

5 ATMOSPHERIC STABILITY

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A **sounding** is the vertical profile of temperature and other variables in the atmosphere over one geographic location. **Stability** refers to the ability of the atmosphere to be turbulent, which you can determine from soundings of temperature, humidity, and wind. Turbulence and stability vary with time and place because of the corresponding variation of the soundings.

We notice the effects of stability by the wind gustiness, dispersion of smoke, refraction of light and sound, strength of thermal updrafts, size of clouds, and intensity of thunderstorms.

Thermodynamic diagrams have been devised to help us plot soundings and determine stability. As you gain experience with these diagrams, you will find that they become easier to use, and faster than solving the thermodynamic equations. In this chapter, we first discuss the different types of thermodynamic diagrams, and then use them to determine stability and turbulence.

BUILDING A THERMO-DIAGRAM

Components

In the Water Vapor chapter you learned how to compute **isohumes** and **moist adiabats**, and in the Thermodynamics chapter you learned to plot **dry adiabats**. You plotted these isopleths on a background graph having temperature along the horizontal axis and log of pressure along the vertical axis. Figs. 5.1a-d show these diagram components.

When these isopleths are combined on a single graph, the result is called a **thermodynamic diagram** or **thermo diagram** (Fig. 5.1e). At first glance, Fig. 5.1e looks like a confusing nest of lines; however, you can use the pattern-recognition capability of your mind to focus on the components as shown in Fig. 5.1a-d. Your efforts to master thermo diagrams now will save you time in the future.

Several types of thermo diagrams are used in meteorology. They all can show the same information, and are used the same way. The thermo diagram we learned so far is called an **Emagram**. We learned this one first because it was easy to create using a computer spreadsheet.



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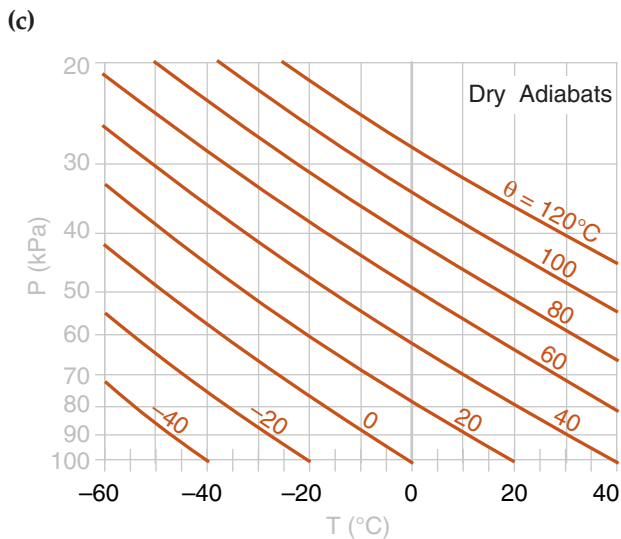
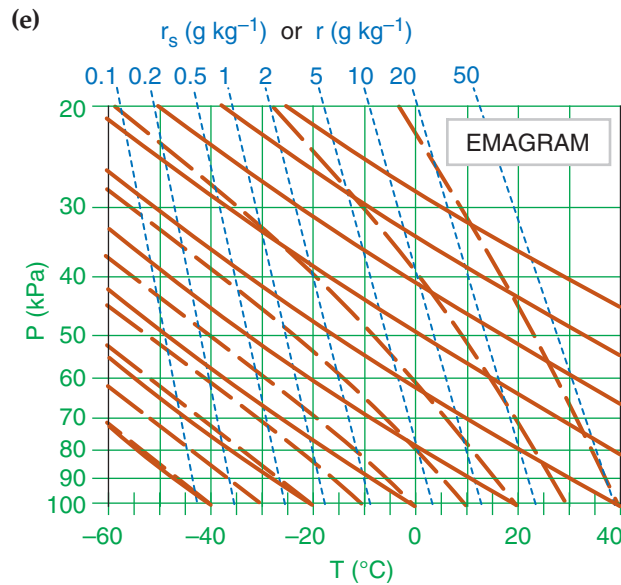
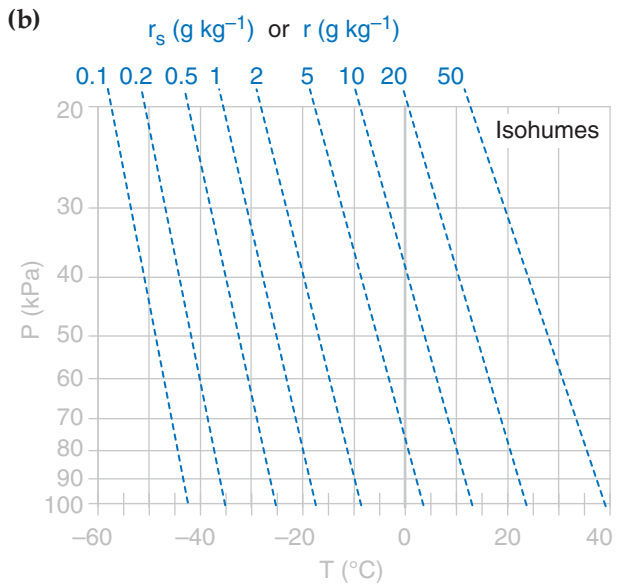
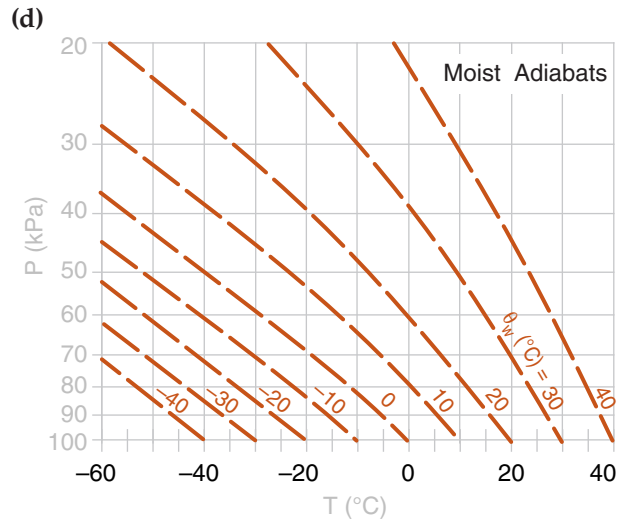
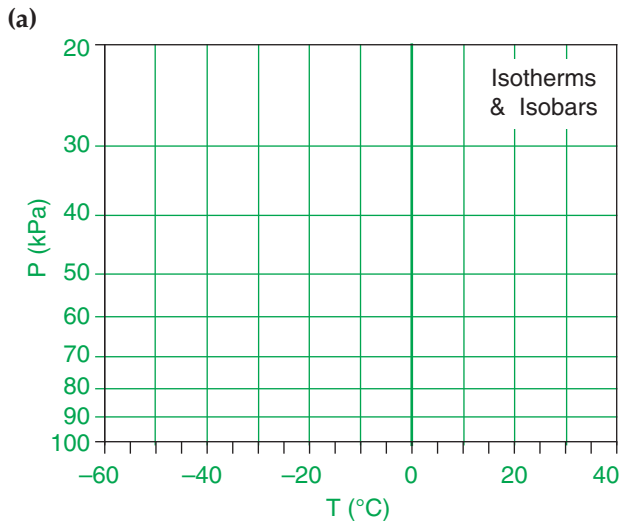


Figure 5.1

Components of an Emagram thermo diagram.

(a) **Isobars** (green thin horizontal lines with logarithmic spacing) and **isotherms** (green thin vertical lines) are used on all these charts as a common background.

(b) **Isohumes** (from the Water Vapor chapter) are dotted light-blue lines.

(c) The dark-orange solid lines are **dry adiabats**.

(d) The dark-orange dashed lines are **moist adiabats**.

(e) Thermo diagram formed by combining parts (a) through (d).

The variables are: pressure (P), temperature (T), mixing ratio (r), saturated mixing ratio (r_s), potential temperature (θ), and wet-bulb potential temperature (θ_w).

Five or more sets of lines are plotted on every thermo diagram, including the Emagram (Fig. 5.1e). Three sets give the **state** of the air (**isotherms, isobars, isohumes**). Two sets (**dry adiabats** and **moist adiabats**) describe **processes** that change the state. **Height contours** are omitted from introductory thermo diagrams, but are included in full thermo diagrams at the end of this chapter.

Pseudoadiabatic Assumption

In the Water Vapor chapter, we assumed an adiabatic process (no heat transfer or mixing to or from the air parcel) when computing the moist adiabats. However, for any of the thermo diagrams, the moist adiabats can be computed assuming either:

- **adiabatic processes** (i.e., **reversible**, where all liquid water is carried with the air parcels), or
- **pseudoadiabatic processes** (i.e., **irreversible**, where all condensed water is assumed to fall out immediately).

Air parcels in the real atmosphere behave between these two extremes, because small droplets and ice crystals are carried with the air parcel while larger ones precipitate out.

When liquid or solid water falls out, it removes from the system some of the sensible heat associated with the temperature of the droplets, and also changes the heat capacity of the remaining air because of the change in relative amounts of the different constituents. The net result is that an air parcel lifted pseudoadiabatically from 100 kPa to 20 kPa will be about 3°C colder than one lifted adiabatically. This small difference between adiabatic and pseudoadiabatic can usually be neglected compared to other errors in measuring soundings.

Complete Thermo Diagrams

Color thermo diagrams printed on large-format paper were traditionally used by weather services for hand plotting of soundings, but have become obsolete and expensive compared to modern plots by computer. The simplified, small-format diagrams presented so far in this chapter are the opposite extreme – useful for education, but not for plotting real soundings. Also, some weather stations have surface pressure greater than 100 kPa, which is off the scale for the simple diagrams presented so far.

As a useful compromise, full-page thermo diagrams in several formats are included at the end of this chapter. They are optimized for you to print on a color printer.

[Hint: Keep the original thermo diagrams in this book clean and unmarked, to serve as master copies.]

INFO • Why so many thermo diagrams?

Meteorological thermo diagrams were originally created as optimized versions of the P vs. α diagrams of classical physics (thermodynamics), where α is the **specific volume** (i.e., $\alpha =$ volume per unit mass $= 1/\rho$, where ρ is air density). A desirable attribute of the P vs. α diagram is that when a cyclic process is traced on this diagram, the area enclosed by the resulting curve is proportional to the specific work done by or to the atmosphere. The disadvantage of P vs. α diagram is that the angle between any isotherm and adiabat is relatively small, making it difficult to interpret atmospheric soundings.

Three meteorological thermo diagrams have been devised that satisfy the “area = work” attribute, and are optimized for meteorology to have greater angles between the isotherms and adiabats:

- Emagram,
- Skew-T Log-P Diagram
- Tephigram.

Meteorologists rarely need to utilize the “area = work” attribute, so they also can use any of three additional diagrams:

- Stüve Diagram,
- Pseudoadiabatic (Stüve) Diagram,
- Theta-Height (θ - z) Diagram.

Why are there so many diagrams that show the same things? Historically, different diagrams were devised somewhat independently in different countries. Nations would adopt one as the “official” diagram for their national weather service, and teach only that one to their meteorologists. For example, the tephigram is used in British Commonwealth countries (UK, Canada, Australia, New Zealand). To this day, many meteorologists feel most comfortable with the diagram they learned first.

For many readers, this myriad of diagrams might make an already-difficult subject seem even more daunting. Luckily, all the diagrams show the same thermodynamic state (T, P, r) and process lines (θ, θ_w), but in different orientations. So once you have learned how to read one diagram, it is fairly easy to read the others.

The skill to read diverse thermo diagrams will serve you well when acquiring weather data via the internet, because they can come in any format. The internet is the main reason I cataloged the different thermo diagrams in this book.

Of all these diagrams, the Skew-T and Tephigram have the greatest angle between isotherms and adiabats, and are therefore preferred when studying soundings and stability. These two diagrams look similar, but the **Skew-T** is growing in popularity because it is easier to create on a computer.

TYPES OF THERMO DIAGRAMS

Emagram

“Emagram” is a contraction for “Energy-per-unit-mass diagram.” This semi-log diagram (Fig. 5.3a) has temperature (T) along the abscissa, and pressure (P) decreasing logarithmically upward along the ordinate. The isotherms and isobars are straight lines that are perpendicular to each other, and form the orthogonal basis for this diagram.

Dry adiabats (θ) are nearly-straight, diagonal lines slanted up towards the left, and curve slightly concave upward. Moist adiabats (θ_w) curve concave downward, but become parallel to the dry adiabats at cold temperatures and high altitudes. Isohumes (r) are nearly-straight, nearly-vertical lines (with a slight tilt upward to the left).

Height contours (on the large Emagram at the end of this chapter) are approximate, and are provided as a rough guide to users. These contours are nearly horizontal (tilting downward slightly to the right) and nearly straight (slightly concave upward). They are often not drawn in thermo diagrams, and indeed are not shown in Figs. 5.1 and 5.3.

Stüve & Pseudoadiabatic Diagrams

The Stüve diagram (Fig. 5.3b) has T along the abscissa, and pressure decreasing upward along the ordinate according to $(P/P_0)^{\mathfrak{R}_d/C_{pd}}$, where $\mathfrak{R}_d/C_{pd} = 0.28571$ (dimensionless) and $P_0 = 100$ kPa. Stüve diagrams look virtually identical to Emagrams, except that the dry adiabats are perfectly straight. These dry adiabats converge to a point at $P = 0$ kPa and $T = 0$ K, which is usually well off of the upper left corner of the graph.

Any thermo diagram can be computed using the pseudoadiabatic assumption for the moist adiabats. However, the particular diagram that is known as a “Pseudoadiabatic Diagram” is often a Stüve plot.

Skew-T Log-P Diagram

Fig. 5.3c is a Skew-T. Although T is labeled linearly along the abscissa, the isotherms are parallel, straight, diagonal lines tilting upward to the right. The Skew-T gets its name because the isotherms are not vertical, but skewed. Pressure decreases logarithmically upward along the ordinate, and the isobars are parallel, horizontal, straight lines.

Dry adiabats are diagonal lines slanted up towards the left, with a pronounced curve concave upward. Moist adiabats are more sharply curved concave left near the bottom of the diagram, changing to less curved, concave to the right, as they merge

into the dry adiabats at higher altitudes and colder temperatures. Isohumes are almost straight lines, tilting upward to the right.

This diagram is designed so that the isotherms and dry adiabats are nearly (but not perfectly) perpendicular to each other.

Tephigram

The name Tephigram is a contraction of Tee-Phi Diagram. The logarithm of potential temperature θ physically represents the **entropy** (ϕ = Greek letter Phi) change, and is plotted along the ordinate of Fig. 5.2. Temperature T is along the abscissa. Thus, it is a temperature-entropy diagram, or T - ϕ diagram. Isobars are curved green lines in Fig. 5.2.

The range of meteorological interest is shown with the bold rectangle in this figure. When this region is enlarged and tilted horizontally, the result is the tephigram as plotted in Fig. 5.3d.

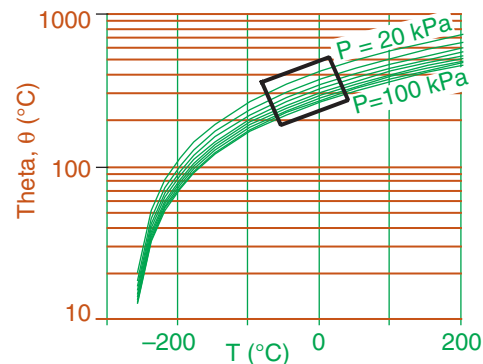


Fig. 5.2
Rectangle shows portion of thermo diagram used for tephigram.

The tephigram looks similar to the Skew-T, with isotherms as straight diagonal lines tilting upward to the right. Dry adiabats are perfectly straight diagonal lines tilting upward left, and are exactly perpendicular to the isotherms. Moist adiabats have strong curvature concave left, and become parallel to the dry adiabats at cold temperatures and high altitudes. Isohumes are gently curved (concave down) nearly-horizontal lines. Isohumes are nearly straight diagonal lines tilting upward right.

Theta-Height (θ - z) Diagrams

The orthogonal basis for this diagram are heights (z) plotted along the ordinate and potential temperature (θ) along the abscissa (Fig. 5.3e). Moist adiabats curve to the right of the dry adiabats, but become parallel to the dry adiabats at high altitudes and cold temperatures. Heights are accurate in this diagram, but the isobars are only approximate. Isohumes tilt steeply upward to the right. This diagram is used extensively in boundary-layer meteorology.

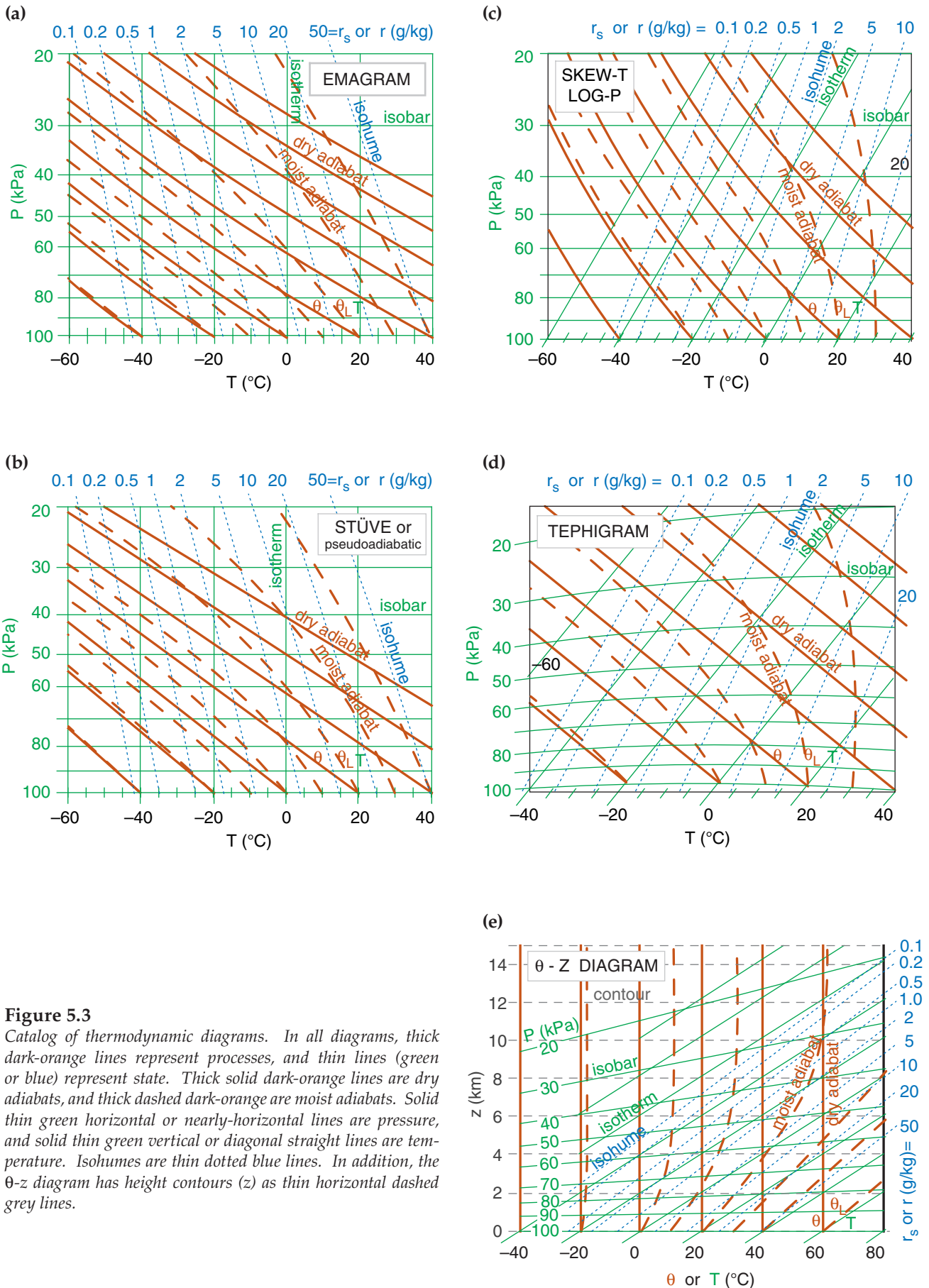
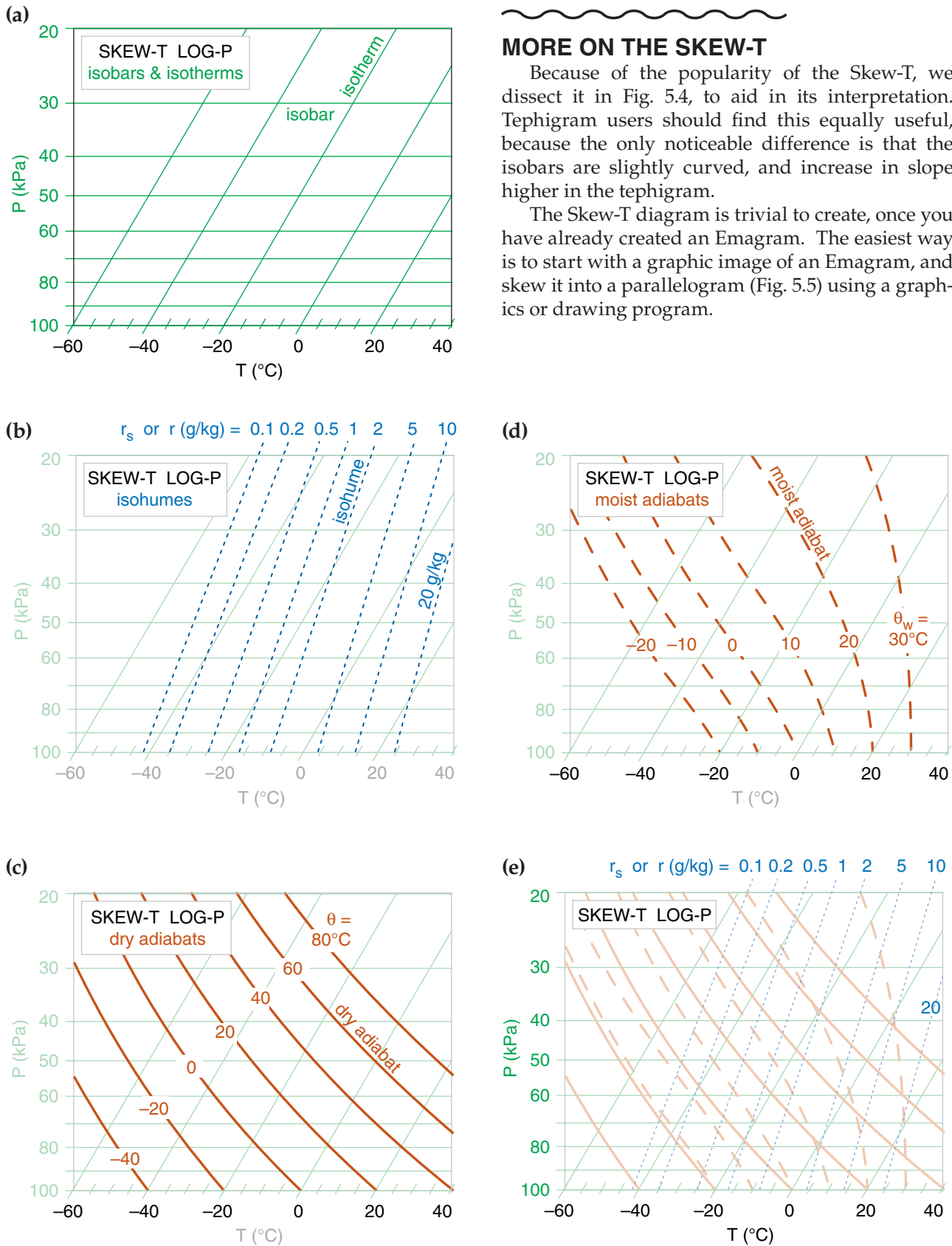


Figure 5.3
 Catalog of thermodynamic diagrams. In all diagrams, thick dark-orange lines represent processes, and thin lines (green or blue) represent state. Thick solid dark-orange lines are dry adiabats, and thick dashed dark-orange are moist adiabats. Solid thin green horizontal or nearly-horizontal lines are pressure, and solid thin green vertical or diagonal straight lines are temperature. Isohumes are thin dotted blue lines. In addition, the θ - z diagram has height contours (z) as thin horizontal dashed grey lines.



MORE ON THE SKEW-T

Because of the popularity of the Skew-T, we dissect it in Fig. 5.4, to aid in its interpretation. Tephigram users should find this equally useful, because the only noticeable difference is that the isobars are slightly curved, and increase in slope higher in the tephigram.

The Skew-T diagram is trivial to create, once you have already created an Emagram. The easiest way is to start with a graphic image of an Emagram, and skew it into a parallelogram (Fig. 5.5) using a graphics or drawing program.

Fig. 5.4

(a)-(d) Components of a Skew-T diagram, drawn to the same scale so they can be superimposed as a learning aid. The state of the air: (a) isobars and isotherms; (b) isohumes as thin dotted lines. Processes: (c) dry adiabats; (d) moist adiabats. (e) Full Skew-T Log-P, shaded lighter to make it easier to write on. The "Skew-T Log-P" diagram is often called a "Skew-T" for short.

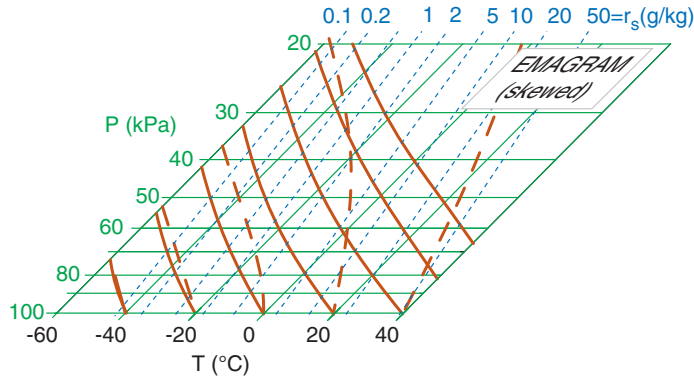


Fig. 5.5
A Skew-T is nothing more than a skewed Emagram.

It is equally simple to draw a Skew-T with a spreadsheet program, especially if you start with the numbers you already calculated to create an Emagram. Let X ($^{\circ}\text{C}$) be the abscissa of a Cartesian graph, and Y (dimensionless) be the ordinate. Let K be the skewness factor, which is arbitrary. A value of $K = 35^{\circ}\text{C}$ gives a nice skewness (such that the isotherms and adiabats are nearly perpendicular), but you can pick any skewness you want.

The coordinates for any point on the graph are:

$$Y = \ln\left(\frac{P_0}{P}\right) \tag{5.1a}$$

$$X = T + K \cdot Y \tag{5.1b}$$

To draw isotherms, set T to the isotherm value, and find a set of X values corresponding to different heights Y . Plot this one line as an isotherm. For the other isotherms, repeat for other values of T . [Hint: it is convenient to set the spreadsheet program to draw a logarithmic vertical axis with the max and min reversed. If you do this, then use P , not Y , as your vertical coordinate. But you still must use Y in the equation above to find X .]

For the isohumes, dry adiabats, and moist adiabats, use the temperatures you found for the Emagram as the input temperatures in the equation above. For example, the $\theta = 20^{\circ}\text{C}$ adiabat has the following temperatures as a function of pressure (recall Thermo Diagrams-Part 1, in the Thermodynamics chapter), from which the corresponding values of X can be found.

$P(\text{kPa}) =$	100	,	80	,	60	,	40	,	20
$T(^{\circ}\text{C}) =$	20	,	1.9	,	-19.8	,	-47.5	,	-88
$X =$	20	,	9.7	,	-1.9	,	-15.4	,	-31.7

Sample Application

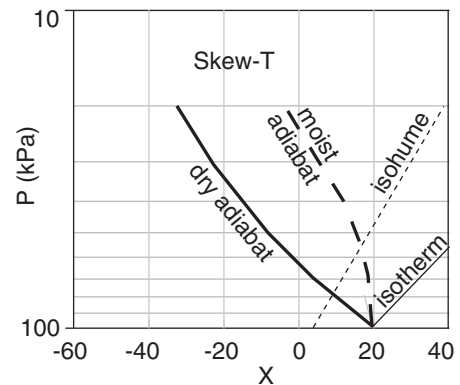
Create a Skew-T diagram with one isotherm ($T = 20^{\circ}\text{C}$), one dry adiabat ($\theta = 20^{\circ}\text{C}$), one moist adiabat ($\theta_w = 20^{\circ}\text{C}$), and one isohume ($r = 5 \text{ g/kg}$). Use as input the temperatures from the Emagram (as calculated in earlier chapters). Use $K = 35^{\circ}\text{C}$, and $P_0 = 100 \text{ kPa}$.

Find the Answer

Given: The table of Emagram numbers below. Use eq. (5.1) in a spreadsheet to find X and Y .

	A	B	C	D	E
1	Create Skew-T from Emagram Numbers				
2					
3		Iso-	Dry	Moist	Iso-
4		therm	Adiabat	Adiabat	hume
5		$T=20^{\circ}\text{C}$	$\theta=20^{\circ}\text{C}$	$\theta_w=20^{\circ}\text{C}$	$r=5\text{g/kg}$
6	Emagram (Given):				
7	P (kPa)	T ($^{\circ}\text{C}$)	T ($^{\circ}\text{C}$)	T ($^{\circ}\text{C}$)	T ($^{\circ}\text{C}$)
8	100	20	20	20	3.7
9	90	20	11.3	16	2.3
10	80	20	1.9	11.4	0.7
11	70	20	-8.4	6	-1.2
12	60	20	-19.8	-0.3	-3.2
13	50	20	-32.6	-8.8	-5.6
14	40	20	-47.5	-20	-8.5
15	30	20	-65.3	-37	-12
16	20	20	-88	-60	-17
17					
18	Skew-T:				
19	Let K=	35 $^{\circ}\text{C}$		Let $P_0 =$	100kPa
20	P	Y	X	X	X
21	100	0.00	20	20	3.7
22	90	0.11	23.7	15.0	6.0
23	80	0.22	27.8	9.7	8.5
24	70	0.36	32.5	4.1	11.3
25	60	0.51	37.9	-1.9	14.7
26	50	0.69	44.3	-8.3	18.7
27	40	0.92	52.1	-15.4	23.6
28	30	1.20	62.1	-23.2	30.1
29	20	1.61	76.3	-31.7	39.3

Using the spreadsheet to plot the result gives:

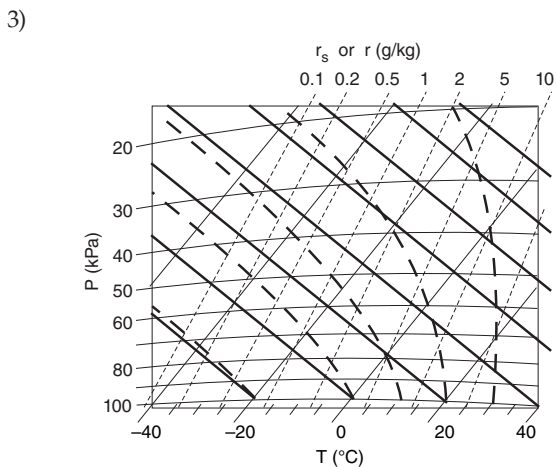
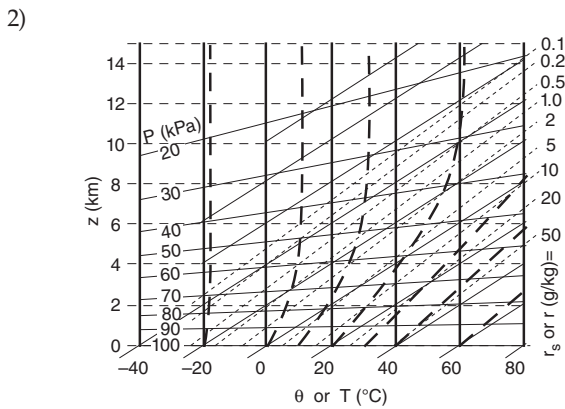
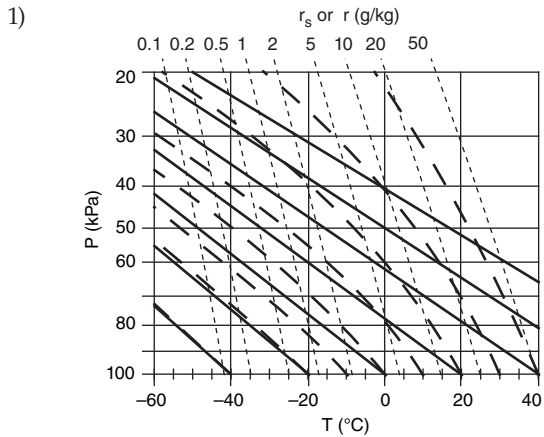


Check: Sketch reasonable. Matches Fig. 5.4e.

Exposition: Experiment with different values of K starting with $K = 0^{\circ}\text{C}$, and see how the graph changes from an Emagram to a Skew-T.

Sample Application

For each thermo diagram numbered below, identify its name.



CAUTION: Thermo diagrams printed or published by other agencies do NOT follow any standard format of line style or color. That is the reason these thermo diagrams are shown only with black lines in this sample application — so you don't use the colors as a crutch.

(continued next page)

GUIDE FOR QUICK IDENTIFICATION OF THERMO DIAGRAMS

If you receive a thermo diagram via the internet without any labels identifying the diagram type, use the following procedure to figure it out:

- 1) if there are some lines from lower left to upper right, then either Skew-T, Tephigram, or θ -Z .
 - a) if the isobars are exactly horizontal and straight, then Skew-T,
 - b) else if the strongly curved lines (moist adiabats) asymptotically approach vertical near the top of the diagram, then θ -Z,
 - c) else Tephigram,
- 2) else either Emagram or Stüve .
 - a) if dry adiabats are slightly curved, then Emagram,
 - b) else Stüve (including Pseudoadiabatic version of Stüve).

When you encounter an unfamiliar thermo diagram, you can take steps to help segregate and identify the various lines. First, identify the isobars, which are horizontal and straight, or nearly so, on virtually all diagrams. Isobars are usually spaced roughly logarithmically with height, and are labeled as either 100, 90, 80, 70 kPa etc., or as 1000, 900, 800, 700 mb or hPa etc. These serve as a surrogate measure of height.

Next, identify the dry and moist adiabats. These are the lines that converge and become parallel at cold temperatures and high altitudes. In all the diagrams, the moist adiabats are the most curved. The adiabats are spaced and labeled roughly linearly, such as 0, 10, 20, 30 °C etc, or often in Kelvin as 273, 283, 293, 303 K, etc. While the dry adiabats are always labeled as potential temperature θ , the moist adiabats can be labeled as either equivalent potential temperature θ_e , liquid-water potential temperature θ_L , or wet-bulb potential temperature θ_w .

An alternative for identifying adiabats is to look at any labeled temperature at the bottom of the graph. Usually at 100 kPa (1000 mb or hPa), you will find that T , θ_w , and θ lines all radiate from the same

Hint: At first glance, a thermo diagram looks like a complicated nest of lines. A way to discriminate one set of lines (e.g., adiabats) from other sets of lines (e.g., isotherms) is to hold the printed diagram so that you can view it from near the edge of the page. For example, if you view an Emagram from the lower right corner, the dry adiabats will stand out as slightly curved lines. Look from different edges to highlight other isopleths.

points. For all diagrams, the T line is rightmost, θ is leftmost, and θ_w is in the middle.

Finally, identify the isotherms and isohumes. These lines cross each other at a small angle, and extend from the bottom to the top of the graph. The isohumes always tilt to the left of the isotherms with increasing height. The isotherms increment linearly (e.g., 0, 10, 20, 30, °C etc.), while the isohumes usually increment logarithmically (e.g., 1, 2, 5, 10, 20 g kg⁻¹, etc.). Isotherms are either vertical, or tilt upward to the right.

THERMO-DIAGRAM APPLICATIONS

Fig. 5.6 shows a simplified, Skew-T diagram. We use this as the common background for the demonstrations that follow.

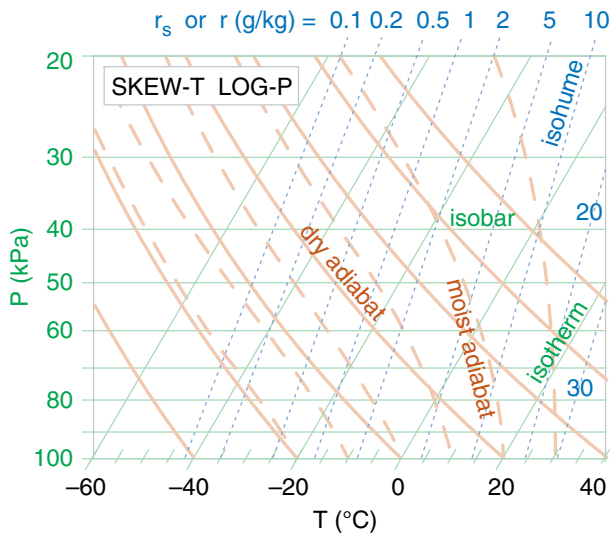
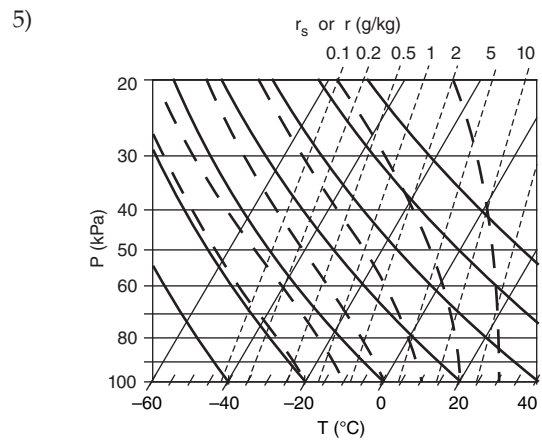
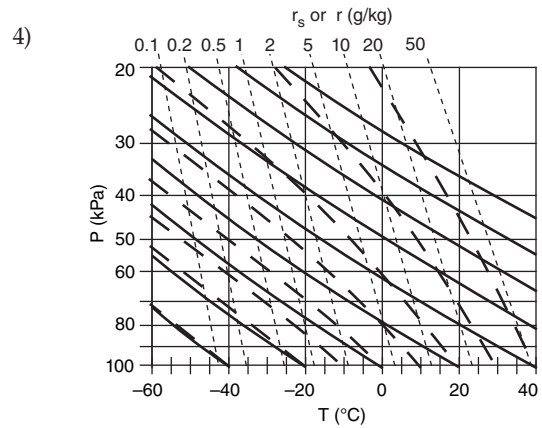


Figure 5.6
Simplified Skew-T diagram. Isoleths are labeled. Colors are lightened to serve as a background for future discussions.

Sample Application (continuation)



Find the Answer: Use the quick guide.

- 1) **Stüve**, because satisfies item 2b in the guide.
- 2) **θ -Z diagram**, because satisfies item 1b.
- 3) **Tephigram**, because satisfies item 1c.
- 4) **Emagram**, because satisfies item 2a.
- 5) **Skew-T**, because satisfies item 1a.

Check: Reasonable, agrees with Fig. 5.3.

Exposition: For practice, pick any one of the diagrams above, and without looking back in Fig. 5.3, try to identify which lines are which.

[Hint: In this book, all thermo diagrams use the same format for the same types of lines: isotherms and isobars are thin solid green, isohumes are thin dotted blue, dry adiabats are thick solid dark-orange, moist adiabats are thick dashed dark-orange, and heights are thin dashed grey (if plotted at all).]

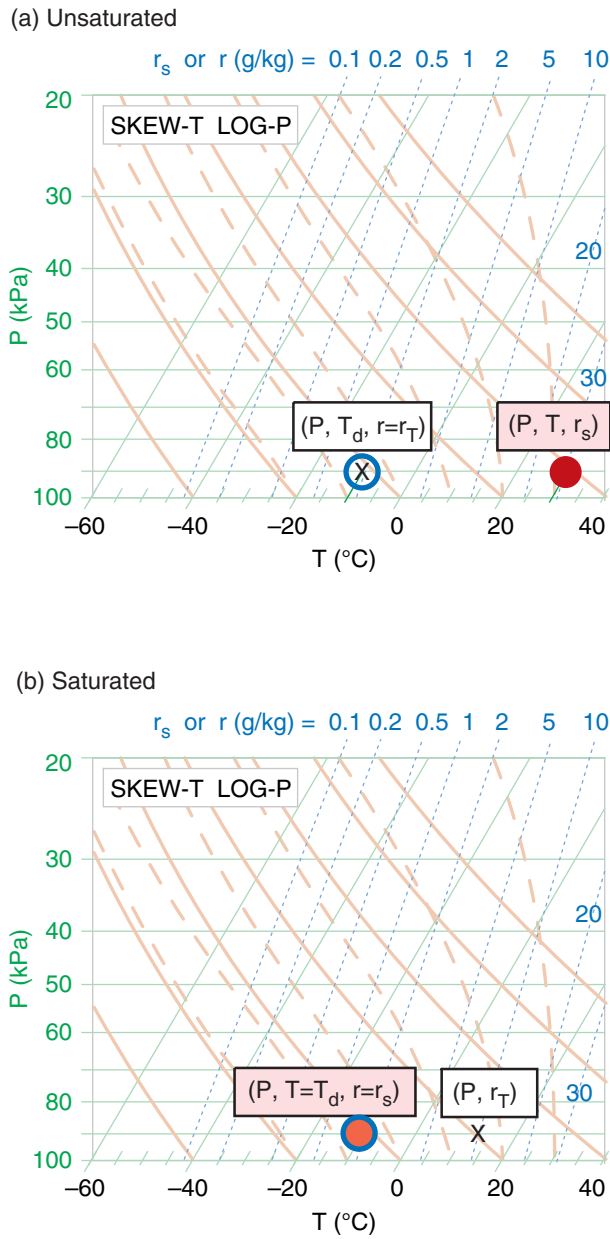


Figure 5.7
 In general, three points are needed to indicate one air parcel on a thermo diagram. The red-dot (red-filled circle) indicates air temperature. The blue circle (with no fill) indicates dew-point temperature (as a measure of the amount of water vapor in the air). The X indicates total water content. (a) Example of an unsaturated air parcel plotted on a skew-T diagram. (b) A different air parcel — this one is saturated. (See the Sample Application for details.)

Thermodynamic State

You need three points on a thermodynamic diagram to indicate air-parcel state (pressure, temperature, humidity). For this one air parcel at one height, all three points are always plotted on the same isobar (P).

One point represents T , the actual air temperature (red dot; red filled circle in Fig. 5.7). Its coordinates in the thermo diagram are given by (P, T, r_s) . Note that r_s is redundant, because it can be determined from P and T . But we include it for convenience, because we use the r_s value for many calculations.

Another point represents T_d , the dew-point temperature (blue circle outline; no fill). Its coordinates are (P, T_d, r) . Again, r is redundant.

The third point gives r_T , the total water mixing ratio in the air parcel (designated by “X” on the thermo diagram). Its coordinates are (P, r_T) .

Often, two of these three points coincide. One example is for unsaturated air (Fig. 5.7a) with no rain falling through it. For this case, the temperature and dew-point circles are plotted as described above. However, the X is plotted inside the dew-point circle because the total water in the air parcel equals the water-vapor mixing ratio ($r_T = r$), which is fully specified by the dew point. Namely, the isohume that passes through the dew-point circle indicates the actual amount of water vapor in the air, and there are no other phases of water in that air parcel.

The isohume that passes through the actual air temperature gives the saturation mixing ratio (r_s , the maximum equilibrium amount of moisture that could be held as vapor). A separate symbol is NOT plotted on the thermo diagram for r_s for two reasons. One, it does not indicate the actual state of the air parcel. Two, it is fully specified as the isohume that passes through the temperature point.

For saturated (cloudy or foggy) air parcels (Fig. 5.7b), the two temperature points coincide ($T = T_d$ and $r = r_s$). In this figure, the red-filled circle still represents air temperature, and the blue circle outline still represents the dew-point temperature (for water vapor). However, the total water X is usually to the right of the temperature circles, because clouds contain liquid water and/or ice in addition to water vapor. This is explained later.

[Tip: On these figures, we use large size circles and X’s to make them easier to see. However, when you plot them on your own thermo diagram, use smaller circles and X’s that you write very precisely on the diagram. Utilize colored pens or pencils if you have them.]

Processes

Unsaturated (Dry) Lifting

Suppose that an air parcel that is initially unsaturated rises without mixing with the surrounding environment, and with no radiative or other heat transfer to or from it. The rising air parcel cools at the dry adiabatic lapse rate, while carrying all the air and water molecules with it. For this case, mixing ratio (r) and potential temperature (θ) are constant.

Starting from the initial state as plotted by the “old” points on the thermo diagram (Fig. 5.8), determine which dry adiabat is under the center of the red filled circle (temperature), then move the red circle up along this adiabat. Similarly for moisture, determine which isohume is under the center of the blue hollow circle and “X”, and follow it up (keeping r constant). Always end the movement of all three points at the same new pressure.

If the points don’t lie directly on a drawn isopleth (which is usually the case), then imagine a new isopleth that goes through your point, and which is parallel to (and maintains its relative distance from) the neighboring drawn isopleths. You can do this by eye, or you may use a straight-edge to draw on the diagram a new isopleth that goes through your point of interest. Then, move the air parcel along this imaginary isopleth. Namely, move the temperature point in a direction parallel to the dry adiabats, and move the humidity point parallel to the isohumes.

Descending unsaturated (non-cloudy) air parcels behave similarly, except moving downward along their appropriate isopleths.

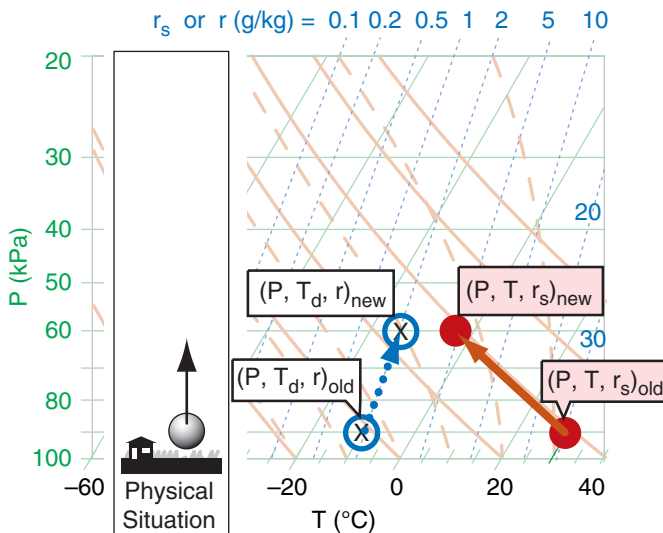


Figure 5.8
Example of the three points (red filled circle for T , open blue circle for T_d , and “X” for r_T) on a skew- T diagram representing the initial (old) state of the air, and how that state changes as the air parcel moves up to a new (lower) pressure level. Note that r_s has changed because the air parcel cooled, but r is unchanged because the air parcel is unsaturated.

Sample Application

- (a) What are the values of pressure, temperature, total water mixing ratio, dew point, saturation mixing ratio, actual mixing ratio, and relative humidity of the “old” air parcel in Fig. 5.8a before it starts to rise?
- (b) Same question, but at its new altitude.

Find the Answer

Given: Use info in Fig. 5.8 to answer this.
Find: $P = ?$ kPa, $T = ?$ °C, $r_T = ?$ g kg⁻¹, $T_d = ?$ °C, $r_s = ?$ g kg⁻¹, $r = ?$ g kg⁻¹, $RH = ?$ %

(a) Get pressure from the isobar (horizontal green line) that passes through the center of both points. The pressure is displayed along the left side of the thermo diagram. $P = 90$ kPa.

Get temperature from the isotherm (green diagonal line) that passes through the center of the red dot. Follow that isotherm diagonally down to the bottom of the graph to read its value. $T = 30$ °C.

Get total water mixing ratio from the isohume (dotted blue line) that passes through the “X”. Read the isohume value at the top of the graph. $r_T = 2$ g kg⁻¹.

Get the dew-point temperature from the isotherm (diagonal green line) that passes through the center of the open blue circle. Follow this line diagonally down to the bottom of the thermo diagram to read its label: $T_d = -10$ °C.

Get the saturation mixing ratio from the isohume (dotted blue line) that passes through the center of the red circle. It is labeled as: $r_s = 30$ g kg⁻¹.

Get the actual mixing ratio from the isohume (dotted blue line) that passes through the center of the open blue circle, and read the label at the top of the graph. $r = 2$ g kg⁻¹.

Relative humidity is not displayed on thermo diagrams. But you can find it from $RH\% = (100\%)(r/r_s) = (100\%)(2/30) = 6.7\%$.

(b) Try this on your own, using the same procedure as described above but for the parcel at the new altitude in Fig. 5.8, and then compare with my answers.

$P = 60$ kPa, $T = -3$ °C, $r_T = 2$ g kg⁻¹, $T_d = -15$ °C, $r_s = 5$ g kg⁻¹, $r = 2$ g kg⁻¹, $RH\% = 40\%$.

Check: Physics and units are reasonable.

Exposition: Although the air was extremely dry initially, as the air parcel was lifted its relative humidity increased. How did this happen, even though the amount of water in the air ($r = 2$ g kg⁻¹) did not change? The reason was that after the air parcel was lifted to its new altitude, it was much colder, and cold air has a smaller capacity to hold water vapor (r_s decreased).

If you were to continue lifting the parcel, eventually it would cool to the point where $r_s = r$, meaning that the air is saturated. The altitude where this happens indicates cloud base for convective clouds (clouds created by local updrafts of air).

You can get more accurate answers using larger thermo diagrams at the end of this chapter.

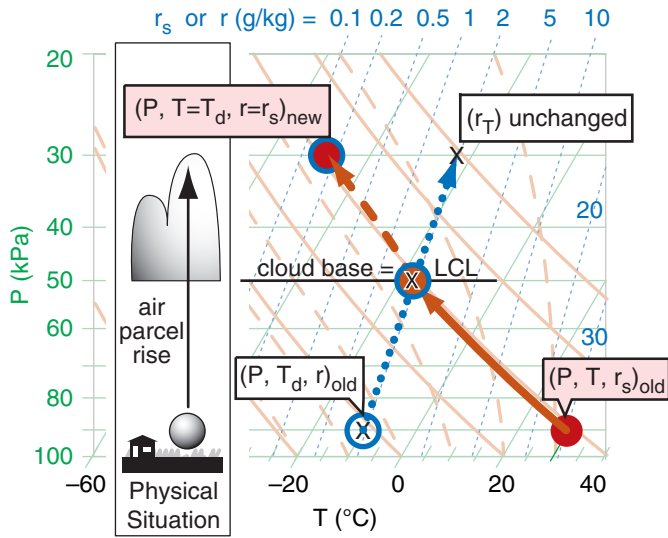


Figure 5.9
 Starting with an initial (old) unsaturated air parcel, if you lift it until the temperature and dew-point are equal (namely, where the dry adiabat intersects the isohume), that height is the LCL (lifting condensation level). At this point, the air parcel is just saturated, hence, this marks cloud base for convective clouds because any further lifting causes water vapor to condense onto dust particles — creating liquid cloud droplets. If the air parcel continues to rise up to a new pressure while carrying all its cloud droplets with it, then the three points on the skew-T diagram move as shown in the skew-T diagram. The amount of liquid water (r_L) tied up in cloud droplets at the ending point equals the total-water mixing ratio (r_T) minus the portion that is still held as vapor (r).

Saturated (Moist) Lifting & Liquid Water

Notice in Fig. 5.8 that the temperature and dew-point curves for the air parcel get closer to each other as the parcel rises. Eventually the dry adiabat and isohume lines cross. The pressure or altitude where they cross (see Fig. 5.9) is defined as the LCL (lifting condensation level), which marks cloud base height for convective clouds (clouds with updrafts from the local region under them, such as cumulus clouds and thunderstorms). This is the altitude where the rising air first becomes saturated — where $T_d = T$, and $r = r_s = r_T$.

Above the LCL the blue circle (T_d) and red dot (T) move together, along a saturated adiabat curve (the orange dashed line that goes through the LCL point). Since saturated air is holding all the vapor that it can, it also means that the actual mixing ratio r equals the saturation value r_s .

If all the condensate (initially as tiny cloud droplets or ice crystals) continues to move with the air parcel and does not fall out as precipitation, then the total-water mixing ratio (r_T) in the parcel is constant. It is easy to estimate the liquid-water mixing ratio (r_L , the relative mass of cloud droplets) as the difference between the total water and the water vapor. In summary, for saturated (cloudy) air: $T = T_d$, $r = r_s$, and $r_L = r_T - r_s$.

Cloudy descending air follows the same lines, but in reverse. Knowing the temperature (red dot with blue circle near the top of Fig. 5.9) and total water content (“X” in that figure) inside the cloud at any one altitude or pressure, then follow the isohume (dotted blue line) down from the “X” and the saturated adiabat (dashed orange line) down from the initial temperature point. As the cloudy air parcel descends it is compressed and warms, allowing more and more of the cloud droplets to evaporate. Eventually, at the LCL, the last droplets have evaporated, so any further descent must follow the dry adiabat (solid orange line) and isohume (dotted blue line) in Fig. 5.9. Namely, the blue circle and the red dot split apart below the LCL.

Precipitation

So far, we assumed that all the liquid water droplets stay inside (move with) the air parcel, meaning $r_T = \text{constant}$. Namely, the “X” in Fig. 5.9 says on one dotted blue line isohume. However, if some of the liquid drops or ice crystals precipitate out of the parcel, then r_T decreases toward r_s , and “X” shifts to a different isohume (Fig. 5.10). The “X” reaches the blue circle ($r_T = r_s$) if all the precipitation falls out.

After precipitation is finished, if there is subsequent ascent or descent, the you must follow the isohume (dotted blue line) that goes through the new location for “X” on the diagram.

Sample Application
 Use Fig. 5.9 to estimate P , T , T_d , r , r_s , r_T , r_L or r_{ice} for the parcel: (a) at the LCL, and (b) at $P = 30$ kPa.

Find the Answer
 Given: The dots, circles, and “X’s” in Fig. 5.9.
 Find: $P = ?$ kPa, $T = ?^\circ\text{C}$, $T_d = ?^\circ\text{C}$,
 $r = ?$ g kg⁻¹, $r_s = ?$ g kg⁻¹, $r_T = ?$ g kg⁻¹, $r_L = ?$ g kg⁻¹

(a) At cloud base (i.e., at the LCL), Fig. 5.9 indicates:
 $P = 50$ kPa, $T = -17^\circ\text{C}$, $T_d = -17^\circ\text{C}$,
 $r = 2$ g kg⁻¹, $r_s = 2$ g kg⁻¹, $r_T = 2$ g kg⁻¹,
 $r_L = r_T - r_s = 0$ g kg⁻¹

(b) At cloud top (assuming no precipitation):
 $P = 30$ kPa, $T = -48^\circ\text{C}$, $T_d = -48^\circ\text{C}$,
 $r = 0.15$ g kg⁻¹, $r_s = 0.15$ g kg⁻¹, $r_T = 2$ g kg⁻¹,
 $r_{ice} = r_T - r_s = 2.0 - 0.15 = 1.85$ g kg⁻¹

Check: Physics and units are reasonable.
Exposition: At temperatures below about -38°C , all hydrometeors are in the form of ice. This is good for aircraft, because ice crystals blow over the wings rather than sticking to them. However, some jet engines have problems when the ice crystals melt and re-freeze on the turbine blades, causing engine problems.

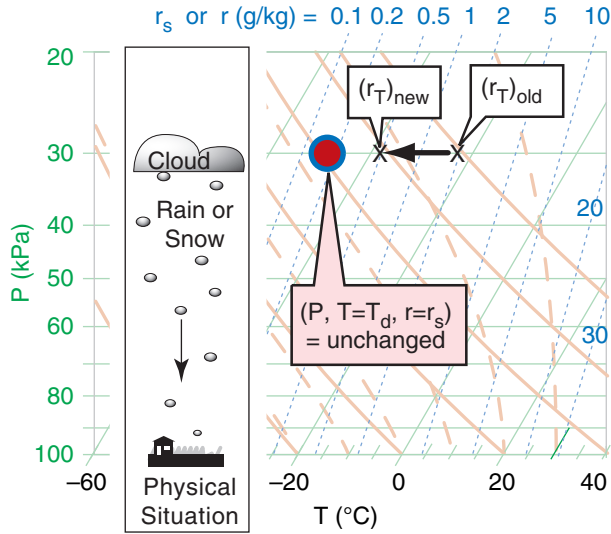


Figure 5.10
Total-water mixing ratio r_T decreases from one isohume (2 g kg^{-1} in this example) to another (0.5 g kg^{-1}) on this skew-T diagram due to precipitation fallout.

Instead of losing water due to precipitation out of an air parcel, sometimes total water is gained when precipitation falls into a parcel from above. Regardless of whether precipitation adds or subtracts water, the process is irreversible, causing a change in the parcel's LCL.

In Fig. 5.10 we illustrate precipitation as happening only while the air parcel is at a fixed pressure or altitude. This is overly simple, as precipitation can also occur while the parcel rises or descends. One way to approximate this on a thermo diagram is to take short adiabatic changes in altitude, and then allow precipitation to occur after each step.

Radiative Heating/Cooling

In the infrared (IR) clouds are blackbodies. They absorb all the IR radiation incident on the outer surface of the cloud, and they emit IR radiation outward in all directions based on the temperature of the cloud droplets on that outer surface.

If the ground and/or atmosphere below cloud base is warmer than the cloud, then there will be more IR radiation absorbed by the cloud base than is re-emitted, and the cloud-base warms. At night, if the ground is colder than cloud base, then the cloud will emit more IR towards the ground than it absorbs, preventing the ground from cooling as much as it would under clear skies, but cooling cloud base.

The atmosphere above the cloud is usually colder than the cloud, and the cloud also “sees” the near-absolute zero temperatures of outer space. Thus, cloud top cools by IR radiation all the time (night and day, see Fig. 5.11). Such cooling creates cold “thermals”

Sample Application

What if ice crystals falling out of the cloud-top air, as illustrated in Fig. 5.10, reduced the total-water mixing ratio from 2 to 0.5 g kg^{-1} . (a) What is its new thermodynamic state at $P = 30 \text{ kPa}$? (b) What is its new P_{LCL} if it were to then descend?

Find the Answer

Given: Fig. 5.10, $r_{T \text{ old}} = 2 \text{ g kg}^{-1}$, and $r_{\text{new}} = 0.5 \text{ g kg}^{-1}$
Find: (a) $P = ? \text{ kPa}$, $T = ?^\circ\text{C}$, $T_d = ?^\circ\text{C}$, $r = ? \text{ g kg}^{-1}$, $r_s = ? \text{ g kg}^{-1}$, $r_T = ? \text{ g kg}^{-1}$, $r_{\text{ice}} = ? \text{ g kg}^{-1}$, $r_{\text{precip}} = ? \text{ g kg}^{-1}$
(b) $P_{LCL} = ? \text{ kPa}$.

(a) New state at cloud top after precipitation:

$$P = 30 \text{ kPa}, T = -48^\circ\text{C}, T_d = -48^\circ\text{C}$$

$$r = 0.15 \text{ g kg}^{-1}, r_s = 0.15 \text{ g kg}^{-1}, r_T = 0.5 \text{ g kg}^{-1}$$

$$r_{\text{ice}} = r_T - r_s = 0.5 - 0.15 = 0.35 \text{ g kg}^{-1}$$

$$r_{\text{precip}} = 2 - 0.5 = 1.5 \text{ g kg}^{-1}$$

(b) New lifting condensation level (LCL):

In Fig. 5.10, from the red dot follow the orange-dashed saturated adiabat downward. (Since no orange dashed line goes through the red dot, we need to interpolate between neighboring dashed lines left and right of the red dot). Also, from the X for the new $r_T = 0.5 \text{ g kg}^{-1}$ value, follow the blue-dotted isohume downward. Where those two lines cross is the new LCL. Doing this on a thermo diagram (not shown) gives roughly $P_{LCL} = 37 \text{ kPa}$.

Check: Physics and units are reasonable.

Exposition: Note that the new LCL is at a higher altitude (lower pressure) than the LCL in Fig. 5.9, because precipitation fallout left the air drier.

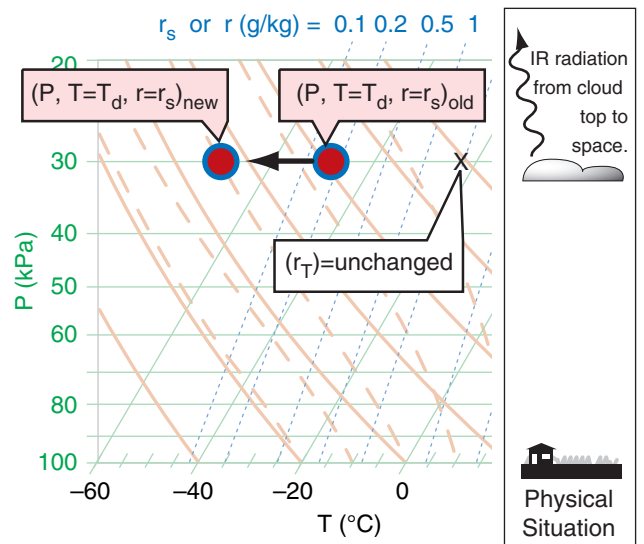


Figure 5.11
Infrared (IR) radiation to space from cloud top can cool the cloud-top air. Such cooling lowers the saturation mixing ratio and forces more water vapor to condense, thus increasing the amount of condensate.

Sample Application

Suppose IR radiation caused the cloud-top thermodynamic state to change as shown in Fig. 5.11, where $T_{new} \approx -71^\circ\text{C}$ and no precipitation falls out. What is the air parcel's new thermodynamic state, and what will be its new P_{LCL} ?

Find the Answer

Given: data in Fig. 5.11

Find: (a) $P = ? \text{ kPa}$, $T = ?^\circ\text{C}$, $T_d = ?^\circ\text{C}$, $r = ? \text{ g kg}^{-1}$,
 $r_s = ? \text{ g kg}^{-1}$, $r_T = ? \text{ g kg}^{-1}$, $r_{ice} = ? \text{ g kg}^{-1}$, $r_{precip} = ? \text{ g kg}^{-1}$
 (b) $P_{LCL} = ? \text{ kPa}$.

(a) Approximate new state, base on visual interpretation of Fig. 5.11:

$$P = \mathbf{30 \text{ kPa}}, \quad T = \mathbf{-71^\circ\text{C}}, \quad T_d = \mathbf{-71^\circ\text{C}},$$

$$r \approx \mathbf{0 \text{ g kg}^{-1}}, \quad r_s \approx \mathbf{0 \text{ g kg}^{-1}}, \quad r_T = \mathbf{2.0 \text{ g kg}^{-1}},$$

$$r_{ice} = r_T - r_s = 2 - 0 = \mathbf{2 \text{ g kg}^{-1}}$$

(b) New lifting condensation level (LCL):

In Fig. 5.11, from the red dot follow the orange-dashed saturated adiabat downward. Also, from the X for $r_T = 2 \text{ g kg}^{-1}$ value, follow the blue-dotted isohume downward. Where those two lines cross is the new LCL. Doing this on a thermo diagram (such as Fig. 5.11) gives roughly $P_{LCL} \approx \mathbf{80 \text{ kPa}}$.

Check: Physics and units are reasonable.

Exposition: The lifting condensation level is at lower altitude (higher pressure) than it was originally.

NOTE: For this and for previous Sample Applications, you might find a slightly different answer if you used a different thermo diagram. This is normal. These slight differences in the new air-parcel state are usually neglected, because there is even greater uncertainty in measured old values of air-parcel state.

in the top 2 cm of cloud-top air that sink lower into the cloud and spread the cooling effect over more of the cloud. This same effect occurs in fog, which tends to make fog denser and more difficult to dissipate.

During daytime light from the sun can reach the clouds, where some is absorbed by the cloud droplets and some is scattered back out (making clouds appear white). The portion absorbed in the tops and sides of clouds is spread over a thicker regions of cloud that extends about 2 m into the cloud from the outer surface. This means that heating due to sunlight is spread over a thicker layer of cloud, as opposed to IR cooling which happens only at the immediate cloud surface.

Recall that the word "adiabatic" means no heat is transferred between the air parcel and the surrounding environment. Radiation is not adiabatic, and hence is called **diabatic**. In Fig. 5.11, the diabatic IR cooling causes the red dot and blue circle to move together ($T = T_d$) to colder temperatures, while the total-water mixing ratio "X" is unchanged because radiation does not create or destroy water molecules. Nonetheless, as temperature drops, $r_s (= r)$ decreases, causing liquid r_L or solid r_{ice} water to increase (according to $r_{L \text{ new OR } r_{ice \text{ new}}} = r_T - r_{s \text{ new}}$) in the absence of precipitation.

Solar heating of the cloud would cause diabatic warming on the thermo diagram.

Radiation is important in cloud evolution and cloud dynamics, particularly for the anvils of mesoscale convective systems (thunderstorms), hurricanes, and frontal stratus clouds (see the Clouds chapter).

INFO • Homeostasis and LeChatelier's Principle

Many systems in nature are in equilibrium as a result of a balance between opposing forcings. For such a system, if one of the forcings is changed, the system responds in a way to reach a new equilibrium that partially undoes the changed forcing.

In chemistry, this is known as **LeChatelier's Principle**. For example, if two reagents are in equilibrium in a beaker, then if you add more of one of the reagents, a reaction occurs that reduces the increase of concentration of the added reagent.

For global climate, the principle is known as **homeostasis**. For example, increased solar radiation will cause the earth-system temperature to increase, thereby causing increased cooling due to outgoing IR radiation (due to the Stefan-Boltzmann law) which partially compensates the original heating.

Many external influences (particularly solar heating) create instabilities in the atmosphere. The atmosphere responds by generating motions on a wide range of scales. These motions transport heat that partially undoes the effect of the external influence. We experience these responses as the global circulation, weather systems and storms.

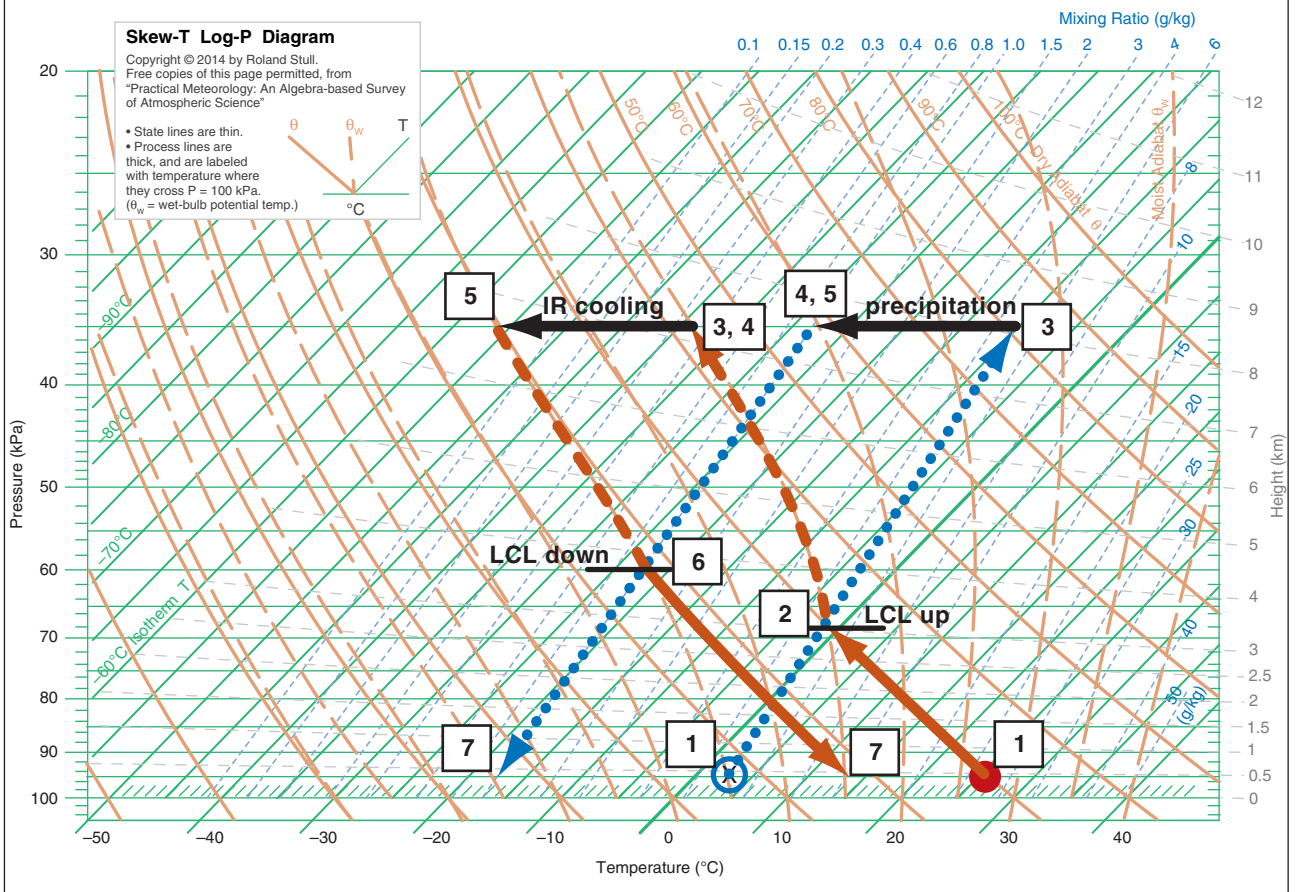
Sample Application

Suppose an air parcel has an initial state of $(P, T, T_d) = (95 \text{ kPa}, 26^\circ\text{C}, 3^\circ\text{C})$. It rises to an altitude where $P = 35 \text{ kPa}$, which represents cloud top. While there, precipitation removes water amount $\Delta r_T = 4 \text{ g kg}^{-1}$, and IR cooling lowers the temperature by $\Delta T = -18^\circ\text{C}$. If the air parcel then returns back to its original pressure, what is its final thermodynamic state? Use the full-size skew-T from the end of this chapter for this exercise. Explain all steps.

Find the Answer

Given: $(P, T, T_d)_{\text{initial}} = (95 \text{ kPa}, 26^\circ\text{C}, 3^\circ\text{C})$. $P_{\text{top}} = 35 \text{ kPa}$. $\Delta r = -4 \text{ g kg}^{-1}$, and $\Delta T = -18^\circ\text{C}$.
 Find: $(P, T, T_d)_{\text{final}} = (95 \text{ kPa}, ?^\circ\text{C}, ?^\circ\text{C})$

- [1] The red dot and blue circle show the initial temperature and dew-point of the air parcel, respectively. Interpolating between the 4 and 6 g kg^{-1} isohumes, it appears that the initial dew point corresponds to roughly $r \approx 5 \text{ g kg}^{-1}$, which is plotted as the line with large blue dots with upward arrow.
- [2] From [1] follow the dry adiabat for T (solid orange line), and follow the isohume for T_d (dotted blue line). They cross at an LCL of about $P_{\text{LCL}} \approx 68 \text{ kPa}$ (given by the horizontal black line through that LCL point, and interpolating between the horizontal green lines to find the pressure). This is cloud base for ascending air.
- [3] As the air parcel continues to rise to cloud top at $P = 35 \text{ kPa}$, the temperature and dew-point follow the saturated adiabat (dashed orange line). Thus, at cloud top, $T = T_d \approx -38^\circ\text{C}$ (which is the isotherm (diagonal green line) that goes through temperature point [3] at the tip of the upward arrowhead. The total-water mixing ratio is conserved and still follows the 5 g kg^{-1} isohume.
- [4] Precipitation removes 4 g kg^{-1} from the parcel, leaving $r_T = 5 - 4 = 1 \text{ g kg}^{-1}$ as indicated by the dotted blue isohume. But the parcel air temperature is unchanged from point [3].
- [5] Infrared (IR) radiative cooling decreases the temperature by 18°C , causing a new cloud-top temperature of $T = T_d \approx -38 - 18 = -56^\circ\text{C}$. The corresponding r_s value is so small that it is below 0.1 g kg^{-1} . But the total water content is unchanged from point [4].
- [6] As this air is saturated (because $r_s < r_T$), the descent follows a saturated adiabat (dashed orange line) down to where the saturated adiabat crosses the isohume (dotted blue line). This intersection marks the LCL for descending air, which is at about $P_{\text{LCL}} = 60 \text{ kPa}$ (interpolate between the horizontal green line to the black line through this point).
- [7] Below this cloud base, T follows the dry adiabat (solid orange line), ending with $T \approx 14^\circ\text{C}$ at $P = 95 \text{ kPa}$. The dew-point follows the 1 g kg^{-1} isohume (dotted blue line), giving a final $T_d \approx -17^\circ\text{C}$. $(P, T, T_d)_{\text{final}} = (95 \text{ kPa}, 14^\circ\text{C}, -17^\circ\text{C})$



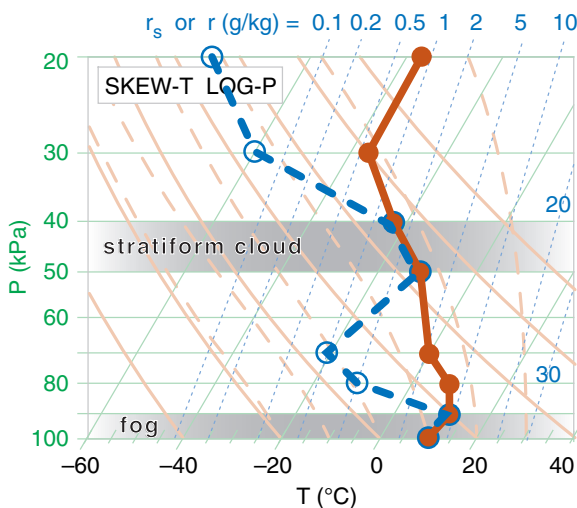
Sample Application

Suppose you made a sounding by releasing a radiosonde balloon, and it reported the following data. Plot it on a skew-T log-P diagram.

P (kPa)	T (°C)	T_d (°C)
20	-36	-80
30	-36	-60
40	-23	-23
50	-12	-12
70	0	-20
80	8	-10
90	10	10
100	10	10

Find the Answer

Here I used red for temperature (T) and blue for dewpoint (T_d). Always connect the data points with straight lines.



Check: Sketches OK. Also, dew point is never greater than temperature, as required.

Exposition: There are two levels where dew-point temperature equals the actual air temperature, and these two levels correspond to saturated (cloudy) air. We would call the bottom layer **fog**, and the elevated one would be a mid-level stratiform (layered) cloud. In the Clouds chapter you will learn that this type of cloud is called **altostratus**. (Note, from the sounding it is impossible to diagnose the detailed structure of that cloud, so it could be **altocumulus**.)

There are also two layers of dry air, where there is a large spread between T and T_d . These layers are roughly 80 to 60 kPa, and 35 to 20 kPa.

AN AIR PARCEL & ITS ENVIRONMENT

Consider the column of air that is fixed (Eulerian framework) over any small area on the globe. Let this air column represent a stationary **environment** through which other things (aircraft, raindrops, air parcels) can move.

For an air parcel that moves through this environment, we can follow the parcel (Lagrangian framework) to determine how its thermodynamic state might change as it moves to different altitudes. As a first approximation, we often assume that there is no mixing between the air parcel and the surrounding environment.

Soundings

As stated at the start of this chapter, the vertical profile of environmental conditions is called an **upper-air sounding**, or just a **sounding**. The word “sounding” is used many ways. It is the:

- activity of collecting the environment data (as in “to make a sounding”).
- data that was so collected (as in “to analyze the sounding data”).
- resulting plot of these data on a thermo diagram (as in “the sounding shows a deep unstable layer”).

A **sonde** is an expendable weather instrument with built-in radio transmitter that can be attached to a platform that moves up or down through the air column to measure its environmental state. Helium-filled latex weather balloons carry **radiosondes** to measure thermodynamic state (P, T, T_d) or carry **rawinsondes** measure (z, P, T, T_d, U, V) by also utilizing GPS or other navigation signals. **Rocketsondes** are lofted by small sounding rockets, and **dropsondes** are dropped by parachute from aircraft. **Unmanned aerial vehicles (UAVs)** and conventional aircraft can carry weather sensors. Radar and satellites are **remote sensors** that can also measure portions of soundings.

The sounding represents a snapshot of the state of the air in the environment, such as in the Sample Application on this page. The plot of any dependent variable vs. height or pressure is called a **vertical profile**. The negative of the vertical temperature gradient ($-\Delta T_e / \Delta z$) is defined as the **environmental lapse rate**, where subscript e means environment.

Normally, temperature and dew point are plotted at each height. Other chemicals such as smoke particles or ozone can also be plotted as a sounding. It is difficult to measure density, so instead its value at any height is calculated from the ideal gas law. For cloudy air, the T_d points coincide with the T points. Most sondes have some imprecision, so we infer clouds in any layer where T_d is near T .

Straight lines are drawn connecting the temperature points (see INFO box on **mandatory** and **significant levels**), and separate straight lines are drawn for the humidity points for unsaturated air. Liquid or ice mixing ratios are not usually measured by radiosondes, but can be obtained by **research aircraft** flying slant ascent or descent soundings.

Buoyant Force

For an object such as an air parcel that is totally immersed in a fluid (air), buoyant force per unit mass (F/m) depends on the density difference between the object (ρ_o) and the surrounding fluid (ρ_f):

$$\frac{F}{m} = \frac{\rho_o - \rho_f}{\rho_o} \cdot g = g' \tag{5.2}$$

where gravitational acceleration (negative downward) is $g = -9.8 \text{ m}\cdot\text{s}^{-2}$. If the immersed object is **positively buoyant** ($F = \text{positive}$) then the buoyant force pulls upward. **Negatively buoyant** objects ($F = \text{negative}$) are pulled downward, while **neutrally buoyant** objects have zero buoyant force.

Recall that weight is the force caused by gravity acting on a mass ($F = m\cdot g$). Since buoyancy changes the net force (makes the weight less), the net effect on the object is that of a **reduced gravity** (g') as defined in eq. (5.2). Thus $F = m\cdot g'$.

Identify the air parcel (subscript p) as the object, and the environment (subscript e) as the surrounding fluid. Because both air parcels and the surrounding environment are made of air, use the ideal gas law ($P = \rho \mathfrak{R} T_v$) to describe the densities in eq. (5.2).

$$\frac{F}{m} = \frac{T_{ve} - T_{vp}}{T_{ve}} \cdot g = g' \tag{5.3a}$$

where the pressure inside the parcel is assumed to always equal that of the surrounding environment at the same height, allowing the pressures from the ideal gas law to cancel each other in eq. (5.3a).

Virtual temperature T_v (in Kelvins) is used to account for the effects of both temperature and water vapor on the buoyancy. For relatively dry air, $T_v \approx T$, giving:

$$\frac{F}{m} \approx \frac{T_p - T_e}{T_e} \cdot |g| = g' \tag{5.3b}$$

Namely, parcels warmer than their environment tend to rise. Colder parcels tend to sink. As air parcels move vertically, their temperatures change as given by adiabatic (dry or saturated) lapse rates, so you need to repeatedly re-evaluate the buoyancy of the parcel relative to its new local environment. Thermo diagrams are extremely handy for this, as sketched in Fig. 5.12.

INFO • Mandatory & Significant Levels

Rising rawinsondes record the weather at $\Delta z \approx 5 \text{ m}$ increments, yielding data at about 5000 heights. To reduce the amount of data transmitted to weather centers, straight-line segments are fit to the sounding, and only the end points of the line segments are reported. These points, called **significant levels**, are at the kinks in the sounding (as plotted in the previous Sample Application). Additional **mandatory levels** at the surface and 100, 92.5, 85, 70, 50, 40, 30, 25, 20, 15, 10, 7, 5, 3, 2, & 1 kPa are also reported, to make it easier to analyze **upper-air charts** at these levels.

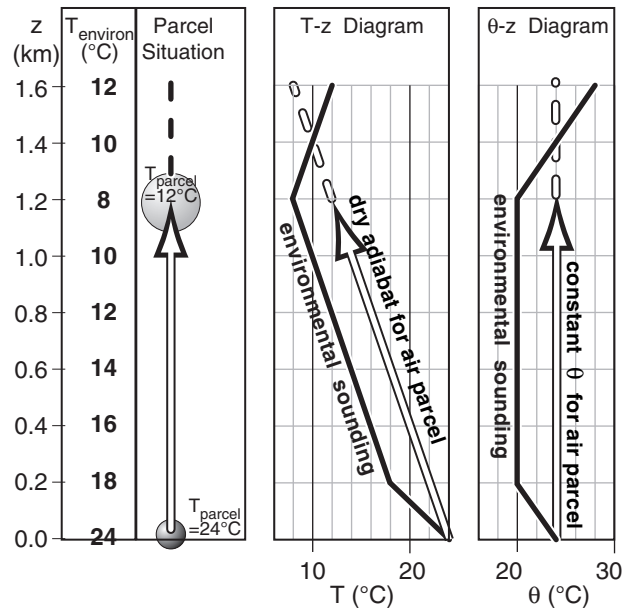


Figure 5.12
 Buoyancy at any one height (z) depends on the temperature (T) or potential-temperature (θ) difference between the environment and the air parcel. A thermo diagram aids this, by allowing us to see the background environment and the air-parcel temperature at every height. In this example for unsaturated air lifted from the surface, the parcel is positively buoyant up to $z = 1.4 \text{ km}$. Above that altitude, the parcel is colder than the environment and is negatively buoyant.

Sample Application

When the air parcel in Fig. 5.12 reaches 1 km altitude, what is its buoyant force/mass? The air is dry.

Find the Answer

Given: $T_e = 10^\circ\text{C} + 273 = 283 \text{ K}$, $T_p = 14^\circ\text{C} + 273 = 287 \text{ K}$
 Find: $F/m = ? \text{ m}\cdot\text{s}^{-2}$

Apply eq. (5.3a) for dry air:

$$F/m = (287 \text{ K} - 283 \text{ K}) \cdot (9.8 \text{ m}\cdot\text{s}^{-2}) / 283 \text{ K} = \mathbf{0.14 \text{ m}\cdot\text{s}^{-2}} = 0.14 \text{ N/kg}$$

(see units in Appendix A)

Check: Physics and units are reasonable.

Exposition: Positive buoyancy force favors rising air.

Sample Application

- a) Use a thermo diagram to estimate the potential-temperature values for each point in the environmental sounding, and plot your answers a new, linear graph of θ vs. P . Assume dry air. Environmental sounding: $[P(\text{kPa}), T(^{\circ}\text{C})] = [100, 20], [95, 10], [85, 0], [75, 0], [70, -4]$
- b) An air parcel at $[P(\text{kPa}), T(^{\circ}\text{C})] = [100, 23]$ moves to a new height of 80 kPa. Plot its path on your θ vs. P graph, and find its new buoyant force/mass at 80 kPa.

Find the Answer

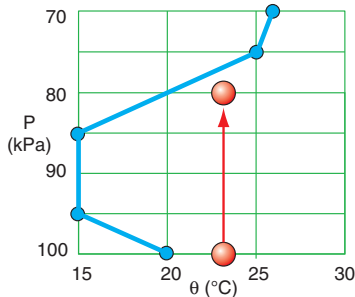
Given: The T vs. P data above. Dry, thus $T_v = T$.
 Find: (a) Estimate θ from thermo diag. & plot vs. P .
 (b) Plot parcel rise on the same graph.
 Calculate buoyant F/m at $P = 80$ kPa.

Method

a) From the large thermo diagrams at the chapter end, the environment potential temperature sounding is:

$[P(\text{kPa}), \theta(^{\circ}\text{C})]$
[100,20]
[95,15]
[85,15]
[75,25]
[70,26]

See blue line in the figure:



b) The parcel initial state is $[P(\text{kPa}), \theta(^{\circ}\text{C})] = [100, 23]$. See red line in figure. At $P = 80$ kPa, the figure shows $\Delta\theta = 23 - 20^{\circ}\text{C} = 3^{\circ}\text{C} = 3$ K. Because dry air: $\Delta\theta_v = \Delta\theta$. $T_v = T \approx 10^{\circ}\text{C} = 283$ K as approximate average T . Apply eq. (5.3d):
 $F/m = (3\text{K}/283\text{K}) \cdot (9.8 \text{ m s}^{-2}) = 0.10 \text{ m s}^{-2} = \mathbf{0.10 \text{ N kg}^{-1}}$

Check: Physics and units are reasonable.
Exposition: The upward buoyancy force causes the air parcel to continue to rise, until it hits the blue line.

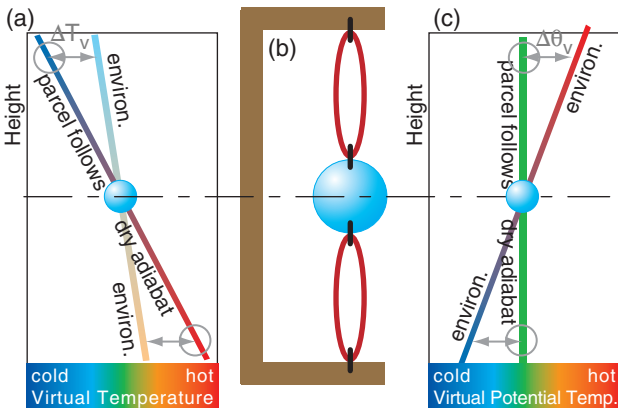


Figure 5.13
 (a) In a statically stable environment, a buoyantly neutral air parcel (blue) becomes colder than the environment when lifted (top grey circle) and warmer if lowered (bottom grey circle). (b) The opposing forces act on the parcel similar to a weight suspended between two elastic bands. (c) Same concept using θ_v .

Sometimes it is more convenient to use virtual potential temperature θ_v (eqs. 3.13, 3.14) to calculate buoyant force, because adiabatic parcel motion has constant θ_{vp} . Buoyant force per mass is then:

$$\frac{F}{m} = \frac{\theta_{ve} - \theta_{vp}}{T_{ve}} \cdot g = g' \tag{5.3c}$$

where gravitational acceleration ($g = -9.8 \text{ m s}^{-2}$) is a negative number in eq. (5.3c) because it acts downward. Subscript e indicates “environment”.

Many meteorologists prefer to use the magnitude of gravitational acceleration as a positive number:

$$\frac{F}{m} = \frac{\theta_{vp} - \theta_{ve}}{T_{ve}} \cdot |g| = g' \tag{5.3d}$$

which more obviously shows that upward buoyancy force occurs for a parcel warmer than the environment at its same altitude or pressure.

Picture the physical situation in the left third of Fig. 5.12 on the previous page. Suppose you grab a small piece of the environmental air near the ground and lift it. As the parcel rises, it cools adiabatically. It is moving through a somewhat static environment, and thus is encountering different surrounding (environmental) temperatures at different altitudes (represented by the colored background). Namely, both the parcel temperature and the environmental temperature are different at different altitudes.

We need to know these different temperatures at each altitude in order to determine the temperature difference and the resulting parcel buoyancy from eqs. (5.3). To make our lives easier, we use thermo diagrams on which we can plot the background environment, and which show dry adiabats as relate to parcel rise. Two examples are the emagram and θ - Z diagrams in Fig. 5.12. These both represent the same parcel and same environment.

Brunt-Väisälä Frequency

Suppose the ambient environment lapse rate Γ is less the dry adiabatic lapse rate ($\Gamma_d = 9.8^{\circ}\text{C km}^{-1}$). Namely, temperature does not decrease as fast with increasing height as Γ_d (see Fig. 5.13a).

Suppose you start with an air parcel having the same initial temperature as the environment (the blue sphere in Fig. 5.13a) at any initial altitude. If you push the air parcel down, it warms at the adiabatic lapse rate, which makes it warmer than the surrounding environment at its new height. This positive buoyant force would tend to push the parcel upward. The parcel’s inertia would cause it to overshoot upward past its initial height, where adiabatic cooling would create a negative buoyant force that reverses the parcel’s upward motion.

In summary, a parcel that is displaced vertically from its initial altitude will oscillate up and down. The environment that enables such an oscillation is said to be statically stable. The oscillation is similar to that experienced by a vertically displaced weight suspended between two elastic bands (Fig. 5.13b).

The air parcel oscillates vertically with frequency (radians s⁻¹)

$$N_{BV} = \sqrt{\frac{|g|}{T_v} \cdot \left(\frac{\Delta T_v}{\Delta z} + \Gamma_d \right)} \quad \bullet(5.4a)$$

where N_{BV} is the Brunt-Väisälä frequency, and $|g| = 9.8 \text{ m s}^{-2}$ is the magnitude of gravitational acceleration. Virtual temperature T_v must be used to account for water vapor, which has lower density than dry air. If the air is relatively dry, then $T_v \approx T$. Use absolute units (K) for T_v in the denominator.

The same frequency can be expressed in terms of virtual potential temperature θ_v (see Fig. 5.13c):

$$N_{BV} = \sqrt{\frac{|g|}{T_v} \cdot \frac{\Delta \theta_v}{\Delta z}} \quad \bullet(5.4b)$$

The oscillation period is

$$P_{BV} = \frac{2\pi}{N_{BV}} \quad (5.5)$$

If the static stability weakens (as the environmental $\Delta T/\Delta z$ approaches $-\Gamma_d$), then the frequency decreases and the period increases toward infinity. The equations above are idealized, and neglect the damping of the oscillation due to air drag (friction).

Sample Application

For a dry standard-atmosphere defined in Chapter 1, find the Brunt-Väisälä frequency & period at $z = 4 \text{ km}$.

Find the Answer

Given: $\Delta T/\Delta z = -6.5 \text{ K/km}$ from eq. (1.16).

$T_v = T$ because dry air.

Use Table 1-5 at $z = 4 \text{ km}$ to get: $T = -11^\circ\text{C} = 262 \text{ K}$

Find: $N_{BV} = ? \text{ rad s}^{-1}$, $P_{BV} = ? \text{ s}$

Apply eq. (5.4a)

$$N_{BV} = \sqrt{\frac{(9.8\text{ms}^{-2})}{262\text{K}} \cdot \left(-6.5 + 9.8 \frac{\text{K}}{\text{km}} \right) \cdot \left(\frac{1\text{km}}{1000\text{m}} \right)}$$

$$= [1.234 \times 10^{-4} \text{ s}^{-2}]^{1/2} = \underline{0.0111 \text{ rad s}^{-1}}$$

Apply eq. (5.5):

$$P_{BV} = \frac{2\pi \text{ radians}}{0.0111 \text{ radians/s}} = \underline{565.5 \text{ s}} = 9.4 \text{ min}$$

Check: Physics and units are reasonable.

Exposition: The Higher Math box at right shows that N_{BV} must have units of (radians s⁻¹) because it is the argument of a sine function. But meteorologists often write the units as (s⁻¹), where the radians are implied.

HIGHER MATH • Brunt-Väisälä Frequency

Create the Governing Equations

Consider a scenario as sketched in Fig. 5.13a, where the environment lapse rate is Γ and the dry adiabatic lapse rate is Γ_d . Define the origin of a coordinate system to be where the two lines cross in that figure, and let z be height above that origin. Consider dry air for simplicity ($T_v = T$). Based on geometry, Fig. 5.13a shows that the temperature difference between the two lines at height z is $\Delta T = (\Gamma - \Gamma_d)z$.

Eq. (5.3b) gives the corresponding vertical force F per unit mass m of the air parcel:

$$F/m = |g| \cdot \Delta T/T_e = |g| \cdot [(\Gamma - \Gamma_d)z]/T_e$$

But $F/m = a$ according to Newton's 2nd law, where acceleration $a \equiv d^2z/dt^2$. Combining these eqs. gives:

$$\frac{d^2z}{dt^2} = \frac{|g|}{T_e} \cdot (\Gamma - \Gamma_d) \cdot z$$

Because T_e is in Kelvin, it varies by only a small percentage as the air parcel oscillates, so approximate it with the average environmental temperature \bar{T}_e over the vertical span of oscillation.

All factors on the right side of the eq. above are constant except for z , so to simplify the notation, use a new variable $N_{BV}^2 \equiv -|g| \cdot (\Gamma - \Gamma_d) / \bar{T}_e$ to represent those constant factors. [This definition agrees with eq. (5.4a) when you recall that $\Gamma = -\Delta T/\Delta z$.]

This leaves us with a second-order differential equation (a hyperbolic differential eq.) known as the wave equation:

$$\frac{d^2z}{dt^2} = -N_{BV}^2 \cdot z \quad (5a)$$

Solve the Differential Equation

For the wave equation, try a wave solution:

$$z = A \cdot \sin(ft) \quad (5b)$$

with unknown amplitude A and unknown frequency of oscillation f (radians s⁻¹). Insert (5b) in (5a) to get:

$$-f^2 \cdot A \cdot \sin(f \cdot t) = -N_{BV}^2 \cdot A \cdot \sin(f \cdot t)$$

After you cancel the $A \cdot \sin(ft)$ from each side, you get:

$$f = N_{BV}$$

Using this in eq. (5b) gives an air parcel height that oscillates in time:

$$z = A \cdot \sin(N_{BV} \cdot t) \quad (5c)$$

Exposition

The amplitude A is the initial distance that you displace the air parcel from the origin. If you plug eq. (5.5) into (5c) you get: $z = A \cdot \sin(2\pi t/P_{BV})$, which gives one full oscillation during a time period of $t = P_{BV}$.

FLOW STABILITY

Stability is a characteristic of how a system reacts to small disturbances. If the disturbance is damped, the system is said to be **stable**. If the disturbance causes an amplifying response (irregular motions or regular oscillations), the system is **unstable**.

For **fluid-flow stability** we will focus on turbulent responses spanning the smallest eddies to deep thunderstorms. The stability characteristics are:

- **Unstable** air becomes, or is, **turbulent** (irregular, gusty, stormy).
- **Stable** air becomes, or is, **laminar** (non-turbulent, smooth, non-stormy).
- **Neutral** air has no tendency to change (disturbances neither amplify or dampen).

Flow stability is controlled by ALL the processes (buoyancy, inertia, wind shear, rotation, etc.) acting on the flow. However, to simplify our understanding of flow, we sometimes focus on just a subset of processes. If you ignore all processes except buoyancy, then you are studying **static stability**. If you include buoyancy and wind-shear processes, then you are studying **dynamic stability**.

Static Stability

One way to estimate static stability is by taking a small piece of the environment and hypothetically displacing it as an air parcel a small distance vertically from its starting point, assuming the surrounding environment is horizontally large and is quasi-stationary. Another is to lift whole environmental layers. We will look at both methods.

Parcel Method for Static Stability

Will a displaced air parcel experience buoyancy forces that push it in the same direction it was displaced (i.e., an amplifying or unstable response), or will buoyancy forces tend to push the parcel back to its starting (equilibrium) height (a stable response)?

The answer to this question is tricky, because the region of unstable response can span large vertical regions. For example, a parcel that is warmer than its environment will keep rising over large distances so long as it remains warmer than its surrounding environment at the same altitude as the parcel.

This type of buoyancy motion, called **convection**, stirs the air and generates turbulence over the whole span of its rise. Namely, a thin region that is locally stable might become turbulent if an air parcel moves through it from some distant source. Such distant effects are known as **nonlocal turbulence**, and must be considered when determining stability.

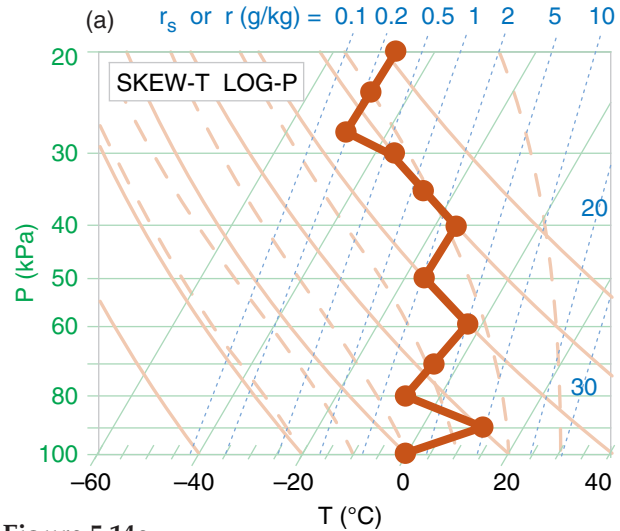


Figure 5.14a
Static stability determination. Step 1: Plot the sounding. (This sounding is contrived to illustrate all stabilities.)

Step 1: Graphical methods are usually best for determining static stability, in order to account for nonlocal effects. Plot the environmental sounding on a thermo diagram (Fig. 5.14a). Any type of thermo diagram will work — they all will give the same stability determination.

Step 2: At every apex (kink in the sounding), hypothetically displace an air parcel from that kink a small distance upward. If the parcel is unsaturated, follow a dry adiabat upward to determine the parcel's new temperature. If saturated, follow a moist adiabat. If that displaced parcel experiences an upward buoyant force (i.e., is warmer than its surrounding environment at its displaced height), then it is locally unstable, so write the letter "U" just above that apex (Fig 5.14b). If the upward displaced

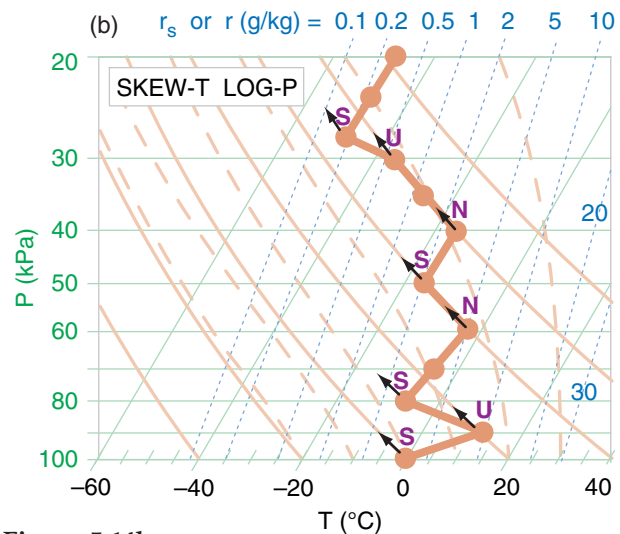


Figure 5.14b
Static stability determination. Step 2: Upward displacements. (Assume this contrived sounding is unsaturated.)

parcel experiences no force (i.e., is nearly the same temperature as the environment at its displaced height), then write “N” just above that apex because it is locally neutral. If the upward displaced parcel experiences a downward force (i.e., is cooler than the environment at its displaced height), then write “S” just above the apex because it is locally stable.

Step 3: Do a similar exercise at every apex, but hypothetically displacing the air parcel downward (Fig. 5.14c). If the parcel is cooler than its environment at its displaced height and experiences a downward force, then it is locally unstable, so write “U” just below that apex. If the parcel is the same temperature as the environment at its displaced height, write “N” just below the apex. If the parcel is warmer than the environment at its displaced height, then write “S” just below the apex.

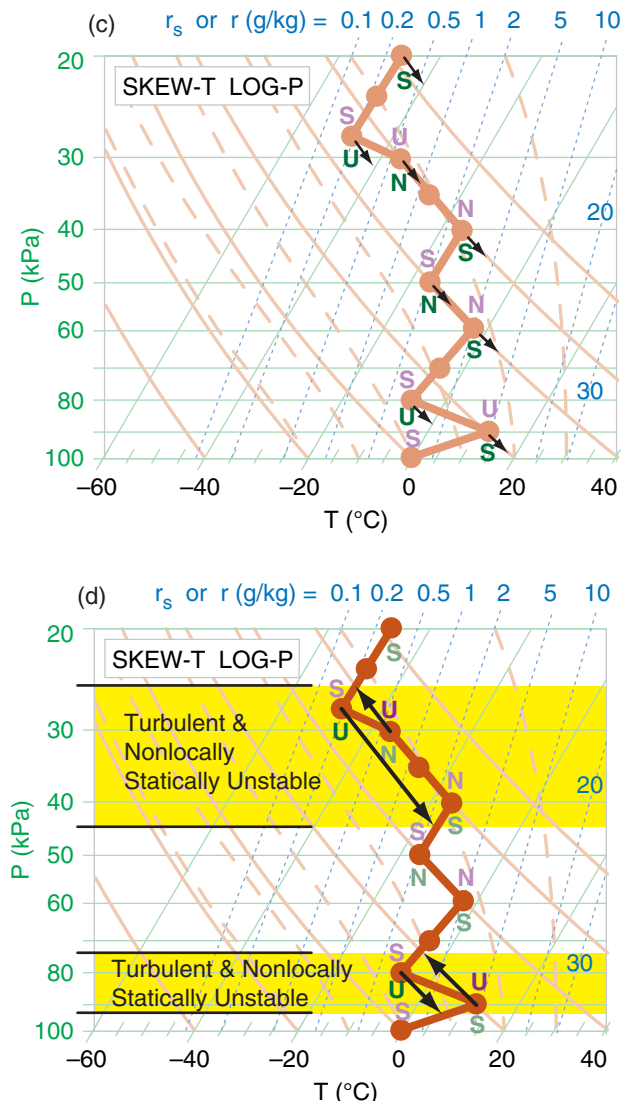


Figure 5.14c & d
 Static stability determination. (c) Step 3: Downward displacement. (d) Step 4: Identify unstable regions (shaded yellow).

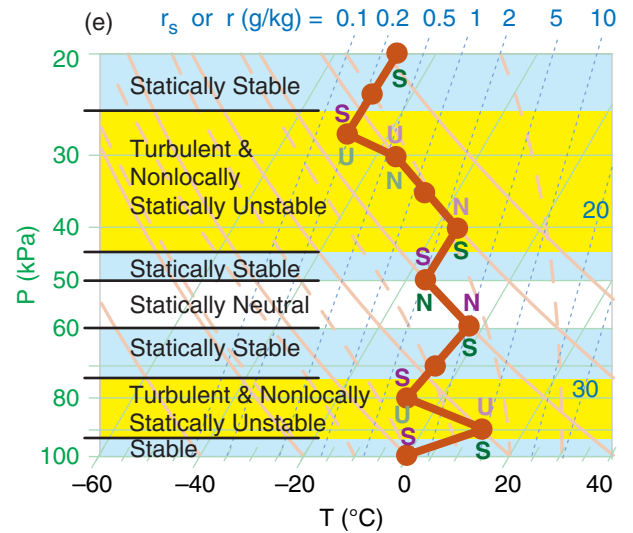


Figure 5.14e
 Static stability determination. Step 5: Identify statically stable regions (shaded light blue) and neutral regions (no shading).

Step 4: From every apex with U above, draw an arrow that follows the adiabat upward until it hits the sounding (Fig. 5.14d). From every apex with U below, draw an arrow that follows the adiabat downward until it hits the sounding. All domains spanned by these “U” arrows are turbulent and are nonlocally statically unstable. Any vertically overlapping “U” regions should be interpreted as one contiguous nonlocally unstable and turbulent region.

Step 5: Any regions outside of the unstable regions from the previous step are stable or neutral, as indicated by the “S” or “N” letters next to those sounding line segments (Fig. 5.14e).

Within statically **neutral** subdomains, the vertical gradients of temperature T or potential temperature θ are:

$$\frac{\Delta T}{\Delta z} \approx - \begin{cases} \Gamma_d & \text{if unsaturated} \\ \Gamma_s & \text{if saturated} \end{cases} \quad \bullet(5.6a)$$

or

$$\frac{\Delta \theta}{\Delta z} \approx \begin{cases} 0 & \text{if unsaturated} \\ \Gamma_d - \Gamma_s & \text{if saturated} \end{cases} \quad \bullet(5.6b)$$

where $\Gamma_d = 9.8 \text{ K km}^{-1}$ is the dry adiabatic lapse rate, and Γ_s is the saturated (or moist) adiabatic lapse rate (which varies, but is always a positive number).

In statically **stable** subdomains, the temperature does not decrease with height as fast as the adiabatic rate (including isothermal layers, and **temperature inversion** layers where temperature increases with height). Thus:

$$\frac{\Delta T}{\Delta z} > - \begin{cases} \Gamma_d & \text{if unsaturated} \\ \Gamma_s & \text{if saturated} \end{cases} \quad \bullet(5.7a)$$

or

$$\frac{\Delta \theta}{\Delta z} > \begin{cases} 0 & \text{if unsaturated} \\ \Gamma_d - \Gamma_s & \text{if saturated} \end{cases} \quad \bullet(5.7b)$$

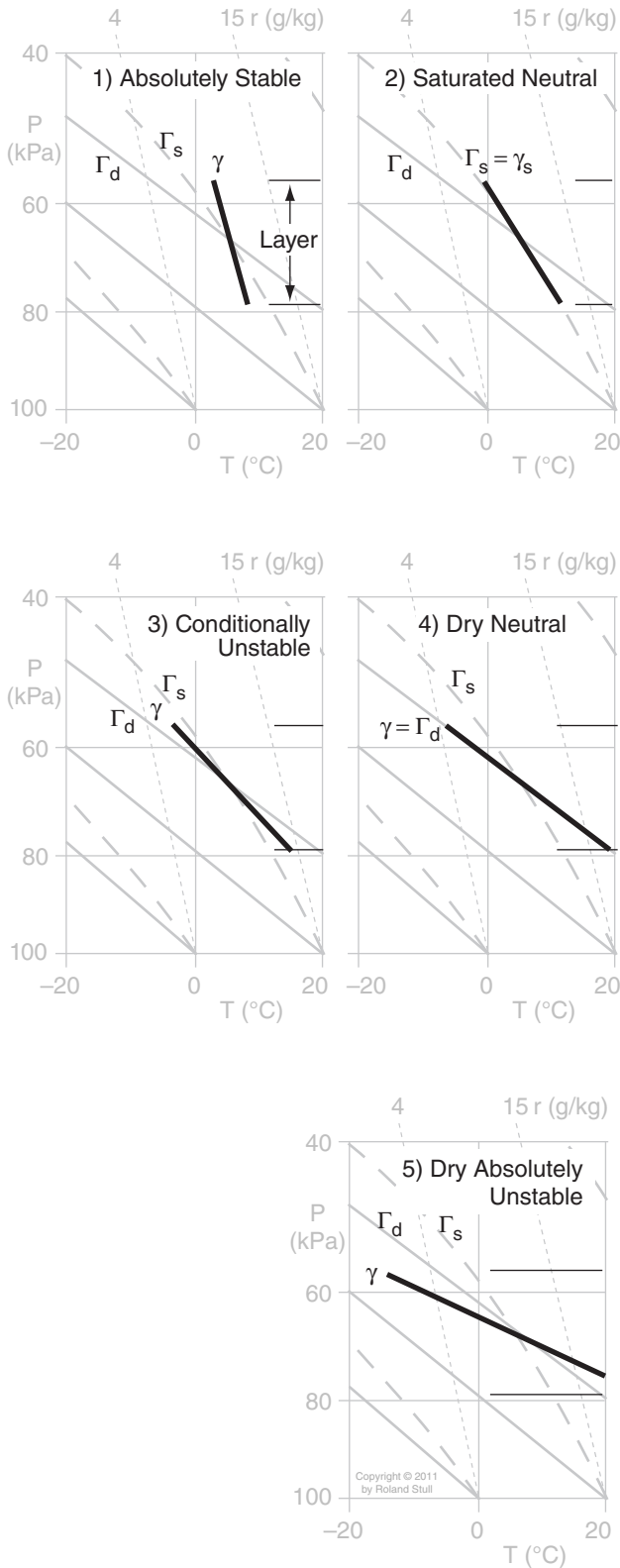


Figure 5.15
 Layer stabilities, shown on a portion of an Emagram. The thick solid line is environmental air temperature within one layer of air (i.e., a line segment from a complete sounding), and the thick dotted line is environmental dew-point temperature. Γ_d and Γ_s are the dry and saturated adiabatic lapse rates.

Layer Method for Static Stability

Sometimes a whole layer of environmental air is lifted or lowered by an outside process. Synoptic-scale warm and cold fronts (see the Fronts & Air-masses chapter) and coherent clusters of thunderstorms (**mesoscale convective complexes**, see the Thunderstorm chapters) are examples of layer-lifting processes. In these cases we cannot use the parcel method to determine static stability, because it assumes a quasi-stationary environment.

Lapse Rate Names

The lapse rate γ of an environmental layer is defined as the temperature decrease with height:

$$\gamma = -\Delta T / \Delta z \quad (5.8)$$

Lapse rates are named as shown in Table 5-1, where isothermal and inversion are both subadiabatic.

Table 5-1. Names of environmental lapse rates, where Γ_d is the dry adiabatic lapse rate.

Name	Layer Lapse Rate	Fig.
adiabatic	$\gamma = \Gamma_d = 9.8^\circ\text{C km}^{-1}$	5.15(4)
superadiabatic	$\gamma > \Gamma_d$	5.15(5)
subadiabatic	$\gamma < \Gamma_d$	5.15(1)
• isothermal	$T = \text{constant with } z$	
• inversion	$T = \text{increases with } z$	

Layer Stability Classes: Five classes of layer stability are listed in Table 5-2. For a layer of air that is already saturated (namely, it is a layer of clouds, with $T = T_d$), then the environmental lapse rate γ , which is still defined by the equation above, is indicated as γ_s to remind us that the layer is saturated. As before, the word “dry” just means unsaturated here, so there could be moisture in the air.

Table 5-2. Layer static stability, where $\Gamma_d = 9.8^\circ\text{C km}^{-1}$ is the dry adiabatic lapse rate, and Γ_s is the saturated adiabatic lapse rate. Γ_s is always $\leq \Gamma_d$.

Name	Layer Lapse Rate
1) absolutely stable	$\gamma < \Gamma_s$
2) saturated neutral	$\gamma_s = \Gamma_s$
3) conditionally unstable	$\Gamma_s < \gamma < \Gamma_d$
4) dry neutral	$\gamma = \Gamma_d$
5) dry absolutely unstable	$\gamma > \Gamma_d$

Fig. 5.15 illustrates these stabilities. The layer of air between the thin horizontal lines has a linear lapse rate shown by the thick line. The slope of this thick line relative to the slopes of the dry Γ_d and moist Γ_s adiabats determines the layer stability class.

Conditional Stability: One of the types of layer stability listed in Table 5-2 is conditional stability. A layer of conditionally unstable air has lapse rate between the dry and moist adiabatic lapse rates. This layer is stable if the air is unsaturated, but is unstable if the air is saturated (cloudy). Hence, the “condition” refers to whether the air is cloudy or not.

Conditionally unstable air, if over a deep enough layer, is a favorable environment for thunderstorms to grow. A special set of **thunderstorm stability indices** has been developed to determine if thunderstorms will form, and how intense they might be. These are discussed in the Thunderstorm chapters of this book. The presence of thunderstorms indicate violently unstable air that extends throughout the depth of the troposphere.

When a whole layer is lifted, the temperature at each level within that layer will change according to whether those levels are saturated or not. Fig. 5.16 shows a sample sounding, and how temperatures change following dry adiabats until reaching their local LCL, above which they follow the moist adiabat. This particular example shows how a layer that starts as conditionally unstable can become absolutely unstable if the bottom of the layer reaches saturation sooner than the top.

Dynamic Stability

Dynamic stability considers both buoyancy and wind shear to determine whether the flow will become turbulent. **Wind shear** is the change of wind speed or direction with height, and can be squared to indicate the kinetic energy available to cause turbulence.

The ratio of buoyant energy to shear-kinetic energy is called the **bulk Richardson number, Ri**, which is dimensionless:

$$Ri = \frac{|g| \cdot (\Delta T_v + \Gamma_d \cdot \Delta z) \cdot \Delta z}{T_v \cdot [(\Delta U)^2 + (\Delta V)^2]} \quad \bullet(5.9a)$$

and

$$Ri = \frac{|g| \cdot \Delta \theta_v \cdot \Delta z}{T_v \cdot [(\Delta U)^2 + (\Delta V)^2]} \quad \bullet(5.9b)$$

where [$\Delta \theta_v$, ΔU , ΔV , ΔT_v] are the change of [virtual potential temperature, east-west wind component, north-south wind component, virtual temperature] across a layer of thickness Δz . As before, $\Gamma_d = 9.8 \cdot K \cdot km^{-1}$ is the dry adiabatic lapse rate, and T_v must be in absolute units (K) in the denominator of eq. (5.9).

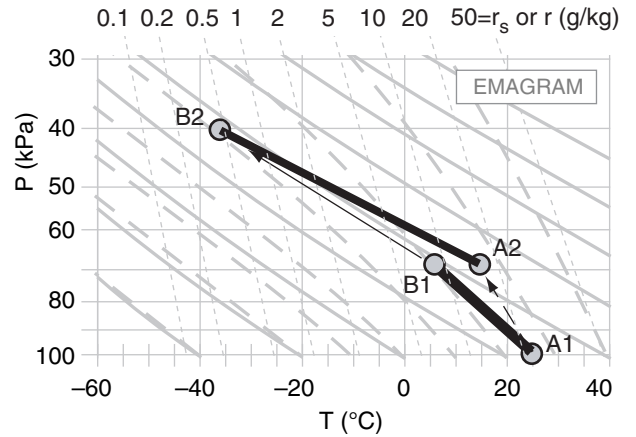


Figure 5.16
Example of lifting a layer to determine layer stability. Conditionally unstable layer (thick black line between points A1 and B1 on this Emagram) is lifted 30 kPa. Suppose that the bottom of the layer is already saturated, but the top is not. Upon lifting, the bottom of the layer will follow a moist adiabat from A1 to A2, while the top follows a dry adiabat from B1 to B2. The layer at its new height (black line A2 to B2) is now absolutely unstable.

Sample Application

Given these data, is the atmosphere turbulent?

z (km)	T_v (°C)	U (m s ⁻¹)	V (m s ⁻¹)
5.5	-23	50	0
5.0	-20	40	8

Find the Answer

Given: $\Delta T = -3^\circ C$, $\Delta V = -8$, $\Delta U = 10 \text{ m s}^{-1}$, $\Delta z = 0.5 \text{ km}$, where all vertical differences are (top - bottom)
Find: Turbulent (yes/no) ?

First, examine static stability.

$$\Delta \theta_v = \Delta T_v + \Gamma_d \cdot \Delta z = (-3^\circ C) + (9.8 \text{ }^\circ C \text{ km}^{-1}) \cdot (0.5 \text{ km}) = 1.9^\circ C$$

Because the top of the layer has warmer θ_v than the bottom, the air is locally statically stable. No nonlocal info to alter this conclusion. Thus, static stability does not make the air turbulent. But need to consider wind.

Apply eq. (5.9b). Use average $T_v = -21.5 + 273 = 251.5 \text{ K}$

$$Ri = \frac{(9.8 \text{ m} \cdot \text{s}^{-2}) \cdot (1.9 \text{ K}) \cdot (500 \text{ m})}{(251.5 \text{ K}) \cdot [(10)^2 + (-8)^2 (\text{m/s})^2]} = \underline{0.226}$$

This Ri below the critical value of 0.25, therefore the flow is turbulent because of dynamic instability.

Check: Physics are reasonable. Ri is dimensionless.

Exposition: Turbulence tends to mix the air, which reduces wind shears and temperature differences, and thus undoes the dynamic instability that caused it, as expected by LeChatelier’s Principle (see INFO box).

Sample Application

For the following sounding, determine the regions of turbulence.

z (km)	T (°C)	U (m s ⁻¹)	V (m s ⁻¹)
2	0	15	0
1.5	0	12	0
1.2	2	6	4
0.8	2	5	4
0.1	8	5	2
0	12	0	0

Find the Answer:

Given: The sounding above.

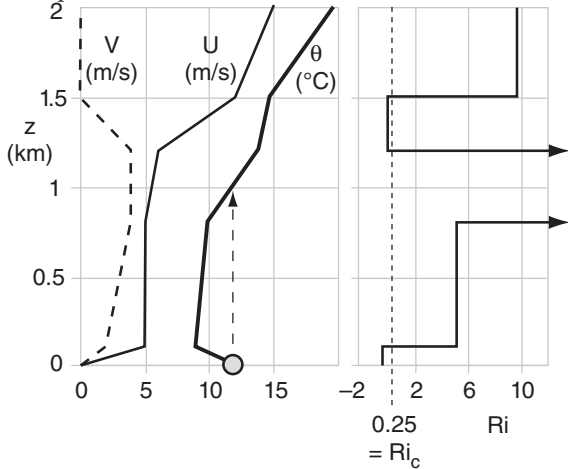
Find: a) Static stability (nonlocal parcel apex method), b) dynamic stability, & (c) identify turbulence.

Assume dry air, so $T = T_p$.

Method: Use spreadsheet to compute θ and Ri . Note that Ri applies to the layers between sounding levels.

z (km)	θ (°C)	z_{layer} (km)	Ri
2	19.6		
1.5	14.7	1.5 to 2.0	9.77
1.2	13.8	1.2 to 1.5	0.19
0.8	9.8	0.8 to 1.2	55.9
0.1	9.0	0.1 to 0.8	5.25
0	12	0 to 0.1	-0.358

Next, plot these results:



- a) Static Stability: Unstable & turbulent for $z = 0$ to 1 km, as shown by nonlocal air-parcel rise in the θ sounding.
- b) Dynamic Stability: Unstable & turbulent for $z = 0$ to 0.1 km, & for $z = 1.2$ to 1.5 km, where $Ri < 0.25$.
- c) Turbulence exists where the air is statically OR dynamically unstable, or both. Therefore:
Turbulence at 0 - 1 km, and 1.2 to 1.5 km.

Check: Physics, sketch & units are reasonable.

Exposition: At $z = 0$ to 1 km is the mixed layer (a type of atmos. boundary layer). At $z = 1.2$ to 1.5 is **clear-air turbulence (CAT)** and perhaps K-H waves.

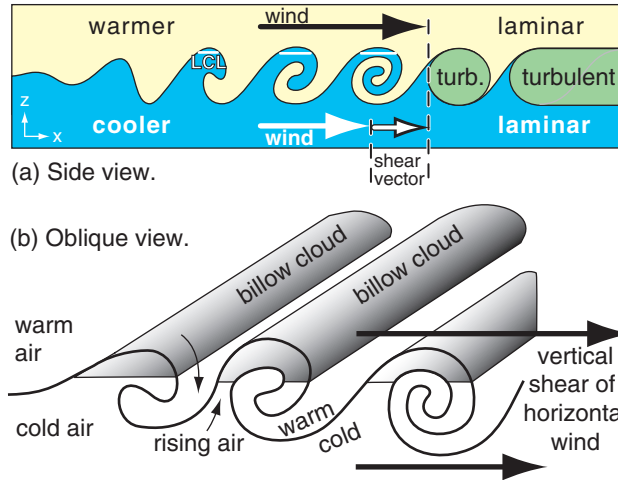


Figure 5.17

(a) Side view of Kelvin-Helmholtz waves growing in a dynamically unstable region: wind shear across a statically stable layer. (b) View showing parallel cloud bands called billow clouds

For the special case of statically stable air:

$$Ri = \frac{N_{BV}^2 \cdot (\Delta z)^2}{(\Delta U)^2 + (\Delta V)^2} \quad (5.9c)$$

N_{BV} is undefined for air that is not statically stable.

For thin layers of atmosphere, the system is **dynamically unstable** if

$$Ri < Ri_c \quad \bullet(5.10)$$

$Ri_c = 0.25$ is the **critical Richardson number**.

When statically stable laminar flow is just becoming dynamically unstable due to an increase in wind shear, the dynamically unstable layer behaves like breaking ocean waves (Fig. 5.17) in slow motion. These waves are called **Kelvin-Helmholtz waves**, or **K-H waves** for short. As these waves break, the dynamically unstable layer becomes turbulent, and is known as **clear-air turbulence (CAT)** if outside thunderstorms and above the boundary layer.

If the bottom layer of cool air is humid enough, then clouds can form in any wave crest that is higher than its lifting condensation level (LCL). Since these clouds form narrow parallel bands perpendicular to the shear vector, the clouds look like billows of an accordion, and are called **billow clouds** (Fig. 5.17).

Existence of Turbulence

Statically stable flows can be dynamically unstable and can become turbulent if the wind shear is strong enough.

Nonlocally statically unstable flow always becomes turbulent regardless of the wind shear. Thus, either dynamic instability or nonlocal static instability are sufficient to enable turbulence.

However, laminar flow exists only if the layer of air is stable both dynamically and statically.

FINDING TROPOPAUSE HEIGHT AND MIXED-LAYER DEPTH

Tropopause

Recall from Chapter 1 (Fig 1.10) that the standard-atmosphere temperature decreases with height within the troposphere, but is isothermal with height in the bottom part of the stratosphere. Isothermal layers are very strongly statically stable. The strong and deep statically stable layer in the stratosphere discourages vertical motion through it, which is why most of our weather (including thunderstorms and hurricanes) is trapped below it. The base of the stratospheric isothermal layer is the tropopause.

Real soundings are more complex than the simple standard atmosphere, and can have many different stable layers (and isothermal layers) at different heights (Fig. 5.18). Nonetheless, if the radiosonde balloon rises high enough before bursting, then it often can enter the stratosphere. To locate the tropopause, we just need to look for the bottom of the very thick isothermal layer that is near a pressure altitude of 40 to 20 kPa.

Strong static stability can be found by regions where potential-temperature lines (**isentropes**) are packed close together. The strongly stable stratosphere has tighter packing of isentropes than the less-stable troposphere, such as sketched in the idealized vertical cross section of Fig. 5.19. Thus, you can locate the tropopause at the bottom of the region of tight isentropes, as sketched in Fig. 5.19. *[CAUTION: The tropopause usually does NOT follow any single isentropes.]*

To illustrate the relationship between stability and isentropes, suppose a rawinsonde (i.e., a helium-filled weather balloon) is released from near the ground and rises along the orange line in Fig. 5.19. While it rises, it measures temperature and pressure of the surrounding environment, allowing one to calculate the vertical profile of potential temperature θ .

It starts in cold air near the ground, where the large spacing between isentropes shows that the static stability is weak near the ground. At about 4 km above the ground it passes through a frontal inversion, where static stability is stronger as shown by the close spacing of the two isentropes ($\theta = 260$ & 270 K). Higher, in the mid troposphere, the larger spacing indicates weaker stability. Starting at about 10 km altitude the sonde enters a region of close spacing of isentropes, indicating that the sonde has entered the stratosphere. The bottom of this strongly stable region is the tropopause.

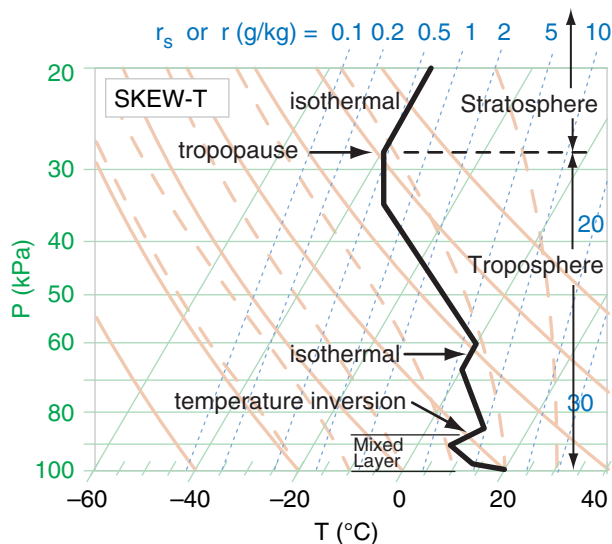


Figure 5.18
Sample atmospheric sounding having two isothermal layers and one temperature inversion.

Sample Application

What is the tropopause height (or pressure) in the sounding of Fig. 5.18, and how is it identified?

Find the Answer

Given: the plotted sounding.
Find: $P_{Trop} = ?$ kPa

The base of the deep isothermal layer near the top of this sounding is at $P_{Trop} \approx 28$ kPa. The other isothermal and inversion layers are too low, and too thin.

Check: Sketch OK.

Exposition: The tropopause is lower near the poles and higher near the equator. It is lower in winter, higher in summer (see Table 6-1 in the Cloud chapter), but varies considerably from day to day.

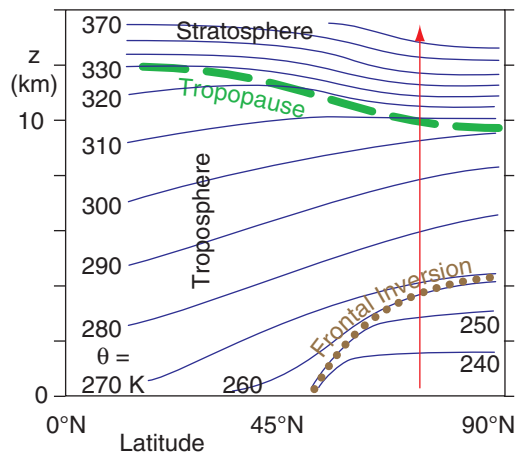


Figure 5.19
Idealized vertical cross section. Isentropes (connecting points of equal θ) are dark blue. Frontal inversion is dotted brown. Tropopause is dashed green. Red arrow indicates rawinsonde.

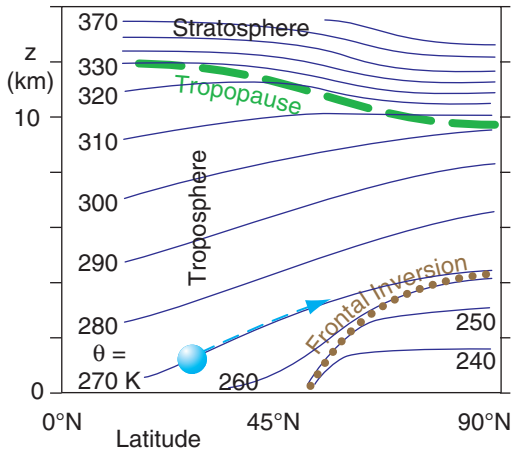


Figure 5.20
Same as Fig. 5.19, but now the cyan sphere represents an air parcel of potential temperature $\theta = 270$ K, which rides along the environmental 270 K isentrope as the parcel is advected toward the North Pole (90°N). Thick green dashed line indicates the tropopause. Thin blue dashed line shows the air-parcel path.

As air parcels advect (move) horizontally, they also tend to move vertically to stay on a surface of constant potential temperature (i.e., an **isentropic surface**), assuming no heat transfer to/from the parcel. The reason is buoyancy. For example, the parcel at 30°N illustrated in Fig. 5.20 has a potential temperature of $\theta = 270$ K, so it would try to stay on the 270 K isentrope (dark blue line in that figure).

If it were to stray slightly [above, below] that surface, it would be surrounded by air that has [warmer, cooler] potential temperature, causing a buoyant force that would tend to [lower, raise] the parcel back to the altitude of the 270 K isentrope. **Isentropic analysis** is a method of plotting weather variables on constant θ surfaces as a way to estimate dynamics under adiabatic conditions.

Mixed-Layer

When the sun heats the ground to be warmer than the adjacent air, or when cold air advects over warmer ground, some of the heat is conducted into the thin layer of air touching the ground (see the Thermodynamics chapter). This warm near-surface air wants to rise as bubbles or vertical plumes called **thermals**. You can sometimes see birds soaring in the updrafts of these thermals.

The thermals create vertical **convective circulations** that stir the bottom portion of the troposphere into a **well-mixed layer**. This **convective mixed layer** is evidence of nonlocal static instability in the boundary layer — the bottom 300 to 3000 m of the troposphere.

The **mixed layer** (ML) is usually capped by a statically stable layer or **temperature inversion** (where temperature increases with height), which prevents the thermals from rising through it and traps pollutants below it. Thus, the ML depth z_i is important for air-quality studies.

To find z_i from a sounding, use the nonlocal methods discussed earlier in this chapter. (1) Draw the measured sounding on a thermo diagram. (2) Estimate the temperature of the near-surface air parcel and plot it on the same diagram. (3) Lift this parcel adiabatically until it hits the environmental sounding, which defines z_i (or P_i if using pressure as the vertical coordinate). θ vs. z thermo diagrams (see end of this chapter) are handy for ML studies.

A less accurate, but quick approach, is to look for the temperature inversion (or similar layer that is statically stable) that is near the ground, but not touching it. A **temperature inversion** is where T increases with height. The top of the mixed layer is usually somewhere within this temperature inversion. In Fig. 5.18, the average height of the temperature inversion is about $z_i = 87$ kPa. See the Boundary Layer and Air Pollution chapters for details.

Sample Application

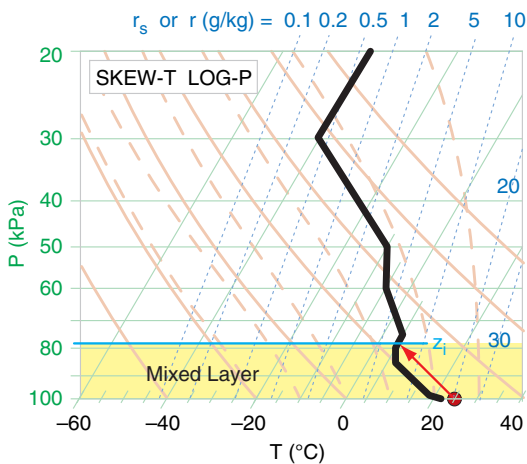
Plot the following sounding on a Skew-T diagram. If the ground heats an air parcel to the following state [$P=100$ kPa, $T = 25^\circ\text{C}$], then find the pressure of the mixed-layer top. Sounding: [P (kPa), T ($^\circ\text{C}$)] = [100,22], [99,19], [85,7], [80,5], [75,5], [60,-5], [50,-10], [30,-40], [20,-40].

Find the Answer

Given: the data above.

Find: P_i (pressure in kPa at mixed-layer top z_i)

See Skew-T below. Red dot and arrow represent the heated near-surface air parcel. Black line is the plotted environmental sounding.



From the diagram, $P_i \approx 78$ kPa.

Check: Sketch agrees with larger thermo diagrams.

Exposition: The yellow-shaded region is nonlocally unstable. It is the turbulent mixed layer.

INFO • A holistic approach to stability

The nonlocal approach is not new. Thunderstorm forecasters usually use the whole sounding, and consider air parcels rising from the surface to the top of the storm to estimate the potential severity of storms. They don't focus on only the local lapse rate in the mid troposphere to estimate thunderstorm intensity.

"Lapse rate" (subadiabatic, adiabatic, superadiabatic) is **not** synonymous with "static stability" (stable, neutral, unstable). If the purpose of a stability definition is to determine whether flow becomes turbulent, then you must consider nonlocal effects within the whole sounding.

REVIEW

Thermo diagrams are useful for estimating thermodynamic state (pressure, temperature, and moisture) of the air without having to solve equations. There are many different types of thermo diagrams: Emagram, Skew-T, Stüve (or Pseudoadiabatic Diagram), Tephigram, and θ -Z diagram. They all serve the same purpose, and all have isotherms, isobars, isohumes, dry and moist adiabats, and height contours (in some cases).

You can also track state changes caused by physical processes acting on the air. In this chapter you read about processes such as radiative heating and cooling, evaporation and condensation, and precipitation falling into or out of an air parcel. You also learned how to estimate cloud-base altitude from the lifting condensation level, and cloud-top altitude from nonlocal stability and buoyancy. You saw how potential temperature is a convenient way to describe the state of vertically moving air parcels.

Thermo diagrams are often used to plot upper-air soundings made with rawinsonde balloons or calculated from computerized forecasts. Soundings plotted on thermo diagrams allow you to find the depth of the mixed layer and the top of the troposphere (i.e., the tropopause). They also allow you to calculate important parameters such as the Brunt-Väisälä frequency and the Richardson number.

Air flow can change from laminar to turbulent if the flow is unstable. Instability can be due to the vertical variation of temperature alone (as quantified by static stability), or be due to the combined effects of wind and temperature variations with height (as quantified by dynamic stability). By plotting soundings and analyzing stability you can anticipate turbulence such as causes rapid dispersion of air pollutants and transport of heat, moisture, and momentum.

HOMEWORK EXERCISES**Broaden Knowledge & Comprehension**

B1. Use the internet to acquire a map of rawinsonde sounding locations for your part of the world, and determine which four sounding locations are closest to your school.

B2. Use the internet to acquire **WMO (World Meteorological Organization)** and **ICAO (International Civil Aviation Organization)** identification codes for the four sounding locations closest to your school or business. For each of those four locations, what is the latitude, longitude, topographic elevation, and other info about the site.

B3. Use the internet to acquire different thermo diagrams (e.g., Emagrams, skew-Ts, Stüve diagrams, tephigrams, etc) for the most recent rawinsonde sounding made at the launch site closest to your school or business. Which web site (URL) has the diagram that is clearest or easiest to read?

B4. Use the internet to acquire today's surface weather data of (P , T , and T_d) for the a weather station that is near you. After plotting those on a skew-T diagram, use graphical methods to estimate the lifting condensation level. How does this height compare to any cumulus cloud bases that might be reported from the same station?

B5. Use the internet to acquire rawinsonde data in text form (i.e., a table of numbers) for an upper-air station near you, or a different station that your instructor assigns. Then plot this sounding by hand on a large thermo diagram (skew-T, unless your instructor indicates a different diagram). Determine the LCL for an air parcel rising from the surface.

B6. Use the internet to acquire a sequence of 4 successive soundings from a single rawinsonde station assigned by your instructor. Describe the evolution of the air at that location (e.g., warming, cooling, getting more humid, getting windier, etc.).

B7. Use the internet to acquire an already-plotted sounding near your location prior to a rain storm. From the plotted sounding, determine the temperature and mixing ratio near the ground, and then move that hypothetical air parcel upward to a pressure of 40 kPa. Use the thermo diagram to estimate the final liquid-water mixing ratio of the parcel. Can you relate the actual precipitation depth that was observed from this storm to your hypothetical liq-

uid-water mixing ratio? Would you get a more-realistic answer if you stopped the parcel at a different pressure?

B8. In the Thunderstorm chapters you will learn additional ways to use soundings on thermo diagrams to help estimate thunderstorm strength, tornado likelihood, and other storm characteristics. As a preview of this application, find on the internet an already-plotted sounding that also shows wind information or Convective Available Potential Energy (CAPE) or other stability indices. See if that web site has a link to a legend that defines those terms. If so, use that legend to interpret the stability indices on the sounding, and explain what they mean.

B9. Use the internet to acquire a sounding for a nearby location (or other site that your instructor assigns) that you can output in both plotted graph and text form. Use the plotted sounding to determine the static stability at all heights, using the apex parcel method. Use the text data to calculate Brunt-Väisälä frequency and bulk Richardson number in the stable layers. Combine your dynamic and static stability info to suggest which regions in the sounding are turbulent. [Hints" (1) For any stable layer with uniform lapse rate within it, use only the temperatures and winds at the top and bottom of the layer to calculate these variables. (2) Ignore any moisture indicated by the sounding, and assume $T_v \approx T$].

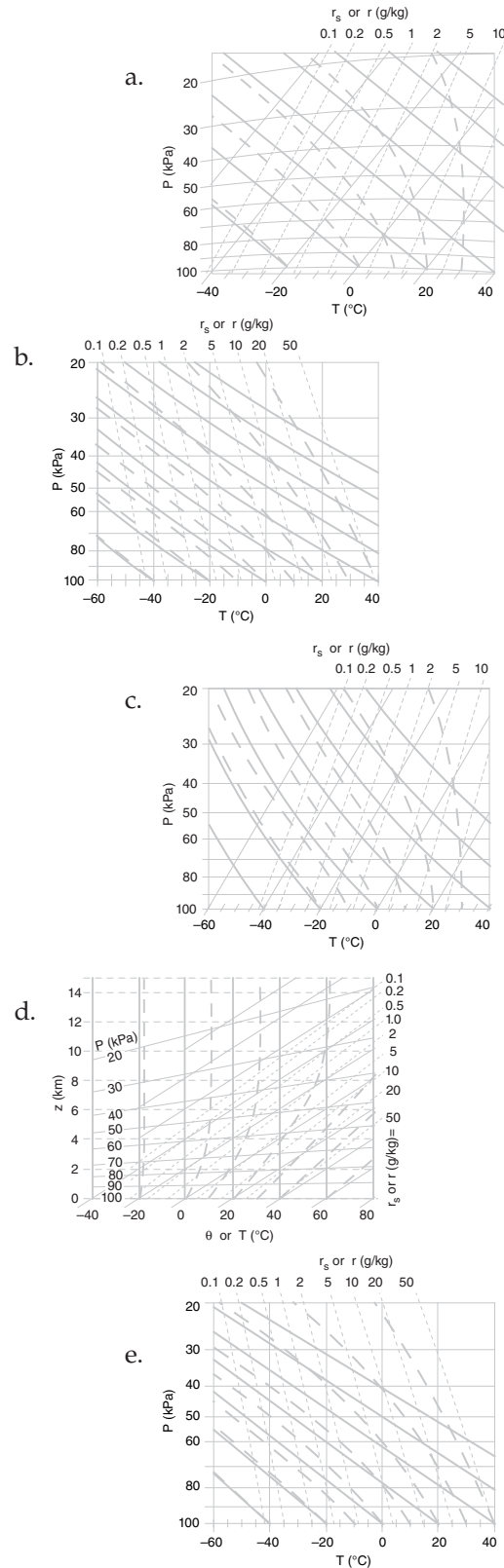
B10. Use the internet to acquire a plotted sounding from early in the morning. Suppose that as the day evolves, the near-surface dew-point temperature doesn't change, but the near-surface air temperature becomes warmer. Considering nonlocal static stability effects, how warm would the near-surface air parcel need to become later that day in order for it to be positively buoyant (dry below the LCL, saturated above the LCL) up to a height where $P = 60$ kPa? Draw that air-parcel path on the sounding diagram, and discuss where cloud base and cloud top would be.

B11. Use the internet to acquire a plotted sounding near you. Estimate the pressures at the boundary-layer top and at the tropopause. Draw these locations on your thermo diagram, and discuss how you found them based on your sounding.

B12. Use the internet to acquire images of Kelvin-Helmholtz waves and billow clouds. Print two examples of each, along with their URLs.

Apply

A1. Identify the thermo diagram type.



A2. On copies of ALL 5 thermo diagrams from A1, find and label one of the:

- a. dry adiabats b. isohumes c. isobars
- d. isentropes e. saturated adiabats
- f. isotherms g. moist adiabats

A3(S). Use a spreadsheet to create the Skew-T, as given in the Sample Application in the Skew-T section of this Chapter, except do it for isopleths listed below. Describe the results of your experiments with different values of the coefficient K , which determines the amount of skewness.

	T , θ , & θ_w (°C)	r (g kg ⁻¹)
a.	30	50
b.	25	20
c.	15	10
d.	10	2
e.	5	1
f.	0	0.5
g.	-5	0.2
h.	-10	0.1
i.	-15	15
j.	-20	25

A4. Make a copy of the page holding the thermo diagrams for exercise A1. Then, on all these diagrams, plot the one thermodynamic state given below. (Note: these are single air parcels, not components of a sounding.)

	P (kPa)	T (°C)	T_d (°C)
a.	50	0	-15
b.	80	-5	-10
c.	35	-25	-30
d.	75	25	16
e.	60	15	-10
f.	90	20	10
g.	65	-25	-40
h.	85	-10	-30
i.	70	10	5
j.	80	15	10
k.	55	-10	-15

A5. Using thermodynamic state of the air parcel given in the previous exercise, plot it on the end-of-chapter large thermo diagram specified by your instructor (if none specified, use the skew-T diagram). Hint: Use a copy of a blank thermo diagram so you can keep the master copy clean.

A6. Plot the one thermodynamic state from exercise A4 on one large thermo diagram specified by your instructor (if none specified, use the skew-T). For that plotted point, use the thermo diagram (not equations) to find the values of the following items:

- i) mixing ratio
- ii) potential temperature

- iii) wet-bulb temperature
- iv) wet-bulb potential temperature
- v) saturation mixing ratio
- vi) LCL
- vii) relative humidity
- viii) equivalent potential temperature

A7. Same as A6, but use only an Emagram or Stüve.

A8. Starting with one assigned air-parcel with state from question A4:

- i) lower it to a pressure of 100 kPa, and find its new temperature and dew point. Use only a tephigram or Skew-T.
- ii) lower it to a pressure of 100 kPa, and find its new temperature and dew point. Use only a Stüve or Emagram.
- iii) raise it to a pressure of 20 kPa, and find its new temperature and dew point. Use only a tephigram or Skew-T.
- iv) raise it to a pressure of 20 kPa, and find its new temperature and dew point. Use only a Stüve or Emagram.

A9. Which thermo diagrams from Fig. 5.3 have

- a. straight isobars?
- b. straight dry adiabats?
- c. straight moist adiabats?
- d. have the greatest angle between isotherms and dry adiabats?
- e. moist adiabats that asymptotically approach dry adiabats near the top of the diagram?
- f. moist adiabats that asymptotically approach dry adiabats near the left of the diagram?

A10. Use the data below as an air-parcel initial state, and plot it on the specified large end-of-chapter thermo diagram. Assume this air parcel is then lifted to a final height where $P = 25$ kPa. Find the final values of the following variables T_d , r_s , T , r_L , and r_T .

	P (kPa)	T (°C)	T_d (°C)	Thermo Diagram
a.	95	30	15	Stüve
b.	95	30	25	Tephigram
c.	95	20	10	Skew-T
d.	95	15	-5	Skew-T
e.	100	25	10	Emagram
f.	100	25	5	Tephigram
g.	100	30	20	Emagram
h.	100	15	5	Stüve
i.	90	20	5	Tephigram
j.	90	15	5	Emagram
k.	90	5	-5	Skew-T
l.	90	10	0	Stüve

A11. Using the same thermo diagram as specified in exercise A10, find the pressure at the LCL for an air

parcel that started with the conditions as specified in that exercise.

A12. Starting with the FINAL state of the air parcel specified from exercise A10, allow all of the liquid water to rain out (except if the liquid water from the final state of A10 was greater than 1 g kg^{-1} , then allow only 1 g kg^{-1} of liquid water to rain out). Then lower the air parcel back to its starting pressure, and determine its new temperature and new dewpoint. Use the specified thermo diagram from A10.

A13. Starting with the FINAL state of the air parcel specified from exercise A10, allow radiative cooling to change the air parcel temperature by $\Delta T = -8^\circ\text{C}$. Then lower the air parcel back to its starting pressure, and determine its new temperature and dew point. Use the specified thermo diagram from A10.

Sounding Data. The data here gives environmental conditions (i.e, the ambient sounding). Use this sounding for the questions that follow. Given $V = 0$.

P(kPa)	T($^\circ\text{C}$)	T _d ($^\circ\text{C}$)	U (m s ⁻¹)
20	-25	-55	45
30	-25	-50	50
40	-20	-20	44
45	-15	-15	40
50	-10	-24	30
70	12	-20	30
80	19	3	20
90	21	15	7
99	29	17	7
100	33	20	2

Definitions: **Significant levels** are altitudes where the sounding has a bend or kink. In between significant levels you must draw straight lines.

Layers are the regions of air that are in between two neighboring levels.

A14. Using the end-of-chapter large thermo diagram specified below, plot the sounding data from the previous column on that diagram. For temperatures, use solid filled circles (red if available), and then connect those temperature-sounding points from bottom to top of the atmosphere with straight line segments. For dewpoints, use an open circles (blue perimeter if possible), and connect those points with dashed straight line segments.

- a. Emagram b. Tephigram c. Skew-T
- d. θ -Z diagram e. Stüve

A15. a. For each pressure and temperature pair in the sounding data, calculate the corresponding potential temperature. Then plot a graph of potential

temperature vs. pressure, using linear scales for both graph axes.

b. Use the result from (a) and the humidity information from the sounding data to find virtual potential temperature, and then plot it vs. pressure using linear scales for both axes.

A16. Given the sounding data. Suppose that you create an air parcel at the pressure-level (kPa) indicated below, where that parcel has the same initial thermodynamic state as the sounding at that pressure. Then move that parcel up through the environment to the next higher significant level (i.e., next lower pressure). What is the value of the buoyant force/mass acting on the parcel at its new level?

- a. 100 b. 99 c. 90 d. 80 e. 70 f. 50 g. 45
- h. 40 i. 30

A17. Given the sounding data. Find N_{BV} and P_{BV} for an air parcel that starts in the middle of the layer indicated below, and for which its initial displacement and subsequent oscillation is contained within that one layer. Use the layer with a bottom pressure-level (kPa) of:

- a. 90 b. 80 c. 70 d. 50 e. 45 f. 40 g. 30

A18. Using the sounding data such as plotted for exercise A14, using the apex parcel method to find static stability for the one environmental layer, where the bottom of that one layer is at P (kPa) =

- a. 100 b. 98 c. 87 d. 80 e. 70 f. 60 g. 40

For this exercise you may assume the air is dry. Also, the stability in any one layer could depend on nonlocal effects from other layers.

A19. Same as A18, but find the layer stability using the sounding data.

A20. Using the sounding data, calculate Richardson number, which you can then use to find a layer's dynamic stability, for the layer that has a layer base at level P (kPa) =

- a. 100 b. 98 c. 87 d. 80 e. 70 f. 60 g. 40

A21. Indicate which portions of the sounding above are likely to be turbulent, and explain why.

A22. Using the sounding data:

a. Does the sounding data indicate the presence of a mixed layer near the ground? What is its corresponding depth, z_i ?

b. Determine the height of the tropopause.

c. Locate any layer (stratiform) clouds that might be present in the environment.

A23. With the sounding data:

a. Compare the LCL pressure heights found using a thermo diagram vs. that found using the equations from the Water Vapor chapter, for an air parcel starting with the initial state as given by the sounding data at 100 kPa.

b. On the same thermo diagram from part (a), at what pressure altitude does the air parcel from (a) stop rising (i.e., what is the pressure at cloud top)? At cloud top in the cloud, find the mixing ratios for water vapor and liquid water.

c. Suppose that all the liquid water that you found in part (b) were to precipitate out, after which the air parcel descends back to a pressure altitude of 100 kPa. Compare the initial and final relative humidities of the air parcel (i.e., before rising from 100 kPa, and after returning to 100 kPa).

A24. At $P = 100$ kPa, $z = 0$ km, an air parcel has initial values of $T = 30^\circ\text{C}$ and $T_d = 0^\circ\text{C}$. Use each of the large thermo diagrams at the end of this chapter to find the LCL. How do these LCL values compare to the theoretical value from the equation in the Water Vapor chapter?

Evaluate & Analyze

E1. If an unsaturated air parcel rises, why does its humidity follow an isohume (constant r) instead of an isodrotherm (constant T_d)?

E2. Suppose a rising dry air parcel cools at a rate different than Γ_d . What process(es) could cause this difference? Why?

E3. From the hypsometric equation one expects that colder air has thinner thickness. So why do the height contour lines on the large Emagram tilt down to the right, rather than down to the left?

E4. The large thermo diagrams at the end of the chapter have mixing-ratio labels near the top. Why don't these label values increase linearly?

E5. Dry adiabats are perfectly straight lines on the Stüve diagram, but have slight curvature on the Emagram (see the large diagrams at the end of the chapter). If the dry adiabatic lapse rate is constant, why are the lines curved in the Emagram?

E6. On a thermo diagram, why are three points needed to represent any single air parcel?

E7. Could you determine the complete thermodynamic state of an air parcel given only its LCL and its initial (P, T)? Explain.

E8. The "air parcel" method of static stability assumes that the ambient environment doesn't change as an air parcel moves through it. Under what conditions is this assumption good? When would the assumption fail?

E9. Given an air parcel of virtual temperature T_v embedded in a uniform environment of the same T_v . Why does it have zero buoyant force, even though gravity is trying to pull the air parcel down?

E10. Can stationary metal vehicles float in air analogous to how metal ships can float on the ocean? If so, how should the metal aircraft be designed?

E11. Recall equations (5.2 & 5.3a) for buoyant force, based on difference between the air parcel (object) and its environment (surrounding fluid). Why does the equation using density difference have a different sign than the equation using virtual temperature?

E12. Compare the slope of the saturated adiabats on the Emagram and the θ -Z diagram. Why do they tilt in opposite directions on these two diagrams?

E13. To compare the many different thermo diagrams at the end of this chapter, first make a copy of each large blank diagram. Then, on each one, plot the same air parcel (P, T, T_d) = (90 kPa, 20°C , 10°C). If the parcel is lifted to a pressure of 50 kPa, find the final thermodynamic state of the parcel, and compare the answers from the different diagrams.

E14. The sounding in Fig. 5.14e has lines connecting the data points. For each line segment, label its lapse rate (subadiabatic, adiabatic, superadiabatic). Is there a perfect relationship between lapse rate and the static stability indicated in that figure? If not, why not?

E15. Discuss the nature of the Brunt-Väisälä frequency in air that is statically unstable.

E16. The equation for the Brunt-Väisälä oscillation period contains 2π . Why?

E17. a. Check the units of the factors in the Richardson number equations (5.9) to confirm that the Richardson number is indeed dimensionless.

b. What is the value of Richardson number for statically unstable air? What does this value imply about turbulence?

c. What is the value of Richardson number for an air layer with no wind shear? What does this value imply about turbulence?

E18. Although the large thermo diagrams at the end of this chapter include pressures greater than 100 kPa, most of the smaller diagrams earlier in the chapter have a max pressure of 100 kPa at the bottom of the diagram. For these smaller diagrams, what methods can you use to plot pressures that are more than 100 kPa?

E19. If mixing ratio is conserved in an unsaturated rising air parcel, why isn't saturated mixing ratio conserved?

E20(S). (This exercise is lengthy.)

For pressures in the range of 100 to 10 kPa, use a computer spreadsheet to create a:

a. Stüve diagram. [Hint: use $(P_o/P)^{\gamma_d/C_{pd}}$ as vertical coordinate.]

b. Skew-T diagram.

c. Tephigram. [Hint: plot it in tilted form, as in Fig. 5.2, but with moist adiabats and isohumes added to the other lines in that figure. This is a very difficult exercise.]

d. θ -Z diagram. [Hint, plot all the isopleths except the isobars.]

E21(S). Use a computer spreadsheet to create an Emagram for a wider range of pressures (120 kPa to 5 kPa) than I had plotted in my small thermo diagrams. Plot only 1 dry adiabat ($\theta = 0^\circ\text{C}$) and 1 isohume ($r = 10 \text{ g kg}^{-1}$) for this exercise.

E22. How would the air parcel line in Fig. 5.12 be different if the rising parcel entrains a small amount of environmental air as it rises? Sketch this new path on a similar diagram.

E23. For a layer of isothermal air, what is the sign of the Richardson number?

E24. How are the gradient, bulk, and flux Richardson numbers related? The chapter on the Atmospheric Boundary Layer defines the flux Richardson number, and also describes K-theory, a tool you might find useful in answering this question.

E25. Create a new conceptual algorithm different from the "Guide" in this chapter to help you identify different thermo diagram types. Test it to ensure that it doesn't falsely identify some diagrams.

E26(S). Use a spreadsheet to reproduce Fig. 5.2, using the relationship for potential temperature as a function of T and P as given in the Heat chapter.

E27. All the dry adiabats in a Stüve diagram converge to a point above the top left of the diagram. Explain why or why not the dry adiabats in an Emagram converge to a point.

E28. Suppose that turbulent mixing in a dynamically unstable, but statically stable, environment causes both $\Delta T/\Delta z$ and $\Delta U/\Delta z$ to be reduced by the same fraction, b . Show how the Richardson number increases as b decreases. Use this to explain why turbulence acts to reduce the dynamic instabilities that caused it, analogous to LeChatelier's principle. (Hint: Assume $V = 0$ for simplicity.)

E29. Fig. 5.10 indicates state changes due to fallout of precipitation.

a. Can the opposite happen? Namely, can precipitation fall INTO an air parcel, with all of the water staying in the air? If so, give an example and show how that process would be plotted on a thermo diagram.

b. What happens if rain from above falls through an air parcel with zero or partial evaporation? Namely, the rainfall does not change during its passage through the air parcel. Indicate this on a thermo diagram.

Synthesize

S1. What if you multiply the numerator and denominator of equation (5.9c) by $0.5m$, where m is air-parcel mass. Describe how the numerator and denominator could be interpreted as potential energy and kinetic energy, respectively.

S2. Use the internet to uncover a brief history of Archimedes. Where did he live? What discoveries did he make, and which were relevant to meteorology? What is he most famous for? What was his role in the wars that were waged at that time?

S3. What if water-vapor condensation caused cooling instead of warming. Describe any possible changes in climate and weather. Draw a rough thermodynamic diagram by eye for this physical situation.

S4. Describe any possible changes in climate and weather if buoyancy force was a function of only air-parcel temperature and not on temperature difference?

S5. Fig. 5.17 showed Kelvin-Helmholtz (K-H) waves that form and break in the atmosphere. These are a type of interfacial waves that form on interfaces between dense (cold) air and less-dense (warm) air. Another type of wave called “internal wave” can exist in statically stable air. Use the internet to learn more about internal waves — waves that can move vertically as well as horizontally. Write a brief summary of internal waves.

S6. Describe how the Richardson number would be different if vertical velocity were included in the denominator. Speculate on why it is not included?

S7. Describe any changes in climate and weather that might occur if only Earth’s air warmed due to global warming without any associated change in surface temperature. Justify your hypothesis in terms of static stability.

S8. Suppose a nuclear war happened on Earth, and that many of the explosions caused small-diameter dust particles to be blown into the stratosphere where they would settle out very slowly. This situation is called nuclear winter. Using the principles you learned in the radiation chapter, describe how stratospheric and lower-tropospheric temperatures might change under this thick layer of dust. Describe any changes in climate and weather associated with the resulting changes in atmospheric static stability.

S9. An isentropic chart shows the altitude (either z or P) of a constant θ surface. Describe how you would use upper-air soundings from different weather stations to get the data needed to draw an isentropic chart.

S10. Devise a new type of thermo diagram that has $\log(r)$ along the abscissa, and $\log(P)$ along the ordinate with scale reversed (max and bottom and min at top). Plot the isotherms and dry adiabats in this diagram. Name the diagram after yourself, and explain the virtues and utility of your diagram.

S11. What if T_d could be greater than T in a sounding. Explain how that might be possible, and describe how convective clouds might be different, if at all.

S12. Suppose the tropopause was touching the ground everywhere on Earth. How would the weather or climate be different, if at all?

LARGE-SIZE THERMO DIAGRAMS

You have our permission to freely reproduce the following seven thermo diagrams for your own personal use, or for education (but not for commercial resale), so long as you retain the author, title, and publisher citations in the copies. The diagrams are printed in faded colors so your own plotted soundings will be easier to see. We recommend that you do NOT write on the following seven pages, so that they remain as clean master copies.

These diagrams share common line formats: thin lines representing state of the air; thick lines representing adiabatic processes that change the state.

- Isotherms (temperature: T) and isobars (pressure: P) are thin solid green lines,
- Isohumes (mixing ratio: r) are thin dotted blue,
- Dry adiabats (potential temperature: θ) are thick solid orange lines,
- Moist adiabats (wet-bulb potential temperature: θ_w) are thick dashed orange lines,
- Contours (height: z) are very thin dashed grey lines (if plotted at all).

The height contours are only approximate. Potential temperatures are based on $P_o = 100$ kPa, where we set $z = 0$ for simplicity at this reference pressure.

The line labels also follow a common format for most of the diagrams in this book:

- Isobar labels are along the left axis. Units: kPa.
- Isohume labels are along top axis (and right axis in one case). Units: g kg^{-1} .
- Height labels are along the right axis. Units: km.
- Isotherms and both dry and moist adiabats are labeled at the bottom. Units: $^{\circ}\text{C}$.

For this last item, the three isopleths sprout from each temperature label like branches from a bush. The branches are always in the same order, although their angles differ from graph to graph. Dry adiabats are the left branch, moist adiabats are the center branch, and isotherms are the right branch.

Two versions of the Skew- T and θ - Z Diagram are included. The standard version spans the range $P = 105$ to 20 kPa. The other version [with “(ABL)” added to the title] is designed for **atmospheric boundary-layer** and air-pollution work, and gives more detail in the bottom third of the troposphere.

On the next pages are these thermo diagrams:

- Emagram
- Stüve Diagram
- Skew- T Log- P Diagram
- Tephigram
- Theta- Z Diagram
- Theta- Z Diagram (ABL)
- Skew- T Log- P Diagram (ABL).