

7 PRECIPITATION PROCESSES

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Hydrometeors are liquid and ice particles that form in the atmosphere. Hydrometeor sizes range from small cloud droplets and ice crystals to large hailstones. **Precipitation** occurs when hydrometeors are large and heavy enough to fall to the Earth's surface. **Virga** occurs when hydrometeors fall from a cloud, but evaporate before reaching the Earth's surface.

Precipitation particles are much larger than cloud particles, as illustrated in Fig. 7.1. One "typical" raindrop holds as much water as a million "typical" cloud droplets. How do such large precipitation particles form?

The **microphysics** of cloud- and precipitation-particle formation is affected by super-saturation, nucleation, diffusion, and collision.

Supersaturation indicates the amount of excess water vapor available to form rain and snow.

Nucleation is the formation of new liquid or solid hydrometeors as water vapor attaches to tiny dust particles carried in the air. These particles are called cloud condensation nuclei or ice nuclei.

Diffusion is the somewhat random migration of water-vapor molecules through the air toward existing hydrometeors. Conduction of heat away from the droplet is also necessary to compensate the latent heating of condensation or deposition.

Collision between two hydrometeors allows them to combine into larger particles. These processes affect liquid water and ice differently.

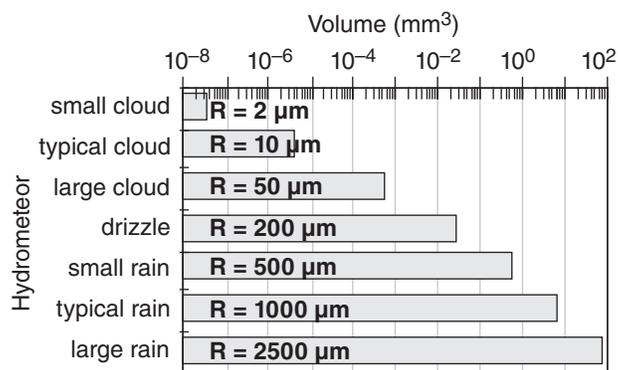


Figure 7.1

Drop volumes and radii, R , for various types of hydrometeor.

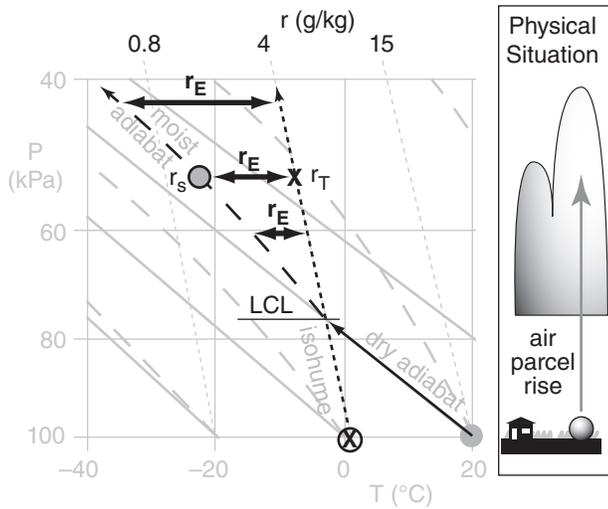


Figure 7.2 Thermo diagram (emagram) showing how excess-water mixing ratio r_E (thick line with arrows at each end) increases with increasing height above cloud base (LCL) for an air parcel rising adiabatically. r_T = total water mixing ratio; r_s = saturation mixing ratio for water vapor. r_E is the amount of water that can form hydrometeors.

Sample Application

An air parcel starts at 100 kPa with temperature 20°C and mixing ratio 4 g kg⁻¹. Use a thermo diagram to find the adiabatic value of excess-water mixing ratio when the parcel reaches a pressure of 60 kPa, and find the max available supersaturation.

Find the Answer

Given: Initially $r = r_T = 4 \text{ g kg}^{-1}$, $T = 20^\circ\text{C}$,
 $P = 100 \text{ kPa}$
 Finally: $P = 60 \text{ kPa}$.

Find: $r_E = ? \text{ g kg}^{-1}$, $S_A = ?$ (dimensionless)

Use a full size thermo diagram from the Atmospheric Stability chapter. Starting with the initial conditions, and following the air parcel up dry-adiabatically from the surface, we find the LCL is at $P = 75 \text{ kPa}$ and $T = -3^\circ\text{C}$, which marks cloud-base. Above the LCL, follow the moist adiabat, which gives $T \approx -15^\circ\text{C}$ at $P = 60 \text{ kPa}$. Assuming no entrainment and mixing, $r_T = 4 \text{ g kg}^{-1}$.

Use eq. (7.5): $r_E = (4 - 2) \text{ g kg}^{-1} = \underline{2 \text{ g kg}^{-1}}$.
 Then use eq. (7.7): $S_A = r_E/r_s = \underline{1} = \underline{100\%}$.

Check: Units OK. Physics OK. Agrees with Figs. 7.2 & 7.3)

Exposition: A supersaturation of 100% corresponds to a relative humidity of $RH = 200\%$ from eq. (7.2). Thus, there is twice as much water in the air as can be held at equilibrium. This excess water vapor drives condensation. For real clouds, we expect r_E in the range of 0.5 to 1.5 g kg⁻¹, based on the curves in Fig. 7.3.

7.1. SUPERSATURATION & WATER AVAILABILITY

7.1.1. Supersaturation

When there is more water vapor in the air than it can hold at equilibrium, the air is said to be **supersaturated**. Supersaturated air has relative humidity RH greater than 100%. Define this excess relative humidity as a **supersaturation** fraction S :

$$S = RH - 1 \quad \bullet(7.1)$$

or as a supersaturation percentage $S\%$:

$$S\% = 100\% \cdot S = RH\% - 100\% \quad \bullet(7.2)$$

Using the definition of relative humidity from the Water Vapor chapter, rewrite supersaturation in terms of total vapor pressure e and saturation vapor pressure e_s :

$$S = \frac{e}{e_s} - 1 \quad \bullet(7.3)$$

for $e > e_s$. It can also be approximated using mixing ratios:

$$S \approx \frac{r}{r_s} - 1 \quad \bullet(7.4)$$

for $r > r_s$.

7.1.2. Water Availability

Suppose that an initially-unsaturated air parcel has total water mixing ratio of r_T . If the air parcel rises adiabatically (i.e., no mixing with the environment, and no precipitation in or out), then it cools and its saturation mixing ratio decreases. You can use a thermo diagram (from the Atmospheric Stability chapter) to find these changing mixing ratios, as sketched here in Fig. 7.2.

Above the parcel's lifting condensation level (LCL) the saturation mixing ratio r_s is less than r_T . This supersaturated air has an **excess water mixing ratio** r_E of:

$$r_E = r_T - r_s \quad \bullet(7.5)$$

The excess water is available to condense onto liquid drops and to deposit onto ice crystals:

$$r_L + r_i = r_E \quad (7.6)$$

where r_L is liquid-water mixing ratio, and r_i is ice mixing ratio. At temperatures T in the range $-40 < T < 0^\circ\text{C}$, supercooled liquid water and ice crystals can co-exist in the air.

Available supersaturation fraction is defined as:

$$S_A = \frac{r_E}{r_s} = \frac{r_T}{r_s} - 1 \tag{7.7}$$

As hydrometeors grow and remove water vapor, r becomes less than r_T , and S becomes less than S_A . Droplets and ice crystals stop growing by condensation and deposition when they have consumed all the available supersaturation (i.e., when $r \rightarrow r_s$ and $S \rightarrow 0$).

The adiabatic estimate of r_E increases with height above the LCL, as sketched in Figs. 7.2 and 7.3. However, in many real clouds, **diabatic** (i.e., non-adiabatic) processes such as entrainment and mixing of clear drier air into the top and sides of the cloud cause the total-water mixing ratio to not be conserved with height. As a result, typical amounts of excess-water mixing ratio are less than adiabatic (dashed lines in Fig. 7.3).

Beware that the adiabatic r_E curve varies from situation to situation, depending on the thermodynamic initial conditions of the air parcel. So Fig. 7.3 is just one example (based on the Fig. 7.2 scenario).

7.1.3. Number and Size of Hydrometeors

Suppose we partition the available excess water equally between all hydrometeors (for example, for all liquid water droplets). In this way, we can estimate the average radius R for each droplet due only to condensation (i.e., before collisions between droplets allow some to merge and grow into larger drops):

$$R = \left[\frac{3}{4\pi} \cdot \frac{\rho_{air}}{\rho_{water}} \cdot \frac{r_E}{n} \right]^{1/3} \tag{7.8}$$

where excess-water mixing ratio r_E is in $\text{kg}_{\text{water}} \text{kg}_{\text{air}}^{-1}$, ρ is density, and n is the number density of hydrometeors (the count of hydrometeors per cubic meter of air). Typical values are $R = 2$ to $50 \mu\text{m}$, which is small compared to the $1000 \mu\text{m}$ separation between droplets, and is too small to be precipitation.

This is an important consideration. Namely, even if we ignore the slowness of the diffusion process (described later), the hydrometeors stop growing by condensation or deposition before they become precipitation. The reason is that there are too many hydrometeors, all competing for water molecules, thus limiting each to grow only a little.

The number density n of hydrometeors is initially controlled by the number density of nuclei upon which they can form, as described next.

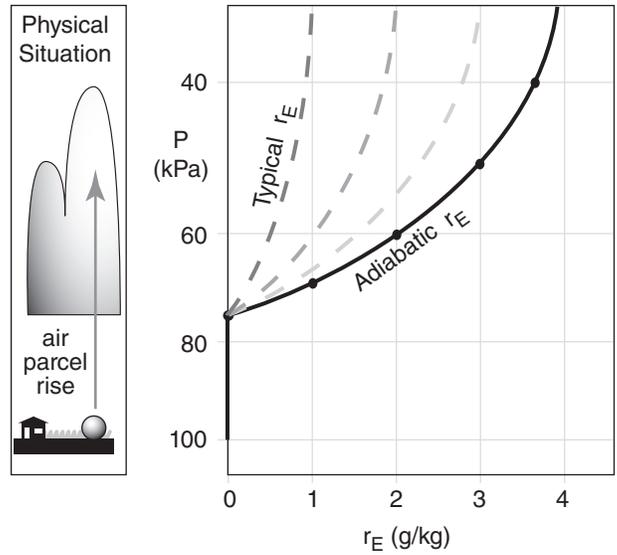


Figure 7.3

Example of the increase of excess water mixing ratio r_E vs. height above cloud base for the scenario in the previous figure. The adiabatic value (solid line) represents the max likely value in non-precipitating clouds, while the dashed curves show ranges of actual max values. P is pressure, a surrogate for height.

Sample Application

Within a cloud, suppose air density is 1 kg m^{-3} and the excess water mixing ratio is 4 g kg^{-1} . Find the final drop radius for hydrometeor counts of (a) 10^8 m^{-3} , and (b) 10^9 m^{-3} .

Find the Answer

Given: $r_E = 0.004 \text{ kg}_{\text{water}} \text{ kg}_{\text{air}}^{-1}$, $\rho_{air} = 1 \text{ kg m}^{-3}$
 $n = \text{(a) } 10^8 \text{ m}^{-3}$, and $\text{(b) } 10^9 \text{ m}^{-3}$.

Find: $R = ? \mu\text{m}$

Assume: $\rho_{water} = 1000 \text{ kg m}^{-3}$, as listed in Appendix A.

Use eq. (7.8). Part (a):

$$R = \left[\frac{3 \cdot (1 \text{ kg}_{\text{air}} / \text{m}^3) \cdot (0.004 \text{ kg}_{\text{water}} / \text{kg}_{\text{air}})}{4\pi \cdot (10^3 \text{ kg}_{\text{water}} / \text{m}^3) \cdot (10^8 \text{ m}^{-3})} \right]^{1/3}$$

$$= 2.12 \times 10^{-5} \text{ m} = \mathbf{21.2 \mu\text{m}}$$

(b) Similarly, $R = \mathbf{9.8 \mu\text{m}}$.

Check: Units OK. Physics OK.

Exposition: Both of these numbers are well within the range of “typical” cloud droplets. Thus, the final drop size is NOT large enough to become precipitation.

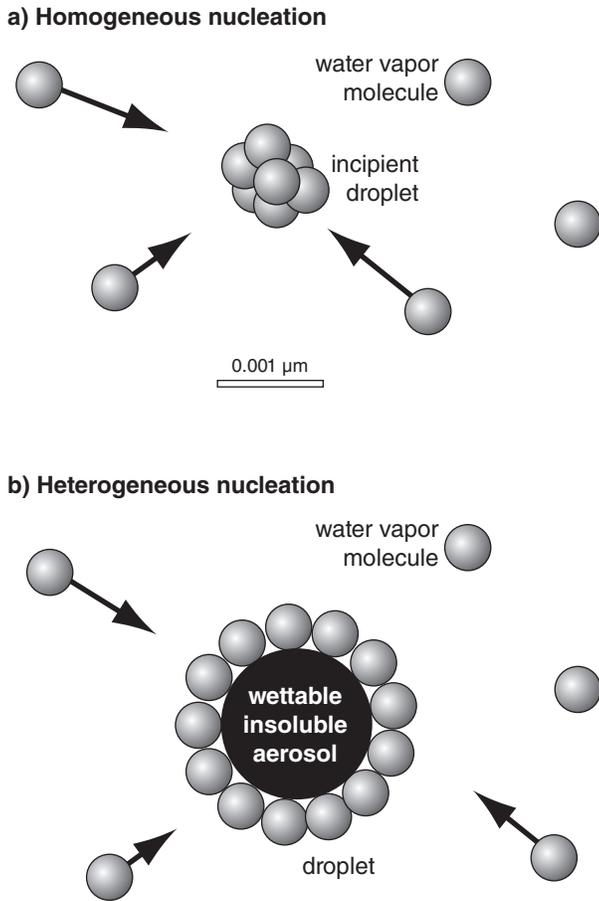


Figure 7.4
 Sketch of differences between (a) homogeneous nucleation of cloud droplets and (b) heterogeneous nucleation around a wettable insoluble aerosol particle.

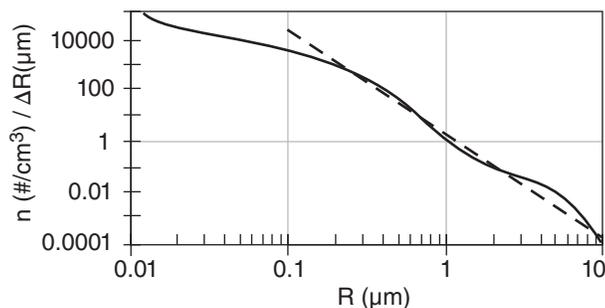


Figure 7.5
 CCN particle count n vs. radius R , for $c = 2,000,000 \mu\text{m}^3 \cdot \text{m}^{-3}$.

7.2. NUCLEATION OF LIQUID DROPLETS

Nucleation (the creation of new droplets) in clean air is called **homogeneous nucleation** (Fig. 7.4a). We will show that homogeneous nucleation is virtually impossible in the real atmosphere and can be neglected. Nucleation of cloud droplets by water vapor condensing on tiny dust particles in the air is called **heterogeneous nucleation** (Fig. 7.4b). Even with heterogeneous nucleation, there is a barrier to droplet formation that must first be overcome.

7.2.1. Cloud Condensation Nuclei (CCN)

An **aerosol** is any tiny solid or liquid particle suspended in the air. The subset of aerosol particles that can nucleate cloud droplets are called **cloud condensation nuclei (CCN)**. To nucleate a droplet, a solid aerosol either must be soluble in water (such as various salt particles), or be sufficiently large in diameter (radius $> 0.1 \mu\text{m}$) and have a wettable surface (i.e., be **hydrophilic**).

Boundary-layer air over oceans has smaller concentrations of aerosols than continental air. Over oceans, of the 150 to 1000 total aerosol particles cm^{-3} of air, only about 90 to 200 particles cm^{-3} are CCN at normal values of relative humidity ($RH \approx 101\%$) inside clouds. Over continents, of the 2,000 to 70,000 total aerosol particles cm^{-3} of air, only about 200 to 700 particles cm^{-3} are CCN. At higher relative humidities, larger percentages of aerosols act as CCN. Exceptionally clean air over the Arctic can have only 30 CCN particles cm^{-3} , while over industrial cities the CCN count can approach 10^6 particles cm^{-3} .

CCN particles can form when pollutant gases (of molecular size: 10^{-4} to $10^{-3} \mu\text{m}$) in the air cluster to form **ultrafine aerosols** (size 10^{-3} to $10^{-2} \mu\text{m}$) or are oxidized in the presence of sunlight. Over the oceans, sulfate and sulfuric acid CCN can form this way from gases such as dimethyl sulfide and methane sulfonic acid, produced by **phytoplankton** (microscopic drifting plant life in the ocean).

Further condensation of more pollutant gases and **coagulation** (sticking together) cause the aerosols to quickly grow to 0.01 to $1 \mu\text{m}$ size, called **fine aerosols**. Beyond this size range they grow more slowly. As a result, aerosols in this size range tend to accumulate — a process called **accumulation mode**. At larger 1 to $10 \mu\text{m}$ sizes, **coarse mode** CCN can form by other processes, such as strong winds that pick up fine dirt from the ground.

Small nuclei are much more abundant than larger ones (thin wiggly line in Fig. 7.5). Instead of a smooth decrease in number of particles as their size increases, the aerosol curve often has two or three

peaks corresponding to the ultrafine, accumulation, and coarse modes.

Over continental regions, the **number density** (n = count of particles per volume of air) of particles with radius between $R - 0.5 \cdot \Delta R$ and $R + 0.5 \cdot \Delta R$ can be approximated by:

$$n(R) = c \cdot R^{-4} \cdot \Delta R \quad (7.9)$$

for particles larger than $0.2 \mu\text{m}$, and for small ΔR . Constant c depends on the total concentration of particles. This distribution, called the **Junge distribution**, is the dashed straight line in Fig. 7.5.

7.2.2. Curvature and Solute Effects

Both droplet curvature and chemical composition affect the evaporation rate, which affect the fate of the droplet.

7.2.2.1. Curvature Effect

The evaporation rate from the curved surface of a droplet is greater than that from a flat water surface, due to surface tension. But droplet growth requires condensation to exceed evaporation. Thus, to be able to grow, smaller droplets need greater RH in the air than larger drops. The resulting equilibrium RH in the air as a function of droplet radius R is described by **Kelvin's equation**

$$RH\% = 100\% \cdot \exp\left[\frac{2 \cdot \sigma}{\rho_m \cdot k_B \cdot T \cdot R}\right] \quad (7.10a)$$

where $\sigma = 0.076 \text{ N m}^{-1}$ is **surface tension** of pure water at 0°C , $\rho_m = 3.3 \times 10^{28} \text{ molecules m}^{-3}$ is the number density of water molecules in liquid at 0°C , $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1} \cdot \text{molecule}^{-1}$ is Boltzmann's constant, and T is absolute temperature (in Kelvin) of the droplet. Kelvin's equation can be abbreviated as

$$RH\% = 100\% \cdot \exp\left[\frac{c_1}{T \cdot R}\right] \quad (7.10b)$$

where $c_1 = 0.3338 \mu\text{m} \cdot \text{K} = 2\sigma / (\rho_m \cdot k_B)$.

Fig. 7.6 illustrates the curvature effect described by Kelvin's equation, where conditions above the line allow droplets to grow. For example, droplets of radius smaller than $0.005 \mu\text{m}$ need an environment having $RH > 128\%$ to grow. But larger droplets of radius $0.1 \mu\text{m}$ need only $RH > 101\%$ to grow.

For homogeneous nucleation in clean air (no aerosols), incipient droplets form when several water-vapor molecules merge (Fig. 7.4a). The resulting droplet has extremely small radius ($\approx 0.001 \mu\text{m}$), causing it to evaporate quickly, given the typical humidities in clouds of $\approx 101\%$ (grey shaded band in Fig. 7.6). How-

Sample Application

If $c = 5 \times 10^7 \mu\text{m}^3 \cdot \text{m}^{-3}$ for the Junge distribution, then how many CCN are expected within radii ranges of (a) $0.45 - 0.55 \mu\text{m}$ and (b) $0.95 - 1.05 \mu\text{m}$?

Find the Answer

Given: $R = 0.5 \mu\text{m}$ & $1.0 \mu\text{m}$, $\Delta R = 0.1 \mu\text{m}$ for both ranges, and $c = 5 \times 10^7 \mu\text{m}^3 \cdot \text{m}^{-3}$
Find: $n = ? \# \text{m}^{-3}$

Use eq. (7.9):

$$(a) n = (5 \times 10^7 \mu\text{m}^3 \cdot \text{m}^{-3}) \cdot (0.5 \mu\text{m})^{-4} \cdot (0.1 \mu\text{m}) = \underline{8 \times 10^7 \text{ m}^{-3}}$$

$$(b) n = (5 \times 10^7 \mu\text{m}^3 \cdot \text{m}^{-3}) \cdot (1 \mu\text{m})^{-4} \cdot (0.1 \mu\text{m}) = \underline{5 \times 10^6 \text{ m}^{-3}}$$

Check: Units OK. Physics OK.

Exposition: Doubling the particle radius reduces the number density by more than tenfold. If each CCN nucleates a cloud droplet, then each m^3 of cloudy air contain tens of millions of cloud droplets. Concentrations in real clouds can be 10s to 1000s of times greater.

Sample Application

What humidity in a cloud at 0°C is needed to allow a droplet of radius $0.03 \mu\text{m}$ to grow?

Find the Answer:

Given: $R = 0.03 \mu\text{m}$. $T = 273 \text{ K}$

Find: $RH = ? \%$

Use eq. (7.10b)

$$RH\% = 100\% \cdot \exp\left[\frac{0.334 \mu\text{m} \cdot \text{K}}{(273 \text{ K}) \cdot (0.03 \mu\text{m})}\right] = \underline{104.16\%}$$

Check: Units OK. Magnitude OK.

Exposition: A stable droplet this small is unlikely in a real cloud, where typically $RH \approx 101\%$ or less.

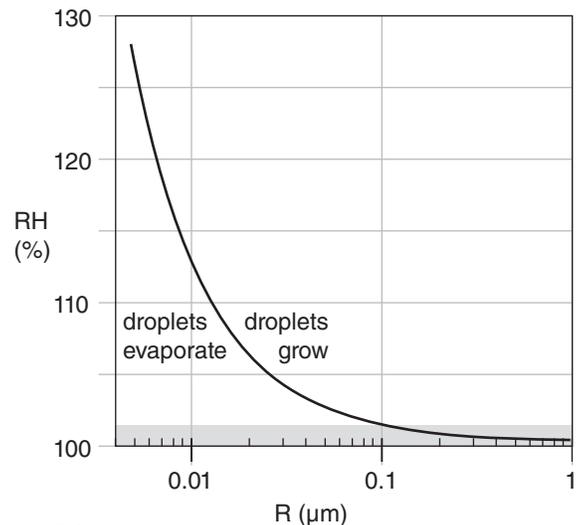


Figure 7.6

Curvature effect, showing the relative humidity (RH) needed in air for a droplet of radius R to grow. The curve is given by Kelvin's equation, for pure water at 0°C . Shaded region highlights the range of RH typically found in clouds.

Table 7-1. Properties of some solutes. M_s is molecular weight, i is approximate ion count.

Solute	Chemistry	M_s	i
salt	NaCl	58.44	2
ammonium sulfate	$(\text{NH}_4)_2\text{SO}_4$	132.13	3
hydrogen peroxide	H_2O_2	34.01	2
sulfuric acid	H_2SO_4	98.07	3
nitric acid	HNO_3	63.01	2

Sample Application

Find the equilibrium relative humidity over a droplet of radius $0.2 \mu\text{m}$, temperature 20°C , containing 10^{-16} g of ammonium sulfate.

Find the Answer

Given: $R = 0.2 \mu\text{m}$, $T = 293 \text{ K}$, $m_s = 10^{-16} \text{ g}$

Find: $RH\% = 100\% \cdot (e_s^*/e_s) = ? \%$

From Table 7-1 for ammonium sulfate:

$M_s = 132.13$, and $i = 3$.

Use eq. (7.11):

$$\frac{e_s^*}{e_s} \approx \frac{\exp\left(\frac{0.3338\text{K} \cdot \mu\text{m}}{(293\text{K}) \cdot (0.2\mu\text{m})}\right)}{1 + \frac{(4.3 \times 10^{12} \mu\text{m}^3 \cdot \text{g}^{-1}) \cdot 3 \cdot (10^{-16} \text{g})}{(132.13) \cdot (0.2\mu\text{m})^3}}$$

$$= 1.00571 / (1+0.00122) = 1.00448$$

$$RH\% = 100\% \cdot (e_s^*/e_s) = \mathbf{100.448\%}$$

Check: Units OK. Physics OK.

Exposition: Fig. 7.7b gives a value of about 100.49% for a temperature of 0°C . Thus, warmer temperatures require less supersaturation of water vapor in the air to prevent the droplet from vaporizing.

Sample Application

In fog at 10°C , the vapor pressure is 1.4 kPa . Find the supersaturation fraction and percentage.

Find the Answer

Given: $T = 10^\circ\text{C}$, $e_s^* = 1.4 \text{ kPa}$

Find: $S = ?$, and $S\% = ?$

First, use Table 4-1 in the Water Vapor chapter to find the saturation vapor pressure at 10°C : $e_s = 1.233 \text{ kPa}$.

Next, use eq. (7.12):

$$S = (1.4 \text{ kPa} / 1.233 \text{ kPa}) - 1 = 1.135 - 1 = \mathbf{0.135}$$

$$S\% = 100\% \cdot S = \mathbf{13.5\%}$$

Check: e_s^* is indeed $> e_s$, thus supersaturated.

Exposition: Unrealistically large, given typical supersaturations in clouds and fog of order $S\%=1\%$.

ever, for heterogeneous nucleation the small number of water molecules can coat the outside of the aerosol particle (Fig. 7.4b), with a resulting radius that is relatively large. Droplets formed by heterogeneous nucleation grow and remove water-vapor molecules from the air, thereby lowering the RH and eliminating the chance for homogeneous nucleation.

7.2.2.2. Solute Effect

Solutions (i.e., water containing dissolved chemicals) evaporate water molecules at a slower rate than does pure water. Solutions occur when condensation occurs on impurities such as certain cloud condensation nuclei (CCN) that dissolve in the nascent water droplet. This can partially compensate the curvature effect.

Recall that the saturation vapor pressure e_s over a flat surface of pure water was given in the Water Vapor chapter by the Clausius-Clapeyron equation. The two opposing effects of curvature and solute can be combined into one equation (**Köhler equation**) for the ratio of actual saturation vapor pressure e_s^* in equilibrium over a solution with a curved surface, to vapor pressure over a flat surface of pure water e_s :

$$RH = \frac{e_s^*}{e_s} \approx \frac{\exp\left(\frac{c_1}{T \cdot R}\right)}{1 + \frac{c_2 \cdot i \cdot m_s}{M_s \cdot R^3}} \quad \bullet(7.11)$$

where RH is the relative humidity fraction, T is absolute temperature, R is drop radius, i is number of ions per molecule in solution (called the **van't Hoff factor**), m_s is mass of solute in the droplet, and M_s is molecular weight of solute. The two parameters are: $c_1 = 0.3338 \text{ K}\cdot\mu\text{m}$, and $c_2 = 4.3 \times 10^{12} \mu\text{m}^3 \cdot \text{g}^{-1}$. Table 7-1 gives properties for common atmospheric solutes.

In eq. (7.11) the relative humidity RH can be greater than 1, corresponding to a relative-humidity percentage ($RH\% = 100\% \cdot e_s^*/e_s$) that is greater than 100%. Similar to eq. (7.3), **supersaturation** relative to the hydrometeor can be defined as a fraction:

$$S = (e_s^*/e_s) - 1 \quad \bullet(7.12)$$

or as a percentage, $S\% = 100\% \cdot S$:

$$S\% = 100\% \cdot [(e_s^*/e_s) - 1] \quad \bullet(7.13)$$

Thus, the left hand side of eq. (7.11) can be easily rewritten as supersaturation.

The numerator of eq. (7.11) describes the **curvature effect**, and together with the left hand side is the **Kelvin equation**. The denominator describes the **solute effect** of impurities in the water. Eq. (7.11) was solved in a spreadsheet to produce the **Köhler**

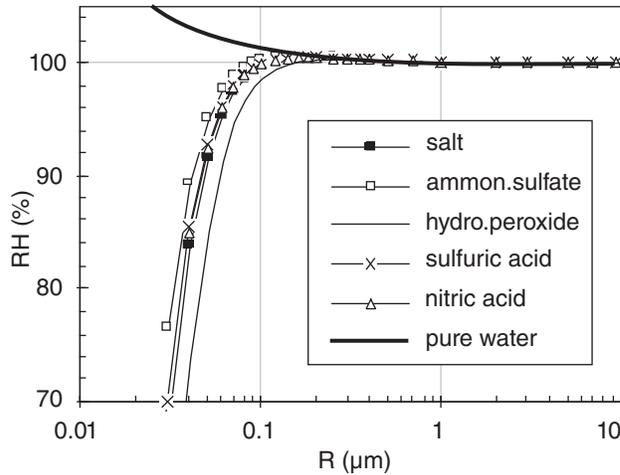


Figure 7.7a
Köhler curves, showing equilibrium relative humidities over droplets of different radius with various solutes. $T = 0^{\circ}\text{C}$. Solute mass = 10^{-16} g.

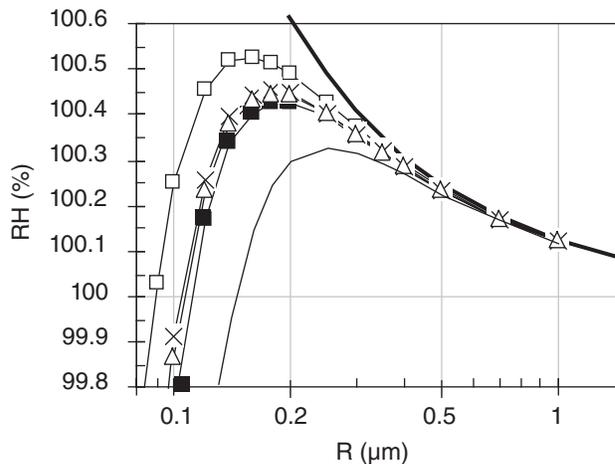


Figure 7.7b
Blow-up of Fig. 7.7a.

curves in Fig. 7.7. Using these Köhler curves, we can study nucleation.

First, the curve in Fig. 7.7 for pure water increases exponentially as drop radius becomes smaller. This curve was already shown in Fig. 7.6, computed from Kelvin’s equation. It was used to explain why homogeneous nucleation is unlikely, because when the first several water-vapor molecules come together to form a condensate, the droplet radius is so small that the droplet explosively evaporates.

Second, solutions of some chemicals can form small droplets even at humidities of less than 100%. Such **hygroscopic** (water attracting) pollutants in the air will grow into droplets by taking water vapor out of the air.

Third, if humidities become even slightly greater than the peaks of the Köhler curves, or if droplet radius becomes large enough (see INFO box), then

INFO • Droplet Growth

To help interpret Fig. 7.7b, consider a droplet containing just one chemical such as salt, in a cloud having a known humidity, such as $RH = 100.3\%$. This is redrawn below.

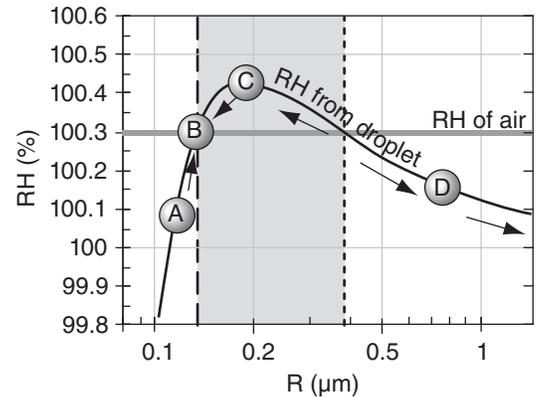


Figure 7.a. Interpretation of a Köhler curve.

Think of the Köhler curve as the RH associated with the droplet, which is trying to drive evaporation from the droplet. The RH in air (horizontal grey line) is trying to drive condensation to the droplet.

If $RH_{droplet} > RH_{air}$ then evaporation > condensation, and the droplet radius becomes smaller as water molecules leave the droplet. This is the situation for any droplet in the light grey band; namely, droplet C suffers net evaporation, causing its radius to decrease toward B.

If $RH_{droplet} < RH_{air}$ then evaporation < condensation, and droplet radius increases due to net condensation. Droplet D has this state, and as its radius increases it moves to the right in the graph, causing $RH_{droplet}$ to decrease further and driving even faster droplet growth. Activated droplets such as these continue growing (shown by right-pointing arrows) until they consume the excess humidity (driving the RH_{air} down toward 100%). Such droplet growth is a first stage in formation of precipitation in warm clouds.

Droplet A also has $RH_{droplet} < RH_{air}$ and would tend to increase in radius due to net condensation. But in this part of the Köhler curve, increasing radius causes increasing $RH_{droplet}$, and causes the net condensation to diminish until the droplet radius reaches that at B.

Thus, the vertical dotted line in Fig. 7.a is an unstable equilibrium. Namely, a droplet on the Köhler curve at the dotted line would either grow or shrink if perturbed slightly from its equilibrium point.

However, the vertical dashed line is a stable equilibrium point. Droplets approach this radius and then stop growing. Namely, they stay as small haze particles, and do not grow into larger cloud or precipitation particles.

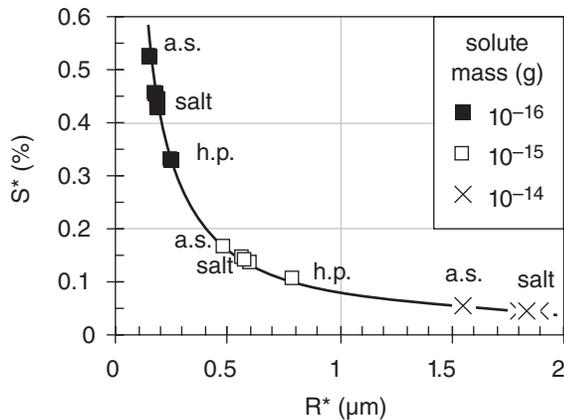


Figure 7.8
Critical values of supersaturation S^* vs. critical drop radius R^* , for various solute masses and chemicals. a.s. = ammonium sulfate, salt = sodium chloride, h.p. = hydrogen peroxide. Nitric acid and sulfuric acid are near the salt data points.

Sample Application

Find the critical radius and supersaturation value for 10^{-15} g of ammonium sulfate at 0°C .

Find the Answer

Given: $m_s = 10^{-15}$ g, $T = 273$ K

Find: $R^* = ? \mu\text{m}$, $S^* = ? \%$.

Use eq. (7.14) & Table 7-1. $R^* =$

$$\sqrt{\frac{(3.8681 \times 10^{13} \text{ K}^{-1} \cdot \text{g}^{-1} \mu\text{m}^2) \cdot 3 \cdot (10^{-15} \text{ g}) \cdot (273 \text{ K})}{132.13}}$$

$$R^* = \mathbf{0.49 \mu\text{m}}$$

Use eq. (7.15) & Table 7-1. $S^* =$

$$S^* = \sqrt{\frac{(1.278 \times 10^{-15} \text{ K}^3 \cdot \text{g}) \cdot 132.13}{3 \cdot (10^{-15} \text{ g}) \cdot (273 \text{ K})^3}} = 0.00166$$

$$S^* = \mathbf{0.166\%}$$

Check: Units OK. Physics OK.

Exposition: Agrees with the point (white square) plotted in Fig. 7.8 for "a.s.". Thus, the larger mass nucleus needs less supersaturation to become activated.

For a parcel of rising air with increasing supersaturation, the larger nuclei will become activated first, followed by the smaller nuclei if the parcel keeps cooling and if the excess vapor is not removed by the larger nuclei first.

droplets can grow unimpeded. CCN reaching this state are said to be **activated**. Growth of droplets from activated nuclei continues until enough vapor (i.e., r_E) is consumed to reduce the supersaturation back toward 100%. Pure droplets cannot form or co-exist in an environment with neighboring solution droplets, because of the low supersaturation remaining in the air after much of the vapor has condensed out onto the solution droplets. That is another reason why homogeneous nucleation can be neglected for practical purposes.

Fourth, although not shown in these curves, the equations allow droplets to form at lesser supersaturations if the mass of dissolved solute is greater. Hence, larger CCN can grow into droplets earlier and can grow faster than smaller CCN.

7.2.3. Critical Radius

The location of the peak of the Köhler curves marks the barrier between the larger, activated droplets that can continue to grow, from the smaller **haze** droplets that reach an equilibrium at small size. The drop radius R^* at this peak is called the **critical radius**, and the corresponding **critical supersaturation** fraction is $S^* = e_s^*/e_s - 1$. They are given by:

$$R^* = \sqrt{\frac{c_3 \cdot i \cdot m_s \cdot T}{M_s}} \quad \bullet(7.14)$$

and

$$S^* = \sqrt{\frac{c_4 \cdot M_s}{i \cdot m_s \cdot T^3}} \quad \bullet(7.15)$$

where $c_3 = 3.8681 \times 10^{13} \mu\text{m}^2 \cdot \text{K}^{-1} \cdot \text{g}^{-1}$, and $c_4 = 1.278 \times 10^{-15} \text{ K}^3 \cdot \text{g}$. For critical supersaturation as a percentage, use $S^* = 100\% \cdot S^*$.

Critical conditions are plotted in Fig. 7.8 for various masses of different chemicals. Obviously S^* is inversely related to R^* , so as solute mass increases, smaller supersaturations are necessary to reach the critical point, and at that point the droplets will be larger. Also, notice that different chemicals will grow to different sizes, which is one factor causing a range of drop sizes in the cloud.

7.2.4. Haze

For conditions left of the peak on any Köhler curve (i.e., $R < R^*$), CCN rapidly grow into small droplets that stop growing at an equilibrium size determined by the humidity, temperature, and solute (see previous INFO box). These small droplets are called **haze** droplets. Thus, tiny droplets can exist even at relative humidities below 100%.

Sample Application

For 10^{-16} g of ammonium sulfate at 0°C , how does haze droplet radius change as RH increases from 70 to 80%? Also, how many molecules are in each aerosol?

Find the Answer

Assume: Same conditions as in Fig. 7.7a.
 Given: $RH = 70\%$, 80% .
 Find: $R = ? \mu\text{m}$, $n = ?$ molecules

Solve eq. (7.11), or use Fig. 7.7a. I will use the Fig.
 $R \approx 0.027 \mu\text{m}$ at 70%; $R \approx 0.032 \mu\text{m}$ at 80%.

The number of molecules is $n = \rho_m \cdot [(4/3) \cdot \pi \cdot R^3]$
 $n = 2.72 \times 10^6$ molecules; $n = 4.53 \times 10^6$ molecules

Check: Units OK. Physics OK.

Exposition: Haze particles indeed become larger as relative humidity increases, thereby reducing visibility. Scattering of light by this size of particles is called **Mie scattering** (see the Atmos. Optics chapter).

Haze droplets are aerosols. When a tiny, dry CCN grows to its equilibrium size by the condensation of water molecules, this process is called **aerosol swelling**. Aerosol swelling is responsible for reducing **visibility** in polluted air as humidities increase above about 75%.

Even haze particles contain many water molecules. Liquid water contains about $\rho_m = 3.3 \times 10^{28}$ molecules m^{-3} . Thus, the smallest haze particles of radius $0.02 \mu\text{m}$ contain roughly $n = \rho_m \cdot [(4/3) \cdot \pi \cdot R^3] = 1.1$ million molecules.

The word **smog** is a contraction of “smoke” and “fog”, which is a reasonable lay description of haze. Many urban smogs are a stew of ingredients including ozone, volatile hydrocarbons such as evaporated gasoline, and various oxides of nitrogen. These react in the atmosphere, particularly in the presence of sunlight, to create sulfates, nitrates, and hydrogen peroxide CCNs. Aerosol swelling and reduced visibilities are quite likely in such urban smogs, particularly when the air is humid.

7.2.5. Activated Nuclei

For conditions to the right of the peak on any Köhler curve (i.e., $R > R^*$), CCN are activated and can continue growing. There is no equilibrium that would stop their growth, assuming sufficient water vapor is present. These droplets can become larger than haze droplets, and are called **cloud droplets**.

Because atmospheric particles consist of a variety of chemicals with a range of masses, we anticipate from the Köhler curves that different CCN will become activated at different amounts of

HIGHER MATH • Critical Radius

Derivation of the critical radius, eq. (7.14).

The critical radius is at $e_s^*/e_s = \text{maximum}$. But at the maximum, the slope is zero: $d(e_s^*/e_s)/dR = 0$. By finding this derivative of eq. (7.11) with respect to R , and setting it to zero, we can solve for R at the maximum. This is R^* by definition.

The right side of eq. (7.11) is of the form a/b . A rule of calculus is:

$$d(a/b)/dR = [b \cdot (da/dR) - a \cdot (db/dR)] / b^2.$$

Also, a is of the form $a = \exp(f)$, for which another rule is:

$$da/dR = a \cdot (df/dR).$$

Combining these 2 rules and setting the whole thing to zero gives:

$$0 = \frac{a}{b} \left[\frac{df}{dR} - \frac{1}{b} \frac{db}{dR} \right]$$

But (a/b) is just the original right side of eq. (7.11), which we know is close to 1.0 at the max, not close to 0. Thus, the eq. above equals 0 only if:

$$\frac{df}{dR} = \frac{1}{b} \frac{db}{dR}$$

Plugging in for f and b and differentiating yields:

$$\frac{-c_1}{T \cdot R^2} = \frac{1}{\left[1 + \frac{c_2 \cdot i \cdot m_s}{M_s \cdot R^3} \right]} \cdot \frac{(-3) \cdot c_2 \cdot i \cdot m_s}{M_s \cdot R^4}$$

Multiply both sides by $(-R^4 \cdot T / c_1)$:

$$R^2 = (3 \cdot T / c_1) \cdot \frac{\left[\frac{c_2 \cdot i \cdot m_s}{M_s} \right]}{\left[1 + \frac{c_2 \cdot i \cdot m_s}{M_s \cdot R^3} \right]}$$

Multiply the numerator and denominator of the right side by $M_s / (c_2 \cdot i \cdot m_s)$, which gives:

$$R^2 = (3 \cdot T / c_1) \cdot \frac{1}{\left[\frac{M_s}{c_2 \cdot i \cdot m_s} + \frac{1}{R^3} \right]}$$

By plugging in typical values, we can show that the $1/R^3$ term is small enough to be negligible compared to the other term in square brackets. This leaves:

$$R^2 \cong \left(\frac{3 \cdot c_2}{c_1} \right) \cdot \left(\frac{T \cdot i \cdot m_s}{M_s} \right)$$

Define $c_3 = 3 \cdot c_2 / c_1$, set $R = R^*$, and take the square root of both sides to get the final answer:

$$R^* \cong \sqrt{\frac{c_3 \cdot i \cdot m_s \cdot T}{M_s}} \tag{7.14}$$

Sample Application

How many nuclei would be activated in continental air of supersaturation percentage 0.5%? Also, how much air surrounds each droplet, and what is the distance between drops?

Find the Answer

Given: $S = 0.005$

Find: $n_{CCN} = ?$ particles m^{-3} ,
 $Vol = ?$ mm^3 , $x = ?$ mm.

(a) Use eq. (7.16):

$$n_{CCN} = (6 \times 10^8 \text{ m}^{-3}) \cdot (0.5)^{0.5} = \mathbf{4.24 \times 10^8 \text{ m}^{-3}}$$

(b) Also:

$$Vol = 1/n_{CCN} = \mathbf{2.36 \text{ mm}^3 \text{ droplet}^{-1}}$$

(c) Using eq. (7.17): $x = Vol^{1/3} = \mathbf{1.33 \text{ mm}}$.

Check: Units OK. Physics OK.

Exposition: If all of these nuclei become cloud droplets, then there are over 40 million droplets within each cubic meter of cloud. But there is a relatively large distance between each drop.

supersaturation. The number density n_{CCN} (# of CCN per m^3) activated as a function of supersaturation fraction S is roughly:

$$n_{CCN} = c \cdot (100 \cdot S)^k \tag{7.16}$$

but varies widely. In maritime air, $c \approx 1 \times 10^8 \text{ m}^{-3}$ and $k \approx 0.7$. In continental air $c \approx 6 \times 10^8 \text{ m}^{-3}$ and $k \approx 0.5$. The number of activated nuclei is in the range of 10^8 to 10^9 m^{-3} , which is usually just a small fraction of the total number of particles in the air.

The distance x between cloud droplets is on the order of 1 mm, and is given by

$$x = n_{CCN}^{-1/3} \tag{7.17}$$

7.3. NUCLEATION OF ICE CRYSTALS

Ice cannot survive at temperatures above 0°C at normal atmospheric pressures. Below that temperature, ice crystals can exist in equilibrium with air that is supersaturated with respect to ice. The saturation curve for ice was plotted in Fig. 4.2, and is close to, but slightly below, the curve for supercooled liquid water.

There is a thermodynamic barrier to ice formation, analogous to the barrier for droplet growth. This barrier can be overcome with either very cold temperatures (colder than -40°C), high supersaturation in the air, or by the presence of ice nuclei.

7.3.1. Processes

Homogeneous freezing nucleation is the name for the spontaneous freezing that occurs within supercooled liquid-water droplets as temperature decreases to near -40°C . No impurities are needed for this. Instead, ice embryos form by chance when clusters of water molecules happen to come together with the correct orientations (Fig. 7.9a). At -40°C , the cluster needs only about 250 molecules. At slightly warmer temperatures the critical embryo size is much larger, and thus less likely to occur. Because larger supercooled droplets contain greater numbers of molecules, they are more likely to form an ice embryo and freeze.

Heterogeneous freezing nucleation can occur a variety of ways (Fig. 7.9b-d), but all require an impurity (Table 7-2), which is generically called an **ice nucleus**. These processes are described next.

Deposition nucleation (Fig. 7.9b) occurs when water vapor deposits directly on a **deposition nucleus**. While the solute effect was important

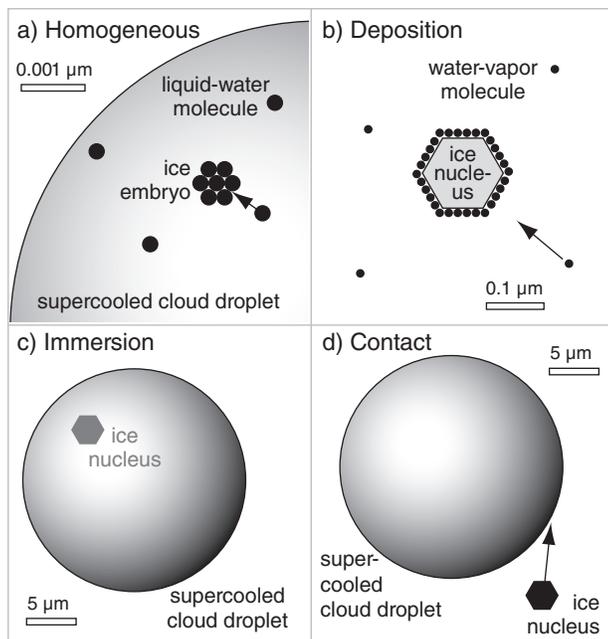


Figure 7.9
 Sketch of (a) homogeneous, and (b-d) heterogeneous ice nucleation processes. Note the different size scales in each sketch.

for liquid droplet nucleation, it does not apply to ice nucleation because salts are excluded from the ice-crystal lattice as water freezes. Thus, the size and crystal structure of the ice nucleus is more important. Deposition is unlikely on particles of size 0.1 μm or less. Colder temperatures and greater supersaturation increase deposition nucleation.

Immersion freezing (Fig. 7.9c) occurs for those liquid drops already containing an undissolved ice nucleus called a **freezing nucleus**. Each nucleus has a critical temperature below 0°C at which it triggers freezing. Thus, as external processes cause such a contaminated drop to cool, it can eventually reach the critical temperature and freeze.

Larger drops contain more freezing nuclei, and have a greater chance of containing a nucleus that triggers freezing at warmer temperatures (although still below 0°C). To freeze half of the drops of radius R , the temperature must drop to T , given statistically by

$$T \approx T_1 + T_2 \cdot \ln(R/R_0) \tag{7.18}$$

where $R_0 = 5 \mu\text{m}$, $T_1 = 235 \text{ K}$, and $T_2 = 3 \text{ K}$.

Condensation freezing is a cross between deposition nucleation and immersion freezing. In this scenario, which occurs below 0°C, nuclei are more attractive as condensation nuclei than as deposition nuclei. Thus, supercooled liquid water starts to condense around the nucleus. However, this liquid water immediately freezes due to the immersion-nucleation properties of the nucleus.

Contact freezing (Fig. 7.9d) occurs when an uncontaminated supercooled liquid drop happens to hit an external **contact nucleus**. If the droplet is cooler than the critical temperature of the ice nucleus, then it will freeze almost instantly. This also happens when (supercooled) **freezing rain** hits and instantly freezes on trees and power lines. Ice crystals in the air are also good contact nuclei for supercooled water.

7.3.2. Ice Nuclei

Only substances with similar molecular structure as ice can serve as ice nuclei. Such substances are said to be **epitaxial** with ice.

Natural ice nuclei include fine particles of clay such as kaolinite stirred up from the soil by the wind. Certain bacteria and amino acids such as l-leucine and l-tryptophan from plants also can nucleate ice. Combustion products from forest fires contain many ice nuclei. Also, ice crystals from one cloud can fall or blow into a different cloud, triggering continued ice formation.

Other substances have been manufactured specifically to seed clouds, to intentionally change the amount or type (rain vs. hail) of precipitation. Silver

Table 7-2. Listed are the warmest ice nucleation threshold temperatures (°C) for substances that act as ice nuclei. Processes are: 1 contact freezing, 2 condensation freezing, 3 deposition, and 4 immersion freezing. (x = not a process for this substance.)

Substance	Process			
	1	2	3	4
silver iodide	-3	-4	-8	-13
cupric sulfide	-6	x	-13	-16
lead iodide	-6	-7	-15	x
cadmium iodide	-12	x	-21	x
metaldehyde	-3	-2	-10	x
1,5-dihydroxynaphlene	-6	-6	-12	x
phloroglucinol	x	-5	-9	x
kaolinite	-5	-10	-19	-32

Sample Application

What cloud temperature is needed to immersion-freeze half of the droplets of 100 μm radius?

Find the Answer

Given: $R = 100 \mu\text{m}$. Find: $T = ? \text{ K}$

Use eq. (7.18)

$$T = 235\text{K} + (3\text{K}) \cdot \ln(100\mu\text{m}/5\mu\text{m}) = 244\text{K} = \underline{-29^\circ\text{C}}$$

Check: Units OK. Physics OK.

Exposition: Smaller droplets can remain unfrozen at much colder temperatures than larger drops, which are thus available to participate in the WBF precipitation growth process described later.

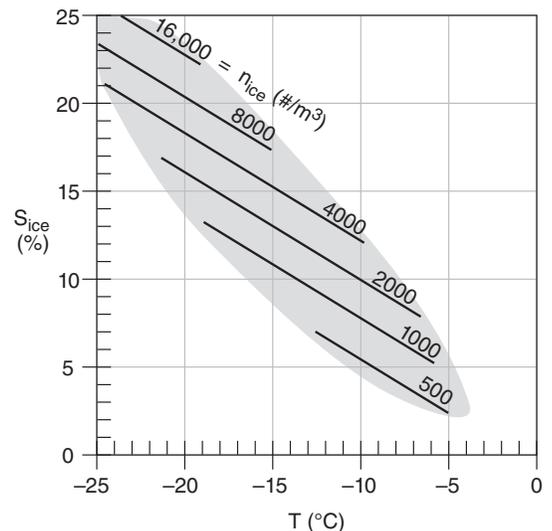


Figure 7.10

Number density n_{ice} of active ice nuclei per m^3 of air as a function of ice supersaturation S and temperature T . Shaded region encloses typically observed combinations of S and T .

Sample Application

Given a supersaturation gradient of 1% per 2 μm near a droplet in a cloud at 4 km altitude where saturated mixing ratio is 1.5 g kg⁻¹ and the diffusivity is 2x10⁻⁵ m²·s⁻¹. Find (a) the mixing-ratio gradient, (b) the kinematic moisture flux, and (c) the dynamic moisture flux.

Find the Answer:

Given: ΔS%/Δx = 1% / 2 μm, r_s = 0.0015 kg_{water}/kg_{air}
 D = 2x10⁻⁵ m²·s⁻¹.

Find: (a) Δr/Δx = ? (kg_{water}/kg_{air})/m, (b) F = ?
 (kg_{water}/kg_{air})·(m s⁻¹), (c) F̄ = ? kg_{water}·m⁻²·s⁻¹

First, convert units in the saturation gradient:
 ΔS/Δx = [1%/2 μm]·(10⁶μm/1m)/100% = 5000 m⁻¹.
 Next, use eq. (7.21):
 Δr/Δx = (0.0015kg/kg)·(5000m⁻¹)
 = 7.5 (kg_{water}/kg_{air})·m⁻¹.

Then use eq. (7.20):
 F = -(2x10⁻⁵ m²·s⁻¹)·(7.5 (kg/kg)·m⁻¹)
 = -1.5x10⁻⁴ (kg_{water}/kg_{air})·(m s⁻¹).

Finally, use the definition of kinematic flux from the Solar & IR Radiation chapter: F = F̄ / ρ_{air}. As a first guess for density, assume a standard atmosphere at z = 2 km, and use Table 1-5 in Chapter 1 to estimate ρ_{air} ≈ 0.82 kg_{air} m⁻³. Thus: F̄ = (ρ_{air})·(F)
 = (0.82kg_{air} m⁻³)·[-1.5x10⁻⁴(kg_{water}/kg_{air})·(m s⁻¹)]
 = -1.23x10⁻⁴ kg_{water}·m⁻²·s⁻¹.

Check: Units OK. Physics OK.

Exposition: Diffusive fluxes are very small, which is why it can take a couple hours for droplets to grow to their maximum drop radius. The negative sign means water vapor flows from high to low humidity.

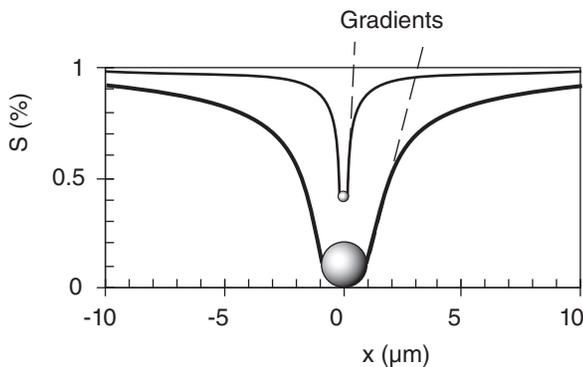


Figure 7.11
 Humidity gradients in supersaturated environment near growing droplets. S% is supersaturation percentage, x is distance. The nucleus for both droplets is salt of mass 10⁻¹⁶ g. Background supersaturation is 1%, the small droplet has radius 0.2 μm, and the large drop 1.0 μm. Equilibrium supersaturation adjacent to the droplets is taken from Fig. 7.7b. Nearest neighbor droplets are roughly 1000 μm distant.

iodide has been a popular chemical for cold-cloud seeding. Other cold-cloud-seeding chemicals are lead iodide, cupric sulfide, phloroglucinol, and met-aldehyde. Table 7-2 lists some ice nuclei and their critical temperatures. Because contact nucleation occurs at the warmest temperatures for most substances, it is the most likely process causing ice nucleation.

Air usually contains a mixture of ice nuclei having a variety of ice nucleation processes that become active at different temperatures and supersaturations. The number density n_{ice} of active ice nuclei per cubic meter of air is shown in Fig. 7.10.

7.4. LIQUID DROPLET GROWTH BY DIFFUSION

In a supersaturated environment, condensation onto a growing droplet removes water vapor from the adjacent air (Fig. 7.11). This lowers the humidity near the droplet, creating a humidity gradient down which water vapor can diffuse.

Diffusion is the process where individual water-vapor molecules meander through air via Brownian motion (i.e., random walk). The net direction of diffusion is always down the humidity gradient toward drier air.

The diffusive moisture flux F̄ in kg_{water}·m⁻²·s⁻¹ is

$$F̄ = -D \cdot \frac{\Delta \rho_v}{\Delta x} \quad \bullet(7.19)$$

where x is distance, D is diffusivity, and ρ_v is absolute humidity (water-vapor density) in kg_{water} m⁻³. To rewrite this in kinematic form, divide by dry-air density ρ_{air} to give:

$$F = -D \cdot \frac{\Delta r}{\Delta x} \quad \bullet(7.20)$$

where r is water-vapor mixing ratio, and kinematic flux F has units of mixing ratio times velocity [(kg_{water} kg_{air}⁻¹)·(m s⁻¹)]. Larger gradients cause larger fluxes, which cause droplets to grow faster.

The mixing ratio gradient is Δr/Δx. This is related to the supersaturation gradient ΔS/Δx by:

$$\frac{\Delta r}{\Delta x} = \frac{\Delta[(1+S) \cdot r_s]}{\Delta x} \approx r_s \cdot \frac{\Delta S}{\Delta x} \quad (7.21)$$

where r_s is the saturation mixing ratio over a flat water surface. For this expression, the supersaturation fraction is defined by eq. (7.4), and supersaturation percentage by eq. (7.2).

Larger drops create a more gentle gradient than smaller drops. The humidity profile is given by:

$$S \approx S_\infty + \frac{R}{|x|} \cdot (S_R - S_\infty) \tag{7.22}$$

where S is supersaturation fraction at distance x from the center of the drop, S_∞ is background supersaturation at a large distance from the droplet, S_R is equilibrium supersaturation adjacent to the drop, and R is droplet radius. Eq. (7.22) was solved on a spreadsheet to create Fig. 7.11.

The **diffusivity** D is approximately

$$D = c \cdot \frac{P_o}{P} \cdot \left(\frac{T}{T_o} \right)^{1.94} \tag{7.23}$$

where $c = 2.11 \times 10^{-5} \text{ m}^2 \cdot \text{s}^{-1}$ is an empirical constant, $P_o = 101.3 \text{ kPa}$, and $T_o = 273.15 \text{ K}$. This molecular diffusivity for moisture is similar to the thermal conductivity for heat, discussed in the Heat Budgets chapter.

During droplet growth, not only must water vapor diffuse through the air toward the droplet, but heat must conduct away from the drop. This is the latent heat that was released during condensation. Without conduction of heat away from the drop, it would become warm enough to prevent further condensation, and would stop growing.

Droplet radius R increases with the square-root of time t , as governed by the combined effects of water diffusivity and heat conductivity:

$$R \approx c_4 \cdot (D \cdot S_\infty \cdot t)^{1/2} \tag{7.24}$$

where S_∞ is the background supersaturation fraction far from the drop. Also, dimensionless constant c_4 is:

$$c_4 = (2 \cdot r_\infty \cdot \rho_{air} / \rho_{liq,water})^{1/2} \tag{7.25}$$

where background mixing ratio is r_∞ . If the supersaturation in air surrounding droplets is great enough, the smaller droplets grow by diffusion faster than larger droplets, because of the greater humidity gradients near the smaller drops (Fig. 7.11 and eq. 7.22). Thus, the small droplets will tend to catch up to the larger droplets.

The result is a drop size distribution that tends to become **monodisperse**, where most of the drops have approximately the same radius. Also, eq. (7.24) suggests that time periods of many days would be necessary to grow rain-size drops by diffusion alone. But real raindrops form in much less time (tens of minutes), and are known to have a wide range of sizes. Hence, diffusion cannot be the only physical process contributing to rain formation.

Sample Application

Find the water vapor diffusivity at $P = 100 \text{ kPa}$ and $T = -10^\circ\text{C}$.

Find the Answer

Given: $P = 100 \text{ kPa}$ and $T = 263 \text{ K}$.

Find: $D = ? \text{ m}^2 \cdot \text{s}^{-1}$.

Use eq. (7.23):

$$D = (2.11 \times 10^{-5} \text{ m}^2 \cdot \text{s}^{-1}) \left(\frac{101.3 \text{ kPa}}{100 \text{ kPa}} \right) \left(\frac{263 \text{ K}}{273.15 \text{ K}} \right)^{1.94} = \underline{1.99 \times 10^{-5} \text{ m}^2 \cdot \text{s}^{-1}}$$

Check: Units OK. Physics OK.

Exposition: Such a small diffusivity means that a large gradient is needed to drive the vapor flux.

Sample Application

Find and plot drop radius vs. time for diffusive growth, for the same conditions as the previous Sample Application. Assume 1% supersaturation.

Find the Answer

Given: $P = 100 \text{ kPa}$, $T = 263 \text{ K}$, $D = 2 \times 10^{-5} \text{ m}^2 \cdot \text{s}^{-1}$.

Find: $R(\mu\text{m})$ vs. t

First get ρ_{air} from the ideal gas law:

$$\rho_{air} = \frac{P}{R_d \cdot T} = \frac{100 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{K}^{-1} \cdot \text{m}^3 \cdot \text{kg}^{-1}) \cdot (263 \text{ K})} = 1.325 \text{ kg} \cdot \text{m}^{-3}$$

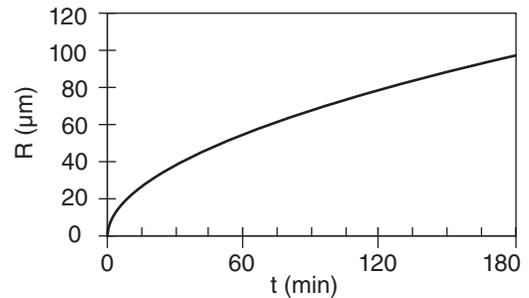
$r_s \approx 1.8 \text{ g kg}^{-1} = 0.0018 \text{ kg kg}^{-1}$ from thermo diagram in Ch. 5. But supersaturation $S = 1\% = 0.01 = [r_\infty / r_s] - 1$. Thus $r_\infty = [1 + S] \cdot r_s$

$$= 1.01 \cdot (0.0018 \text{ kg kg}^{-1}) = 0.00182 \text{ kg kg}^{-1}$$

Using this in eq. (7.25): $c_4 = [2 \cdot (0.00182 \text{ kg}_w / \text{kg}_a) \cdot (1.325 \text{ kg}_a \cdot \text{m}^{-3}) / (1000 \text{ kg}_w \cdot \text{m}^{-3})]^{1/2}$
 $c_4 = 0.0022$ (dimensionless)

Finally solve eq. (7.24) on a spreadsheet:

$$R \approx 0.0022 \cdot [(2 \times 10^{-5} \text{ m}^2 \cdot \text{s}^{-1}) \cdot (0.01) \cdot t]^{1/2}$$



Check: Units OK. Physics OK.

Exposition: The droplet radius increases with the square root of time — fast initially and slower later. After 3 hours, it has a size on the borderline between cloud and rain drops. Thus, it is virtually impossible to grow full-size rain drops solely by diffusion.

INFO • Cubic Ice

Fourteen **phases** of ice have been identified, and are labeled using Roman numerals I–XIV (see Fig. 7.b). More phases might be discovered in the future. Each phase is a preferred arrangement of molecules having uniform chemical composition and physical state.

At normal atmospheric temperatures and pressures, ice I is most prevalent. However, it comes in two variants: **hexagonal ice (Ih)**, and **cubic ice (Ic)**. Ice Ih is the form that is thermodynamically stable in the troposphere. Both forms of ice I have a tetrahedral arrangement of water molecules.

Hexagonal ice Ih forms crystals that are hexagonal plates, hexagonal columns with flat ends, hexagonal columns with pyramidal ends, and **dendrites** (snowflakes with 6 arms). Samples of these crystal shapes have been collected in the atmosphere, and are frequently observed. This is the normal ice that we see.

Cubic ice Ic is believed to be able to form as cubes, square columns capped by pyramids, and octahedrons (equal to two pyramids with their bases merged). Natural crystals of ice Ic have been detected in the lower stratosphere, but never been successfully captured in the lower troposphere partly because it is metastable with respect to ice Ih, and at warmer temperatures ice Ic rapidly converts to Ih. Cubic ice has been created in the lab under atmospheric conditions, and its existence in the atmosphere has been inferred from certain halos observed around the sun (because ice crystals act like prisms; see the Optics chapter).

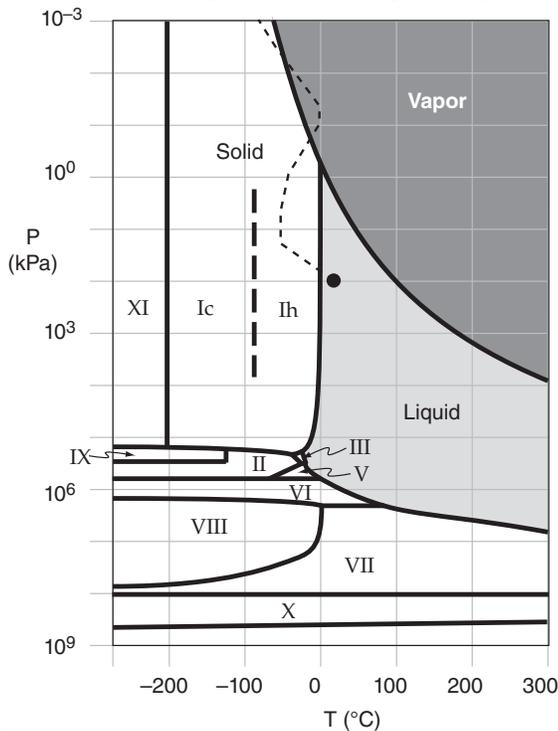


Figure 7.b
Phase diagram for water, as a function of temperature (*T*) and pressure (*P*). Standard atmosphere is thin dotted line, with circle at Earth-surface standard conditions. (Not all phases are plotted.) See exercise U26 and Table 7-7.

7.5. ICE GROWTH BY DIFFUSION

7.5.1. Ice Crystal Habits

In the troposphere, the normal ice crystal shape that forms from direct deposition of water vapor is hexagonal (see INFO Box at left). The particular hexagonal shape that grows depends on temperature and supersaturation (Fig. 7.12). These shapes are called **habits**. Supersaturation is sometimes given as **water-vapor density excess** $\rho_{ve} = \rho_v - \rho_{vs}$, where ρ_v is absolute humidity and ρ_{vs} is the saturation value of absolute humidity.

As ice crystals fall and move by wind and turbulence, they pass through regions of different temperature and vapor-density excess in the cloud. This allows individual crystals to grow into complex combinations of habits (Fig. 7.13). For example, a crystal that starts growth as a column might later be capped on each end by large plates. Because each crystal travels through a slightly different path through the cloud, each snowflake has a unique shape.

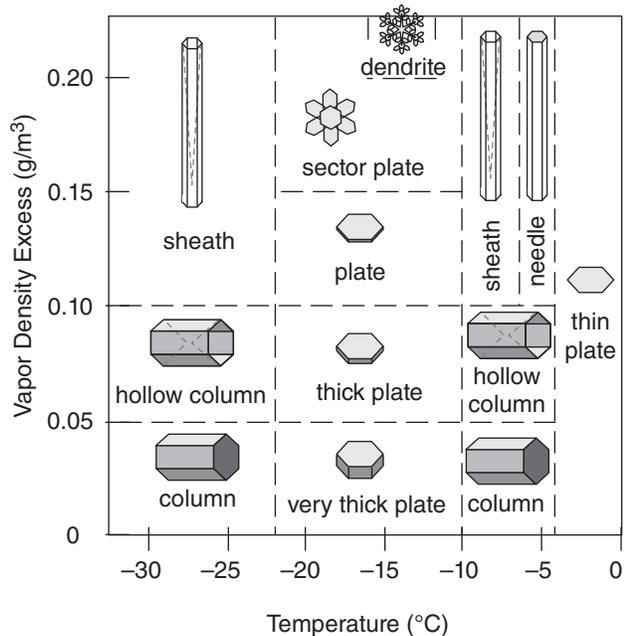


Figure 7.12
Ice crystal habits (idealized). Sheaths are hollow needles, and needles are long, narrow, solid columns. Outside the range of this figure, solid-column rosettes are found at temperatures below -40°C .

	A	B	C	D	E
1	 Simple Prisms	 Solid Columns	 Sheaths	 Scrolls on Plates	 Triangular Forms
2	 Hexagonal Plates	 Hollow Columns	 Cups	 Columns on Plates	 12-branched Stars
3	 Stellar Plates	 Bullet Rosettes	 Capped Columns	 Split Plates & Stars	 Radiating Plates
4	 Sectored Plates	 Isolated Bullets	 Multiply Capped Columns	 Skeletal Forms	 Radiating Dendrites
5	 Simple Stars	 Simple Needles	 Capped Bullets	 Twin Columns	 Irregulars
6	 Stellar Dendrites	 Needle Clusters	 Double Plates	 Arrowhead Twins	 Rimed
7	 Fernlike Stellar Dendrites	 Crossed Needles	 Hollow Plates	 Crossed Plates	 Graupel

Figure 7.13
Some of the observed ice crystal shapes. [Courtesy of Kenneth Libbrecht, www.SnowCrystals.com. Used with permission.]

Sample Application

For each snow crystal class in Table 7-A (see INFO Box at right), identify (by row and column label) all crystal shapes from Fig. 7.13 that are in that class.

Find the Answer

- | | |
|-----------------|--|
| <u>Class</u> | <u>Shapes</u> |
| 1a Columns: | 1A, 1B, 2B, 5D |
| 1b Needles: | 1C, 5B, 6B, 7B |
| 1c Plates: | 1E, 2A, 3A, 3D, 4A, 4D, 6C, 7C, 7D |
| 1d Stellars: | 2E, 3A, 3D, 4A, 4D, 4E, 5A, 6A, |
| 1e Irregular: | 1D, 1E, 2C, 2D, 3C, 3E, 4C, 5C, 5E, 6D |
| 1f Graupel: | 6E, 7E |
| 1g Hail: | (none) |
| 1h Ice Pellets: | (none) |

Exposition: Identifying ice-crystal class is somewhat subjective. Different experts might give a slightly different classification than the solution above.

INFO • Snow Grain Classification

Snow avalanches are often associated with weak snow layers buried under stronger layers of snow. Field observations of snow crystal shape and size are important for detecting the different snow layers. The International Commission on Snow and Ice (ICSI) developed in 1990 a standard symbology (Table 7-A & Table 7-B) to use when logging snow data.

Table 7-A. Morphological (shape-based) classification of precipitation particles. T = temperature, ρ_{ve} = excess water vapor density. [Colbeck et al, 1990: *The International Classification for Seasonal Snow on the Ground*, ICSI. 37pp. Available from http://www.crrel.usace.army.mil/techpub/CRREL_Reports/reports/Seasonal_Snow.pdf]

	Name	Symbol	Shape & Formation
1a	Columns		Short, prismatic crystal, solid or hollow. See Fig. 7.12 for T & ρ_{ve} conditions.
1b	Needles		Needle-like, approximately cylindrical. See Fig. 7.12 for T & ρ_{ve} conditions.
1c	Plates		Plate-like, mostly hexagonal. See Fig. 7.12 for T & ρ_{ve} conditions.
1d	Stellars, Dendrites		Six-fold star-like, planar or spatial. See Fig. 7.12 for T & ρ_{ve} conditions.
1e	Irregular Crystals		Clusters of very small crystals. Polycrystals growing at varying environmental conditions.
1f	Graupel		Heavily rimed particles. Caused by accretion of supercooled water.
1g	Hail		Laminar internal structure, translucent or milky, glazed surface. Growth by accretion of supercooled water.
1h	Ice Pellets		Transparent, mostly small spheroids. Frozen rain.

Table 7-B. Snow-grain classification (ICSI).

Term	Size (mm)
Very fine	< 0.2
Fine	0.2 to 0.5
Medium	0.5 to 1.0
Coarse	1.0 to 2.0
Very coarse	2.0 to 5.0
Extreme	> 5.0

Sample Application

Given a mixed-phase cloud (i.e., having both ice crystals and supercooled liquid water droplets) at -14°C that is saturated with respect to water (and thus supersaturated with respect to ice; see Fig. 4.2 in the Water Vapor chapter). If the water vapor diffusivity is $1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, then what is the relative mass of ice crystals after 1 hour of growth, for (a) a 3-D crystal and (b) a 2-D crystal that is $15 \mu\text{m}$ thick?

Find the Answer

Given: $D = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, $t = 1 \text{ h} = 3600 \text{ s}$, $T = 259\text{K}$
 $d = 15 \mu\text{m} = 1.5 \times 10^{-5} \text{ m}$.

Find: $m = ? \text{ kg}$

Assume: initial mass m is negligible

First, use Fig. 4.2 in the Water Vapor chapter to estimate the supersaturation. The insert in that fig. shows that $(e_{\text{water}} - e_{\text{ice}}) \approx 0.0275 \text{ kPa}$ at $T = -14^{\circ}\text{C}$, and $e_{\text{ice}} \approx 0.175 \text{ kPa}$. The supersaturation is:

$$S = (e_{\text{water}} - e_{\text{ice}}) / e_{\text{ice}} = (0.0275 \text{ kPa}) / (0.175 \text{ kPa})$$

$$S = 0.157 \text{ (dimensionless)}$$

Use the ideal gas law for water vapor (eq. 4.10) to estimate vapor density from vapor pressure (which for this case equals $e_{\text{water}} \approx 0.20 \text{ kPa}$, from previous paragraph):

$$\rho_v = e_{\text{water}} / (R_v \cdot T) =$$

$$= (0.2 \text{ kPa}) / [(0.461 \text{ kPa}\cdot\text{K}^{-1}\cdot\text{m}^3\cdot\text{kg}^{-1}) \cdot (259 \text{ K})]$$

$$= 1.68 \times 10^{-3} \text{ kg m}^{-3}$$

(a) Use eq. (7.26). The factor in parenthesis is $(\rho_v^3 / \rho_i)^{1/2} = [(1.68 \times 10^{-3} \text{ kg m}^{-3})^3 / (916.8 \text{ kg m}^{-3})]^{1/2} = 2.27 \times 10^{-6} \text{ kg m}^{-3}$

and the term in square brackets of eq. (7.26) is

$$[] = [(1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}) \cdot (0.157) \cdot (3600 \text{ s})]^{3/2}$$

$$[] = (8.478 \times 10^{-3} \text{ m}^2 \text{ s}^{-3/2})^{3/2} = 7.8 \times 10^{-4} \text{ m}^3$$

Thus, solving eq. (7.26):

$$m = 11.85 \cdot (2.27 \times 10^{-6} \text{ kg m}^{-3}) \cdot (7.8 \times 10^{-4} \text{ m}^3)$$

$$= 2.1 \times 10^{-8} \text{ kg} = \underline{2.1 \times 10^{-5} \text{ g}}$$

(b) Use eq. (7.27):

$$m = \frac{5.09}{1.5 \times 10^{-5} \text{ m}} \cdot \left(\frac{(0.00168 \text{ kg/m}^3)^2}{916.8 \text{ kg/m}^3} \right) \cdot [8.478 \times 10^{-3} \text{ m}^2]^2$$

$$= 7.51 \times 10^{-8} \text{ kg} = \underline{7.51 \times 10^{-5} \text{ g}}$$

Check: Units OK. Physics OK. Agrees with Fig. 7.14.

Exposition: Typically observed ice-crystal mass is about $3 \times 10^{-5} \text{ g}$. Typical **snowflakes** that fall to Earth are often aggregates of hundreds of ice crystals stuck together, with a total mass of about $3 \text{ mg snowflake}^{-1}$.

7.5.2. Growth Rates

Because of the diversity of shapes, it is better to measure crystal size by its mass m rather than by some not-so-representative radius. Rate of growth by diffusion depends on crystal habit.

Columns and very thick plates have an **aspect ratio** (height-to-width ratio) of roughly 1. If the aspect ratio remains constant during growth, then the growth equation is:

$$m \approx c_3 \cdot (\rho_v^3 / \rho_i)^{1/2} \cdot [D \cdot S \cdot t]^{3/2} \tag{7.26}$$

where $c_3 = 11.85$ (dimensionless), ρ_v is the density of water vapor (=absolute humidity, see eq. 4.10), ρ_i is ice density (= 916.8 kg m^{-3} at 0°C), D is diffusivity, S is supersaturation fraction, and t is time. If the crystal were spherical with radius R , then its mass would be $m = \rho_{\text{liq.water}} \cdot (4 \cdot \pi / 3) \cdot R^3$. Taking the cube root of both sides of eq. (7.26) gives an equation similar to eq. (7.24). Thus growth rate of a 3-D crystal is very similar to growth of a liquid droplet.

For 2-D growth, such as dendrites or plates of constant thickness d , the growth equation changes to

$$m \approx \frac{c_2}{d} \cdot \left(\frac{\rho_v^2}{\rho_i} \right) \cdot [D \cdot S \cdot t]^2 \tag{7.27}$$

where $c_2 = 5.09$ (dimensionless). For 1-D growth of needles and sheaths of constant diameter, the growth equation is

$$m \propto \exp \left[(D \cdot S \cdot t)^{1/2} \right] \tag{7.28}$$

These three growth rates are sketched in Fig. 7.14.

Evidently 2-D crystals increase mass faster than 3-D ones, and 1-D crystals increase mass faster still. Those crystals that gain the mass fastest are the ones that will precipitate first.

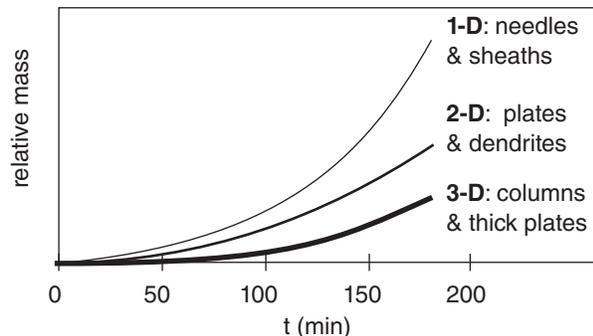


Figure 7.14
 Relative growth rates of crystals of different habits.

7.5.3. The Wegener-Bergeron-Findeisen (WBF) Process

Recall from the Water Vapor chapter that ice has a lower saturation vapor pressure than liquid water at the same temperature. Fig. 7.15 shows an enlargement of the saturation vapor-pressure curves for liquid water and ice.

Suppose that initially (time 1, on the time line in Fig. 7.15) there are only supercooled liquid water droplets in a cloudy air parcel. These droplets exist in a supersaturated environment and therefore grow as the excess water vapor diffuses toward the droplets. As the air parcel rises and cools within the cloud, some ice nuclei might become activated at time 2, causing ice crystals to form and grow. The excess water vapor now deposits on both the liquid and solid hydrometeors.

Both the ice crystals and liquid droplets continue to grow, because both are in a supersaturated environment (time 3). However, the ice crystal grows a bit faster because it is further from its ice saturation line (i.e., more supersaturated) than the liquid droplet is from liquid saturation line.

As both hydrometeors grow, water vapor is removed from the air, reducing the supersaturation. Eventually, near point 4 on the time line, so much vapor has been consumed that the relative humidity has dropped below 100% with respect to liquid water. Hence, the liquid droplet begins to evaporate into the unsaturated air. However, at point 4 the ice crystal continues to grow because the air is still supersaturated with respect to ice.

The net result is that the ice crystals grow at the expense of the evaporating liquid droplets, until the liquid droplets disappear (point 5). This is called the **Wegener-Bergeron-Findeisen (WBF) process**.

The difference between ice and liquid saturation vapor pressures is greatest in the range -8°C to -16°C , as shown in Fig. 7.16 (from the insert in Fig. 4.2). This is the temperature range where we expect the maximum effect from the WBF growth process, also known as the **cold-cloud process** because temperatures below freezing are needed.

If a large number of ice nuclei exist in the air, then a large number of ice crystals will form that are each too small to precipitate. For a very small number of ice nuclei, those few ice crystals will rapidly grow and precipitate out, leaving behind many small liquid cloud droplets in the cloud. Both of these scenarios lead to relatively little precipitation.

Only with a medium concentration (1 to 10) ice nuclei per liter (compared to about a million liquid droplets in the same volume) will the ice nuclei be able to scavenge most of the condensed water before precipitating out. This scenario causes the maximum precipitation for the WBF processes. But a

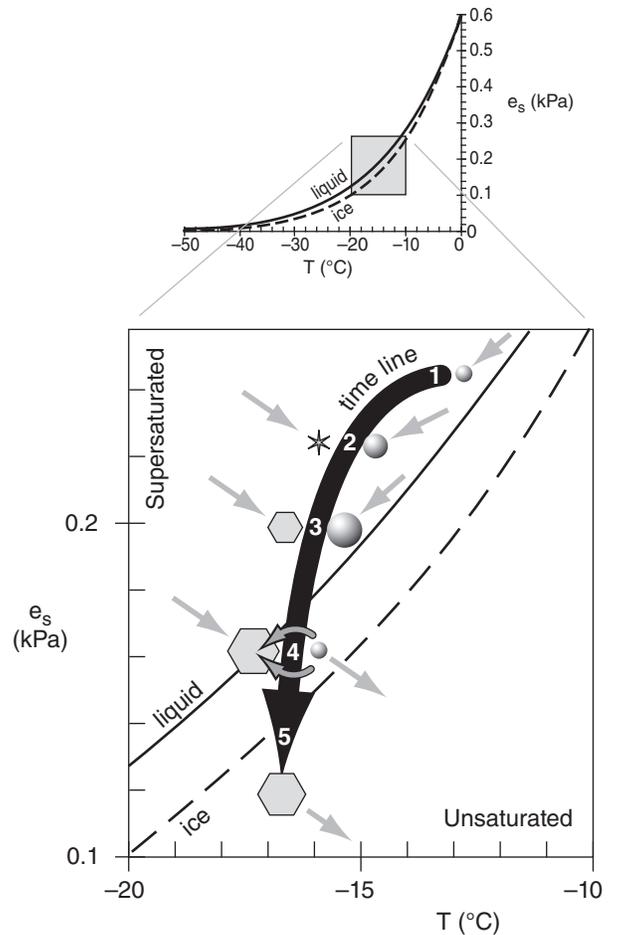


Figure 7.15
Enlargement of part of the saturation curve from Fig. 4.2, illustrating the WBF (cold-cloud) ice-growth process in a rising, cooling air parcel. Shown are saturation vapor pressure (e_s) vs. temperature (T) over liquid water and ice. Spheres represent cloud droplets, and hexagons represent ice crystals. Small grey arrows indicate movement of water vapor toward or away from the hydrometeors. Thick black arrow indicates time evolution.

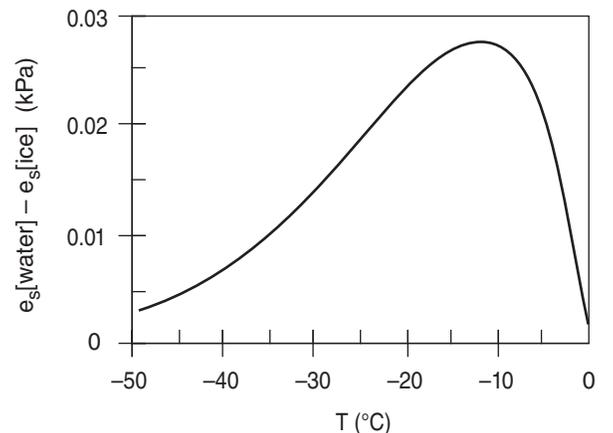


Figure 7.16
Saturation vapor pressure difference over water vs. over ice (copied from Fig. 4.2).

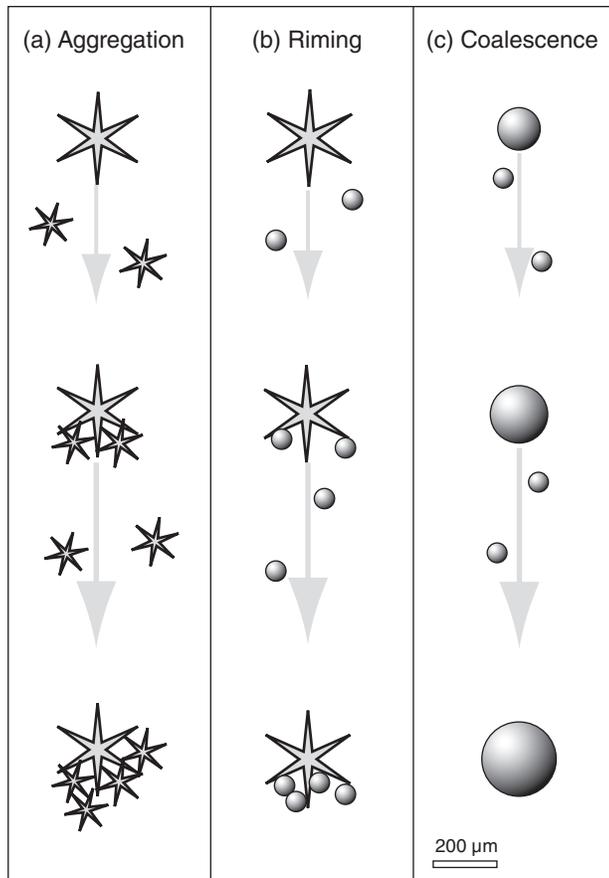


Figure 7.17
Collision and collection of hydrometeors in (a) very cold clouds with no liquid water; (b) cold clouds with both supercooled liquid-water droplets that freeze quickly when they hit ice crystals; and (c) warm clouds. Grey arrows show the fall velocity of the larger hydrometeor *relative* to the smaller ones.

restriction on this precipitation formation process is that it happens only in cold clouds (clouds colder than 0°C).

As was discussed in the nucleation sections, there are indeed fewer ice nuclei than CCN, hence the cold-cloud process can be an important first step in getting hydrometeors large enough to begin to fall out of the cloud as precipitation.

The cold-cloud process can occur even in summer, but higher in the troposphere where the air is colder. As these ice particles fall into warmer air at lower altitude, they melt into raindrops to create summer convective rain showers (see Fig. 7.21 later in this chapter).

7.6. COLLISION AND COLLECTION

Larger hydrometeors (ones with a greater mass/drag ratio) fall the fastest. As a result, different hydrometeors move at different speeds, allowing some to **collide** (hit each other). Not all collisions result in the merging of two hydrometeors. Those particles that do merge form a particle that is even heavier, falls faster, and collides with even more particles (Fig. 7.17). Hence, this positive feedback can cause hydrometeors to rapidly grow large enough to precipitate.

7.6.1. Terminal Velocity of Hydrometeors

Everything including cloud and rain drops is pulled by gravity. The equilibrium velocity resulting when gravity balances frictional drag is called the **terminal velocity**.

7.6.1.1. Cloud Droplets

For particles of radius $R < 40 \mu\text{m}$, which includes most cloud droplets and aerosols, **Stokes Drag Law** gives the terminal velocity w_T as

$$w_T \approx -k_1 \cdot R^2 \tag{7.29}$$

where $k_1 = 1.19 \times 10^8 \text{ m}^{-1} \cdot \text{s}^{-1}$. The negative sign on indicates the droplets are falling.

When drops fall at their terminal velocity, the gravitational pull on the drops is transmitted by frictional drag to the air. In other words, the weight of the air includes the weight of the drops within it. Hence, droplet-laden air is heavier than cloud-free air, and behaves as if it were colder (see virtual temperature, eq. 1.22). Falling rain also tends to drag air with it.

Sample Application

What updraft wind is needed to keep a typical cloud droplet ($R = 10 \mu\text{m}$) from falling?

Find the Answer

Given: $R = 10 \mu\text{m}$

Find: $w_T = ? \text{ m s}^{-1}$, and use: $w_{up} = -w_T$

Use eq. (7.29):

$$w_T = (-1.19 \times 10^8 \text{ m}^{-1} \text{ s}^{-1}) \cdot (10 \times 10^{-6} \text{ m})^2 =$$

$$= -0.012 \text{ m s}^{-1} = \underline{\underline{-1.2 \text{ cm s}^{-1}}}$$

$$w_{up} = \underline{\underline{1.2 \text{ cm s}^{-1}}}$$

Check: Units OK. Physics OK.

Exposition: The required updraft velocity is positive 1.2 cm s^{-1} , which is a very gentle movement of air. Recalling that most clouds form by adiabatic cooling within updrafts of air, these updrafts also keep cloud droplets aloft.

Typical terminal velocities of these smallest droplets and aerosols are mm s^{-1} to cm s^{-1} relative to the air. However, the rising air in clouds often has updraft velocities (cm s^{-1} to m s^{-1}) that are greater than the terminal velocity of the particles. The net result is that cloud droplets and aerosols are carried upward inside the cloud.

7.6.1.2. Rain Drops

Rain drops are sufficiently large and fall fast enough that Stokes drag law is not appropriate. If raindrops were perfect spheres, then

$$w_T \approx -k_2 \cdot \left(\frac{\rho_o}{\rho_{air}} \cdot R \right)^{1/2} \tag{7.30}$$

where $k_2 = 220 \text{ m}^{1/2}\cdot\text{s}^{-1}$, $\rho_o = 1.225 \text{ kg}\cdot\text{m}^{-3}$ is air density at sea level, and ρ_{air} is air density at the drop altitude. Again, the negative sign in the equation means a downward velocity.

However, the larger raindrops become flattened as they fall due to the drag (see polarimetric radar section of the Satellites & Radar chapter). They do not have a tear-drop shape. This flattening increases air drag even further, and reduces their terminal velocity from that of a sphere. Fig. 7.18 shows raindrop terminal velocities. For the smallest drops, the curve has a slope of 2, corresponding to Stokes law. For intermediate sizes $R = 500$ to $1000 \mu\text{m}$, the slope is 0.5, which corresponds to eq. (7.30). At the larger sizes about $R = 2.5 \text{ mm}$, the terminal velocity curve has near zero slope as the droplet becomes so deformed that it begins to look like a parachute. Drops larger than about 2.5 mm radius tend to break up. The largest raindrops rarely exceed 4 mm radius.

Let R be the **equivalent radius** of a sphere having the same volume as the deformed drop. An empirical curve for terminal velocity (relative to air) over range $20 \leq R \leq 2500 \mu\text{m}$ is:

$$w_T = -c \cdot \left[w_o - w_1 \cdot \exp\left(\frac{R_o - R}{R_1} \right) \right] \tag{7.31}$$

where $w_o = 12 \text{ m s}^{-1}$, $w_1 = 1 \text{ m s}^{-1}$, $R_o = 2500 \mu\text{m}$, and $R_1 = 1000 \mu\text{m}$. This curve gives a maximum terminal velocity of 11 m s^{-1} for the largest drops. The density correction factor is $c = (\rho_{70\text{kPa}}/\rho_{air})^{1/2} \approx (70 \text{ kPa}/P)^{1/2}$, where P is ambient pressure. Rain falls faster where the air is thinner (less dense).

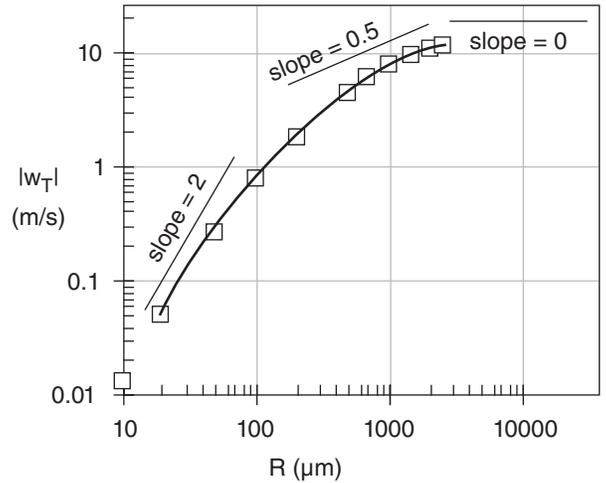


Figure 7.18
Raindrop terminal speed $|w_T|$ at $P = 70 \text{ kPa}$ and $T = 0^\circ\text{C}$. R is the equivalent radius of a spherical drop. Curve is eq. (7.31).

Sample Application

Find the terminal velocity of a droplet of equivalent radius $1500 \mu\text{m}$, at $P = 70 \text{ kPa}$.

Find the Answer

Given: $R = 1500 \mu\text{m}$, $c = 1$ at $P = 70 \text{ kPa}$.
Find: $w_T = ? \text{ m s}^{-1}$.

Use eq. (7.31):
 $w_T = -1 \cdot [(12 \text{ m s}^{-1}) - (1 \text{ m s}^{-1}) \cdot \exp\{(2500\mu\text{m} - 1500\mu\text{m}) / (1000\mu\text{m})\}]$
 $= \underline{\underline{-9.3 \text{ m s}^{-1}}}$

Check: Units OK. Physics OK. Agrees with Fig. 7.18. Negative sign means falling downward.

Exposition: This $w_T \approx 34 \text{ km h}^{-1}$. Updrafts in thunderstorms are fast, and keep these large drops aloft.

Sample Application

Plot the terminal velocity of hailstones vs. equivalent diameter (0.01 to 0.1 m) at 5 km altitude.

Find the Answer

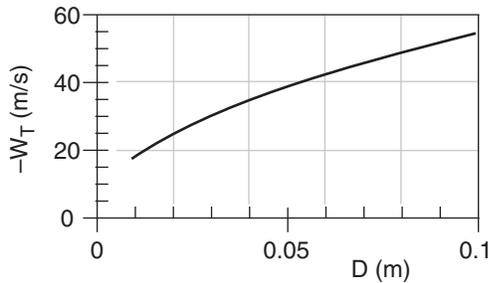
Given: $z = 5 \text{ km}$

Find: $w_T \text{ (m s}^{-1}\text{) vs. } D \text{ (m), where } D = 2R \text{ .}$

Assume: std. atmosphere. Thus $\rho_{air} = 0.7361 \text{ kg m}^{-3}$ at $z = 5 \text{ km}$, from Chapter 1.

Assume: $C_D = 0.55$, and $\rho_i = 900 \text{ kg m}^{-3}$, calm air.

Use a spreadsheet to solve eq. (7.32).



Check: Magnitudes agree with plot in the Thunderstorm chapters. Sign is negative (implying downward fall).

Exposition: Massive hailstones falling at these fast velocities can kill people, strip foliage and small branches off trees, destroy crops, and kill animals.

7.6.1.3. Hailstones

The terminal velocity w_T of a hailstone relative to the air is approximated by:

$$w_T = - \left[\frac{8 |g| \rho_i}{3 C_D \rho_{air}} R \right]^{1/2} \quad \bullet(7.32)$$

where $|g| = 9.8 \text{ m s}^{-2}$ is gravitational acceleration magnitude, $C_D \approx 0.55$ (dimensionless) is a **drag coefficient** of the hailstone through air, $\rho_i \approx 900 \text{ kg m}^{-3}$ is the density of the hailstone, ρ_{air} is air density, and R is hailstone radius. The negative sign means the hailstone falls downward.

The drag coefficient varies between 0.4 and 0.8, because hailstones have different shapes, surface roughnesses, and tumblings. The hailstone ice density can be less than the density of pure ice, because of varying amounts of imbedded air bubbles. Air density decreases with increasing altitude (see Chapter 1); hailstones fall faster in thinner air. For non-spherical hailstones, R is taken as the equivalent radius of a sphere that has the same volume as the actual hailstone.

Large hailstones form only in thunderstorms with strong updrafts. Thus, the hailstone terminal velocity relative to the ground is the sum of the air updraft speed (a positive number) and the hailstone terminal velocity relative to the air (a negative number).

7.6.2. Collection & Aggregation Processes

7.6.2.1. Warm-cloud Process

The merging of two liquid droplets (Fig. 7.17c) is called **coalescence**. This is the only process for making precipitation-size hydrometeors that can happen in warm clouds (clouds warmer than 0°C), and is thus called the **warm-cloud process**.

When droplets of different size approach each other, they do not always merge. One reason is that the smaller droplet partly follows the air as it flows around the larger droplet, and thus may not collide with the larger drop (Fig. 7.19). This is quantified by a **collision efficiency** (E), which is small ($0.02 < E < 0.1$) when the smaller droplet is very small ($2 < R < 5 \mu\text{m}$). But if both droplets are relatively large (such as when the smaller droplet has $R > 10 \mu\text{m}$, and the larger droplet has $R > 30 \mu\text{m}$), then efficiencies can be $0.5 \leq E \leq 1$.

Even if two droplets collide, they might not merge because a thin layer of air can be trapped between the two droplets (Fig. 7.20b). For this situation, the two droplets bounce off of each other and do not coalesce. The **coalescence efficiency** (E') is

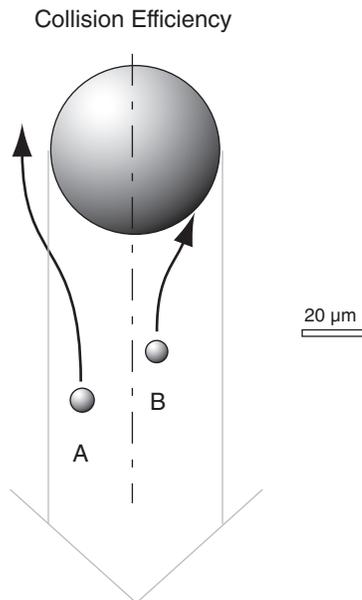


Figure 7.19

Smaller droplets A & B are both in the path of the falling larger drop. Relative to the large drop, small droplet A is pushed aside by the air flowing around the large drop, and does not collide. But droplet B, closer to the centerline of the larger drop, does collide.

very small ($0.1 \leq E' \leq 0.3$) when both drops are large ($300 \leq R \leq 500 \mu\text{m}$). The efficiency is greater ($E' > 0.8$) when both droplets are small ($R < 150 \mu\text{m}$, as in Fig. 7.20a).

The product of both efficiencies is the **collection efficiency**: $E_c = E \cdot E'$. The maximum efficiency possible is 1.0, but usually efficiencies are smaller.

7.6.2.2. Cold-cloud Processes

When ice particles collide and stick to other ice particles (Fig. 7.17a), the process is called **aggregation**. Such aggregation is aided if the colliding particles are dendrites, for which the snowflake arms can interlock. Also, if the ice particles are warmer than -5°C , then the ice surface becomes sticky, allowing multiple ice crystals to aggregate into soft little irregular clumps of snow.

The growth of ice particles by collection and instant freezing of supercooled liquid droplets (Fig. 7.17b) in mixed-phase clouds is called **accretion** or **riming**. Hydrometeors that become so heavily rimed as to completely cover and mask the original habit are called **graupel**. Graupel has the consistency of a sugar cube (i.e., many separate solid grains stuck together), but often in the shape of a cone or a sphere. For an aggregate to be called graupel, its diameter must be no larger than 5 mm.

If the collected water does not freeze instantly upon contacting the ice particle, but instead flows around it before freezing, then **hail** can form. Graupel contains a lot of air trapped between the frozen droplets on the graupel, and thus is often softer and less dense than hail. See the Thunderstorm chapters for more information about hail.

7.6.3. Precipitation Formation

7.6.3.1. Warm Clouds

How do terminal velocity and collection efficiency relate to the formation of large, precipitation-size particles in warm clouds? Recall that: (1) the atmosphere has an excessively large number of CCN; (2) this causes the available condensate in a rising cooling air parcel to be partitioned into a large number of small droplets (droplets too small to rain out); (3) droplets tend to become monodisperse (nearly the same size) due to diffusion; (4) droplets of the same size have the same terminal velocity, and thus would be unlikely to collide with each other as they are kept aloft in the updraft; and (5) with no collisions we would not expect larger precipitation drops to form in warm clouds.

Yet warm-cloud rain happens quite nicely in the real atmosphere, especially in the tropics. Why?

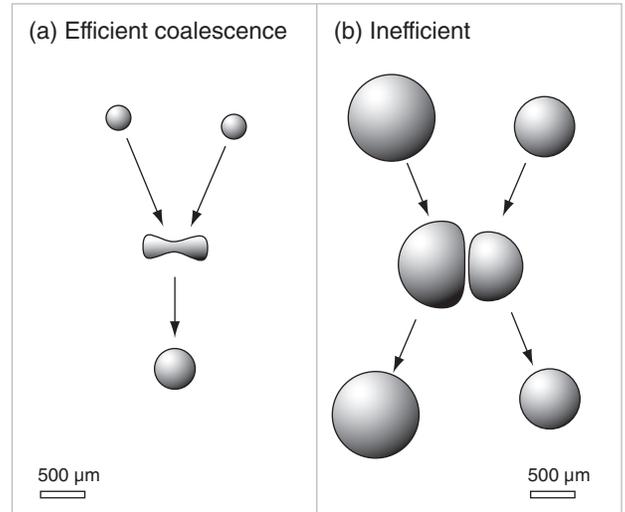


Figure 7.20

Colliding small droplets in (a) coalesce into a single drop. Colliding larger droplets in (b) do not coalesce, because of a film of air trapped between them that cannot completely escape before the two droplets bounce off of each other.

INFO • Meteors and Meteorology

Ancient Greeks defined “meteors” as anything in the sky. They were particularly concerned about missiles the gods might toss down, such as bits of rock, ice, or lightning bolts.

Only much later did scientists discriminate between missiles from space (bits of rock called **meteoroids**) and missiles from the atmosphere (bits of just about anything else from the sky). But by then “**meteorology**” was firmly entrenched as the name for **atmospheric science**.

According to the *Glossary of Meteorology*, meteorologists study the following meteors:

- **hydrometeors** – wet: clouds, rain, snow, fog, dew, frost, etc.
- **lithometeors** – dry: dust, sand, smoke, haze
- **igneous meteors** – lightning, corona
- **electrometeors** – lightning (again), thunder
- **luminous meteors** – rainbows, halos, etc.

Except for “hydrometeors”, these terms are seldom used any more.

Five factors can help make warm-cloud rain:

- First, by random chance a small number of collisions do occur, which starts to broaden the spectrum of drop sizes. This broadens the range of terminal velocities to allow more collisions, which accelerates via positive feedback (with help from breakup of larger drops).
- Second, not all CCN are the same size — some are called **giant CCN** (particles $> 3 \mu\text{m}$ radius, with a wettable surface) and can create a small number of larger cloud droplets that fall relative to (and collide with) the other cloud droplets.
- Third, turbulence can entrain outside clear air into the top and sides of a cloud, causing some droplets to partly evaporate, thereby broadening the spectrum of droplet sizes, again allowing collisions.
- Fourth, IR radiation from individual drops near cloud top and sides can cool the drops slightly below the ambient air temperature, allowing greater condensation growth of those drops relative to interior drops.
- Fifth, electrical charge build-up in cumuliform clouds (see the Thunderstorm chapters) can draw droplets together of different charge, and can cause sparks between nearby droplets to allow them to coalesce more efficiently.

7.6.3.2. Cold Clouds

In cold clouds ($T < 0^\circ\text{C}$), the smaller number of ice nuclei in the atmosphere allows the available condensate to deposit onto a small number of larger ice particles. Even in mixed-phase clouds, the WBF process can remove water molecules from the large number of droplets and deposit them onto a small number of ice crystals. Thus, the ice crystals are larger, and can fall as precipitation. Also, the crystals often have a wide range of sizes and shapes so they can collide and aggregate easily, which also creates large-enough particles to fall as precipitation.

Cumuliform clouds including thunderstorms can be deep enough to have their bases in warm air and their tops in cold air (Fig. 7.21). Thus, ice particles can grow to large size (order of 1-5 mm) via the WBF process, aggregation, and riming in the cold part of the cloud, and then melt into large raindrops as they pass through warmer air closer to the ground. Most rain from mid-latitude thunderstorms forms this way. See the Thunderstorm chapters for more information about heavy rain.

Also, on those rare occasions when thunderstorms can be triggered in late Fall or early Spring, boundary-layer temperatures can be cold enough to allow snow from thunderstorms to reach the ground as large snow clusters (snow balls) without totally melting, accompanied by lightning and thun-

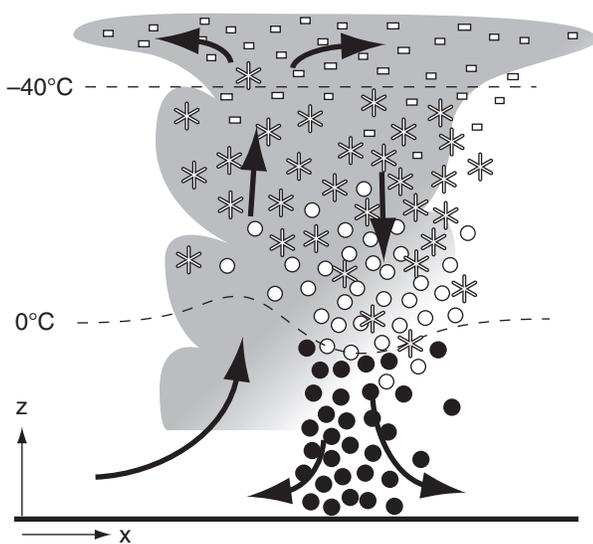


Figure 7.21

Thunderstorms can have cold-cloud tops and warm-cloud bases, allowing efficient growth of ice precipitation that can melt into large raindrops on the way down.

der from the storm. This is sometimes nicknamed **thundersnow**.

Regardless of whether clouds are warm or cold, a simple rule of precipitation is that thicker clouds can cause heavier precipitation rates with larger size drops. The main reason is that hydrometeors take a longer time to fall the greater distance through thicker clouds, giving them more time to grow.

7.7. PRECIPITATION CHARACTERISTICS

7.7.1. Rain-Drop Size Distribution

Small raindrops outnumber large ones. Classically, the rain-drop spectrum has been fit by an exponential function, known as the **Marshall-Palmer distribution**:

$$N = \frac{N_o}{\Lambda} \exp(-\Lambda \cdot R) \quad \bullet(7.33)$$

where N is the number of drops of radius greater than R within each cubic meter of air, and $N_o \approx 1.6 \times 10^7 \text{ m}^{-4}$. Parameter $\Lambda(\text{m}^{-1}) = 8200 \cdot (RR)^{-0.21}$, where RR is rainfall rate in mm h^{-1} . Fig. 7.22 shows the Marshall-Palmer distribution. While there are on the order of 1000 drops of drizzle size and larger in each cubic meter of air, there are only tens of drops of typical raindrop size and larger.

Fig. 7.23 shows rain-drop distributions for a variety of rain rates. Larger drop diameters are associated with heavier rainfall rates. Drop diameters in the range of 4 to 8 mm diameters have been observed for the heaviest rain events. Because large liquid drops tend to break up into smaller ones as they fall, it is possible that some of the largest-diameter drops reported in Fig. 7.23 consisted of still-melting graupel or aggregated/rimed snow.

Beware that these curves vary significantly from storm to storm. Many thunderstorms have much greater numbers of large 1000 to 2500 μm radius drops. Real clouds are very heterogeneous in their generation of rain, causing a patchiness in the rainfall rates. This patchiness causes the raindrop size distributions to continually change with time and place; namely, they are not the simplistic, steady distributions given above. This patchiness is also observed by weather radar (see the Satellites & Radar chapter), and in the rainfall observed by rain gauges on the ground. You have probably also experienced it when driving or walking through rain.

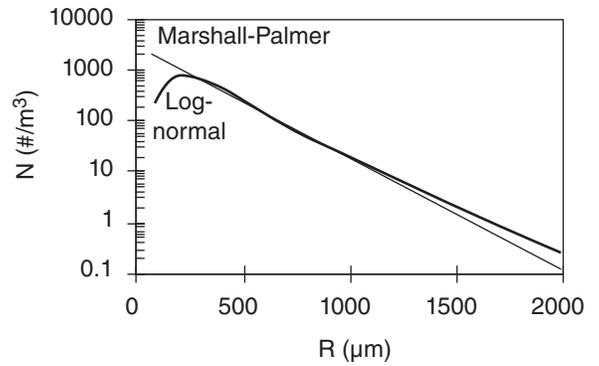


Figure 7.22
Rain drop size distributions. Rainfall rate was $RR = 10 \text{ mm h}^{-1}$.

Sample Application

Find the number of raindrops larger than 1000 μm , using the parameters given below.

Find the Answer

Given: $RR = 10 \text{ mm h}^{-1}$, $N_o = 1.6 \times 10^7 \text{ m}^{-4}$, $R = 1000 \mu\text{m}$

Find: $N = ? \text{ m}^{-3}$.

First, find $\Lambda = 8200 \cdot (10 \text{ mm h}^{-1})^{-0.21} = 5056 \text{ m}^{-1}$.

Next, use eq. (7.33) for Marshall-Palmer:

$$N = [(1.6 \times 10^7 \text{ m}^{-4}) / (5056 \text{ m}^{-1})] \cdot \exp[-(5056 \text{ m}^{-1}) \cdot (10^{-3} \text{ m})] = \underline{20.2 \text{ m}^{-3}}.$$

Check: Units OK. Physics OK. Agrees with Fig. 7.22.

Exposition: Don't forget that N is like 1 – cumulative distribution. It doesn't give the count of drops of size equal to R , but counts all drops of size R and greater.

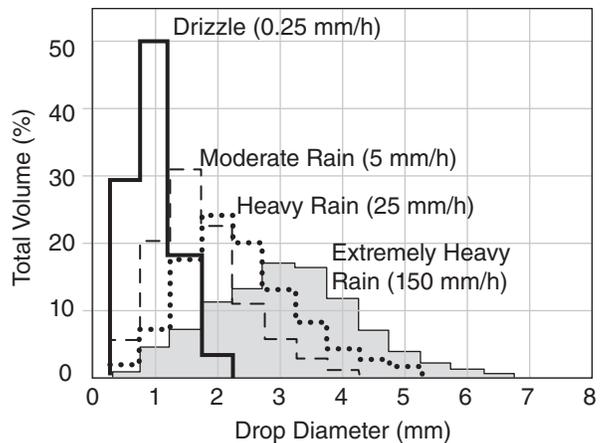


Figure 7.23
Typical distribution of rain-drop sizes during different rain rates. Area under each curve equals 100% of the total volume of rain accumulated in the rain gauge for that rain rate.

Table 7-3. Rain intensity criteria (from USA Fed. Meteor. Handbook No. 1, Sep 2005), and their corresponding weather-map symbols and Meteorological Aviation Report (METAR) codes for continuous, non-freezing rain.

Rain Intensity	Rainfall rate		Symbol on Map
	inches h ⁻¹	≈mm h ⁻¹	METAR code
heavy	> 0.30	> 7.6	 +RA
moderate	0.11 – 0.30	2.6 – 7.6	 RA
light	0* – 0.10	0* to 2.5	 -RA
trace	< 0.005	<0.1	

Sample Application

Is depth-envelope eq. (7.34) valid for: (a) 206 mm/20 min at Curtea-de-Arges, Romania on 7 July 1889; (b) 304.8 mm/42 min at Holt, MO, USA on 22 July 1947?

Find the Answer

Given: (a) $d_{obs} = 206 \text{ mm}$, $P_{rain} = 20 \text{ minutes} = 0.333 \text{ h}$
 (b) $d_{obs} = 304.8 \text{ mm}$, $P_{rain} = 42 \text{ min} = 0.7 \text{ h}$
 Find: if d_{obs} is approx. equal or less than $d_{max \text{ rain}}$

Use eq. (7.34): (a) $d_{max \text{ rain}} = \mathbf{209.6 \text{ mm}}$
 (b) $d_{max \text{ rain}} = \mathbf{303.7 \text{ mm}}$

Check: Units look OK. Magnitude is reasonable.
Exposition: Yes, envelope is approx. valid for both.

Sample Application

What intensity precipitation for Edmonton would be expected for a return period of 50 years and a duration 15 min?

Find the Answer

Given: $RP = 50 \text{ y}$, $D = 0.25 \text{ h}$. Edmonton, Canada
 Find: $I = ? \text{ (mm h}^{-1}\text{)}$

Use eq. (7.36): $I_o = 15.8(\text{mm h}^{-1}) \cdot [1 + 0.5 \cdot \ln(50/2) - 0.015 \cdot \ln^2(50/2)] = 38.77 \text{ (mm h}^{-1}\text{)}$
 Use eq. (7.35): $I = 38.77 \text{ (mm h}^{-1}\text{)} \cdot (0.25/1)^{-0.6425} = \mathbf{94.5 \text{ (mm h}^{-1}\text{)}}$

Check: Units good. Agrees with Fig. 7.24.
Exposition: This is a very heavy rainfall rate over a short time, but luckily it doesn't happen frequently.

7.7.2. Rainfall Rates

Sometimes **rainfall rate** (i.e., **precipitation intensity**) is classified as **light** (or **slight**), **moderate**, or **heavy**. Different countries set different thresholds for these rainfall categories. Table 7-3 shows rainfall intensity criteria used in the USA to determine weather symbols in station plots on surface weather maps. A **trace** amount of precipitation is a very small amount of rain that might wet the ground, but is too small to be detected in a rain gauge (i.e., < 0.1 mm). See the Thunderstorm chapters for more information about very heavy rain and downpours from thunderstorms.

Drizzle is precipitation of very small drops (diameter < 0.5 mm) that are closely spaced and uniform. Although precipitation rates (mm h⁻¹) from drizzle are usually very small, drizzle can reduce visibility (Table 7-4).

World-record rainfall total accumulated depth $d_{max \text{ rain}}$ in a rain gauge over any storm period P_{rain} is approximately contained under the following envelope:

$$d_{max \text{ rain}} = a \cdot P_{rain}^{1/2} \tag{7.34}$$

where $a = 363 \text{ mm}_{rain} \cdot \text{h}^{-1/2}$. For example, on 25 May 1920 Fussen, Germany received 126 mm in 8 minutes. On 18 July 1942 Smethport, PA, USA received 780 mm in 6 hours. On 15 March 1952, Cilaos, La Réunion Island received 1,830 mm in 1 day. Cherrapunji, India received 2,493 mm in 48 h on 15-16 Jun 1995, and 9,300 mm for the month of July 1861, and received 26,470 mm in the year ending 31 July 1861.

Rain **intensity-duration-frequency (IDF)** relationships are used when designing dikes, bridges, and drainage systems. One simple IDF model is

$$\frac{I}{I_o} = \left(\frac{D}{D_o} \right)^p \tag{7.35}$$

where

$$\frac{I_o}{I_r} = 1 + b \ln \left(\frac{RP}{RP_r} \right) + a \left[\ln \left(\frac{RP}{RP_r} \right) \right]^2 \tag{7.36}$$

$RP = \text{return period}$ (years; the average time between events), $I = \text{precipitation intensity}$ (mm h⁻¹), D is precipitation duration (h), and subscripts o and r indicate reference values.

For example, for Edmonton, Canada, $a = -0.015$, $b = 0.5$, $p = -0.6425$, $D_o = 1 \text{ h}$, $RP_r = 2 \text{ years}$, and $I_r = 15.8 \text{ (mm h}^{-1}\text{)}$. For any return period of interest (e.g., $RP = 50 \text{ years}$), first use eq. (7.36) to find I_o . Then use that I_o with any duration (e.g., $D = 0.25 \text{ h}$) in eq. (7.35) to find the corresponding intensity of precipitation. Fig. 7.24 shows the IDF curves for Edmonton. Civil engineers can use that figure to find the return period for a rainstorm of any intensity and duration.

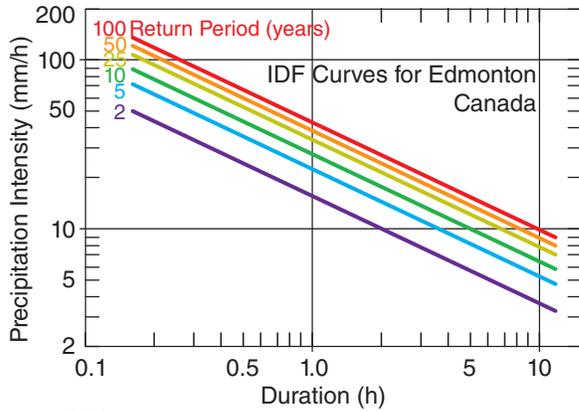


Figure 7.24
Sample intensity-duration-frequency (IDF) curves. (Data from Jobin, Jolly & Chan, 2013: *CMOS Bulletin*, 41(2), 45-53.)

7.7.3. Snowfall Rates & Snow Accumulation

Snowfall rates in the US are classified using the same visibility criterion as drizzle (Table 7-4). Low visibility due to heavy snowfall or blowing loose snow, when accompanied by strong winds, is classified as a **blizzard** if it persists for 3 to 4 hours (Table 7-5). If you are outside in a blizzard, you could easily get disoriented and not be able to return to a shelter, because of **white-out** conditions, where the snow makes the ground and sky look uniformly white so you cannot discern any features or landmarks.

When it is snowing, the precipitation rate is usually measured as **liquid-water equivalent** in units of mm h⁻¹. Namely, it is the precipitation rate after all precipitation is melted. Heated rain gauges accomplish this.

You can also estimate the snowfall rate by periodically measuring the depth of snow on the ground using a meter stick or other metric. However, melting of the snow on warm ground, and compression of snow by the weight of snow above cause large errors in these estimates. Thus, liquid-water equivalent is used instead as a more accurate measure.

New fallen snow has a density roughly 10% of that of liquid water. Thus, a rough first guess is that new-fallen snow depth is about 10 times the liquid-water equivalent depth.

Actual snow densities vary widely, as listed in Table 7-6. The density of freshly falling dry snow is very small because of the air between branches of each ice crystal, and because of air trapped between ice crystals as they accumulate on the ground. After snow has fallen, **metamorphosis** takes place where the tips of the crystals evaporate and re-deposit near the crystal centers. Such snow gradually changes into **snow grains** (similar to sugar grains), and becomes more compact and dense. Snow can be further modified by partial melting and refreezing (on a diurnal cycle, and also on an annual cycle for

Table 7-4. Drizzle and snow intensity criteria (from USA Fed. Meteor. Handbook No. 1, Sep 2005), and their weather-map symbols and Meteorological Aviation Report (METAR) codes for continuous precipitation.

Precip. Intensity	Visibility (x_v)		Symbol on Map	
	miles	≈km	METAR Code	
heavy	$x_v \leq 0.25$	$x_v \leq 0.4$	’ ’	* * *
			+DZ	+SN
moderate	$0.25 < x_v \leq 0.5$	$0.4 < x_v \leq 0.8$	’ ’	* *
			DZ	SN
light	$x_v > 0.5$	$x_v > 0.8$	’ ’	* *
			-DZ	-SN

Table 7-5. Blizzard criteria.

Weather Condition	Criteria (all must be met)	
	USA	Canada
visibility	≤ 0.25 mile	≤ 1 km
wind speed	≥ 35 mi h ⁻¹	≥ 40 km h ⁻¹
duration	≥ 3 h	≥ 4 h

Table 7-6. Snow density.

Density (kg m ⁻³)	Characteristics
50 - 100	Fresh falling snow.
100 - 200	New top snow. Uncompacted. Called “ powder ” by skiers.
200 - 300	Settled snow on ground. Self-compacted after several days.
300 - 500	Snow compacted by grooming machines. Some target densities (kg m ⁻³) for groomed ski slopes are: 450 for cross-country (nordic) tracks, 530-550 for snowboard and downhill (alpine) runs, and 585 - 620 for slalom. Also forms naturally in deep layers of snow, such as during glacier formation.
500 - 550	Called “ névé ”. Snow that has been partially melted, refrozen, & compacted.
550 - 830	Called “ firn ”. Naturally compacted and aged over 1 year. A form of ice still containing air channels, observed during glacier formation.
830 - 917	Ice with bubbles, typical in the top 1000 m of old glaciers.
917	Solid ice (no bubbles). Typical of glacier ice below 1000 m depth.

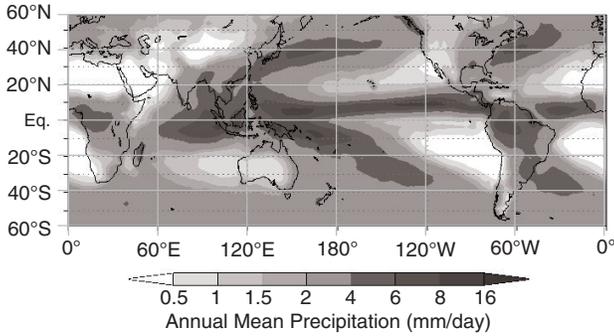


Figure 7.25
 Annual mean precipitation during years 1979 to 2000.
 [Adapted from image courtesy of US Climate Prediction Center (NOAA/ National Weather Service, National Centers for Environmental Prediction). <http://www.cpc.ncep.noaa.gov/products/precip/CWlink/wayne/annual.precip.html>]

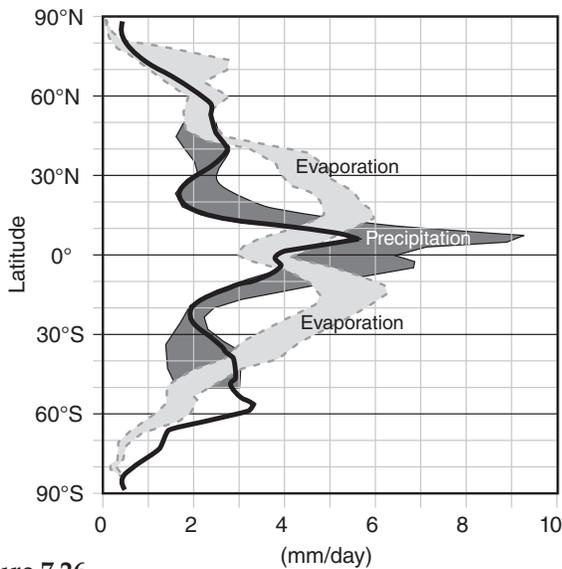


Figure 7.26
 Zonally averaged rates of evaporation (light grey with dotted outline) and precipitation (dark grey with solid black outline). Estimated from a combination of satellite observations, numerical simulations, and surface observations. Spread of the grey areas indicates uncertainty (disagreement among the different methods). Data courtesy of NASA GSFC and the Global Precipitation Climatology Project (plotted as the thick black line).

glacier snow). The weight of additional snow on top can further compact deeper older snow.

Piste is the name for a ski run where the snow has been compacted by grooming machines. Density and strength can be increased by mechanically chopping and compacting the snow, by adding liquid water (that later freezes), and by adding chemicals such as nitrate fertilizers or urea.

7.7.4. Precipitation Distribution

Combining rain-gauge data over land with satellite observations over oceans gives the annual precipitation distribution shown in Fig. 7.25. The heaviest rain is in the tropics, where the warm sea surface causes copious amounts of evaporation (Fig. 7.26), where the warm air can hold a large amount of precipitable water, and where the general circulation contains updrafts. Rain is suppressed at 30° north and south due to extensive regions of downdraft in the Hadley-cell circulation. This circulation is discussed in the General Circulation chapter.

7.8. PRECIPITATION MEASUREMENT

The simplest precipitation instrument is a **rain gauge**, which is a cylindrical bucket into which the rain falls. By using a measuring stick to manually read the water depth in the bucket at successive times such as every hour, you can determine rainfall rate. For greater sensitivity, a funnel can be placed over the bucket to collect rain faster, but the depth of water in the bucket must be reduced by the ratio of the horizontal cross-section areas of the bucket to the funnel opening. To get the **liquid-water equivalent** of the snowfall, some rain gauges are heated to melt snow, and others are painted black to passively melt snow by absorbing sunlight. Some gauges are surrounded by a segmented **wind shield** to reduce errors due to blowing precipitation.

Automated rain gauges exist. **Weighing rain gauges** weigh the rain-filled bucket over successive intervals, inferring rain accumulation by weight increase, knowing the density of liquid water.

Tipping-bucket rain gauges direct the captured rain into a tiny bucket on one side of a lever. When the bucket gets full, it tips the lever like a seesaw (teeter-totter), emptying that bucket while simultaneously moving under the funnel an empty bucket from the other end of the lever. Each tip can be counted digitally, and the frequency of tips during an hour gives the rainfall rate.

Drip rain gauges funnel rainwater into a small orifice, under which individual drips of water form

by surface tension. As each drip separates from the funnel orifice, it touches two conductors as it falls toward a drain, allowing each drip to be counted.

An **evaporative rain gauge** has two metal plates, one above the other, each oriented horizontally (one facing up, and the other facing down). Each plate is heated electrically to maintain the same specified temperature warmer than ambient air. Precipitation falling on the hot top plate evaporates quickly, thereby removing heat from that plate. By measuring the amount of extra electricity needed to keep the top plate at the same temperature as the bottom, and knowing the latent heat of vaporization, the rainfall rate can be determined.

Attenuation rain gauges have a light beam that shines horizontally across an open air path exposed to precipitation. The attenuation of the light beam is related to precipitation intensity, but errors can be due to air pollution, fog, and different absorption cross-sections of liquid vs. solid precipitation.

A **disdrometer** measures size distribution of rain drops via the momentum imparted to a horizontal plate by each falling drop. Another method is a particle imager that sends light from an array of light-emitting diodes to an array of tiny photodetectors. Each hydrometeor casts a shadow that can be detected, where the size and the shape of the shadows are used together to estimate precipitation rate, hydrometeor type and size. A **Knollenberg probe** uses this imaging method, and can be mounted on aircraft flying through clouds and precipitation.

A **liquid-water content (LWC) probe** consists of an electrically heated wire. When mounted on an aircraft flying through a cloud, the rain and cloud droplets evaporate upon hitting the hot wire. By measuring the electrical power needed to maintain a constant wire temperature against the evaporative cooling, the LWC can be inferred.

Snow amount on the ground can be measured by placing a liquid antifreeze-filled thin-skin metal **snow pillow** on the ground before the winter snow season. As snow accumulates during the season, the weight of the snow squeezes the pillow and displaces some of the fluid. Pressure sensors measure the weight of the displaced fluid to infer snow weight.

Downward-pointing **ultrasonic snow-depth sensors** mounted on a tall pole measure the travel time for an emitted pulse of sound to reach the ground and echo back to the sensor on the pole. This gives the distance between the top of the snow and the sensor, which can be subtracted from the sensor height above bare ground to give the snow depth. Similar sensors use travel time for IR or visible light pulses.

Remote sensors (see the Satellites & Radar chapter) can also be used to measure rain rate or accumu-

lation. Ground-based **weather radar** actively emits microwaves, and can estimate rainfall rate from the echo intensity and polarization characteristics of the microwave signal that is scattered back to the radar from the precipitation particles. **Passive microwave sensors** on some weather satellites can measure the **brightness temperature** of the minute amounts of microwaves emitted from the Earth's surface and atmosphere. With this info one can infer the atmospheric total water content in a column of the atmosphere (used to estimate **tropical rainfall** over the oceans), and can infer snow depth on the ground (over high-latitude regions).



7.9. REVIEW

Cloud droplets that form on cloud condensation nuclei (CCN) overcome a formation barrier caused by the surface tension of the curved surface. However, because there are so many CCN between which the available water is partitioned, the result is a large number of very small droplets. These drops grow slowly by diffusion, and develop a monodisperse droplet-size distribution. Such a distribution reduces droplet collisions, and does not favor droplet growth into precipitation hydrometeors. Hence, we get pretty clouds, but no rain.

Warm-cloud ($T > 0^{\circ}\text{C}$) rain can happen in the tropics, particularly over oceans where there are fewer CCN allow formation of a smaller number of larger drops. Several other processes can cause the droplet sizes to have more diversity, resulting in different terminal velocities for different drops. This encourages collision and coalescence to merge smaller droplets into ones that are large enough to precipitate out.

In clouds colder than 0°C , ice nuclei trigger ice crystals to grow. Ice crystals can exist in the air along with supercooled liquid drops. Because of the difference between liquid and ice saturation humidities, the ice can grow at the expense of evaporating liquid droplets. If the ratio of water to ice hydrometeors is about a million to one, then most of the water will be transferred to ice crystals, which are then heavy enough to fall as precipitation.

As larger ice particles fall and hit smaller supercooled liquid droplets, the droplets can freeze as rime onto the ice crystals, causing the hydrometeors to grow even faster. This process can create graupel and hail. Also, ice crystals can aggregate (collide and stick together) to make larger clumps of snow. Most rain at midlatitudes results from melted snow that form from this "cold cloud" process.

7.10. HOMEWORK EXERCISES

7.10.1. Broaden Knowledge & Comprehension

B1. Search the web for any journal articles, conference papers, or other technical reports that have pictures of droplet or ice-crystal growth or fall processes (such as photos taken in vertical wind tunnels).

B2. Can you find any satellite photos on the web showing haze? If so, which satellites and which channels on those satellites show haze the best? Do you think that satellites could be used to monitor air pollution in urban areas?

B3. Search the web for microphotographs of ice crystals and snow flakes with different habits. What habits in those photos were not given in the idealized Fig. 7.12?

B4. Find on the web a clear microphotograph of a dendrite snow flake. Print it out, and determine the fractal dimension of the snow flake. (Hint, see the Clouds chapter for a discussion of fractals.)

B5. Search various government air-pollution web sites (such as the U.S. Environmental Protection Agency: <http://www.epa.gov/>) for sizes of aerosol pollutants. Based on typical concentrations (or on concentrations specified in the air-quality standards) of these pollutants, determine the number density, and compare with Fig. 7.5.

B6. Search the web for information about surface tension, and how it relates to **Gibbs free energy**.

B7. Search the web for info on snow & snowflakes.

B8. Search the web for IDF precipitation curves for your area, or for a location assigned by the teacher.

B9. Search the web for climate statistics of actual annual precipitation last year worldwide (or for your country or region), and compare with Fig. 7.25.

B10. Search the web for photos and diagrams of precipitation measurement instruments, and discuss their operation principles.

7.10.2. Apply

A1. Using Fig. 7.1, how many of the following droplets are needed to fill a large rain drop.

- small cloud droplet
- typical cloud droplet
- large cloud droplet

- drizzle droplet
- small rain droplet
- typical rain droplet

A2. Find the supersaturation fraction and supersaturation percentage, given relative humidities (%) of: a. 100.1 b. 100.2 c. 100.4 d. 100.5 e. 100.7 f. 101 g. 101.2 h. 101.8 i. 102 j. 102.5 k. 103.3 l. 104.0 m. 105 n. 107 o. 110

A3. For air at $T = -12^{\circ}\text{C}$, find the supersaturation fraction, given a vapor pressure (kPa) of:

- 0.25
- 0.26
- 0.28
- 0.3
- 0.31
- 0.38
- 0.5
- 0.7
- 0.9
- 1.1
- 1.2
- 1.4
- 1.6
- 1.8
- 1.9

Hint. Get saturation vapor pressure from the Water Vapor chapter.

A4. For air at $P = 80$ kPa and $T = -6^{\circ}\text{C}$, find the supersaturation percentage, given a mixing ratio (g kg^{-1}) of: a. 6 b. 5.8 c. 5.6 d. 5.4 e. 5.2 f. 5.1 g. 5

- 4.9
- 4.7
- 4.5
- 4.3
- 4.1
- 3.8
- 3.6

Hint. Get saturation mixing ratio from a thermo-diagram (at the end of the Atmospheric Stability chapter).

A5. For the previous problem, assume the given mixing ratios represent total water mixing ratio. Find the excess water mixing ratio (g kg^{-1}).

A6. For an air parcel with excess water mixing ratio of 10 g kg^{-1} at a geopotential height of 5 km above mean sea level, find the average radius (μm) of the hydrometeor assuming growth by condensation only, given a hydrometeor number density ($\# \text{ m}^{-3}$) of: a. 1×10^8 b. 2×10^8 c. 3×10^8 d. 4×10^8 e. 5×10^8

- 6×10^8
- 7×10^8
- 8×10^8
- 9×10^8
- 1×10^9
- 2×10^9
- 3×10^9
- 4×10^9
- 5×10^9
- 8×10^9

A7. If $c = 5 \times 10^6 \mu\text{m}^3 \cdot \text{m}^{-3}$, use the Junge distribution to estimate the number density of CCN ($\# \text{ m}^{-3}$) within a $\Delta R = 0.2 \mu\text{m}$ range centered at R (μm) of:

- 0.2
- 0.3
- 0.4
- 0.5
- 0.6
- 0.8
- 1.0
- 2
- 3
- 4
- 5
- 6
- 8
- 10

A8. For pure water at temperature -20°C , use Kelvin's equation to find the equilibrium $RH\%$ in air over a spherical droplet of radius (μm):

- 0.005
- 0.006
- 0.008
- 0.01
- 0.02
- 0.03
- 0.04
- 0.05
- 0.06
- 0.08
- 0.09
- 0.1
- 0.2
- 0.3
- 0.5

A9 (§). Produce Köhler curves such as in Fig. 7.7b, but only for salt of the following masses (g) at 0°C :

- 10^{-18}
- 10^{-17}
- 10^{-16}
- 10^{-15}
- 10^{-14}

- f. 10^{-13} g. 10^{-12} h. 10^{-11} i. 5×10^{-18} j. 5×10^{-17}
 k. 5×10^{-16} l. 5×10^{-15} m. 5×10^{-14} n. 5×10^{-13}

A10 (§). Produce Köhler curves for a solute mass of 10^{-16} g of salt for the following temperatures ($^{\circ}\text{C}$):

- a. -35 b. -30 c. -25 d. -20 e. -15
 f. -10 g. -5 h. 2 i. 5 j. 10 k. 15
 l. 20 m. 25 n. 30

A11. Find the critical radii (μm) and supersaturations at a temperature of -10°C , for

- a. 5×10^{-17} g of hydrogen peroxide
 b. 5×10^{-17} g of sulfuric acid
 c. 5×10^{-17} g of nitric acid
 d. 5×10^{-17} g of ammonium sulfate
 e. 5×10^{-16} g of hydrogen peroxide
 f. 5×10^{-16} g of sulfuric acid
 g. 5×10^{-16} g of nitric acid
 h. 5×10^{-16} g of ammonium sulfate
 j. 5×10^{-15} g of hydrogen peroxide
 k. 5×10^{-15} g of sulfuric acid
 l. 5×10^{-15} g of nitric acid
 m. 5×10^{-15} g of ammonium sulfate
 n. 5×10^{-14} g of hydrogen peroxide
 o. 5×10^{-14} g of sulfuric acid
 p. 5×10^{-14} g of nitric acid
 q. 5×10^{-14} g of ammonium sulfate

A12. For the nuclei of the previous exercise, find the equilibrium haze droplet radius (μm) for the following relative humidities (%):

- (i) 70 (ii) 72 (iii) 74 (iv) 76 (v) 78
 (vi) 80 (vii) 82 (viii) 84 (ix) 86 (x) 88
 (xi) 90 (xii) 92 (xiii) 94 (xiv) 96 (xv) 98

A13. How many CCN will be activated in maritime air at supersaturations (%) of

- a. 0.2 b. 0.3 c. 0.4 d. 0.5 e. 0.6 f. 0.8 g. 1.0
 h. 2 i. 3 j. 4 k. 5 l. 6 m. 8 n. 10

A14. Find the average separation distances (μm) between cloud droplets for the previous problem.

A15. What temperature is needed to immersion-freeze half the droplets of radius (μm)

- a. 10 b. 20 c. 30 d. 40 e. 50 f. 60 g. 70
 h. 80 i. 90 j. 125 k. 150 l. 200 m. 200 n. 300

A16. Estimate the number density of active ice nuclei for the following combinations of temperature and supersaturation [$T(^{\circ}\text{C})$, $S_{ice}(\%)$].

- a. -5, 3 b. -5, 5 c. -10, 5 d. -10, 10
 e. -15, 10 f. -10, 12 g. -15, 12 h. -20, 13
 i. -15, 18 j. -20, 20 k. -25, 20 l. -20, 23
 m. -25, 23 n. -23, 25

A17. For a supersaturation gradient of 1% per $2 \mu\text{m}$, find the kinematic moisture flux due to diffusion. Given $T = -20^{\circ}\text{C}$, and $P = 80 \text{ kPa}$. Use $D \text{ (m}^2\text{s}^{-1}\text{)}$ of:

- a. 1×10^{-6} b. 2×10^{-6} c. 3×10^{-6} d. 4×10^{-6}
 e. 5×10^{-6} f. 6×10^{-6} g. 7×10^{-6} h. 8×10^{-6}
 i. 9×10^{-6} j. 1×10^{-5} k. 3×10^{-5} l. 4×10^{-5}
 m. 5×10^{-5} n. 6×10^{-5} o. 7×10^{-5} p. 8×10^{-5}

A18 (§). Compute and plot supersaturation (%) vs. distance (μm) away from drops of the following radii, given a background supersaturation of 0.5%

- a. $0.15 \mu\text{m}$ containing 10^{-16} g of salt
 b. $0.15 \mu\text{m}$ containing 10^{-16} g of ammon.sulfate
 c. $0.15 \mu\text{m}$ containing 10^{-16} g of sulfuric acid
 d. $0.15 \mu\text{m}$ containing 10^{-16} g of nitric acid
 e. $0.15 \mu\text{m}$ containing 10^{-16} g of hydrogen perox.
 f. $0.3 \mu\text{m}$ containing 10^{-16} g of salt
 g. $0.3 \mu\text{m}$ containing 10^{-16} g of ammon.sulfate
 h. $0.3 \mu\text{m}$ containing 10^{-16} g of sulfuric acid
 i. $0.3 \mu\text{m}$ containing 10^{-16} g of nitric acid
 j. $0.3 \mu\text{m}$ containing 10^{-16} g of hydrogen perox.
 k. $0.5 \mu\text{m}$ containing 10^{-16} g of salt
 l. $0.5 \mu\text{m}$ containing 10^{-16} g of ammon.sulfate
 m. $0.5 \mu\text{m}$ containing 10^{-16} g of sulfuric acid
 n. $0.5 \mu\text{m}$ containing 10^{-16} g of nitric acid
 o. $0.5 \mu\text{m}$ containing 10^{-16} g of hydrogen perox.
 p. $2 \mu\text{m}$ containing 10^{-16} g of salt
 q. $1 \mu\text{m}$ containing 10^{-16} g of ammon.sulfate
 r. $1 \mu\text{m}$ containing 10^{-16} g of sulfuric acid
 s. $1 \mu\text{m}$ containing 10^{-16} g of nitric acid
 t. $1 \mu\text{m}$ containing 10^{-16} g of hydrogen perox.

A19. Find the diffusivity (m^2s^{-1}) for water vapor, given [$P(\text{kPa})$, $T(^{\circ}\text{C})$] of:

- a. 80, 0 b. 80, -5 c. 80, -10 d. 80, -20
 e. 70, 0 f. 70, -5 g. 70, -10 h. 70, -20
 i. 60, 0 j. 60, -5 k. 60, -10 l. 60, -20
 m. 50, 0 n. 50, -5 o. 50, -10 p. 50, -20

A20 (§). For the previous exercise, plot droplet radius (μm) vs. time (minutes) for diffusive growth.

A21. What phase (I - XIV) of ice is expected at the following locations in a standard atmosphere?

- a. Earth's surface b. mid-troposphere
 c. tropopause d. mid stratosphere
 e. stratopause f. mid-mesosphere
 g. mesopause

A22. What phase (I - XIV) of ice is expected for the following conditions of [$P(\text{kPa})$, $T(^{\circ}\text{C})$]?

- a. 1, -250 b. 1, -150 c. 1, -50 d. 1, 50
 e. 10^3 , -250 f. 10^3 , -150 g. 10^3 , -50 h. 10^3 , 50
 i. 5×10^5 , -250 j. 5×10^5 , -150 k. 5×10^5 , -30
 l. 5×10^5 , 50 m. 10^7 , -250 n. 10^7 , -50 o. 10^7 , 50

A23. What crystal habit could be expected for the following combinations of $[\rho_{ve} \text{ (g m}^{-3}\text{)}, T\text{ (}^\circ\text{C)}]$:

- a. 0.22, -25 b. 0.22, -20 c. 0.22, -13
 d. 0.22, -8 e. 0.22, -5 f. 0.22, -2
 g. 0.12, -25 h. 0.12, -20 i. 0.12, -13
 j. 0.12, -8 k. 0.12, -5 l. 0.12, -2
 m. 0.08, -25 n. 0.08, -20 o. 0.08, -13
 p. 0.08, -8 q. 0.08, -5 r. 0.08, -2

A24. Suppose the following ice crystals were to increase mass at the same rate. Find the rate of increase with time of the requested dimension.

- a. effective radius of column growing in 3-D
 b. diameter of plate growing in 2-D
 c. length of needle growing in 1-D

A25 (§). Given $D = 2 \times 10^{-5} \text{ m}^2\text{-s}^{-1}$ and $\rho_v = 0.003 \text{ kg}\cdot\text{m}^{-3}$, plot ice-crystal mass (g) vs. time (minutes) for 3-D growth such as a hexagonal column. Use the following supersaturation fraction:

- a. 0.001 b. 0.002 c. 0.003 d. 0.004 e. 0.005
 f. 0.006 g. 0.007 h. 0.008 i. 0.009 j. 0.01
 k. 0.012 l. 0.014 m. 0.016 n. 0.018 o. 0.020

A26 (§). Same as the previous problem, but for 2-D growth of a thin flat plate of thickness $10 \mu\text{m}$.

A27 (§). Use the Clausius-Clapeyron equation from the Water Vapor chapter to calculate the saturation vapor pressure over liquid water and ice for $-50 \leq T \leq 0^\circ\text{C}$, and use that data to calculate and plot the difference. Namely, reproduce Fig. 7.16 with your own calculations.

A28. Find the terminal velocity of cloud droplets of radius (μm):

a. 0.2 b. 0.4 c. 0.6 d. 0.8
 e. 1.0 f. 2 g. 3 h. 4 i. 5 j. 7
 k. 10 l. 15 m. 20 n. 30 o. 40 p. 50

A29. Find the terminal velocity of rain drops of radius (μm):

a. 100 b. 150 c. 200 d. 300
 e. 400 f. 500 g. 600 h. 700 i. 800
 j. 900 k. 1000 l. 1200 m. 1500 n. 2000

A30. Calculate the terminal velocity of hailstones of radius (cm):

a. 0.25 b. 0.5 c. 0.75 d. 1 e. 1.25
 f. 1.5 g. 1.75 h. 2 i. 2.5 j. 3 k. 3.5
 l. 4 m. 4.5 n. 5 o. 5.5 p. 6

A31. What type of "meteor" is:

- a. a rainbow b. lightning c. corona
 d. dust e. a cloud f. a halo g. sand
 h. rain i. smoke j. snow k. fog
 l. haze m. dew n. frost

A32. For a Marshall-Palmer rain-drop size distribution, if the rainfall rate is

(i) 10 mm h^{-1} , or (ii) 20 mm h^{-1} ,
 how many droplets are expected of radius (μm) greater than:

- a. 100 b. 200 c. 300 d. 400 e. 500
 f. 700 g. 1000 h. 1200 i. 1500 j. 2000

A33. What is the rain intensity classification and the weather map symbol for rainfall rates (mm h^{-1}) of:

- a. 0.02 b. 0.05 c. 0.1 d. 0.2 e. 0.5
 f. 1.0 g. 2 h. 3 i. 4 j. 5
 k. 6 l. 7 m. 8 n. 9 o. 10

A34. For precipitation in the form of

(i) drizzle, or (ii) snow,
 what is the precipitation intensity classification and weather map symbol for visibility (km) of:

- a. 0.1 b. 0.2 c. 0.3 d. 0.4 e. 0.5 f. 0.6 g. 0.7
 h. 0.8 i. 0.9 j. 1.0 k. 1.2 l. 1.5 m. 2 n. 5

A35. Find the rainfall intensity (mm h^{-1}) associated with the following return period (RP in years) and duration (D in hours) values. $RP, D =$

- a. 2, 0.2 b. 2, 0.5 c. 2, 2 d. 2, 6 e. 2, 12
 f. 10, 0.2 g. 10, 0.5 h. 10, 2 i. 10, 6 j. 10, 12
 k. 100, 0.2 l. 100, 0.5 m. 100, 2 n. 100, 6

A36. For a liquid-water equivalent precipitation value of 5 cm, find the snow depth if the snow density (kg m^{-3}) is:

a. 50 b. 75 c. 100 d. 150 e. 200
 f. 250 g. 300 h. 350 i. 400 j. 450 k. 500
 l. 550 m. 600 n. 650 o. 700 p. 800 q. 900

A37. Find the mean annual precipitation for the following locations, given their longitudes, latitudes:

- a. $120^\circ\text{W}, 50^\circ\text{N}$ b. $120^\circ\text{W}, 25^\circ\text{N}$ c. $120^\circ\text{W}, 10^\circ\text{N}$
 d. $120^\circ\text{W}, 10^\circ\text{S}$ e. $120^\circ\text{W}, 30^\circ\text{S}$ f. $60^\circ\text{W}, 0^\circ\text{N}$
 g. $60^\circ\text{W}, 20^\circ\text{N}$ h. $60^\circ\text{W}, 40^\circ\text{N}$ i. $0^\circ\text{W}, 50^\circ\text{N}$
 j. $0^\circ\text{W}, 25^\circ\text{N}$ k. $0^\circ\text{W}, 5^\circ\text{N}$ l. $0^\circ\text{W}, 20^\circ\text{S}$
 m. $120^\circ\text{E}, 25^\circ\text{S}$ n. $120^\circ\text{E}, 0^\circ\text{N}$ o. $120^\circ\text{E}, 30^\circ\text{N}$

A38. What are the values of zonally averaged evaporation and precipitation rates at latitude:

- a. 70°N b. 60°N c. 50°C d. 40°N e. 30°N
 f. 20°N g. 10°N h. 0° i. 10°S j. 20°S
 k. 30°S m. 40°S n. 50°S o. 60°S p. 70°S

7.10.3. Evaluate & Analyze

E1. If saturation is the maximum amount of water vapor that can be held by air at equilibrium, how is supersaturation possible?

E2. Fig. 7.2 shows how excess-water mixing ratio can increase as a cloudy air parcel rises. Can excess-wa-

ter mixing ratio increase with time in a cloudy or foggy air parcel that doesn't rise? Explain.

E3. What can cause the supersaturation S in a cloud to be less than the available supersaturation S_A ?

E4. Fig. 7.3 applies to cumulus clouds surrounded by clear air. Would the curves be different for a uniform stratus layer? Why?

E5. An air parcel contains CCN that allow 10^9 m^{-3} hydrometeors to form. If the air parcel starts at $P = 100 \text{ kPa}$ with $T = 20^\circ\text{C}$ and $T_d = 14^\circ\text{C}$, find the average radius of cloud droplets due to condensation only, after the air parcel rises to P (kPa) of:

- a. 85 b. 82 c. 80 d. 78 e. 75 f. 72 g. 70
h. 67 i. 63 j. 60 k. 58 l. 54 m. 50 n. 45

Hint. Use a thermo diagram to estimate r_E .

E6. Derive eq. (7.8), stating and justifying all assumptions. (Hint: consider the volume of a spherical drop.)

E7. Rewrite eq. (7.8) for hydrometeors that form as cubes (instead of spheres as was used in eq. 7.8). Instead of solving for radius R , solve for the width s of a side of a cube.

E8. In Fig. 7.5, consider the solid curve. The number density ($\# \text{ cm}^{-3}$) of CCN between any two radii R_1 and R_2 is equal to the average value of $n/\Delta R$ within that size interval times $\Delta R (=R_2 - R_1)$. This works best when R_1 and R_2 are relatively close to each other. For larger differences between R_1 and R_2 , just sum over a number of smaller intervals. Find the number density of CCN for droplets of the following ranges of radii (μm):

- a. $0.02 < R < 0.03$ b. $0.03 < R < 0.04$
c. $0.04 < R < 0.05$ d. $0.05 < R < 0.06$
e. $0.06 < R < 0.08$ f. $0.08 < R < 0.1$
g. $0.2 < R < 0.3$ h. $0.3 < R < 0.4$
i. $0.4 < R < 0.5$ j. $0.5 < R < 0.6$
k. $0.6 < R < 0.8$ l. $0.8 < R < 1$
m. $0.1 < R < 0.2$ n. $1 < R < 2$
o. $0.1 < R < 1$ p. $0.1 < R < 10$
q. $0.02 < R < 0.1$ r. $0.02 < R < 10$

E9. a. Find a relationship between the number density of CCN particles n , and the corresponding mass concentration c ($\mu\text{g}\cdot\text{m}^{-3}$), using the dashed line in Fig. 7.5 and assuming that the molecular weight is M_s .

b. Use Table 7-1 to determine the molecular weight of sulfuric acid (H_2SO_4) and nitric acid (HNO_3), which are two contributors to acid rain. Assuming tiny droplets of these acids are the par-

ticles of interest for Fig. 7.5, find the corresponding values or equations for mass concentration of these air pollutants.

E10. a. From the solid curve in Fig. 7.5, find the number density of CCN at $R = 1 \mu\text{m}$, and use this result to determine the value and units of parameter c in the CCN number density eq. (7.9)

b. Plot the curve resulting from this calibrated eq. (7.9) on a linear graph, and on a **log-log graph**.

c. Repeat (a) and (b), but for $R = 0.1 \mu\text{m}$.

d. Repeat (a) and (b), but for $R = 0.01 \mu\text{m}$.

e. Why does eq. (7.9) appear as a straight line in Fig. 7.5? What is the slope of the straight line in Fig. 7.5? [Hint: for log-log graphs, the **slope** is the number of decades along the vertical axis (ordinate) spanned by the line, divided by the number of decades along the horizontal axis (abscissa) spanned by the line. **Decade** means a factor of ten; namely, the interval between major tic marks on a logarithmic axis.]

f. For number densities less than 1 cm^{-3} , what does it mean to have less than one particle (but greater than zero particles)? What would be a better way to quantify such a number density?

g. As discussed in the Atmospheric Optics chapter, air molecules range in diameter between roughly 0.0001 and 0.001 mm . If the dashed line in Fig. 7.5 were extended to that small of size, would the number density indicated by the Junge distribution agree with the actual number density of air molecules? If they are different, discuss why.

E11. Show how the Köhler equation reduces to the Kelvin equation for CCNs that don't dissolve.

E12 (§). Surface tension σ for a cloud droplet of pure water squeezes the droplet, causing the pressure inside the droplet to increase. The resulting pressure difference ΔP between inside and outside the droplet can be found from the **Young-Laplace equation** ($\Delta P = \sigma \cdot dA/dV$, where A is surface area and V is volume of the droplet). For a spherical droplet of radius R this yields:

$$\Delta P = 2\sigma / R$$

Plot this pressure difference (kPa) vs. droplet radius over the range $0.005 \leq R \leq 1 \mu\text{m}$.

E13. a. Using the equation from the previous exercise, combine it with Kelvin's equation to show how the equilibrium relative humidity depends on the pressure excess inside the droplet.

b. If greater pressure inside the drop drives a greater evaporation rate, describe why RH greater than 100% is needed to reach an equilibrium where

condensation from the air balances evaporation from the droplet.

E14. Using the full Köhler equation, discuss how supersaturation varies with:

- a. temperature.
- b. molecular weight of the nucleus chemical.
- c. mass of solute in the incipient droplet.

E15. Consider a Köhler curve such as plotted in the INFO box on Droplet Growth. But let the RH = 100.5% for air. Starting with an aerosol with characteristics of point A on that figure, (a) discuss the evolution of drop size, and (b) explain why haze particles are not possible for that situation.

E16. Considering Fig. 7.7, which type of CCN chemical would allow easier formation of cloud droplets: (a) a CCN with larger critical radius but lower peak supersaturation in the Köhler curve; or (b) a CCN with smaller critical radius but higher supersaturation? Explain.

E17. What is so special about the critical radius, that droplets larger than this radius continue to grow, while smaller droplets remain at a constant radius?

E18. The Kelvin curve (i.e., the Köhler curve for pure water) has no critical radius. Hence, there is no barrier droplets must get across before they can grow from haze to cloud droplets. Yet cloud droplets are easier to create with heterogeneous nucleation on solute CCN than with homogeneous nucleation in clean air. Explain this apparent paradox.

E19. In air parcels that are rising toward their LCL, aerosol swelling increases and visibility decreases as the parcels get closer to their LCL. If haze particles are at their equilibrium radius by definition, why could they be growing in the rising air parcel? Explain.

E20. How high above the LCL must air be lifted to cause sufficient supersaturation to activate CCN for liquid-droplet nucleation?

E21. Suppose a droplet contained all the substances listed in Table 7-2. What is the warmest temperature (°C) that the droplet will freeze due to:

- a. contact freezing b. condensation freezing
- c. deposition freezing d. immersion freezing

E22. Discuss the differences in nucleation between liquid water droplets and ice crystals, assuming the air temperature is below freezing.

E23. Discuss the differences in abundance of cloud vs. ice nuclei, and how this difference varies with atmospheric conditions.

E24. Can some chemicals serve as both water and ice nuclei? For these chemicals, describe how cloud particles would form and grow in a rising air parcel.

E25. If droplets grow by diffusion to a final average radius given by eq. (7.8), why do we even care about the diffusion rate?

E26. In the INFO box on Cubic Ice, a phase diagram was presented with many different phases of ice. The different phases have different natural crystal shapes, as summarized in the table below. In the right column of this table, draw a sketch of each of the shapes listed. (Hint, look in a geometry book or on the internet for sketches of geometric shapes.)

Table 7-7. Crystal shapes for ice phases.

Shape	Phases	Sketch
hexagonal	Ih	
cubic	Ic, VII, X	
tetragonal	III, VI, VIII, IX, XII	
rhombic	II, IV	
ortho-rhombic	XI	
monoclinic	V	

E27. The droplet growth-rate equation (7.24) considers only the situation of constant background supersaturation. However, as the droplet grows, water vapor would be lost from the air causing supersaturation to decrease with time. Modify eq. (7.24) to include such an effect, assuming that the temperature of the air containing the droplets remains constant. Does the droplet still grow with the square root of time?

E28. If ice particles grew as spheres, which growth rate equation would best describe it? Why?

E29. Verify that the units on the right sides of eqs. (7.26) and (7.27) match the units of the left side.

E30. Manipulate the mass growth rate eq. (7.26) to show that the effective radius of the ice particle grows as the square root of time. Show your work.

E31. Considering the different mass growth rates of different ice-crystal shapes, which shape would grow fastest in length of its longest axis?

E32. If the atmosphere were to contain absolutely no CCN, discuss how clouds and rain would form, if at all.

E33. In Fig. 7.15 the thick black line follows the state of a rising air parcel that is cooling with time. Why does that curve show e_s decreasing with time?

E34. If both supercooled liquid droplets and ice crystals were present in sinking cloudy air that is warming adiabatically, describe the evolutions of both types of hydrometeors relative to each other.

E35. On one graph similar to Fig. 7.18, plot 3 curves for terminal velocity. One for cloud droplets (Stokes Law), another for rain drops, and a third for hail. Compare and discuss.

E36. Can ice crystals still accrete smaller liquid water droplets at temperatures greater than freezing? Discuss.

E37. Large ice particles can accumulate smaller supercooled liquid water drops via the aggregation process known as accretion or riming. Can large supercooled liquid water drops accumulate smaller ice crystals? Discuss.

E38. Is it possible for snow to reach the ground when the atmospheric-boundary-layer temperature is warmer than freezing? Discuss.

E39. Compare CCN size spectra with raindrop size spectra, and discuss.

E40. For warm-cloud precipitation, suppose that all 5 formation factors are working simultaneously. Discuss how the precipitation drop size distribution will evolve with time, and how it can create precipitation.

E41. Use the info in Fig. 7.24, or use the associated equations, to calculate and plot the total depth of rain (mm) vs. duration for any one return period.

E42. Is it possible to have blizzard conditions even with zero precipitation rate? Discuss.

E43. Rearrange eqs. (7.35-7.36) to solve for return period for any given precipitation intensity and duration. Hint: If you let $x = \ln(RP/RP_r)$, then eq. (7.36) is a quadratic eq. with a simple algebraic solution.

E44. Precipitation falling out of a column of atmosphere implies that there was net latent heating in that column. Use the annual mean precipitation of Fig. 7.25 to discuss regions of the world having the greatest latent heating of the atmosphere.

E45. According to Fig. 7.26, some latitudes have an imbalance between evaporation and precipitation. How can that be maintained?

E46. Suppose that a disdrometer gives you information on the size of each hydrometeor that falls, and how many of each size hydrometeor falls per hour. Derive an equation to relate this information to total rainfall rate.

7.10.4. Synthesize

S1. What if the saturation vapor pressure over supercooled water and ice were equal.

a. Discuss the formation of clouds and precipitation.

b. Contrast with those processes in the real atmosphere.

c. Discuss how the weather and climate might change, if at all.

S2. In Fig. 7.15, suppose that the saturation vapor pressure over ice were greater, not less, than that over water. How would the WBF process change, if at all? How would precipitation and clouds change, if at all?

S3. What if you were hired to seed warm ($T > 0^\circ\text{C}$) clouds (i.e., to add nuclei), in order to create or enhance precipitation. Which would work better: (a) seeding with 10^5 salt particles cm^{-3} , each with identical of radius $0.1 \mu\text{m}$; or (b) seeding with 1 salt particle cm^{-3} , each with identical radius of $0.5 \mu\text{m}$; or (c) seeding with a range of salt particles sizes? Discuss, and justify.

S4. What if you were hired to seed cold ($T < 0^\circ\text{C}$) clouds (i.e., to add nuclei), in order to create or enhance precipitation. Would seeding with water or ice nuclei lead to the most precipitation forming most rapidly? Explain and justify.

S5. Is it possible to seed clouds (i.e., add nuclei) in such a way as to reduce or prevent precipitation? Discuss the physics behind such weather modification.

S6. What if all particles in the atmosphere were **hydrophobic** (i.e., repelled water). How would the weather and climate be different, if at all?

S7. What if the concentration of cloud nuclei that could become activated were only one-millionth of what currently exists in the atmosphere. How would the weather and climate change, if at all?

S8. Eq. (7.29) indicates that smaller droplets and aerosol particles fall slower. Does Stoke's law apply to particles as small as air molecules? What other factors do air molecules experience that would affect their motion, in addition to gravity?

S9. What if Stoke's law indicated that smaller particles fall faster than larger particles. Discuss the nature of clouds for this situation, and how Earth's weather and climate might be different.

S10. What if rain droplet size distributions were such that there were more large drops than small drops. Discuss how this could possibly happen, and describe the resulting weather and climate.

S11. Suppose that large rain drops did not break up as they fell. That is, suppose they experienced no drag, and there was no upper limit to rain drop size. How might plant and animal life on Earth have evolved differently? Why?

S12. What if cloud and rain drops of all sizes fell at exactly the same terminal velocity. Discuss how the weather and climate might be different.

S13. What if condensation and deposition absorbed latent heat (i.e., caused cooling) instead of releasing latent heat. How would clouds, precipitation, weather and climate be different, if at all.

S14. Weather modification is as much a social issue as a scientific/technical issue. Consider a situation of **cloud seeding** (adding nuclei) to enhance precipitation over arid farm land in county X. If you wanted to make the most amount of money, would you prefer to be the:

- meteorologist organizing the operation,
- farmer employing the meteorologist,
- company insuring the farmer's crop,
- company insuring the meteorologist, or
- lawyer in county Y downwind of county X,

suing the meteorologist, farmer, and insurance companies?

Justify your preference.

S15. Suppose that you discovered how to control the weather via a new form of cloud seeding (adding nuclei). Should you ...

a. keep your results secret and never publish or utilize them, thereby remaining impoverished and unknown?

b. publish your results in a scientific journal, thereby achieving great distinction?

c. patent your technique and license it to various companies, thereby achieving great fortune?

d. form your own company to create tailored weather, and market weather to the highest bidders, thereby becoming a respected business leader?

e. modify the weather in a way that you feel is best for the people on this planet, thereby achieving great power?

f. allow a government agency to hold hearings to decide who gets what weather, thereby achieving great fairness and inefficiency?

g. give your discovery to the military in your favorite country, thereby expressing great patriotism? (Note: the military will probably take it anyway, regardless of whether you give it willingly.)

Discuss and justify your position. (Hint: See the "A SCIENTIFIC PERSPECTIVE" box at the end of this chapter before you answer this question.)

A SCIENTIFIC PERSPECTIVE • Consequences

The scenario of exercise S15 is not as far-fetched as it might appear. Before World War II, American physicists received relatively little research funding. During the war, the U.S. Army offered a tremendous amount of grant money and facilities to physicists and engineers willing to help develop the atomic bomb as part of the Manhattan Project.

While the work they did was scientifically stimulating and patriotic, many of these physicists had second thoughts after the bomb was used to kill thousands of people at the end of the war. These concerned scientists formed the "Federation of Atomic Scientists", which was later renamed the "Federation of American Scientists" (FAS).

The FAS worked to discourage the use of nuclear weapons, and later addressed other environmental and climate-change issues. While their activities were certainly worthy, one has to wonder why they did not consider the consequences before building the bomb.

As scientists and engineers, it is wise for us to think about the moral and ethical consequences before starting each research project.