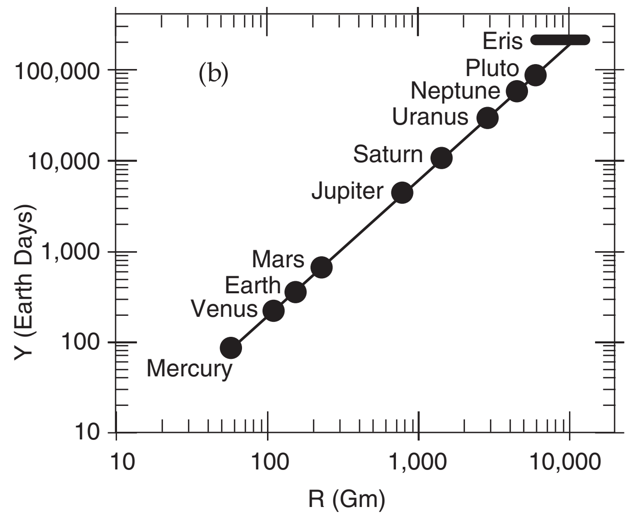


Figure 2.1 a (linear) & b (log-log)
Planetary orbital periods versus distance from sun. Eris (136199) and Pluto (134340) are dwarf planets. Eris, larger than Pluto, has a very elliptical orbit. (a) Linear graph. (b) Log-log graph. (See Appendix A for comparison of various graph formats.)

Sample Application
Verify that eq. (2.1) gives the correct orbital period of one Earth year.

Find the Answer:
Given: R = 149.6 Gm avg. distance sun to Earth.
Find: Y = ? days, the orbital period for Earth

Use eq. (2.1):
Y = (0.1996 d·(Gm)$^{-3/2}$) · [(149.6 Gm)$^{1.5}$]
= **365.2 days**.

Check: Units OK. Sketch OK. Almost 1 yr.
Exposition: In 365.0 days, the Earth does not quite finish a complete orbit. After four years this shortfall accumulates to nearly a day, which we correct using a leap year with an extra day (see Chapter 1).

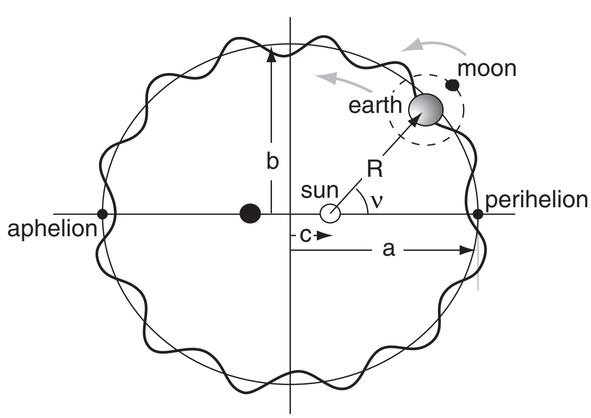


Figure 2.2
Geometry of the Earth's orbit, as viewed from above Earth's North Pole. (Not to scale.) Dark wavy line traces the Earth-center path, while the thin smooth ellipse traces the barycenter path.

the time between new moons is 29.5 days.) Because the mass of the moon (7.35x10^{22} kg) is only 1.23% of the mass of the Earth (Earth mass is 5.9726x10^{24} kg), the barycenter is much closer to the center of the Earth than to the center of the moon. This barycenter is 4671 km from the center of the Earth, which is below the Earth's surface (Earth radius is 6371 km).

To a first approximation, the Earth-moon barycenter orbits around the sun in an **elliptical** orbit (Fig. 2.2, thin-line ellipse) with sidereal period of P = 365.256363 days. Length of the **semi-major axis** (half of the longest axis) of the ellipse is a = 149.598 Gm. This is almost equal to an **astronomical unit** (au), where 1 au = 149,597,870.691 km.

Semi-minor axis (half the shortest axis) length is b = 149.090 Gm. The center of the sun is at one of the foci of the ellipse, and half the distance between the two foci is c = 2.5 Gm, where $a^2 = b^2 + c^2$. The orbit is close to circular, with an **eccentricity** of only about $e \approx c/a = 0.0167$ (a circle has zero eccentricity).

The closest distance (**perihelion**) along the major axis between the Earth and sun is $a - c$ = 146.96 Gm and occurs at about $d_p \approx$ 4 January. The farthest distance (**aphelion**) is $a + c$ = 151.96 Gm and occurs at about 5 July. The dates for the perihelion and aphelion jump a day or two from year to year because the orbital period is not exactly 365 days. Figs. 2.3 show these dates at the **prime meridian** (Greenwich), but the dates will be slightly different in your own time zone. Also, the dates of the perihelion and aphelion gradually become later by 1 day every 58 years, due to **precession** (shifting of the location of the major and minor axes) of the Earth's orbit around the sun (see the Climate chapter).

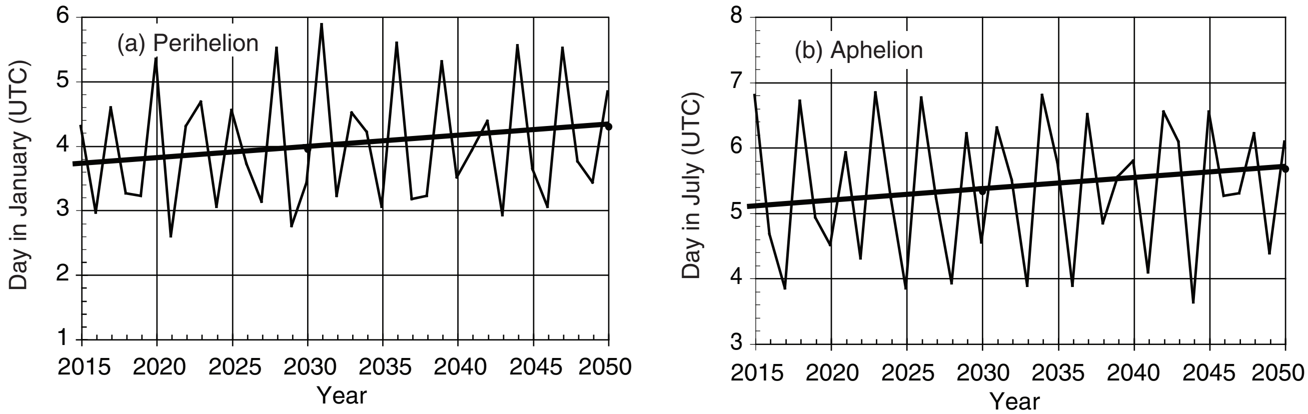


Figure 2.3
Dates (UTC) of the (a) perihelion and (b) aphelion, & their trends (thick line). From the US Naval Observatory. http://aa.usno.navy.mil/data/docs/EarthSeasons.php

Because the Earth is rotating around the Earth-moon barycenter while this barycenter is revolving around the sun, the location of the center of the Earth traces a slightly wiggly path as it orbits the sun. This path is exaggerated in Fig. 2.2 (thick line).

Define a relative **Julian Day**, d, as the day of the year. For example, 15 January corresponds to d = 15. For 5 February, d = 36 (which includes 31 days in January plus 5 days in February).

The angle at the sun between the perihelion and the location of the Earth (actually to the Earth-moon barycenter) is called the **true anomaly** ν (see Fig. 2.2). This angle increases during the year as the day d increases from the perihelion day (about d_p = 4; namely, about 4 January). According to **Kepler's second law**, the angle increases more slowly when the Earth is further from the sun, such that a line connecting the Earth and the sun will sweep out equal areas in equal time intervals.

An angle called the **mean anomaly** M is a simple, but good approximation to ν. It is defined by:

$$M = C \cdot \frac{d - d_p}{P} \tag{2.2}$$

where P = 365.256363 days is the (sidereal) orbital period and C is the angle of a full circle ($C = 2\cdot\pi$ radians = 360°. Use whichever is appropriate for your calculator, spreadsheet, or computer program.)

Because the Earth's orbit is nearly circular, $\nu \approx M$. A better approximation to the **true anomaly** for the elliptical Earth orbit is

$$\nu \approx M + [2e - (e^3/4)]\cdot\sin(M) + [(5/4)\cdot e^2]\cdot\sin(2M) + [(13/12)\cdot e^3]\cdot\sin(3M) \tag{2.3a}$$

or

$$\nu \approx M + 0.0333988\cdot\sin(M) + 0.0003486\cdot\sin(2\cdot M) + 0.0000050\cdot\sin(3\cdot M) \tag{2.3b}$$

for both ν and M in radians, and e = 0.0167.

Sample Application(§)

Use a spreadsheet to find the true anomaly and sun-Earth distance for several days during the year.

Find the Answer

Given: d_p = 4 Jan. P = 365.25 days.
Find: ν = ?° and R = ? Gm.

Sketch: (same as Fig 2.2)
For example, for 15 Feb, d = 46
Use eq. (2.2):
$M = (2 \cdot 3.14159)\cdot(46-4)/\ 365.256363 = \underline{\mathbf{0.722\ radians}}$
Use eq. (2.3b):
$\nu = 0.722 + 0.0333988\cdot\sin(0.722) + 0.0003486\cdot\sin(1.444) + 0.000005\cdot\sin(2.166) = \underline{\mathbf{0.745\ radians}}$
Use eq. (2.4):
$R = (149.457\text{Gm})\cdot(1 - 0.0167^2)/[1+0.0167\cdot\cos(0.745)]$
$= (149.457\text{Gm}) \cdot 0.99972\ /\ 1.012527 = \underline{\mathbf{147.60\ Gm}}$

Repeating this calculation on a spreadsheet for several days, and comparing M vs. ν and R(M) vs. R(ν) , gives:

Date	d	M (rad)	ν (rad)	R(M) (Gm)	R(ν) (Gm)
4 Jan	4	0	0	146.96	146.96
18 Jan	18	0.241	0.249	147.03	147.04
1 Feb	32	0.482	0.497	147.24	147.25
15 Feb	46	0.722	0.745	147.57	147.60
1 Mar	60	0.963	0.991	148.00	148.06
15 Mar	74	1.204	1.236	148.53	148.60
29 Mar	88	1.445	1.487	149.10	149.18
12 Apr	102	1.686	1.719	149.70	149.78
26 Apr	116	1.927	1.958	150.29	150.36
21 Jun	172	2.890	2.898	151.87	151.88
23 Sep	266	4.507	4.474	149.93	150.01
22 Dec	356	6.055	6.047	147.02	147.03

Check: Units OK. Physics OK.
Exposition: Because M and ν are nearly equal, you can use M instead of ν in eq. (2.4), with good accuracy.

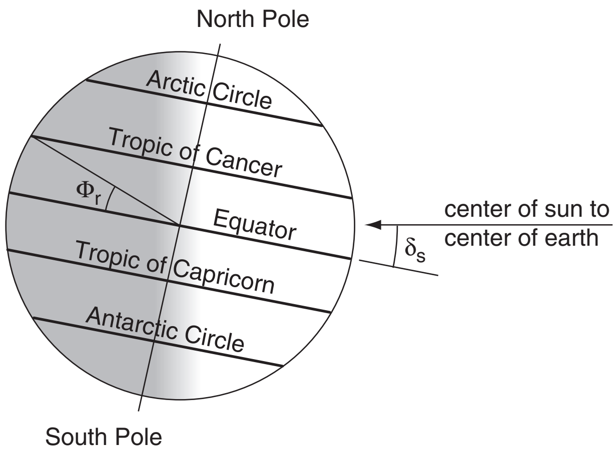


Figure 2.4
Relationship of declination angle δ_s to tilt of the Earth's axis, for a day near northern-hemisphere summer.

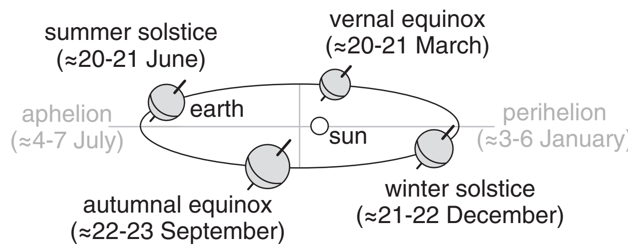


Figure 2.5
Dates (UTC) of northern-hemisphere seasons relative to Earth's orbit.

Table 2-1 . Dates and times (UTC) for northern hemisphere equinoxes and solstices. Format: dd hhmm gives day (dd), hour (hh) and minutes (mm). *From the US Naval Observatory. http://aa.usno.navy.mil/data/docs/EarthSeasons.php*

Year	Spring Equinox (March)	Summer Solstice (June)	Fall Equinox (Sept.)	Winter Solstice (Dec.)
2015	20 2245	21 1638	23 0820	22 0448
2016	20 0430	20 2234	22 1421	21 1044
2017	20 1028	21 0424	22 2002	21 1628
2018	20 1615	21 1007	23 0154	21 2222
2019	20 2158	21 1554	23 0750	22 0419
2020	20 0349	20 2143	22 1330	21 1002
2021	20 0937	21 0332	22 1921	21 1559
2022	20 1533	21 0914	23 0104	21 2148
2023	20 2124	21 1458	23 0650	22 0327
2024	20 0306	20 2051	22 1244	21 0921
2025	20 0901	21 0242	22 1819	21 1503

The distance R between the sun and Earth (actually to the Earth-moon barycenter) as a function of time is

$$R = a \cdot \frac{1 - e^2}{1 + e \cdot \cos(\nu)} \tag{2.4}$$

where $e = 0.0167$ is eccentricity, and $a = 149.457$ Gm is the semi-major axis length. If the simple approximation of $\nu \approx M$ is used, then angle errors are less than 2° and distance errors are less than 0.06%.

Seasonal Effects

The tilt of the Earth's axis relative to a line perpendicular to the **ecliptic** (i.e., the orbital plane of the Earth around the sun) is presently $\Phi_r = 23.44° = 0.409$ radians. The direction of tilt of the Earth's axis is not fixed relative to the stars, but wobbles or precesses like a top with a period of about 25,781 years. Although this is important over millennia (see the Climate chapter), for shorter time intervals (up to a century) the precession is negligible, and you can assume a fixed tilt.

The **solar declination angle** δ_s is defined as the angle between the ecliptic and the plane of the Earth's equator (Fig. 2.4). Assuming a fixed orientation (tilt) of the Earth's axis as the Earth orbits the sun, the solar declination angle varies smoothly as the year progresses. The north pole points partially toward the sun in summer, and gradually changes to point partially away during winter (Fig. 2.5).

Although the Earth is slightly closer to the sun in winter (near the perihelion) and receives slightly more solar radiation then, the Northern Hemisphere receives substantially less sunlight in winter because of the tilt of the Earth's axis. Thus, summers are warmer than winters, due to Earth-axis tilt.

The solar declination angle is greatest (+23.44°) on about 20 to 21 June (**summer solstice** in the Northern Hemisphere, when daytime is longest) and is most negative (–23.44°) on about 21 to 22 December (**winter solstice**, when nighttime is longest). The **vernal equinox** (or **spring equinox**, near 20 to 21 March) and **autumnal equinox** (or **fall equinox**, near 22 to 23 September) are the dates when daylight hours equal nighttime hours ("eqi nox" literally translates to "equal night"), and the solar declination angle is zero.

Astronomers define a **tropical year** (= 365.242190 days) as the time from vernal equinox to the next vernal equinox. Because the tropical year is not an integral number of days, the **Gregorian calendar** (the calendar adopted by much of the western world) must be corrected periodically to prevent the seasons (dates of summer solstice, etc.) from shifting into different months.

To make this correction, a **leap day** (29 Feb) is added every 4th year (i.e., **leap years**, are years divisible by 4), except that years divisible by 100 don't have a leap day. However, years divisible by 400 do have a leap day (for example, year 2000). Because of all these factors, the dates of the solstices, equinoxes, perihelion, and aphelion jump around a few days on the Gregorian calendar (Table 2-1 and Figs. 2.3), but remain in their assigned months.

The solar declination angle for any day of the year is given by

$$\delta_s \approx \Phi_r \cdot \cos\left[\frac{C\cdot(d-d_r)}{d_y}\right] \qquad \bullet(2.5)$$

where d is Julian day, and d_r is the Julian day of the summer solstice, and d_y = 365 (or = 366 on a leap year) is the number of days per year. For years when the summer solstice is on 21 June, d_r = 172. This equation is only approximate, because it assumes the Earth's orbit is circular rather than elliptical. As before, $C = 2\cdot\pi$ radians = 360° (use radians or degrees depending on what your calculator, spreadsheet, or computer program expects).

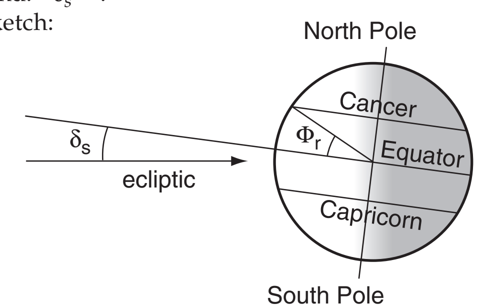
By definition, Earth-tilt angle (Φ_r = 23.44°) equals the latitude of the **Tropic of Cancer** in the Northern Hemisphere (Fig. 2.4). Latitudes are defined to be positive in the Northern Hemisphere. The **Tropic of Capricorn** in the Southern Hemisphere is the same angle, but with a negative sign. The **Arctic Circle** is at latitude 90° – Φ_r = 66.56°, and the **Antarctic Circle** is at latitude –66.56° (i.e., 66.56°S). During the Northern-Hemisphere summer solstice: the solar declination angle equals Φ_r ; the sun at noon is directly overhead (90° elevation angle) at the Tropic of Cancer; and the sun never sets that day at the Arctic Circle.

Sample Application

Find the solar declination angle on 5 March.

Find the Answer

Assume: Not a leap year.
Given: d = 31 Jan + 28 Feb + 5 Mar = 64.
Find: δ_s = ?°
Sketch:



Use eq. (2.5):
δ_s = 23.44° · cos[360°·(64–172)/365]

= 23.44° · cos[–106.521°] = <u>**–6.67°**</u>

Check: Units OK. Sketch OK. Physics OK.
Exposition: On the **vernal equinox** (21 March), the angle should be zero. Before that date, it is winter and the declination angle should be negative. Namely, the ecliptic is below the plane of the equator. In spring and summer, the angle is positive. Because 5 March is near the end of winter, we expect the answer to be a small negative angle.

Sample Application

Find the local elevation angle of the sun at 3 PM local time on 5 March in Vancouver, Canada.

Find the Answer

Assume: Not a leap year. Not daylight savings time.
Also, Vancouver is in the Pacific time zone, where $t_{UTC} = t + 8$ h during standard time.

Given: t = 3 PM = 15 h. Thus, t_{UTC} = 23 h.
ϕ = 49.25°N, λ_e = 123.1°W for Vancouver.
δ_s = –6.665° from previous Sample Application.

Find: Ψ = ?°

Use eq. (2.6): sin(Ψ) =
= sin(49.25°)·sin(–6.665°) –
cos(49.25°)·cos(–6.665°)·cos[360°·(23h/24h) – 123.1°]
= 0.7576 · (–0.1161) – 0.6528 · 0.9932 · cos(221.9°)
= –0.08793 + 0.4826 = 0.3946
Ψ = arcsin(0.3946) = **23.24°**

Check: Units OK. Physics OK.
Exposition: The sun is above the local horizon, as expected for mid afternoon.
Beware of other situations such as night that give negative elevation angle.

Daily Effects

As the Earth rotates about its axis, the **local elevation angle** Ψ of the sun above the local horizon rises and falls. This angle depends on the latitude ϕ and longitude λ_e of the location:

$$\sin(\Psi) = \sin(\phi)\cdot\sin(\delta_s) - \cos(\phi)\cdot\cos(\delta_s)\cdot\cos\left[\frac{C\cdot t_{UTC}}{t_d} - \lambda_e\right] \qquad \bullet(2.6)$$

Time of day t_{UTC} is in UTC, $C = 2\pi$ radians = 360° as before, and the length of the day is t_d. For t_{UTC} in hours, then t_d = 24 h. Latitudes are positive north of the equator, and longitudes are positive west of the prime meridian. The sin(Ψ) relationship is used later in this chapter to calculate the daily cycle of solar energy reaching any point on Earth.
[CAUTION: Don't forget to convert angles to radians if required by your spreadsheet or programming language.]

Sample Application(§)

Use a spreadsheet to plot elevation angle vs. time at Vancouver, for 21 Dec, 23 Mar, and 22 Jun. Plot these three curves on the same graph.

Find the Answer

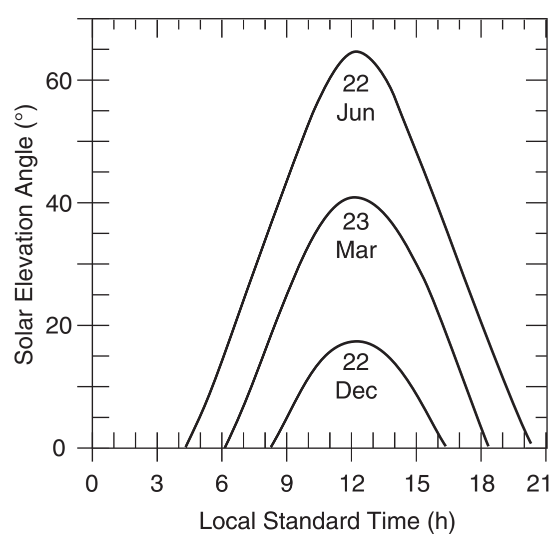
Given: Same as previous Sample Application, except d = 355, 82, 173.
Find: Ψ = ?°. Assume not a leap year.

A portion of the tabulated results are shown below, as well as the full graph.

	Ψ (°)		
t (h)	21 Dec	23 Mar	22 Jun
3	0.0	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.0	6.6
6	0.0	0.0	15.6
7	0.0	7.8	25.1
8	0.0	17.3	34.9
9	5.7	25.9	44.5
10	11.5	33.2	53.4
11	15.5	38.4	60.5
12	17.2	40.8	64.1
13	16.5	39.8	62.6

(continues in next column)

Sample Application *(continuation)*



Check: Units OK. Physics OK. Graph OK.
Exposition: Summers are pleasant with long days. The peak elevation does not happen precisely at local noon, because Vancouver is not centered within its time zone.

The local **azimuth angle** α of the sun relative to north is

$$\cos(\alpha) = \frac{\sin(\delta_S) - \sin(\phi)\cdot\cos(\zeta)}{\cos(\phi)\cdot\sin(\zeta)} \qquad (2.7)$$

where $\zeta = C/4 - \Psi$ is the **zenith angle**. After noon, the azimuth angle might need to be corrected to be $\alpha = C - \alpha$, so that the sun sets in the west instead of the east. Fig. 2.6 shows an example of the elevation and azimuth angles for Vancouver (latitude = 49.25°N, longitude = 123.1°W) during the solstices and equinoxes.

Sunrise, Sunset & Twilight

Geometric sunrise and sunset occur when the center of the sun has zero elevation angle. **Apparent sunrise/set** are defined as when the top of the sun crosses the horizon, as viewed by an observer on the surface. The sun has a finite radius corresponding to an angle of 0.267° when viewed from Earth. Also, **refraction** (bending) of light through the atmosphere allows the top of the sun to be seen even when it is really 0.567° below the horizon. Thus, apparent sunrise/set occurs when the center of the sun has an elevation angle of –0.833°.

When the apparent top of the sun is slightly below the horizon, the surface of the Earth is not receiving direct sunlight. However, the surface can still receive indirect light scattered from air molecules higher in the atmosphere that are illuminated by the sun. The interval during which scattered light is present at the surface is called **twilight**. Because twilight gradually fades as the sun moves lower below the horizon, there is no precise definition of the start of sunrise twilight or the end of sunset twilight.

Arbitrary definitions have been adopted by different organizations to define twilight. **Civil twilight** occurs while the sun center is no lower than –6°, and is based on the ability of humans to see objects on the ground. **Military twilight** occurs while the sun is no lower than –12°. **Astronomical twilight** ends when the skylight becomes sufficiently dark to view certain stars, at solar elevation angle –18°.

Table 2-2 summarizes the solar elevation angle Ψ definitions used for sunrise, sunset and twilight. All of these angles are at or below the horizon.

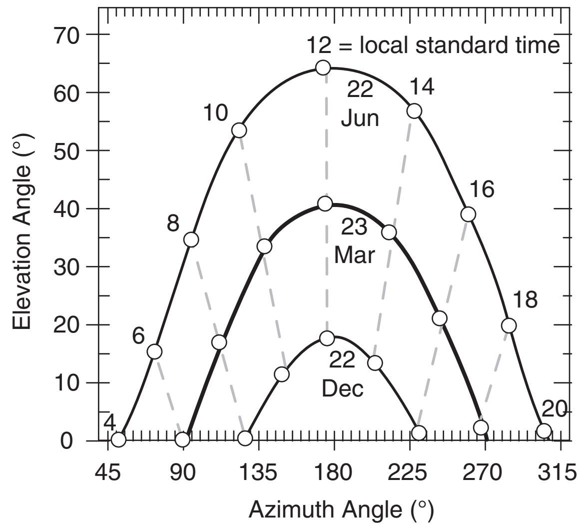


Figure 2.6
Position (solid lines) of the sun for Vancouver, Canada for various seasons. September 21 and March 23 nearly coincide. Isochrones are dashed. All times are Pacific standard time.

Sample Application(§)

Use a spreadsheet to find the Pacific standard time (PST) for all the events of Table 2-2, for Vancouver, Canada, during 22 Dec, 23 Mar, and 22 Jun.

Find the Answer

Given: Julian dates 355, 82, & 173.
Find: t = ? h (local standard time)
Assume: Pacific time zone: $t_{UTC} = t + 8$ h.

Use eq. (2.8a) and Table 2-2:

	22Dec	23Mar	22Jun
Morning:	PST (h)		
geometric sunrise	8.22	6.20	4.19
apparent sunrise	8.11	6.11	4.09
civil twilight starts	7.49	5.58	3.36
military twilight starts	6.80	4.96	2.32
astron. twilight starts	6.16	4.31	n/a
Evening:			
geometric sunset	16.19	18.21	20.22
apparent sunset	16.30	18.30	20.33
civil twilight ends	16.93	18.83	21.05
military twilight ends	17.61	19.45	22.09
astron. twilight ends	18.26	20.10	n/a

Check: Units OK. Physics OK.
Exposition: During the summer solstice (22 June), the sun never gets below –18°. Hence, it is astronomical twilight all night in Vancouver in mid summer.

During June, Vancouver is on daylight time, so the actual local time would be one hour later.

Table 2-2. Elevation angles for diurnal events.

Event	Ψ (°)	Ψ (radians)
Sunrise & Sunset:		
Geometric	0	0
Apparent	–0.833	–0.01454
Twilight:		
Civil	–6	–0.10472
Military	–12	–0.20944
Astronomical	–18	–0.31416

INFO • Astronomical Values for Time

The constants $a = 2{\cdot}e/\omega_E$, $b = [\tan^2(\varepsilon/2)]/\omega_E$, and $c = 2{\cdot}\varpi$ in the Equation of Time are based on the following astronomical values: $\omega_E = 2\pi/24\text{h} = 0.0043633$ radians/minute is Earth's rotation rate about its axis, $e = 0.01671$ is the eccentricity of Earth's orbit around the sun, $\varepsilon = 0.40909$ radians = 23.439° is the obliquity (tilt of Earth's axis), $\varpi = 4.9358$ radians = 282.8° is the angle (in the direction of Earth's orbit around the sun) between a line from the sun to the vernal (Spring) equinox and a line drawn from the sun to the moving perihelion (see Fig. 2.5, and Fig. 21.10 in Chapter 21).

Sample Application (§)

Plot time correction vs. day of the year.

Find the Answer:

Given: d = 0 to 365

Find: Δt_a = ? minutes

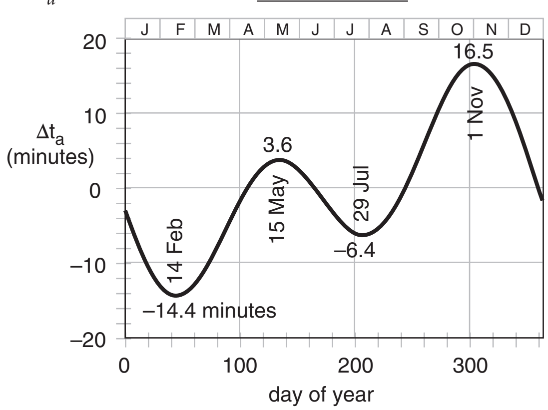
Use a spreadsheet. For example, for d = 45 (which is 14 Feb), first use eq. (2.2) find the mean anomaly:

$M = 2\pi{\cdot}(45\text{-}4)/365.25 = 0.705 \text{ rad} = 40.4°$

Then use the **Equation of Time** (2.8b) in

$\Delta t_a = -(7.659 \text{ min}){\cdot}\sin(0.705 \text{ rad})$
$\quad + (9.836 \text{ min}){\cdot}\sin(2{\cdot}\ 0.705 \text{ rad} + 3.588 \text{ rad})$

$\Delta t_a = -4.96 - 9.46 =$ **<u>–14.4 minutes</u>**



Check: Units OK. Magnitude OK.

Exposition: Because of the negative sign in eq. (2.8c), you need to **<u>ADD</u>** 14.4 minutes to the results of eq. (2.8a) to get the correct sunrise and sunset times.

Approximate (sundial) time-of-day corresponding to these events can be found by rearranging eq. (2.6):

$$t_{UTC} = \frac{t_d}{C}\cdot\left\{\lambda_e \pm \arccos\left[\frac{\sin\phi\cdot\sin\delta_s - \sin\Psi}{\cos\phi\cdot\cos\delta_s}\right]\right\} \tag{2.8a}$$

where the appropriate elevation angle is used from Table 2-2. Where the ± sign appears, use + for sunrise and – for sunset. If any of the answers are negative, add 24 h to the result.

To correct the time for the tilted, elliptical orbit of the earth, use the approximate **Equation of Time**:

$$\Delta t_a \approx -a\cdot\sin(M) + b\cdot\sin(2M + c) \tag{2.8b}$$

where a = 7.659 minutes, b = 9.863 minutes, c = 3.588 radians = 205.58°, and where the mean anomaly M from eq. (2.2) varies with day of the year. This time correction is plotted in the Sample Application. The corrected (mechanical-clock) time t_{eUTC} is:

$$t_{eUTC} = t_{UTC} - \Delta t_a \tag{2.8c}$$

which corrects sundial time to mechanical-clock time. Don't forget to convert the answer from UTC to your local time zone.

FLUX

A **flux density**, $\mathbb{F}$, called a **flux** in this book, is the transfer of a quantity per unit area per unit time. The area is taken perpendicular (normal) to the direction of flux movement. Examples with metric (SI) units are mass flux ($\text{kg}\cdot \text{m}^{-2}\cdot\text{s}^{-1}$) and heat flux, ($\text{J}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$). Using the definition of a watt ($1 \text{ W} = 1 \text{ J}\cdot\text{s}^{-1}$), the heat flux can also be given in units of ($\text{W}\cdot\text{m}^{-2}$). A flux is a measure of the amount of inflow or outflow such as through the side of a fixed volume, and thus is frequently used in Eulerian frameworks (Fig. 2.7).

Because flow is associated with a direction, so is flux associated with a direction. You must account for fluxes $\mathbb{F}_x$, $\mathbb{F}_y$, and $\mathbb{F}_z$ in the x, y, and z directions, respectively. A flux in the positive x-direction (eastward) is written with a positive value of $\mathbb{F}_x$, while a flux towards the opposite direction (westward) is negative.

The total amount of heat or mass flowing through a plane of area A during time interval Δt is given by:

$$Amount = \mathbb{F}\cdot A\cdot\Delta t \tag{2.9}$$

For heat, $Amount \equiv \Delta Q_H$ by definition.

Fluxes are sometimes written in **kinematic form**, F, by dividing by the air density, ρ_{air} :

$$F = \frac{\mathbb{F}}{\rho_{air}} \qquad (2.10)$$

Kinematic mass flux equals the wind speed, M. Kinematic fluxes can also be in the 3 Cartesian directions: F_x, F_y, and F_z.

[CAUTION: Do not confuse the usage of the symbol M. Here it means "wind speed". Earlier it meant "mean anomaly". Throughout this book, many symbols will be re-used to represent different things, due to the limited number of symbols. Even if a symbol is not defined, you can usually determine its meaning from context.]

Heat fluxes $\mathbb{F}_H$ can be put into kinematic form by dividing by both air density ρ_{air} and the specific heat for air C_p, which yields a quantity having the same units as temperature times wind speed (K·m·s^{-1}).

$$F_H = \frac{\mathbb{F}_H}{\rho_{air} \cdot C_p} \qquad \text{for heat only} \qquad (2.11)$$

For dry air (subscript "$_d$") at sea level:

$$\rho_{air} \cdot C_{pd} = 1231\ (\text{W·m}^{-2}) \ / \ (\text{K·m·s}^{-1})$$

$$= 12.31\ \text{mb·K}^{-1}$$

$$= 1.231\ \text{kPa·K}^{-1} .$$

The reason for sometimes expressing fluxes in kinematic form is that the result is given in terms of easily measured quantities. For example, while most people do not have "Watt" meters to measure the normal "dynamic" heat flux, they do have thermometers and anemometers. The resulting temperature times wind speed has units of a kinematic heat flux (K·m·s^{-1}). Similarly, for mass flux it is easier to measure wind speed than kilograms of air per area per time.

Heat fluxes can be caused by a variety of processes. **Radiative fluxes** are radiant energy (electromagnetic waves or photons) per unit area per unit time. This flux can travel through a vacuum. **Advective flux** is caused by wind blowing through an area, and carrying with it warmer or colder temperatures. For example a warm wind blowing toward the east causes a positive heat-flux component F_{Hx} in the x-direction. A cold wind blowing toward the west also gives positive F_{Hx}. **Turbulent fluxes** are caused by eddy motions in the air, while **conductive fluxes** are caused by molecules bouncing into each other.

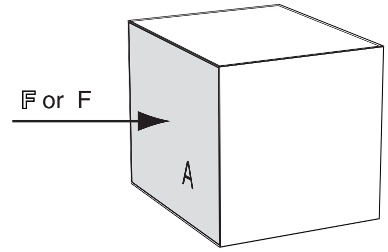


Figure 2.7
Flux F through an area A into one side of a volume.

Sample Application

The mass flux of air is 1 kg·m^{-2}·s^{-1} through a door opening that is 1 m wide by 2.5 m tall. What amount of mass of air passes through the door each minute, and what is the value of kinematic mass flux?

Find the Answer

Given: area A = (1 m) · (2.5 m) = 2.5 m^2
$\mathbb{F}$ = 1 kg·m^{-2}·s^{-1}
Find: (a) Amount = ? kg, and (b) F = ? m·s^{-1}
Sketch: (see Fig. 2.7)

(a) Use eq. (2.9):
Amount = (1 kg·m^{-2}·s^{-1})·(2.5 m^2)·(1 min)·(60 s min^{-1}) = **<u>150 kg</u>**.

(b) Assume: ρ = 1.225 kg·m^{-3} at sea-level
Use eq. (2.10):
F = (1 kg·m^{-2}·s^{-1}) / (1.225 kg·m^{-3})
= **<u>0.82 m·s</u>**$^{-1}$.

Check: Units OK. Sketch OK. Physics OK.
Exposition: The kinematic flux is equivalent to a very light wind speed of less than 1 m s^{-1} blowing through the doorway, yet it transports quite a large amount of mass each minute.

Sample Application

If the heat flux from the sun that reaches the Earth's surface is 600 W m^{-2}, find the flux in kinematic units.

Find the Answer:

Given: $\mathbb{F}_H$ = 600 W m^{-2}
Find: F_H = ? K·m s^{-1}
Assume: sea level.

Use eq. (2.11)
F_H = (600 W m^{-2}) / [1231 (W·m^{-2}) / (K·m·s^{-1})]
= 0.487 K·m s^{-1}

Check: Units OK. Physics OK.
Exposition: This amount of radiative heat flux is equivalent to an advective flux of a 1 m s^{-1} wind blowing air with temperature excess of about 0.5°C.

Sample Application

Red light has a wavelength of 0.7 µm. Find its frequency, circular frequency, and wavenumber in a vacuum.

Find the Answer

Given: c_o = 299,792,458 m s^{-1}, λ = 0.7 µm
Find: ν = ? Hz, ω = ? s^{-1}, σ = ? m^{-1} .

Use eq. (2.12), solving for ν:
$\nu = c_o/\lambda$ = (3x10^8 m s^{-1}) / (0.7x10^{-6} m cycle^{-1})
= **4.28x10^{14} Hz**.

ω = $2\pi\cdot\nu$ = 2·(3.14159)·(4.28x10^{14} Hz)
= **2.69x10^{15}** s^{-1}.

σ = $1/\lambda$ = 1 / (0.7x10^{-6} m cycle^{-1})
= **1.43x10^6** m^{-1}.

Check: Units OK. Physics reasonable.
Exposition: Wavelength, wavenumber, frequency, and circular frequency are all equivalent ways to express the "color" of radiation.

Sample Application

Find the blackbody monochromatic radiant exitance of green light of wavelength 0.53 µm from an object of temperature 3000 K.

Find the Answer

Given: λ = 0.53 µm, T = 3000 K
Find: $E_\lambda{}^*$ = ? W·m^{-2}· µm^{-1} .

Use eq. (2.13):

$$E_\lambda{}^* = \frac{c_1}{\lambda^5\cdot\left[\exp(c_2/(\lambda\cdot T))-1\right]}$$

$$E_\lambda{}^* = \frac{(3.74\text{x}10^8\,\text{Wm}^{-2}\mu\text{m}^4)/(0.53\mu\text{m})^5}{\exp\left[(1.44\text{x}10^4\,\text{K}\mu\text{m})/(0.53\mu\text{m}\cdot 3000\text{K})\right]-1}$$

= **1.04 x 10^6** W·m^{-2}· µm^{-1}

Check: Units OK. Physics reasonable.
Exposition: Because 3000 K is cooler than the sun, about 50 times less green light is emitted. The answer is the watts emitted from each square meter of surface per µm wavelength increment.

RADIATION PRINCIPLES

Propagation

Radiation can be modeled as electromagnetic waves or as photons. Radiation propagates through a vacuum at a constant speed: c_o = 299,792,458 m·s^{-1}. For practical purposes, you can approximate this **speed of light** as $c_o \approx$ 3x10^8 m·s^{-1}. Light travels slightly slower through air, at roughly c = 299,710,000 m·s^{-1} at standard sea-level pressure and temperature, but the speed varies slightly with thermodynamic state of the air (see the Optics chapter).

Using the wave model of radiation, the **wavelength** λ (m·cycle^{-1}) is related to the **frequency**, ν (Hz = cycles·s^{-1}) by:

$$\lambda \cdot \nu = c_o \tag{2.12}$$

Wavelength units are sometimes abbreviated as (m). Because the wavelengths of light are so short, they are often expressed in units of micrometers (µm).

Wavenumber is the number of waves per meter: σ (cycles·m^{-1}) = $1/\lambda$. Its units are sometimes abbreviated as (m^{-1}). **Circular frequency** or **angular frequency** is ω (radians s^{-1}) = $2\pi\cdot\nu$. Its units are sometimes abbreviated as (s^{-1}).

Emission

Objects warmer than absolute zero can emit radiation. An object that emits the maximum possible radiation for its temperature is called a **blackbody**. **Planck's law** gives the amount of blackbody monochromatic (single wavelength or color) radiant flux leaving an area, called **emittance** or **radiant exitance**, $E_\lambda{}^*$:

$$E_\lambda{}^* = \frac{c_1}{\lambda^5\cdot\left[\exp(c_2/(\lambda\cdot T))-1\right]} \tag{•2.13}$$

where T is absolute temperature, and the asterisk indicates blackbody. The two constants are:
c_1 = 3.74 x 10^8 W·m^{-2} · µm^4 , and
c_2 = 1.44 x 10^4 µm·K.

Eq. (2.13) and constant c_1 already include all directions of exiting radiation from the area. $E_\lambda{}^*$ has units of W·m^{-2} µm^{-1} ; namely, flux per unit wavelength. For radiation approaching an area rather than leaving it, the radiant flux is called **irradiance**.

Actual objects can emit less than the theoretical blackbody value: $E_\lambda = e_\lambda\cdot E_\lambda{}^*$, where $0 \le e_\lambda \le 1$ is **emissivity**, a measure of emission efficiency.

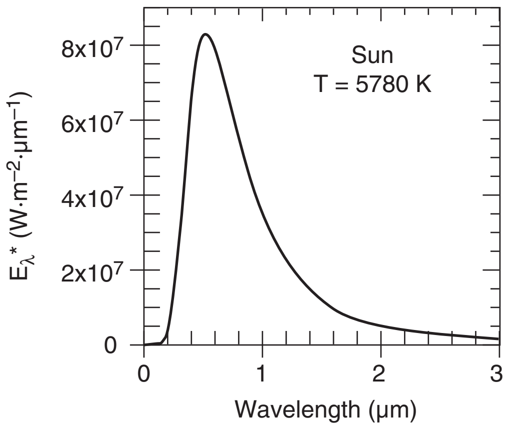


Figure 2.8
Planck radiant exitance, E_λ, from a blackbody approximately the same temperature as the sun.*

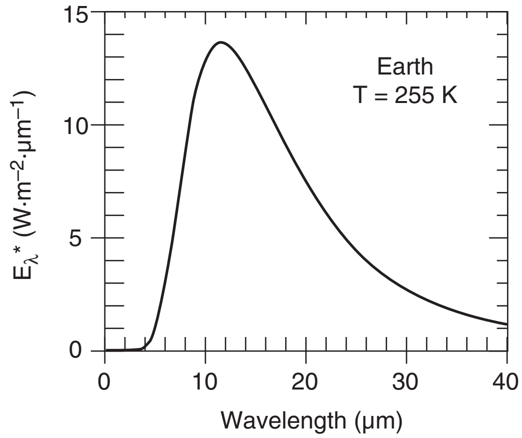


Figure 2.9
Planck radiant exitance, E_λ, from a blackbody approximately the same temperature as the Earth.*

The Planck curve (eq. 2.13) for emission from a blackbody the same temperature as the sun (T = 5780 K) is plotted in Fig. 2.8. Peak emissions from the sun are in the visible range of wavelengths (0.38 – 0.74 µm, see Table 2-3). Radiation from the sun is called **solar radiation** or **short-wave radiation**.

The Planck curve for emission from a blackbody that is approximately the same temperature as the whole Earth-atmosphere system ($T \approx 255$ K) is plotted in Fig. 2.9. Peak emissions from this idealized average Earth system are in the infrared range 8 to 18 µm. This radiation is called **terrestrial radiation**, **long-wave radiation**, or infrared (**IR**) radiation.

The wavelength of the peak emission is given by **Wien's law**:

$$\lambda_{max} = \frac{a}{T} \tag{2.14}$$

where a = 2897 µm·K.

The total amount of emission (= area under Planck curve = total emittance) is given by the **Stefan-Boltzmann law**:

$$E^* = \sigma_{SB} \cdot T^4 \qquad \bullet(2.15)$$

where σ_{SB} = 5.67x10^{-8} W·m^{-2}·K^{-4} is the **Stefan-Boltzmann constant**, and E^* has units of W·m^{-2}. More details about radiation emission are in the Satellites & Radar chapter in the sections on weather satellites.

Table 2-3. Ranges of wavelengths λ of visible colors. Approximate center wavelength is in **boldface**. For more info, see Chapter 22 on Atmospheric Optics.

Color	λ (µm)
Red	0.625 - **0.650** - 0.740
Orange	0.590 - **0.600** - 0.625
Yellow	0.565 - **0.577** - 0.590
Green	0.500 - **0.510** - 0.565
Cyan	0.485 - **0.490** - 0.500
Blue	0.460 - **0.475** - 0.485
Indigo	0.425 - **0.445** - 0.460
Violet	0.380 - **0.400** - 0.425

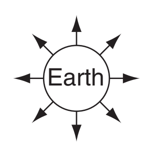
Sample Application

What is the total radiant exitance (radiative flux) emitted from a blackbody Earth at T = 255 K, and what is the wavelength of peak emission?

Find the Answer

Given: T = 255 K
Find: E^* = ? W·m^{-2} , λ_{max} = ? µm

Sketch:



Use eq. (2.15):
E^*= (5.67x10^{-8} W·m^{-2}·K^{-4})·(255 K)^4 = **240 W·m^{-2}**.
Use eq. (2.14):
λ_{max} = (2897 µm·K) / (255 K) = **11.36 µm**

Check: Units OK. λ agrees with peak in Fig. 2.9.
Exposition: You could create the same flux by placing a perfectly-efficient 240 W light bulb in front of a parabolic mirror that reflects the light and IR radiation into a beam that is 1.13 m in diameter.

For comparison, the surface area of the Earth is about 5.1x10^{14} m^2, which when multiplied by the flux gives the total emission of 1.22x10^{17} W. Thus, the Earth acts like a large-wattage IR light bulb.

HIGHER MATH • Incremental Changes

What happens to the total radiative exitance given by eq. (2.15) if temperature increases by 1°C? Such a question is important for climate change.

Find the Answer using calculus:

Calculus allows a simple, elegant way to find the solution. By taking the derivative of both the left and right sides of eq. (2.15), one finds that:

$$\mathrm{d}E^* = 4 \cdot \sigma_{SB} \cdot T^3 \mathrm{d}T$$

assuming that σ_{SB} is constant. It can be written as

$$\Delta E^* = 4 \cdot \sigma_{SB} \cdot T^3 \cdot \Delta T$$

for small ΔT.

Thus, a fixed increase in temperature of $\Delta T = 1°C$ causes a much larger radiative exitance increase at high temperatures than at cold, because of the T^3 factor on the right side of the equation.

Find the Answer using algebra:

This particular problem could also have been solved using algebra, but with a more tedious and less elegant solution. First let

$$E_2 = \sigma_{SB} \cdot T_2^4 \qquad \text{and} \qquad E_1 = \sigma_{SB} \cdot T_1^4$$

Next, take the difference between these two eqs:

$$\Delta E = E_2 - E_1 = \sigma \cdot \left[T_2^4 - T_1^4 \right]$$

Recall from algebra that $(a^2 - b^2) = (a - b)\cdot(a + b)$

$$\Delta E = \sigma_{SB} \cdot \left(T_2{}^2 - T_1{}^2 \right) \cdot \left(T_2{}^2 + T_1{}^2 \right)$$

$$\Delta E = \sigma_{SB} \cdot (T_2 - T_1) \cdot (T_2 + T_1) \cdot \left(T_2{}^2 + T_1{}^2 \right)$$

Since $(T_2 - T_1) / T_1 << 1$, then $(T_2 - T_1) = \Delta T$, but $T_2 + T_1 \approx 2T$, and $T_2{}^2 + T_1{}^2 \approx 2 \cdot T^2$. This gives:

$$\Delta E = \sigma_{SB} \cdot \Delta T \cdot 2T \cdot 2T^2$$

or

$$\Delta E \approx \sigma_{SB} \cdot \left[4T^3 \Delta T \right]$$

which is identical to the answer from calculus.

We were lucky this time, but it is not always possible to use algebra where calculus is needed.

A SCIENTIFIC PERSPECTIVE • Scientific Laws – The Myth

There are no scientific laws. Some theories or models have succeeded for every case tested so far, yet they may fail for other situations. Newton's "Laws of Motion" were accepted as laws for centuries, until they were found to fail in quantum mechanical, relativistic, and galaxy-size situations. It is better to use the word "relationship" instead of "law". In this textbook, the word "law" is used for sake of tradition.

Einstein said "No amount of experimentation can ever prove me right; a single experiment can prove me wrong."

Because a single experiment can prove a relationship wrong, it behooves us as scientists to test theories and equations not only for reasonable values of variables, but also in the limit of extreme values, such as when the variables approach zero or infinity. These are often the most stringent tests of a relationship.

Example

Query: The following is an approximation to Planck's law.

$$E_\lambda{}^* = c_1 \cdot \lambda^{-5} \cdot \exp[\, -c_2 / (\lambda \cdot T) \,] \qquad \text{(a)}$$

Why is it not a perfect substitute?

Find the Answer: If you compare the numerical answers from eqs. (2.13) and (a), you find that they agree very closely over the range of temperatures of the sun and the Earth, and over a wide range of wavelengths. But...

a) What happens as temperature approaches absolute zero? For eq. (2.13), T is in the denominator of the argument of an exponential, which itself is in the denominator of eq. (2.13). If $T = 0$, then $1/T = \infty$. $\text{Exp}(\infty) = \infty$, and $1/\infty = 0$. Thus, $E_\lambda{}^* = 0$, as it should. Namely, no radiation is emitted at absolute zero (according to classical theory).

For eq. (a), if $T = 0$, then $1/T = \infty$, and $\exp(-\infty) = 0$. So it also agrees in the limit of absolute zero. Thus, both equations give the expected answer.

b) **What happens as temperature approaches infinity?** For eq. (2.13), if $T = \infty$, then $1/T = 0$, and $\exp(0) = 1$. Then $1 - 1 = 0$ in the denominator, and $1/0 = \infty$. Thus, $E_\lambda{}^* = \infty$, as it should. Namely, infinite radiation is emitted at infinite temperature.

For eq. (a), if $T = \infty$, then $1/T = 0$, and $\exp(-0) = 1$. Thus, $E_\lambda{}^* = c_1 \cdot \lambda^{-5}$, which is not infinity. Hence, this approaches the wrong answer in the $\infty\, T$ limit.

Conclusion: Eq. (a) is not a perfect relationship, because it fails for this one case. But otherwise it is a good approximation over a wide range of normal temperatures.

Distribution

Radiation emitted from a spherical source decreases with the square of the distance from the center of the sphere:

$$E_2^* = E_1^* \cdot \left(\frac{R_1}{R_2}\right)^2 \qquad \bullet(2.16)$$

where R is the radius from the center of the sphere, and the subscripts denote two different distances from the center. This is called the **inverse square law**.

The reasoning behind eq. (2.16) is that as radiation from a small sphere spreads out radially, it passes through ever-larger conceptual spheres. If no energy is lost during propagation, then the total energy passing across the surface of each sphere must be conserved. Because the surface areas of the spheres increase with the square of the radius, this implies that the energy flux density must decrease at the same rate; i.e., inversely to the square of the radius.

From eq. (2.16) we expect that the radiative flux reaching the Earth's orbit is greatly reduced from that at the surface of the sun. The solar emissions of Fig. 2.8 must be reduced by a factor of 2.167×10^{-5}, based on the square of the ratio of solar radius to Earth-orbital radius from eq. (2.16). This result is compared to the emission from Earth in Fig. 2.10.

The area under the solar-radiation curve in Fig. 2.10 is the **total** (all wavelengths) **solar irradiance (TSI)**, S_o, reaching the Earth's orbit. We call it an irradiance here, instead of an emittance, because relative to the Earth it is an incoming radiant flux. This quantity was formerly called the **solar constant** but we now know that it varies slightly. The average value of solar irradiance measured at the Earth's orbit by satellites is about

$$S_o = 1366 \ (\pm 7) \ \mathrm{W \cdot m^{-2}} \ . \qquad (2.17)$$

In kinematic units (based on sea-level density), the solar irradiance is roughly $S_o = 1.11$ K·m s^{-1}.

About ±4 W·m^{-2} of the ±7 W·m^{-2} error bars are due to radiometer calibration errors between the various satellites. Also, during the 11-year sunspot cycle the solar irradiance normally varies by about 1.4 W·m^{-2}, with peak values during sunspot maxima. One example of a longer term variation in solar activity is the **Maunder Minimum** in the late 1600s, during which the solar irradiance was perhaps 2.7 to 3.7 W·m^{-2} less than values during modern solar minima. Such irradiance changes could cause subtle changes (0.3 to 0.4°C) in global climate. See the Climate chapter for more info on solar variability.

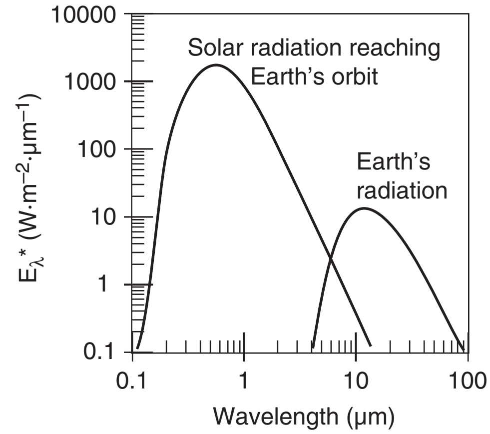


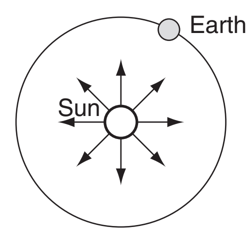
Figure 2.10
Blackbody radiance E reaching top of Earth's atmosphere from the sun and radiance of terrestrial radiation leaving the top of the atmosphere, plotted on a log-log graph.*

Sample Application

Estimate the value of the solar irradiance reaching the orbit of the Earth, given a sun surface temperature (5780 K), sun radius (6.96×10^5 km), and orbital radius (1.495×10^8 km) of the Earth from the sun.

Find the Answer

Given: T_{sun} = 5780 K
$R_{sun} = 6.96 \times 10^5$ km = solar radius
$R_{Earth} = 1.495 \times 10^8$ km = Earth orbit radius
Find: S_o = ? W·m^{-2}
Sketch:



First, use eq. (2.15):
$E_1^* = (5.67 \times 10^{-8} \ \mathrm{W \cdot m^{-2} \cdot K^{-4}}) \cdot (5780 \ \mathrm{K})^4$
$= 6.328 \times 10^7 \ \mathrm{W \cdot m^{-2}}$.
Next, use eq. (2.16), with $R_1 = R_{sun}$ & $R_2 = R_{Earth}$
$S_o = E_2^* = (6.328 \times 10^7 \ \mathrm{W \cdot m^{-2}}) \cdot$
$(6.96 \times 10^5 \ \mathrm{km} / 1.495 \times 10^8 \ \mathrm{km})^2 = \underline{\mathbf{1372 \ W \cdot m^{-2}}}$.

Check: Units OK, Sketch OK. Physics OK.
Exposition: Answer is nearly equal to that measured by satellites, as given in eq. (2.17). The error is due to a poor estimate of effective sun-surface temperature.

Sample Application(§)

Using the results from an earlier Sample Application that calculated the true anomaly and sun-Earth distance for several days during the year, find the solar radiative forcing for those days.

Find the Answer

Given: R values from previously Sample Application
Find: S = ? W·m^{-2}

Sketch: (same as Fig 2.2)
Use eq. (2.18).

Date	d	R(ν) (Gm)	S (W/m^2)
4 Jan	4	146.96	1418
18 Jan	18	147.04	1416
1 Feb	32	147.25	1412
15 Feb	46	147.60	1405
1 Mar	60	148.06	1397
15 Mar	74	148.60	1386
29 Mar	88	149.18	1376
12 Apr	102	149.78	1365
26 Apr	116	150.36	1354
21 Jun	172	151.88	1327
23 Sep	266	150.01	1361
22 Dec	356	147.03	1416

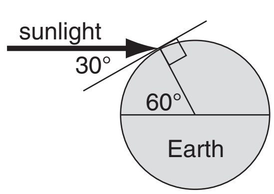
Check: Units OK. Physics OK.
Exposition: During N. Hemisphere <u>winter</u>, solar radiative forcing is up to 50 W·m^{-2} <u>larger</u> than average.

Sample Application

During the equinox at noon at latitude ϕ =60°, the solar elevation angle is Ψ = 90° – 60° = 30°. If the atmosphere is perfectly transparent, then how much radiative flux is absorbed into a perfectly black asphalt parking lot?

Find the Answer

Given: Ψ = 30° = elevation angle
$E = S_o$ = 1366 W·m^{-2}. solar irradiance
Find: $\mathbb{F}_{rad}$ = ? W·m^{-2}
Sketch:



Use eq. (2.19):
$\mathbb{F}_{rad}$ = (1366 W·m^{-2})·sin(30°) = **<u>683 W·m</u>**$^{-2}$.

Check: Units OK. Sketch OK. Physics OK.
Exposition: Because the solar radiation is striking the parking lot at an angle, the radiative flux into the parking lot is half of the solar irradiance.

According to the inverse-square law, variations of distance between Earth and sun cause changes of the **solar radiative forcing**, S, that reaches the top of the atmosphere:

$$S = S_o \cdot \left(\frac{\overline{R}}{R}\right)^2 \tag{2.18}$$

where S_o = 1366 W·m^{-2} is the average total solar irradiance measured at an average distance $\overline{R}$ = 149.6 Gm between the sun and Earth, and R is the actual distance between Earth and the sun as given by eq. (2.4). Remember that the solar irradiance and the solar radiative forcing are the fluxes across a surface that is <u>perpendicular</u> to the solar beam, measured above the Earth's atmosphere.

Let irradiance E be any radiative flux crossing a unit area that is <u>perpendicular</u> to the path of the radiation. If this radiation strikes a surface that is not perpendicular to the radiation, then the radiation per unit surface area is reduced according to the **sine law**. The resulting flux, $\mathbb{F}_{rad}$, at this surface is:

$$\mathbb{F}_{rad} = E \cdot \sin(\Psi) \tag{2.19}$$

where Ψ is the **elevation angle** (the angle of the sun above the surface). In kinematic form, this is

$$F_{rad} = \frac{E}{\rho \cdot C_p} \cdot \sin(\Psi) \qquad \bullet(2.20)$$

where $\rho \cdot C_p$ is given under eq. (2.11).

Average Daily Insolation

The acronym "**insolation**" means "incoming solar radiation" at the top of the atmosphere. The average daily **insolation** $\overline{E}$ takes into account both the solar elevation angle (which varies with season and time of day) and the duration of daylight. For example, there is more total insolation at the poles in summer than at the equator, because the low sun angle near the poles is more than compensated by the long periods of daylight.

$$\overline{E} = \frac{S_o}{\pi} \cdot \left(\frac{a}{R}\right)^2 \cdot \left[h_o' \cdot \sin(\phi) \cdot \sin(\delta_S) + \cos(\phi) \cdot \cos(\delta_S) \cdot \sin(h_o) \right] \tag{2.21}$$

where S_o =1366 W m^{-2} is the solar irradiance, a = 149.457 Gm is Earth's semi-major axis length, R is the actual distance for any day of the year, from eq.

(2.4). In eq. (2.21), h_o' is the sunset and sunrise hour angle in radians.

The hour angle h_o at sunrise and sunset can be found using the following steps:

$$\alpha = -\tan(\phi)\cdot\tan(\delta_s) \qquad (2.22a)$$

$$\beta = \min[1, (\max(-1, \alpha)] \qquad (2.22b)$$

$$h_o = \arccos(\beta) \qquad (2.22c)$$

Eq. (2.22b) truncates the argument of the arccos to be between –1 and 1, in order to account for high latitudes where there are certain days when the sun never sets, and other days when it never rises. *[CAUTION. When finding the arccos, your answer might be in degrees or radians, depending on your calculator, spreadsheet, or computer program. Determine the units by experimenting first with the arccos(0), which will either give 90° or π/2 radians. If necessary, convert the hour angle to units of radians, the result of which is h_o'.]*

Fig. 2.11 shows the average incoming solar radiation vs. latitude and day of the year, found using eq. (2.21). For any one hemisphere, $\overline{E}$ has greater difference between equator and pole during winter than during summer. This causes stronger winds and more active extratropical cyclones in the winter hemisphere than in the summer hemisphere.

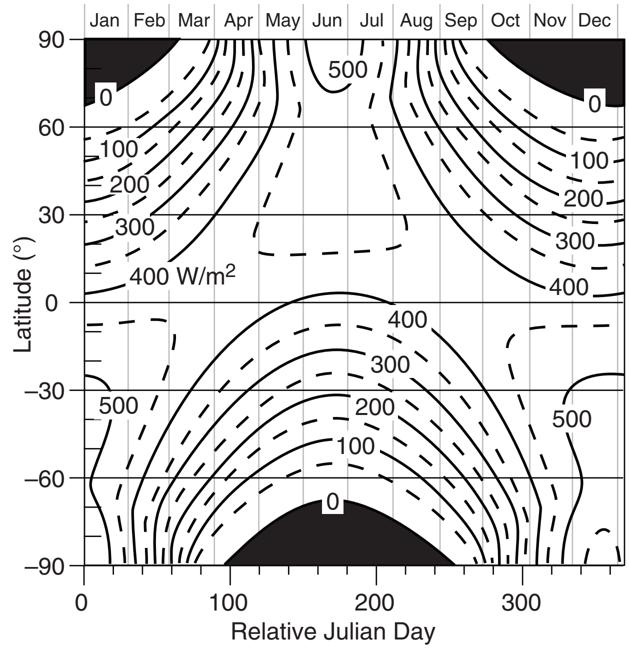


Figure 2.11
Average daily insolation $\overline{E}$ (W m^{-2}) over the globe.

Absorption, Reflection & Transmission

The emissivity, e_λ, is the fraction of blackbody radiation that is actually emitted (see Table 2-4) at any wavelength λ. The absorptivity, a_λ, is the fraction of radiation striking a surface of an object that is **absorbed** (i.e., stays in the object as a different form of energy). **Kirchhoff's law** states that the **absorptivity** equals the **emissivity** of a substance at each wavelength, λ. Thus,

$$a_\lambda = e_\lambda \qquad (2.23)$$

Some substances such as dark glass are semi-**transparent** (i.e., some radiation passes through). A fraction of the incoming (incident) radiation might also be **reflected** (bounced back), and another portion might be **absorbed**. Thus, you can define the efficiencies of reflection, absorption, and transmission as:

$$r_\lambda = \frac{E_{\lambda\ reflected}}{E_{\lambda\ incident}} = \textbf{reflectivity} \qquad (2.24)$$

$$a_\lambda = \frac{E_{\lambda\ absorbed}}{E_{\lambda\ incident}} = \textbf{absorptivity} \qquad (2.25)$$

Sample Application
Find the average daily insolation over Vancouver during the summer solstice.

Find the Answer:
Given: $d = d_r = 173$ at the solstice,
$\phi = 49.25°N$, $\lambda_e = 123.1°W$ for Vancouver.
Find: $\overline{E}$ = ? W m^{-2}

Use eq. (2.5): $\delta_s = \Phi_r = 23.45°$
Use eq. (2.2): $M = 167.55°$, and assume ν ≈ M.
Use eq. (2.4): $R = 151.892$ Gm.
Use eq. (2.22):
$h_o = \arccos[-\tan(49.25°)\cdot\tan(23.45°)] = 120.23°$
$h_o' = h_o \cdot 2\pi/360° = 2.098$ radians
Use eq. (2.21):

$$\overline{E} = \frac{(1368\text{W}\cdot\text{m}^{-2})}{\pi}\cdot\left(\frac{149\text{Gm}}{151.892\text{Gm}}\right)^2 \cdot [2.098\cdot\sin(49.25°)\cdot\sin(23.45°) + \cos(49.25°)\cdot\cos(23.45°)\cdot\sin(120.23°)]$$

$$\overline{E} = (1327 \text{ W m}^{-2})\cdot[2.098(0.3016)+0.5174]$$

$$= \underline{\textbf{486 W m}^{-2}}$$

Check: Units OK. Physics OK. Agrees with Fig. 2.11.
Exposition: At the equator on this same day, the average daily insolation is less than 400 W m^{-2}.

Table 2-4. Typical infrared emissivities.

Surface	e	Surface	e
alfalfa	0.95	iron, galvan.	0.13-0.28
aluminum	0.01-0.05	leaf 0.8 µm	0.05-0.53
asphalt	0.95	leaf 1 µm	0.05-0.6
bricks, red	0.92	leaf 2.4 µm	0.7-0.97
cloud, cirrus	0.3	leaf 10 µm	0.97-0.98
cloud, alto	0.9	lumber, oak	0.9
cloud, low	1.0	paper	0.89-0.95
concrete	0.71-0.9	plaster, white	0.91
desert	0.84-0.91	sand, wet	0.98
forest, conif.	0.97	sandstone	0.98
forest, decid.	0.95	shrubs	0.9
glass	0.87-0.94	silver	0.02
grass	0.9-0.95	snow, fresh	0.99
grass lawn	0.97	snow, old	0.82
gravel	0.92	soils	0.9-0.98
human skin	0.95	soil, peat	0.97-0.98
ice	0.96	urban	0.85-0.95

Sample Application

If 500 W m^{-2} of visible light strikes a translucent object that allows 100 W m^{-2} to shine through and 150 W m^{-2} to bounce off, find the transmissivity, reflectivity, absorptivity, and emissivity.

Find the Answer

Given: $E_{\lambda\ incoming}$ = 500 W m^{-2} ,
$E_{\lambda\ transmitted}$ = 100 W m^{-2} , $E_{\lambda\ reflected}$ = 150 W m^{-2}
Find: a_λ = ? , e_λ = ? , r_λ = ? , and t_λ = ?

Use eq. (2.26): t_λ = (100 W m^{-2}) / (500 W m^{-2}) = **0.2**
Use eq. (2.24): r_λ = (150 W m^{-2}) / (500 W m^{-2}) = **0.3**
Use eq. (2.27): a_λ = 1 – 0.2 – 0.3 = **0.5**
Use eq. (2.23): $e_\lambda = a_\lambda$ = **0.5**

Check: Units dimensionless. Physics reasonable.
Exposition: By definition, **translucent** means partly transparent, and partly absorbing.

$$t_\lambda = \frac{E_{\lambda\ transmitted}}{E_{\lambda\ incident}} = \textbf{transmissivity} \qquad (2.26)$$

Values of e_λ, a_λ, r_λ, and t_λ are between 0 and 1.

The sum of the last three fractions must total 1, as 100% of the radiation at any wavelength must be accounted for:

$$1 = a_\lambda + r_\lambda + t_\lambda \qquad (2.27)$$

or

$$E_{\lambda\ incoming} = E_{\lambda\ absorbed} + E_{\lambda\ reflected} + E_{\lambda\ transmitted} \qquad (2.28)$$

For opaque (t_λ = 0) substances such as the Earth's surface, you find: $a_\lambda = 1 - r_\lambda$.

The reflectivity, absorptivity, and transmissivity usually vary with wavelength. For example, clean snow reflects about 90% of incoming solar radiation, but reflects almost 0% of IR radiation. Thus, snow is "white" in visible light, and "black" in IR. Such behavior is crucial to surface temperature forecasts.

Instead of considering a single wavelength, it is also possible to examine the net effect over a range of wavelengths. The ratio of total reflected to total incoming solar radiation (i.e., averaged over all solar wavelengths) is called the **albedo**, A :

$$A = \frac{E_{reflected}}{E_{incoming}} \qquad \bullet(2.29)$$

The average global albedo for solar radiation reflected from Earth is A = 30% (see the Climate chapter). The actual global albedo at any instant varies with ice cover, snow cover, cloud cover, soil moisture, topography, and vegetation (Table 2-5). The Moon's albedo is only 7%.

The surface of the Earth (land and sea) is a very strong absorber and emitter of radiation.

Table 2-5. Typical albedos (%) for sunlight.

Surface	A (%)	Surface	A (%)
alfalfa	23-32	forest, decid.	10-25
buildings	9	granite	12-18
clay, wet	16	grass, green	26
clay, dry	23	gypsum	55
cloud, thick	70-95	ice, gray	60
cloud, thin	20-65	lava	10
concrete	15-37	lime	45
corn	18	loam, wet	16
cotton	20-22	loam, dry	23
field, fallow	5-12	meadow, green	10-20
forest, conif.	5-15	potatoes	19

Table 2-5 *(continuation)*. Typical albedos (%).

Surface	A (%)	Surface	A (%)
rice paddy	12	soil, red	17
road, asphalt	5-15	soil, sandy	20-25
road, dirt	18-35	sorghum	20
rye winter	18-23	steppe	20
sand dune	20-45	stones	20-30
savanna	15	sugar cane	15
snow, fresh	75-95	tobacco	19
snow, old	35-70	tundra	15-20
soil, dark wet	6-8	urban, mean	15
soil, light dry	16-18	water, deep	5-20
soil, peat	5-15	wheat	10-23

Within the air, however, the process is a bit more complicated. One approach is to treat the whole atmospheric thickness as a single object. Namely, you can compare the radiation at the top versus bottom of the atmosphere to examine the total emissivity, absorptivity, and reflectivity of the whole atmosphere. Over some wavelengths called **windows** there is little absorption, allowing the radiation to "shine" through. In other wavelength ranges there is partial or total absorption. Thus, the atmosphere acts as a filter. Atmospheric windows and transmissivity are discussed in detail in the Satellites & Radar and Climate chapters.

Beer's Law

Sometimes you must examine radiative **extinction** (reduction of radiative flux) across a short path length Δs within the atmosphere (Fig. 2.12). Let n be the **number density** of radiatively important particles in the air (particles m^{-3}), and b be the **extinction cross section** of each particle (m^2 particle^{-1}), where this latter quantity gives the area of the shadow cast by each particle.

Extinction can be caused by absorption and scattering of radiation. If the change in radiation is due only to absorption, then the absorptivity across this layer is

$$a = \frac{E_{incident} - E_{transmitted}}{E_{incident}} \tag{2.30}$$

Beer's law gives the relationship between incident radiative flux, $E_{incident}$, and transmitted radiative flux, $E_{transmitted}$, as

$$E_{transmitted} = E_{incident} \cdot e^{-n \cdot b \cdot \Delta s} \tag{2.31a}$$

Beer's law can be written using an **extinction coefficient**, k:

$$E_{transmitted} = E_{incident} \cdot e^{-k \cdot \rho \cdot \Delta s} \tag{2.31b}$$

where ρ is the density of air, and k has units of $m^2 \, g_{air}^{-1}$. The total extinction across the whole path can be quantified by a dimensionless **optical thickness** (or **optical depth** in the vertical), τ, allowing Beer's law to be rewritten as:

$$E_{transmitted} = E_{incident} \cdot e^{-\tau} \tag{2.31c}$$

To simplify these equations, sometimes a **volume extinction coefficient** γ is defined by

$$\gamma = n \cdot b = k \cdot \rho \tag{2.32}$$

Visual range (V, one definition of **visibility**) is the distance where the intensity of transmitted light has decreased to 2% of the incident light. It estimates the max distance Δs (km) you can see through air.

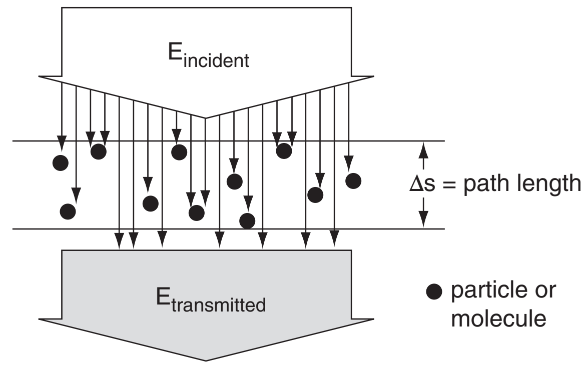


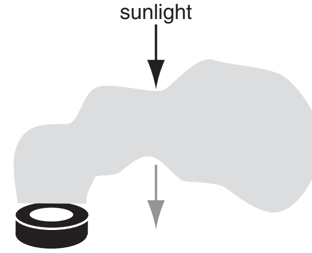
Figure 2.12
Reduction of radiation across an air path due to absorption and scattering by particles such as air-pollution aerosols or cloud droplets, illustrating Beer's law.

Sample Application

Suppose that soot from a burning automobile tire has number density $n = 10^7 \, m^{-3}$, and an extinction cross section of $b = 10^{-9} \, m^2$. (a) Find the radiative flux that was not attenuated (i.e., reduction from the solar constant) across the 20 m diameter smoke plume. Assume the incident flux equals the solar constant. (b) Find the optical depth. (c) If this soot fills the air, what is the visual range?

Find the Answer

Given: $n = 10^7 \, m^{-3}$, $b = 10^{-9} \, m^2$, $\Delta s = 20$ m
$E_{incident} = S = 1366 \, W \cdot m^{-2}$ solar constant
Find: E_{tran} =? $W \cdot m^{-2}$, τ =? (dimensionless), V =? km
Sketch:



Use eq. (2.32): $\gamma = (10^7 \, m^{-3}) \cdot (10^{-9} \, m^2) = 0.01 \, m^{-1}$

(a) Use eq. (2.31a)
$E_{tran} = 1366 \, W \cdot m^{-2} \cdot \exp[-(0.01 \, m^{-1}) \cdot (20 \, m)]$
$= 1366 \, W \cdot m^{-2} \cdot \exp[-0.2] = \underline{\mathbf{1118 \, W \cdot m^{-2}}}$

(b) Rearrange eq. (2.31c):
$\tau = \ln(E_{in}/E_{transmitted}) = \ln(1366/1118) = \underline{\mathbf{0.20}}$

(c) Rearrange eq. (2.31a): $\Delta s = [\ln(E_{incident}/E_{trans})]/\gamma$.
$V = \Delta s = [\ln(1/0.02)]/0.01 \, m^{-1} = \underline{\mathbf{0.391 \, km}}$

Check: Units OK. Sketch OK. Physics OK.
Exposition: Not much attenuation through this small smoke plume; namely, 1366 – 1118 = 248 $W \cdot m^{-2}$ was absorbed by the smoke. However, if this smoke fills the air, then visibility is very poor.

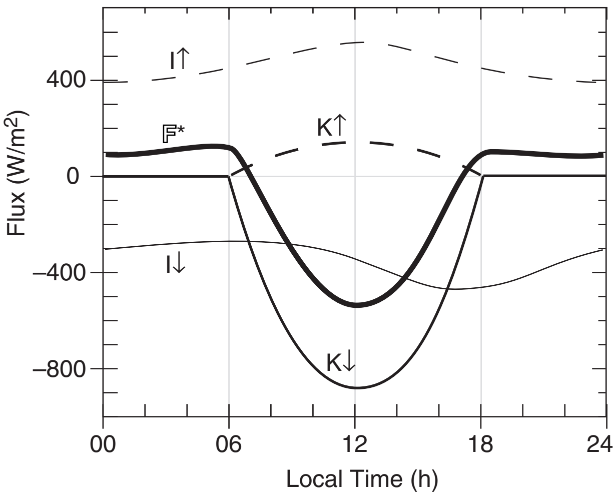


Figure 2.13
Typical diurnal variation of radiative fluxes at the surface. Fluxes are positive upward.

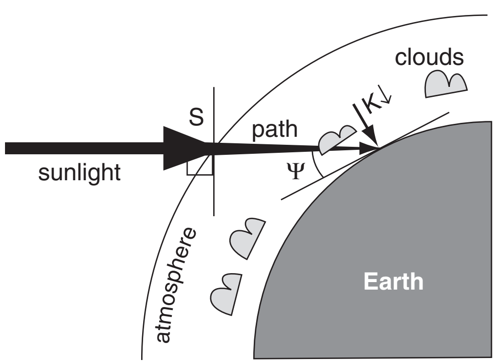


Figure 2.14
Fate of sunlight en route to the Earth's surface.

Sample Application

Downwelling sunlight shines through an atmosphere with 0.8 net sky transmissivity, and hits a ground surface of albedo 0.5, at a time when sin(Ψ) = 0.3. The surface emits 400 W m^{-2} IR upward into the atmosphere, and absorbs 350 W m^{-2} IR coming down from the atmosphere. Find the net radiative flux.

Find the Answer

Given: $Tr = 0.8$, $S_o = 1366$ W m^{-2}, $A = 0.5$.
$I\uparrow = 400$ W m^{-2}, $I\downarrow = 350$ W m^{-2}

Find: $\mathbb{F}^* = ?$ W m^{-2}

Use eq. (2.34):
$K\downarrow = -(1366 \text{ W m}^{-2})\cdot(0.8)\cdot(0.3) = -381 \text{ W m}^{-2}$

Use eq. (2.36): $K\uparrow = -(0.5)\cdot(-381 \text{ W m}^{-2}) = 164 \text{ W m}^{-2}$

Use eq. (2.33): $\mathbb{F}^* = (-381)+(164)+(-350)+(400) \text{ W m}^{-2}$
$\mathbb{F}^* = \underline{-167} \text{ W m}^{-2}$

Check: Units OK. Magnitude and sign OK.
Exposition: Negative sign means net inflow to the surface, such as would cause daytime warming.

SURFACE RADIATION BUDGET

Define $\mathbb{F}^*$ as the **net radiative flux** (positive upward) perpendicular to the Earth's surface. This net flux has contributions (Fig. 2.13) from **downwelling solar** radiation $K\downarrow$, reflected **upwelling solar** $K\uparrow$, **downwelling longwave** (**IR**) radiation emitted from the atmosphere $I\downarrow$, and **upwelling longwave** emitted from the Earth $I\uparrow$:

$$\mathbb{F}^* = K\downarrow + K\uparrow + I\downarrow + I\uparrow \qquad \bullet(2.33)$$

where $K\downarrow$ and $I\downarrow$ are negative because they are downward.

Solar

Recall that the solar irradiance (i.e., the solar constant) is $S_o \approx 1366 \pm 7$ W·m^{-2} (equivalent to 1.11 K·m s^{-1} in kinematic form after dividing by $\rho\cdot C_p$) at the top of the atmosphere. Some of this radiation is attenuated between the top of the atmosphere and the surface (Fig. 2.14). Also, the sine law (eq. 2.19) must be used to find the component of downwelling solar flux $K\downarrow$ that is perpendicular to the surface. The result for daytime is

$$K\downarrow = -S_o \cdot T_r \cdot \sin(\Psi) \qquad (2.34)$$

where T_r is a **net sky transmissivity**. A negative sign is incorporated into eq. (2.34) because $K\downarrow$ is a downward flux. Eq. (2.6) can be used to find sin(Ψ). At night, the downwelling solar flux is zero.

Net transmissivity depends on path length through the atmosphere, atmospheric absorption characteristics, and cloudiness. One approximation for the net transmissivity of solar radiation is

$$T_r = (0.6 + 0.2\sin\Psi)(1-0.4\sigma_H)(1-0.7\sigma_M)(1-0.4\sigma_L) \qquad (2.35)$$

where cloud-cover fractions for high, middle, and low clouds are σ_H, σ_M, and σ_L, respectively. These cloud fractions vary between 0 and 1, and the transmissivity also varies between 0 and 1.

Of the sunlight reaching the surface, a portion might be reflected:

$$K\uparrow = -A \cdot K\downarrow \qquad (2.36)$$

where the surface albedo is A.

Longwave (IR)

Upward emission of IR radiation from the Earth's surface can be found from the Stefan-Boltzmann relationship:

$$I\uparrow = e_{IR} \cdot \sigma_{SB} \cdot T^4 \tag{2.37}$$

where e_{IR} is the surface emissivity in the IR portion of the spectrum (e_{IR} = 0.9 to 0.99 for most surfaces), and σ_{SB} is the Stefan-Boltzmann constant (= $5.67\text{x}10^{-8}$ W·m^{-2}·K^{-4}).

However, downward IR radiation from the atmosphere is much more difficult to calculate. As an alternative, sometimes a **net longwave flux** is defined by

$$I^* = I\downarrow + I\uparrow \tag{2.38}$$

One approximation for this flux is

$$I^* = b \cdot (1 - 0.1\sigma_H - 0.3\sigma_M - 0.6\sigma_L) \tag{2.39}$$

where parameter b = 98.5 W·m^{-2} , or b = 0.08 K·m·s^{-1} in kinematic units.

Net Radiation

Combining eqs. (2.33), (2.34), (2.35), (2.36) and (2.39) gives the **net radiation** ($\mathbb{F}^*$, defined positive upward):

$$\mathbb{F}^* = -(1-A) \cdot S \cdot T_r \cdot \sin(\Psi) + I^* \quad \text{daytime} \quad \bullet(2.40a)$$

$$= I^* \quad \text{nighttime} \quad \bullet(2.40b)$$

ACTINOMETERS

Sensors designed to measure electromagnetic radiative flux are generically called **actinometers** or **radiometers**. In meteorology, actinometers are usually oriented to measure downwelling or upwelling radiation. Sensors that measure the difference between down- and up-welling radiation are called **net actinometers**.

Special categories of actinometers are designed to measure different wavelength bands:

- **pyranometer** – broadband solar (short-wave) irradiance, viewing a hemisphere of solid angle, with the radiation striking a flat, horizontal plate (Fig. 4.15).
- **net pyranometer** – difference between top and bottom hemispheres for short-wave radiation.

Sample Application

Find the net radiation at the surface in Vancouver, Canada, at noon (standard time) on 22 Jun. Low clouds are present with 30% coverage.

Find the Answer

Assume: Grass lawns with albedo A = 0.2 .
No other clouds.
Given: σ_L = 0.3
Find: $\mathbb{F}^*$ = ? W·m^{-2}

Use ψ = 64.1° from an earlier Sample Application.
Use eq. (2.35) to find the transmissivity:
T_r = [0.6+0.2·sinψ]·(1–0.4·σ_L)
= [0.6 + 0.2·sin(64.1°)]·[1–(0.4·0.3)]
= [0.80]·(0.88) = 0.686

Use eq. (2.39) to find net IR contribution:
I^* = b·(1 – 0.6·σ_L)
= (98.5 W·m^{-2})·[1–(0.6·0.3)] = 80.77 W·m^{-2}

Use eq. (2.40a):
$\mathbb{F}^*$ = –(1–A) · S · T_r · sin(ψ) + I^*
= –(1–0.2) · (1366 W·m^{-2}) · 0.686 · sin(64.1°)
+ 80.77 W·m^{-2}
=(–674.36 + 80.77) W·m^{-2} = **–593.59 W·m^{-2}**

Check: Units OK. Physics OK.
Exposition: The surface flux is only about 43% of that at the top of the atmosphere, for this case. The negative sign indicates a net inflow of radiation to the surface, such as can cause warming during daytime.

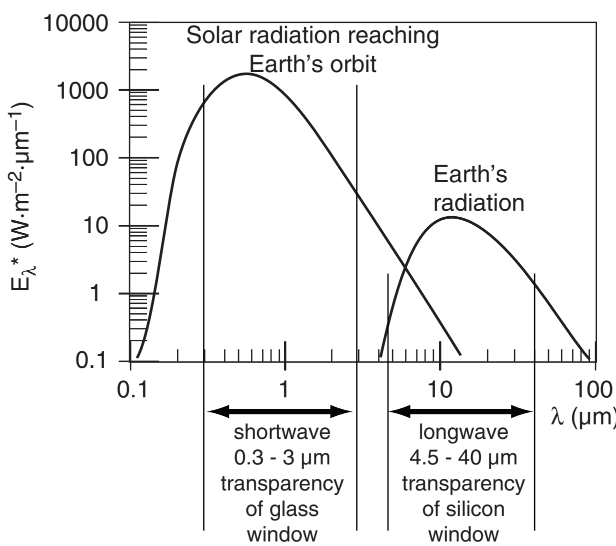


Figure 4.15
The wavelength bands observed by pyranometers (shortwave radiation) and pyrgeometers (longwave radiation) depends on the transparency of the windows used in those instruments.

A SCI. PERSPECTIVE • Seek Solutions

Most differential equations describing meteorological phenomena cannot be solved analytically. They cannot be integrated; they do not appear in a table of integrals; and they are not covered by the handful of mathematical tricks that you learned in math class.

But there is nothing magical about an analytical solution. **Any reasonable solution is better than no solution.** Be creative.

While thinking of creative solutions, also **think of ways to check your answer**. Is it the right order of magnitude, right sign, right units, does it approach a known answer in some limit, must it satisfy some other physical constraint or law or budget?

Example

Find the irradiance that can pass through an atmospheric "window" between wavelengths λ_1 and λ_2.

Find the Answer:

Approach: Integrate Planck's law between the specified wavelengths. This is the area under a portion of the Planck curve.

Check: The area under the whole spectral curve should yield the Stefan-Boltzmann (SB) law. Namely, the answer should be smaller than the SB answer, but should increase and converge to the SB answer as the lower and upper λ limits approach 0 and ∞, respectively.

Methods:

• Pay someone else to get the answer (Don't do this in school!), but be sure to check it yourself.
• Look up the answer in a Table of Integrals.
• Integrate it using the tricks you learned in math class.
• Integrate it using a symbolic equation solver on a computer, such as Mathematica or Maple.
• Find an approximate solution to the full equation. For example, integrate it numerically on a computer. (Trapezoid method, Gaussian integration, finite difference iteration, etc.)
• Find an exact solution for an approximation to the eq., such as a model or idealization of the physics. Most eqs. in this textbook have used this approach.
• Draw the Planck curve on graph paper. Count the squares under the curve between the wavelength bands, and compare to the value of each square, or to the area under the whole curve. (We will use this approach extensively in the Thunderstorm chapter.)
• Draw the curve, and measure area with a planimeter.
• Draw the Planck curve on cardboard or thick paper. Cut out the whole area under the curve. Weigh it. Then cut the portion between wavelengths, & weigh again.
• ...and there are probably many more methods.

• **pyrheliometer** – solar (short wave) direct-beam radiation normal to a flat surface (and shielded from diffuse radiation).
• **diffusometer** – a pyranometer that measures only diffuse solar radiation scattered from air, particles, and clouds in the sky, by using a device that shades the sensor from direct sunlight.
• **pyrgeometer** – infrared (long-wave) radiation from a hemisphere that strikes a flat, horizontal surface (Fig. 4.15).
• **net pyrgeometer** – difference between top and bottom hemispheres for infrared (long-wave) radiation.
• **radiometer** – measure all wavelengths of radiation (short, long, and other bands).
• **net radiometer** – difference between top and bottom hemispheres of radiation at all wavelengths.
• **spectrometers** – measures radiation as a function of wavelength, to determine the spectrum of radiation.

Inside many radiation sensors is a **bolometer**, which works as follows. Radiation strikes an object such as a metal plate, the surface of which has a coating that absorbs radiation mostly in the wavelength band to be measured. By measuring the temperature of the radiatively heated plate relative to a non-irradiated reference, the radiation intensity can be inferred for that wavelength band. The metal plate is usually enclosed in a glass or plastic hemispheric chamber to reduce error caused by heat conduction with the surrounding air.

Inside other radiation sensors are **photometers**. Some photometers use the **photoelectric effect**, where certain materials release electrons when struck by electromagnetic radiation. One type of photometer uses **photovoltaic cells** (also called **solar cells**), where the amount of electrical energy generated can be related to the incident radiation. Another photometric method uses **photoresistor,** which is a high-resistance semiconductor that becomes more conductive when irradiated by light.

Other photometers use **charge-coupled devices** (**CCD**s) similar to the image sensors in digital cameras. These are semiconductor integrated circuits with an array of tiny capacitors that can gain their initial charge by the photoelectric effect, and can then transfer their charge to neighboring capacitors to eventually be "read" by the surrounding circuits.

Simple spectrometers use different filters in front of bolometers or photometers to measure narrow wavelength bands. Higher spectral-resolution spectrometers use **interferometry** (similar to the Michelson inteferometer described in physics books), where the fringes of an interference pattern can be measured and related to the spectral intensities. These are also sometimes called **Fourier-transform spectrometers**, because of the mathematics used to extract the spectral information from the spacing of the fringes.

You can learn more about radiation, including the radiative transfer equation, in the weather-satellite section of the Satellites & Radar chapter. Satellites use radiometers and spectrometers to remotely observe the Earth-atmosphere system. Other satellite-borne radiometers are used to measure the global radiation budget (see the Climate chapter).

REVIEW

The variations of temperature and humidity that you feel near the ground are driven by the diurnal cycle of solar heating during the day and infrared cooling at night. Both diurnal and seasonal heating cycles can be determined from the geometry of the Earth's rotation and orbit around the sun. The same orbital mechanics describes weather-satellite orbits, as is discussed in the Satellites & Radar chapter.

Short-wave radiation is emitted from the sun and propagates through space. It illuminates a hemisphere of Earth. The portion of this radiation that is absorbed is the heat input to the Earth-atmosphere system that drives Earth's weather.

IR radiation from the atmosphere is absorbed at the ground, and IR radiation is also emitted from the ground. The IR and short-wave radiative fluxes do not balance, leaving a net radiation term acting on the surface at any one location. But when averaged over the whole globe, the earth-atmosphere system is approximately in radiative equilibrium.

Instruments to measure radiation are called actinometers or radiometers. Radiometers and spectrometers can be used in remote sensors such as weather satellites.

HOMEWORK EXERCISES

Broaden Knowledge & Comprehension

(Don't forget to cite each web address you use.)

B1. Access a full-disk visible satellite photo image of Earth from the web. What visible clues can you use to determine the current solar declination angle? How does your answer compare with that expected for your latitude and time of year.

B2. Access "web cam" camera images from a city, town, ski area, mountain pass, or highway near you. Use visible shadows on sunny days, along with your knowledge of solar azimuth angles, to determine the direction that the camera is looking.

B3. Access from the web the exact time from military (US Navy) or civilian (National Institute of Standards and Technology) atomic clocks. Synchronize your clocks at home or school, utilizing the proper time zone for your location. What is the time difference between local solar noon (the time when the sun is directly overhead) and the official noon according to your time zone. Use this time difference to determine the number of degrees of longitude that you are away from the center of your time zone.

B4. Access orbital information about one planet (other than Earth) that most interests you (or a planet assigned by the instructor). How elliptical is the orbit of the planet? Also, enjoy imagery of the planet if available.

B5. Access runway visual range reports from surface weather observations (METARs) from the web. Compare two different locations (or times) having different visibilities, and calculate the appropriate volume extinction coefficients and optical thickness. Also search the web to learn how runway visual range (RVR) is measured.

B6. Access both visible and infrared satellite photos from the web, and discuss why they look different. If you can access water-vapor satellite photos, include them in your comparison.

B7. Search the web for information about the sun. Examine satellite-based observations of the sun made at different wavelengths. Discuss the structure of the sun. Do any of the web pages give the current value of the solar irradiance (i.e., the solar constant)? If so, how has it varied recently?

B8. Access from the web daytime visible photos of the whole disk of the Earth, taken from geostationary weather satellites. Discuss how variations in the apparent brightness at different locations (different latitudes; land vs. ocean, etc.) might be related to reflectivity and other factors.

B9. Some weather stations and research stations report hourly observations on the web. Some of these stations include radiative fluxes near the surface. Use this information to create surface net radiation graphs.

B10. Access information from the web about how color relates to wavelength. Also, how does the range of colors that can be perceived by eye compare to the range of colors that can be created on a computer screen?

B11. Search the web for information about albedos and IR emissivities for substances or surfaces that are not already listed in the tables in this chapter.

B12. Find on the web satellite images of either forest-fire smoke plumes or volcanic ash plumes. Compare the intensity of reflected radiation from the Earth's surface as it shines through these plumes with earlier satellite photos when the plumes were not there. Use these data to estimate the extinction coefficient.

B13. Access photos and diagrams from the web that describe how different actinometers are constructed and how they work. Also, list any limitations of these instruments that are described in the web.

Apply

(Students, don't forget to put a box around each answer.)

A1. Given distances R between the sun and planets compute the orbital periods (Y) of:

a. Mercury (R = 58 Gm)
b. Venus (R = 108 Gm)
c. Mars (R = 228 Gm)
d. Jupiter (R = 778 Gm)
e. Saturn (R = 1,427 Gm)
f. Uranus (R = 2,869 Gm)
g. Neptune (R = 4,498 Gm)
h. Pluto (R = 5,900 Gm)
i. Eris: given Y = 557 Earth years, estimate the distance R from the sun assuming a circular orbit. (Note: Eris' orbit is highly eccentric and steeply tilted at 44° relative to the plane of the rest of the solar system, so our assumption of a circular orbit was made here only to simplify the exercise.)

A2. This year, what is the date and time of the:

a. perihelion	b. vernal equinox
c. summer solstice	d. aphelion
e. autumnal equinox	f. winter solstice

A3. What is the relative Julian day for:

a. 10 Jan	b. 25 Jan	c. 10 Feb	d. 25 Feb
e. 10 Mar	f. 25 Mar	g. 10 Apr	h. 25 Apr
i. 10 May	j. 25 May	k. 10 Jun	l. 25 Jun
m. 10 Jul	n. 25 Jul	o. 10 Aug	p. 25 Aug
q. 10 Sep	r. 25 Sep	s. 10 Oct	t. 25 Oct
u. 10 Nov	v. 25 Nov	w. 10 Dec	x. 25 Dec

y. today's date z. date assigned by instructor

A4. For the date assigned from exercise A3, find:

(i) mean anomaly
(ii) true anomaly
(iii) distance between the sun and the Earth
(iv) solar declination angle
(v) average daily insolation

A5(§). Plot the local solar elevation angle vs. local time for 22 December, 23 March, and 22 June for the following city:

a. Seattle, WA, USA
b. Corvallis, OR, USA
c. Boulder, CO, USA
d. Norman, OK, USA
e. Madison, WI, USA
f. Toronto, Canada
g. Montreal, Canada
h. Boston, MA, USA
i. New York City, NY, USA
j. University Park, PA, USA
k. Princeton, NJ, USA
l. Washington, DC, USA
m. Raleigh, NC, USA
n. Tallahassee, FL, USA
o. Reading, England
p. Toulouse, France
q. München, Germany
r. Bergen, Norway
s. Uppsala, Sweden
t. DeBilt, The Netherlands
u. Paris, France
v. Tokyo, Japan
w. Beijing, China
x. Warsaw, Poland
y. Madrid, Spain
z. Melbourne, Australia
aa. Your location today.
bb. A location assigned by your instructor

A6(§). Plot the local solar azimuth angle vs. local time for 22 December, 23 March, and 22 June, for the location from exercise A5.

A7(§). Plot the local solar elevation angle vs. azimuth angle (similar to Fig. 2.6) for the location from exercise A5. Be sure to add tic marks along the resulting curve and label them with the local standard times.

A8(§). Plot local solar elevation angle vs. azimuth angle (such as in Fig. 2.6) for the following location:

a. Arctic Circle b. 75°N c. 85°N d. North Pole
e. Antarctic Circle f. 70°S g. 80°S h. South Pole

for each of the following dates:

(i) 22 Dec (ii) 23 Mar (iii) 22 Jun

A9(§). Plot the duration of evening civil twilight (difference between end of twilight and sunset times) vs. latitude between the south and north poles, for the following date:

a. 22 Dec b. 5 Feb c. 21 Mar d. 5 May
e. 21 Jun f. 5 Aug g. 23 Sep h. 5 Nov

A10. On 15 March for the city listed from exercise A5, at what local standard time is:

a. geometric sunrise
b. apparent sunrise
c. start of civil twilight
d. start of military twilight
e. start of astronomical twilight
f. geometric sunset
g. apparent sunset
h. end of civil twilight
i. end of military twilight
j. end of astronomical twilight

A11. Calculate the Eq. of Time correction for:

a. 1 Jan b. 15 Jan c. 1 Feb d. 15 Feb e. 1 Mar
f. 15 Mar g. 1 Apr h. 15 Apr i. 1 May j. 15 May

A12. Find the mass flux ($kg \cdot m^{-2} \cdot s^{-1}$) at sea-level, given a kinematic mass flux ($m\ s^{-1}$) of:

a. 2 b. 5 c. 7 d. 10 e. 14 f. 18 g. 21
h. 25 i. 30 j. 33 k. 47 l. 59 m. 62 n. 75

A13. Find the kinematic heat fluxes at sea level, given these regular fluxes ($W \cdot m^{-2}$):

a. 1000 b. 900 c. 800 d. 700 e. 600
f. 500 g. 400 h. 300 i. 200 j. 100
k. 43 l. –50 m. –250 n. –325 o. –533

A14. Find the frequency, circular frequency, and wavenumber for light of color:

a. red b. orange c. yellow d. green
e. cyan f. blue f. indigo g. violet

A15(§). Plot Planck curves for the following blackbody temperatures (K):

a. 6000 b. 5000 c. 4000 d. 3000 e. 2500
f. 2000 g. 1500 h. 1000 i. 750 j. 500
k. 300 l. 273 m. 260 n. 250 h. 240

A16. For the temperature of exercise A15, find:

(i) wavelength of peak emissions

(ii) total emittance (i.e., total amount of emissions)

A17. Estimate the value of solar irradiance reaching the orbit of the planet from exercise A1.

A18(§). a. Plot the value of solar irradiance reaching Earth's orbit as a function of relative Julian day.

b. Using the average solar irradiance, plot the radiative flux (reaching the Earth's surface through a perfectly clear atmosphere) vs. latitude. Assume local noon.

A19(§). For the city of exercise A1, plot the average daily insolation vs. Julian day.

A20. What is the value of IR absorptivity of:

a. aluminum b. asphalt c. cirrus cloud
d. conifer forest e. grass lawn f. ice
g. oak h. silver i. old snow
j. urban average k. concrete average
l. desert average m. shrubs n. soils average

A21. Suppose polluted air reflects 30% of the incoming solar radiation. How much ($W\ m^{-2}$) is absorbed, emitted, reflected, and transmitted? Assume an incident radiative flux equal to the solar irradiance, given a transmissivity of:

a. 0 b. 0.05 c. 0.1 d. 0.15 e. 0.2 f. 0.25
g. 0.3 h. 0.35 i. 0.4 j. 0.45 k. 0.5 l. 0.55
m. 0.6 n. 0.65 o. 0.7

A22. What is the value of albedo for the following land use?

a. buildings b. dry clay c. corn d. green grass
e. ice f. potatoes g. rice paddy h. savanna
i. red soil j. sorghum k. sugar cane
l. tobacco

A23. What product of number density times absorption cross section is needed in order for 50% of the incident radiation to be absorbed by airborne volcanic ash over the following path length (km)?

a. 0.2 b. 0.4 c. 0.6 d. 0.8 e. 1.0 f. 1.5 g. 2
h. 2.5 i. 3 j. 3.5 k. 4 l. 4.5 m. 5 n. 7

A24. What fraction of incident radiation is transmitted through a volcanic ash cloud of optical depth:

a. 0.2 b. 0.5 c. 0.7 d. 1.0 e. 1.5 f. 2 g. 3
h. 4 j. 5 k. 6 l. 7 m. 10 n. 15 o. 20

A25. What is the visual range (km) for polluted air that has volume extinction coefficient (m^{-1}) of:

a. 0.00001 b. 0.00002 c. 0.00005 d. 0.0001
e. 0.0002 f. 0.0005 g. 0.001 h. 0.002
i. 0.005 j. 0.01 k. 0.02 l. 0.05

A26. (i) What is the value of solar downward direct radiative flux reaching the surface at the city from exercise A5 at noon on 4 July, given 20% coverage of cumulus (low) clouds.

(ii) If the albedo is 0.5 in your town, what is the reflected solar flux at that same time?

(iii) What is the approximate value of net longwave radiation at that time?

(iv) What is the net radiation at that time, given all the info from parts (i) - (iii)?

A27. For a surface temperature of 20°C, find the emitted upwelling IR radiation (W m^{-2}) over the surface-type from exercise A20.

Evaluate & Analyze

E1. At what time of year does the true anomaly equal:

a. 45° b. 90° c. 135° d. 180° e. 225°
f. 270° g. 315° h. 360°

E2(§) a. Calculate and plot the position (true anomaly and distance) of the Earth around the sun for the first day of each month.

b. Verify Kepler's second law.

c. Compare the elliptical orbit to a circular orbit.

E3. What is the optimum angle for solar collectors at your town?

E4. Design a device to measure the angular diameter of the sun when viewed from Earth. (Hint, one approach is to allow the sun to shine through a pin hole on to a flat surface. Then measure the width of the projected image of the sun on this surface divided by the distance between the surface and the pin hole. What could cause errors in this device?)

E5. For your city, plot the azimuth angle for apparent sunrise vs. relative Julian day. This is the direction you need to point your camera if your want to photograph the sunrise.

E6. a. Compare the length of daylight in Fairbanks, AK, vs Miami, FL, USA.

b. Why do vegetables grow so large in Alaska?

c. Why are few fruits grown in Alaska?

E7. How would Fig. 2.6 be different if daylight (summer) time were used in place of standard time during the appropriate months?

E8(§). Plot a diagram of geometric sunrise times and of sunset times vs. day of the year, for your location.

E9(§). Using apparent sunrise and sunset, calculate and plot the hours of daylight vs. Julian day for your city.

E10. a. On a clear day at your location, observe and record actual sunrise and sunset times, and the duration of twilight.

b. Use that information to determine the day of the year.

c. Based on your personal determination of the length of twilight, and based on your latitude and season, is your personal twilight most like civil, military, or astronomical twilight?

E11. Given a flux of the following units, convert it to a kinematic flux, and discuss the meaning and/or advantages of this form of flux.

a. Moisture flux: $g_{water} \cdot m^{-2} \cdot s^{-1}$

b. Momentum flux: $(kg_{air} \cdot m \cdot s^{-1}) \cdot m^{-2} \cdot s^{-1}$

c. Pollutant flux: $g_{pollutant} \cdot m^{-2} \cdot s^{-1}$

E12. a. What solar temperature is needed for the peak intensity of radiation to occur at 0.2 micrometers?

b. Remembering that humans can see light only between 0.38 and 0.74 microns, would the sun look brighter or dimmer at this new temperature?

E13. A perfectly black asphalt road absorbs 100% of the incident solar radiation. Suppose that its resulting temperature is 50°C. How much visible light does it emit?

E14. If the Earth were to cool 5°C from its present radiative equilibrium temperature, by what percentage would the total emitted IR change?

E15(§). Evaluate the quality of the approximation to Planck's Law [see eq. (a) in the " A Scientific Perspective • Scientific Laws — the Myth" box] against the exact Planck equation (2.13) by plotting both curves for a variety of typical sun and Earth temperatures.

E16. Find the solar irradiance that can pass through an atmospheric "window" between λ_1 = 0.3 μm and λ_2 = 0.8 μm. (See the " A Scientific Perspective • Seek Solutions" box in this chapter for ways to do this without using calculus.)

E17. How much variation in Earth orbital distance from the sun is needed to alter the solar irradiance by 10%?

E18. Solar radiation is a diffuse source of energy, meaning that it is spread over the whole Earth rather than being concentrated in a small region. It has been proposed to get around the problem of the inverse square law of radiation by deploying very large mirrors closer to the sun to focus the light as collimated rays toward the Earth. Assuming that all the structural and space-launch issues could be solved, would this be a viable method of increasing energy on Earth?

E19. The "sine law" for radiation striking a surface at an angle is sometimes written as a "cosine law", but using the zenith angle instead of the elevation angle. Use trig to show that the two equations are physically identical.

E20. Explain the meaning of each term in eq. (2.21).

E21. a. Examine the figure showing average daily insolation. In the summer hemisphere during the few months nearest the summer solstice, explain why the incoming solar radiation over the pole is nearly equal to that over the equator.

b. Why are not the surface temperatures near the pole nearly equal to the temperatures near the equator during the same months?

E22. Using Table 2-5 for the typical albedos, speculate on the following:

a. How would the average albedo will change if a pasture is developed into a residential neighborhood.

b. How would the changes in affect the net radiation budget?

E23. Use Beer's law to determine the relationship between visual range (km) and volume extinction coefficient (m^{-1}). (Note that extinction coefficient can be related to concentration of pollutants and relative humidity.)

E24(§). For your city, calculate and plot the noontime downwelling solar radiation every day of the year, assuming no clouds, and considering the change in solar irradiance due to changing distance between the Earth and sun.

E25. Consider cloud-free skies at your town. If 50% coverage of low clouds moves over your town, how does net radiation change at noon? How does it change at midnight?

E26. To determine the values of terms in the surface net-radiation budget, what actinometers would you use, and how would you deploy them (i.e., which directions does each one need to look to get the data you need)?

Synthesize

(Don't forget to state and justify all assumptions.)

S1. What if the eccentricity of the Earth's orbit around the sun changed to 0.2 ? How would the seasons and climate be different than now?

S2. What if the tilt of the Earth's axis relative to the ecliptic changed to 45° ? How would the seasons and climate be different than now?

S3. What if the tilt of the Earth's axis relative to the ecliptic changed to 90° ? How would the seasons and climate be different than now?

S4. What if the tilt of the Earth's axis relative to the ecliptic changed to 0° ? How would the seasons and climate be different than now?

S5. What if the rotation of the Earth about its axis matched its orbital period around the sun, so that one side of the Earth always faced the sun and the other side was always away. How would weather and climate be different, if at all?

S6. What if the Earth diameter decreased to half of its present value? How would sunrise and sunset time, and solar elevation angles change?

S7. Derive eq. (2.6) from basic principles of geometry and trigonometry. This is quite complicated. It can be done using plane geometry, but is easier if you use spherical geometry. Show your work.

S8. What if the perihelion of Earth's orbit happened at the summer solstice, rather than near the winter solstice. How would noontime, clear-sky values of insolation change at the solstices compared to now?

S9. What if radiative heating was caused by the magnitude of the radiative flux, rather than by the radiative flux divergence. How would the weather or the atmospheric state be different, if at all?

S10. Linearize Planck's Law in the vicinity of one temperature. Namely, derive an equation that gives a straight line that is tangent to any point on the Planck curve. (Hint: If you have calculus skills, try using a Taylor's series expansion.) Determine over what range of temperatures your equation gives reasonable answers. Such linearization is sometimes used retrieving temperature soundings from satellite observations.

S11. Suppose that Kirchhoff's law were to change such that $a_\lambda = 1 - e_\lambda$. What are the implications?

S12. What if Wien's law were to be repealed, because it was found instead that the wavelength of peak emissions increases as temperature increases.
a. Write an equation that would describe this. You may name this equation after yourself.
b. What types of radiation from what sources would affect the radiation budget of Earth?

S13. What if the solar "constant" were even less constant than it is now. Suppose the solar constant randomly varies within a range of ±50% of its present value, with each new value lasting for a few years before changing again. How would weather and climate be different, if at all?

S14. What if the distance between sun and Earth was half what it is now. How would weather and climate be different, if at all?

S15. What if the distance between sun and Earth was double what it is now. How would weather and climate be different, if at all?

S16. Suppose the Earth was shaped like a cube, with the axis of rotation perpendicular to the ecliptic, and with the axis passing through the middle of the top and bottom faces of the cube. How would weather and climate be different, if at all?

S17. Suppose the Earth was shaped like a narrow cylinder, with the axis of rotation perpendicular to the ecliptic, and with the axis passing through the middle of the top and bottom faces of the cylinder. How would weather & climate be different, if at all?

S18. Derive eq. (2.21) from the other equations in this chapter. Show your work. Discuss the physical interpretation of the hour angle, and what the effect of truncating it is.

S19. Suppose that the Earth's surface was perfectly reflective everywhere to short-wave radiation, but that the atmosphere absorbed 50% of the sunlight passing through it without reflecting any. What percentage of the insolation would be reflected to space? Also, how would the weather and climate be different, if at all?

S20. Suppose that the atmosphere totally absorbed all short wave radiation that was incident on it, but also emitted an exactly equal amount of short wave radiation as it absorbed. How would weather and climate be different, if at all?

S21. Consider Beer's law. If there are n particles per cubic meter of air, and if a vertical path length in air is Δs, then multiplying the two gives the number of particles over each square meter of ground. If the absorption cross section b is the area of shadow cast by each particle, then multiplying this times the previous product would give the shadowed area divided by the total area of ground. This ratio is just the absorptivity a. Namely, by this reasoning, one would expect that $a = n \cdot b \cdot \Delta s$.

However, Beer's law is an exponential function. Why? What was wrong with the reasoning in the previous paragraph?

S22. What if the atmosphere were completely transparent to IR radiation. How would the surface net radiation budget be different?

S23. Existing radiometers are based on a bolometer, photovoltaic cell, or charge-coupled device. Design a new type of actinometer that is based on a different principle. Hint, think about what is affected in any way by radiation or sunlight, and then use that effect to measure the radiation.