Chapter 8: A1e, A4e, A7e, A8e
Chapter 11: A11e, A22e, A23e
Chapter 12: A1e, A7e, A8e, A11e, E21
Total marks out of 54

## Chapter 8

A1e)
(2.5 marks)

Using Fig. 8.4, identify whether the following ( $\mu \mathrm{m}$ ) are in a window, dirty window, shoulder, or opaque part of the transmittance spectrum, and identify which sketch in Fig. 8.2 shows how the Earth would look at that wavelength. [Hint: transmittance of $\geq 80 \%$ indicates a window.]
e) 1.33

## Solution:

$1.33 \mu \mathrm{~m}$ is at a window.

This wavelength has high transmittance so would look like Fig 8.2 a.

Discussion: A $1.33 \mu \mathrm{~m}$ wavelength is in the infrared spectrum. There is no molecular emittance and very little molecular scattering at this wavelength, leading to transmittance of $\sim 80 \%$. This means that the satellite sees the earth and clouds quite clearly.

A4e)
(2.5 marks)

Find the brightness temperature for the following wavelengths ( $\mu \mathrm{m}$ ), given a radiance of $10^{\wedge}(-15) \mathrm{W} /\left(\mathrm{m}^{\wedge} \mathbf{2}^{*} \mu \mathrm{~m}\right.$ *sr): e) 4.4

Given: $\quad \lambda(\mu \mathrm{m})=\quad 4.4$

$$
\mathrm{B} \lambda=\quad 1 \mathrm{E}-15 \mathrm{~W} /\left(\mathrm{m}^{\wedge} 2 * \mu \mathrm{~m} * \mathrm{sr}\right)
$$


where: $\quad \mathrm{c} 1 \mathrm{~B}=\quad 1.19 \mathrm{E}+08 \mathrm{~W}^{*} \mathrm{~m}-2^{*} \mu \mathrm{~m} 4^{*} \mathrm{sr}-1$

$$
c 2=\quad 1.44 \mathrm{E}+04 \mu \mathrm{~m} * \mathrm{~K}
$$

| TB $=$ |
| ---: |
|  |
|  |

Check: Units ok. Physics ok.

Discussion: The radiance given in this question is very small so the resulting brightness temperature is also small. If the wavelength was smaller, we would get a larger brightness temperature because shorter wavelengths carry more energy.

A7e)
(3 marks)
For the following altitudes (km) above the Earth's surface, find the satellite orbital periods: e) 5,000.

Given: satellite altitude $=\quad$| 5000 km |
| ---: |
|  |
|  |
|  |
|  |

Find: $\quad \mathrm{t}$ _orbit $=\quad$ ? $\quad \mathrm{s}$
Use eqn. 8.8: $\quad t_{\text {orbit }}=\frac{2 \pi \cdot R^{3 / 2}}{\sqrt{G \cdot M}}$

| where: | $\mathrm{G}=$ <br> $\mathrm{M}=$ | $6.67 \mathrm{E}-11 \mathrm{~N}^{*} \mathrm{~m}^{\wedge} 2 / \mathrm{kg}^{\wedge} 2$ <br> $5.97 \mathrm{E}+24 \mathrm{~kg}$ |
| :--- | :--- | :--- |
|  |  |  |
| r_Earth $=$ |  | 6378 km |

R = radius of Earth + satellite altitude = 11378000 m

| t_orbit $=$ |
| ---: |
|  |

Check: Units ok. Physics ok.

Discussion: Satellites at higher altitudes move slower because they need less energy to stay in orbit.

A8e)
(3.5 marks)

What shade of grey would the following clouds appear in visible, IR, and water-vapor satellite images? e) altostratus

Given: altostratus cloud

Find: $\quad$ Shade of grey in visible, IR, and water-vapor satellite images.

| VIS: | White during the day. |
| :--- | :--- |
| IR: | Medium gray becase mid-altitude and medium temperature |
| WV: | Medium grey because altostratus clouds carry some moisture |

Discussion: The different shades of grey in different satellite images is indicative of the different wavelengths being picked up by that satellite channel.

## Chapter 11

A11e)
(5.5 marks)

Find the magnitude of the thermal wind ( $\mathrm{m} / \mathrm{s}$ ) for the following thickness gradients: e) $\Delta \mathrm{TH}(\mathrm{km}) / \Delta x(\mathrm{~km})=-0.2 / 600, \Delta \mathrm{TH}(\mathrm{km}) / \Delta \mathrm{y}(\mathrm{km})=-0.1 / 400$.

Given:

| $\Delta T H \_x=$ | -0.2 km | -200 m |
| :--- | :--- | ---: |
| $\Delta x=$ | 600 km | 600000 m |
| $\Delta T H \_y=$ | -0.1 km | -100 m |
| $\Delta y=$ | 400 km | 400000 m |

Find: $\quad \mathrm{MTH}=\quad$ ? $\quad \mathrm{m} / \mathrm{s}$
First, use eqns 11.15a and 11.15b:

$$
\begin{aligned}
& u_{T H}=u_{G 2}-u_{G 1}=-\frac{|g|}{f_{c}} \frac{\Delta T H}{\Delta y} \\
& V_{T H}=V_{G 2}-V_{G 1}=+\frac{|g|}{f_{c}} \frac{\Delta T H}{\Delta x}
\end{aligned}
$$

| where $\|\mathrm{g}\|=$ | $9.8 \mathrm{~m} / \mathrm{s}^{\wedge} 2$ |
| :--- | ---: |
| assume $\mathrm{fc}=$ | $1.10 \mathrm{E}-04 / \mathrm{s}$ |


| UTH $=$ | $22.27 \mathrm{~m} / \mathrm{s}$ |
| :--- | ---: |
| VTH $=$ | $-29.70 \mathrm{~m} / \mathrm{s}$ |

Then, use eqn. 11.16: $\quad M_{T H}=\sqrt{U_{T H}{ }^{2}+V_{T H}{ }^{2}}$
MTH $=\quad 37.12 \mathrm{~m} / \mathrm{s}$

Check: Units ok. Physics ok.
Discussion: The thermal wind speed is the difference in the geostrophic wind
speeds at the top and bottom of the layer. Thickness (TH) is analagous to the mean temperature of the layer. From the given thickness gradients, we know that the temperature is colder to the east, and to the north. Thermal wind points parallel to the thickness isolines, with the cold air to its left. This matches with our numerical answers for UTH and VTH.

A22e)
(2.5 marks)

For the latitude given below, what is the value of the beta parameter ( $1 / \mathrm{ms}$ ): e) $70^{\circ}$.

Given: $\quad \phi=\quad 70^{\circ} \quad 1.2217305$ radians
Find: $\quad \beta=\quad$ ? $\quad 1 /(m * s)$
Use eqn. 11.35:

$$
\beta=\frac{\Delta f_{c}}{\Delta y}=\frac{2 \cdot \Omega}{R_{\text {earth }}} \cdot \cos \phi
$$

where: $\quad 2^{*} \Omega /$ Rearth $=\quad 2.29 \mathrm{E}-111 /\left(m^{*} \mathrm{~s}\right)$
$\beta=\quad 7.83 \mathrm{E}-121 /\left(\mathrm{m}^{*} \mathrm{~s}\right)$

Check: Units ok. Physics ok.
Discussion: When we assume that the Coriolis parameter fc changes linearly with latitude (i.e. beta = constant), we are using the "beta plane". This approximation is valid when we are only looking at a small latitude belt.

A23e)
(6 marks)
Suppose the average wind speed is $60 \mathrm{~m} / \mathrm{s}$ from the west at the tropopause For a barotropic Rossby wave at $50^{\circ}$ latitude, find both the intrinsic phase speed ( $\mathrm{m} / \mathrm{s}$ ) and the phase speed ( $\mathrm{m} / \mathrm{s}$ ) relative to the ground for wavelength (km) of: e) 3000.

Given: $\lambda=3000 \mathrm{~km} \quad 3000000 \mathrm{~m}$

$$
\begin{array}{lll}
\mathrm{co}= & ? & \mathrm{~m} / \mathrm{s} \\
\mathrm{c}= & ? & \mathrm{~m} / \mathrm{s}
\end{array}
$$

First, find the beta parameter using eqn. 11.35:

$$
\beta=\frac{\Delta f_{c}}{\Delta y}=\frac{2 \cdot \Omega}{R_{\text {earth }}} \cdot \cos \phi
$$

$$
\begin{array}{ll}
\text { where : } \quad 2 * \Omega / \text { Rearth }= & 2.29 \mathrm{E}-111 /\left(\mathrm{m}^{*} \mathrm{~s}\right) \\
\begin{array}{lll}
\beta= & 1.47 \mathrm{E}-111 /\left(\mathrm{m}^{*} \mathrm{~s}\right) \\
\hline
\end{array}
\end{array}
$$

To find the intrinsic phase speed use eqn. 11.37:

$$
c_{o}=-\beta \cdot\left(\frac{\lambda}{2 \pi}\right)^{2}
$$

$\mathrm{co}=\quad-3.36 \mathrm{~m} / \mathrm{s}$

To find the phase speed use eqn 11.38:

$$
c=U_{o}+c_{o}
$$

$\mathrm{c}=\quad 56.64 \mathrm{~m} / \mathrm{s}$

Check: Units ok. Physics ok.

Discussion: The intrinsic phase speed tells us how fast the wave is moving relative to the mean flow. A negative intrinsic phase speed means that the waves are actually moving westwards if there is no mean flow in the background. The phase speed is the speed of the wave relative to the ground. This wave travels fairly fast (near mean flow wind speed), which is reasonable as it has a fairly short wavelength for a barotropic wave.

## Chapter 12

A1e)
Identify typical characteristics of the following airmass: e) cE.
(2.5 marks)

Given: Airmass $=\mathrm{CE}=$ continental equatorial.

Characteristics: Hot and dry. Formed over land.

## Discussion:

A continental equatorial airmass is not an airmass we talk about in North America. Forming over equatorial South America or Africa, these airmasses are very hot and dry

A7e)
(4.5 marks)

Find the external Rossby radius of deformation at $60^{\circ}$ latitude for a cold airmass of thickness 500 m and $\Delta \theta\left({ }^{\circ} \mathrm{C}\right)$ of: e) 10.
Assume a background temperature of 300 K .

Given: $\quad \Delta \theta\left({ }^{\circ} \mathrm{C}\right)=\quad 10$ or 10K because it's a difference
$\phi\left({ }^{\circ}\right)=60$
$\mathrm{H}(\mathrm{m})=\quad 500$
$\mathrm{T}(\mathrm{K})=300$

Find: $\quad \lambda R=\quad$ ?

Use eqn. 12.5: $\quad a=\lambda_{R}=\frac{\sqrt{|g| \cdot H \cdot \Delta \boldsymbol{\theta}_{v} / \overline{T_{v}}}}{f_{c}}$
where $\mathrm{g}=$ $9.8 \mathrm{~m} / \mathrm{s}^{\wedge} 2$

To find fc use eqn 10.16: $\mathrm{fc}=2^{*} \Omega^{*} \sin \phi$
where $\Omega=\quad 7.29 \mathrm{E}-05 / \mathrm{s}$
$\mathrm{fc}=\quad 1.26 \mathrm{E}-04 / \mathrm{s}$

Assume a dry air mass so that $\Delta \theta{ }^{\sim}=\Delta \theta \mathrm{v}$, and $\mathrm{T}^{\sim}=\mathrm{Tv}$.

| $\lambda R(m)=$ | $1.01 \mathrm{E}+05 \mathrm{~m}$ |
| ---: | ---: |
| 101.22 km |  |

Check: Units ok. Physics ok.
Discussion: above the external Rossby radius of deformation is where the jet
stream would be. The fact that cold air masses cannot redistribute cold air to the equator means some other forces will try to redistribute this heat.

A8)
(10.5 marks)

Find and plot the airmass depth and geostrophic wind as a function of distance from the front for the cases of the previous exercise. Assume a background potential temperature of 300K.

Given:

| $\Delta \theta\left({ }^{\circ} \mathrm{C}\right)=$ | 10 |
| :--- | ---: |
| $\phi\left({ }^{\circ}\right)=$ | 60 |
| $\mathrm{H}(\mathrm{m})=$ | 500 |
| $\mathrm{~T}(\mathrm{~K})=$ | 300 |

From A7d the final spillage distance of the front $a=\lambda R=101.22 \mathrm{~km}$.

$$
a(m)=\quad 1.01 \mathrm{E}+05
$$

Use eqn 12.6:

$$
u_{g}=-\sqrt{|g| \cdot H \cdot\left(\Delta \theta_{v} / \overline{T_{v}}\right)} \cdot \exp \left(-\frac{y+a}{a}\right)
$$

where y is the distance behind a .
and $g=\quad 9.8 \mathrm{~m} / \mathrm{s}^{\wedge} 2$
Assume a dry air mass so that $\Delta \theta{ }^{\sim}=\Delta \theta \mathrm{v}$, and $\mathrm{T}^{\sim}=\mathrm{Tv}$.

Use eqn 12.7:

$$
h=H \cdot\left[1-\exp \left(-\frac{y+a}{a}\right)\right]
$$

| $\mathbf{y}(\mathbf{k m})$ | $\mathbf{U g}(\mathbf{m} / \mathbf{s})$ | $\boldsymbol{h}(\mathbf{k m})$ |
| :---: | ---: | :--- |
| -90 | -11.44 | 0.0524 |
| -80 | -10.36 | 0.0946 |
| -70 | -9.39 | 0.1327 |
| -60 | -8.51 | 0.1672 |
| -50 | -7.71 | 0.1986 |
| -25 | -6.02 | 0.2645 |
| 0 | -4.70 | 0.3161 |
| 25 | -3.67 | 0.3563 |


| 50 | -2.87 | 0.3878 |
| ---: | ---: | ---: |
| 75 | -2.24 | 0.4123 |
| 100 | -1.75 | 0.4315 |
| 125 | -1.37 | 0.4465 |
| 150 | -1.07 | 0.4582 |
| 175 | -0.83 | 0.4674 |
| 200 | -0.65 | 0.4745 |
| 225 | -0.51 | 0.4801 |
| 250 | -0.40 | 0.4844 |
| 275 | -0.31 | 0.4878 |
| 300 | -0.24 | 0.4905 |
| 325 | -0.19 | 0.4926 |
| 350 | -0.15 | 0.4942 |
| 375 | -0.12 | 0.4955 |
| 400 | -0.09 | 0.4965 |
| 425 | -0.07 | 0.4972 |
| 450 | -0.06 | 0.4978 |
| 475 | -0.04 | 0.4983 |
| 500 | -0.03 | 0.4987 |
| 525 | -0.03 | 0.4990 |
| 550 | -0.02 | 0.4992 |
| 575 | -0.02 | 0.4994 |

Check: Units ok. Physics ok.

Discussion: this h vs y plot confirms Fig. 12.17b, where the depth of the cold air mass increases with distance into the cold air mass. Fig 12.17b also shows Ug being strongest at the air mass boundary ( $\mathrm{y}=-\mathrm{a}$ ) and then decreasing in strength further back into the cold air mass. Negative values indicate Ug is blowing to the West.


A11e)
(7 marks)
Plot dryline movement with time, given the following conditions. Surface heat flux is constant with time at kinematic rate $0.2 \mathrm{~K} * \mathrm{~m} / \mathrm{s}$. The vertical gradient of potential temperature in the initial sounding is $\gamma$. Terrain slope is $s=\Delta z / \Delta x . \quad$ e) $\Upsilon(K / k m)=12, s=1 / 500$.

| Given: | $\mathrm{r}=$ | $1 / 500=$$12 \mathrm{~K} / \mathrm{km}$ <br>  <br> $\mathrm{s}=$ <br> $\mathrm{FH}=$$\quad 0.002$ |
| :--- | :--- | :--- |

Plot $\Delta x$ vs $\Delta t$ (the dryline movement with time).

From Sample Application, $\mathrm{QAK}=\mathrm{FH}^{*} \Delta \mathrm{t}$.

Use eqn 12.15:

$$
\Delta x=\frac{1}{s} \cdot\left(\frac{2 \cdot Q_{A k}}{\gamma}\right)^{1 / 2}
$$



| 12.5 | 612.37 |
| ---: | ---: |
| 13 | 624.50 |
| 13.5 | 636.40 |
| 14 | 648.07 |
| 14.5 | 659.55 |
| 15 | 670.82 |
| 15.5 | 681.91 |
| 16 | 692.82 |

Check: Units ok. Physics ok.

Discussion: This plot does not take into account night time, when convective turbulence ceases and prevailing low altitude easterlies advect moist air back towards the west.

## E21)

(4 marks)
Background: Recall that a frontal zone separates warmer and cooler airmasses. The warm airmass side of this zone is where the front is drawn on a weather map. This is true for both cold and warm fronts. Issue: AFTER passage of the cold front is when significant temperature decreases are observed. BEFORE passage of a warm front is when significant warming is observed. Question: Why does this difference exist (ie. AFTER vs BEFORE) for the passage of these two fronts?

## Solution:

Fronts on a weather map are always drawn on the warm side of the surface frontal zone.
For a warm front, the baroclinic zone (the region with the strongest horizontal temperature gradient indicated by the tightest isotherms)
is ahead of the warm front. For this reason, you will feel the rapid warming before the warm front approaches.
For a cold front the baroclinic zone is behind the cold front, so you will feel the rapid cooling after the cold front has passed.

