Chapter 9: A1e, A7a, A8e, A10all
Chapter 13: A3e, A7e, A14e, A16e

## Chapter 9

## A1e)

(4 marks)

| Find the pressure "reduced to sea level" using the following station observations of pressure, height, and virtual temperature. Assume no temperature change over the past 12 hours: e) $P=94 \mathrm{kPa}, \mathrm{z}=610 \mathrm{~m}, \mathrm{Tv}=20 \mathrm{C}$. |  |  |  |
| :---: | :---: | :---: | :---: |
| Given: | Pstn (kPa) $=$ |  | 94 |
|  | zstn (m) = |  | 610 |
|  | $\mathrm{Tv}(\operatorname{deg} \mathrm{C})=$ |  | 20 |
|  | Tv (K) = |  | 293.15 |
| Find: | PMSL $=$ | ? | kPa |

First, use eqn 9.2:

$$
\overline{T_{v}^{*}}=0.5 \cdot\left[T_{v}\left(t_{o}\right)+T_{v}\left(t_{o}-12 \mathrm{~h}\right)+\gamma_{s a^{*}} z_{s t n}\right]
$$

where $\mathrm{Ysa}=\quad 0.0065 \mathrm{~K} / \mathrm{m}$
and $\operatorname{Tv}($ to $-12 \mathrm{~h})=\operatorname{Tv}($ to $)=293.15 \mathrm{~K}$
av Tv*= 295.13 K

Use eqn. 9.1:

$$
P_{M S L}=P_{s t n} \cdot \exp \left(\frac{z_{s t n}}{a \cdot \overline{T_{v}^{*}}}\right)
$$

where $\mathrm{a}=$
$29.3 \mathrm{~m} / \mathrm{K}$

PMSL =
100.9 kPa

Check: Units ok. Physics ok.
Discussion: At some stations, the PMSL may be located below the ground.

A7a)
(5 marks)
Photocopy the USA weather map in Fig. 9.19, and analyze it by drawing isopleths for: a) temperature (isotherms) every 5F


USA surface weather map. Units: $T$ and $T_{d}\left({ }^{\circ} F\right)$, visibility (miles), speed (knots), pressure and 3-hour tendency (see text), 6-hour precipitation (hundredths of inches). Extracted from a "Daily Weather Map" courtesy of the US National Oceanic and Atmospheric Administration (NOAA), National Weather Service (NWS), National Centers for Environmental Prediction (NCEP), Hydrometeorological Prediction Center (HPC). The date/time of this map is omitted to discourage cheating during map-analysis exercises, and the station locations are shifted slightly to reduce overlap.

Check: Shapes of the isotherms make sense. They don't cross each other.

Discussion: There is a strong temperature gradient in the SW.

A8e)
8

> Using the Canadian weather map of Fig. 9.20 , decode the weather data for the station assigned by your instructor: e).


Discussion: Using a map with multiple station plots, we can easily draw the isobars, isotherms, cloud coverage, and precipitation regions. We can then use analysis to infer where the low pressure centre and its associated cold and warm fronts are.

A10all)
(15 marks)
A10. Both of the weather maps of Fig. 9.21 corre- spond to the same weather. Do the following work on a photocopy of these charts:
a. Draw isotherms and identify warm and cold centers. Label isotherms every $2^{\circ} \mathrm{C}$.
b. Draw isobars every 0.2 kPa and identify high and low pressure centers.
c. Add likely wind vectors to the pressure chart.
d. Identify the frontal zone(s) and draw the frontal boundary on the temperature chart.
e. Use both charts to determine the type of front (cold, warm), and draw the appropriate frontal symbols on the front.
f. Indicate likely regions for clouds and suggest cloud types in those regions.
g. Indicate likely regions for precipitation.
h. For which hemisphere are these maps?

Diagrams on next pages.

Discussion: Weather forecasters in training at environment canada have to draw diagrams like this every morning!


- isotherms
- frontal boundary
a A cold front
( warm front

Scattered showers associated w cold frent (both charts) (6) (0) scattexted cumulus or
stratocumulus (beth charts)


## -isobars

low pressure center
䮠 high pressure center
$\Rightarrow$ wind vectors

## Chapter 13

A3e)
(4 marks)
A3. Given a tropospheric depth of 12 km at latitude
$45^{\circ} \mathrm{N}$, what is the meridional (north-south) ampli-
tude (km) of upper-atmosphere (Rossby) waves trig-
gered by mountains, given an average mountain-
range height (km) of:

| a. 0.4 | b. 0.6 | c. 0.8 | d. 1.0 | e. 1.2 | f. 1.4 |
| :--- | :--- | :--- | :--- | :--- | :--- |


| h. 1.8 | i. 2.0 | j. 2.2 | k. 2.4 |
| :--- | :--- | :--- | :--- |
| 1.2 .6 | m. 2.8 | n. 3.0 |  |

Given:

$$
\begin{aligned}
& \Delta z_{-} \mathrm{mtn}= \\
& \Delta z_{-} \mathrm{T}=
\end{aligned}
$$

$$
1.2 \text { km }
$$

$$
12 \text { km }
$$

$$
\phi=\quad 45 \mathrm{deg}
$$

Find:
$A=$
?
km

Use eqn 13.3: $\quad A \approx \frac{f_{c}}{\beta} \cdot \frac{\Delta z_{m+n}}{\Delta z_{T}}$
where $\mathrm{fc} / \beta=$ Rearth*tan $(\phi)$ from page 444 below eqn 13.3
Rearth $=\quad 6371$ km
$\mathrm{fc} / \beta=\quad 6371 \mathrm{~km}$
$\mathrm{A}=\quad 637.10 \mathrm{~km}$

Check: Units ok. Physics ok.
Discussion: The higher the mountains, the shorter the column of air will be, and with this greater change, relative vorticity will need to decrease more to conserve potential vorticity. This decreasing relative vorticity is what initiates the Rossby wave.

A7e)
(7 marks)
A7. When air at latitude $60^{\circ} \mathrm{N}$ flows over a mountain range of height 2 km within a troposphere of depth 12 km , find the radius of curvature $(\mathrm{km})$ at location

> " C " in Fig. 13.20 given an average wind speed $\left(\mathrm{m} \mathrm{s}^{-1}\right)$
> of: a. 20
> b. 25
> c. 30
> d. 35
> e. 40
> f. 45
> g. 50
> h. 55
> i. 60
> j. 65
> k. 70
> 1. 75
> m. 80 n. 85

Given:

$$
\begin{aligned}
& \phi= \\
& \Delta z_{-} \mathrm{mtn}= \\
& \Delta z_{-} \mathrm{T}= \\
& \mathrm{M}=
\end{aligned}
$$

$60^{\circ}$
2 km
12 km
$40 \mathrm{~m} / \mathrm{s}$
1.04719755 radians
2000 m
12000 m
$0.04 \mathrm{~km} / \mathrm{s}$

Find: $\mathrm{Rc}=\quad$ ? km

Can assume that the crest after location c is at the same latitude as loc. a Therefore can use equation 13.4 to find the amplitude of the Rossby wave.

$$
\begin{equation*}
A \approx \frac{\Delta z_{m+n}}{\Delta z_{T}} \cdot R_{\text {earth }} \tag{13.4}
\end{equation*}
$$

where R_earth (km) =

$$
\begin{equation*}
(\mathrm{m})= \tag{6371}
\end{equation*}
$$

$A=$
1061.83 km
$2 A=$
2123.66667 km

$$
1.06 \mathrm{E}+06 \mathrm{~m}
$$

Now that we have A, we can find the change in latitude (and therefore the change in fc ) from point a to point c , i.e. over a north-south distance of 2A.
we know $111 \mathrm{~km}=1^{\circ}$ lat
111
so $2 \mathrm{~A} / 111 \mathrm{~km}=\Delta \phi=$
$19.1321321^{\circ}$ Lat
so $\phi(\mathrm{c})=\quad \phi(\mathrm{a})-\Delta \phi$
$\phi(\mathrm{c})=\quad 40.87{ }^{\circ} \mathrm{N} \quad 0.71327885$ radians
use eqn 10.16
$\mathrm{fc}=2^{*} \Omega^{*} \sin (\phi) \quad$ where $2 \Omega=\quad 1.46 \mathrm{E}-041 / \mathrm{s}$
$\mathrm{fc}($ at c$)=\quad 9.54 \mathrm{E}-051 / \mathrm{s}$
$\mathrm{fc}($ at a$)=\quad 1.26 \mathrm{E}-041 / \mathrm{s}$
Use eqn 13.6 and 13.5
$\zeta_{p}=\frac{f_{c . a}}{\Delta z_{T . a}}$

$$
\begin{equation*}
\zeta_{p}=\frac{(M / R)+f_{c}}{\Delta z}=\text { constant } \tag{13.5}
\end{equation*}
$$

Knowing that potential vorticity is conserved and that $\Delta z T($ at $a)=\Delta z T(a t ~ c)$ we can rearrage these to be:
$R($ at $c)=M /(f c(a t a)-f c(a t c))$

| $R($ at $c)=$ | 1295877.65 m |
| ---: | ---: |
|  | 1295.88 km |

Check: Units ok. Physics ok.

Discussion: The radius of curvature at location c must be positive because it is turning cyclonically to keep potential vorticity constant.

A14e)
(3.5 marks)

| A14. At an altitude where the ambient pressure is 85 |
| :--- |
| kPa, convert the following vertical velocities $\left(\mathrm{m} \mathrm{s}^{-1}\right)$ |
| into omega (Pa s s |


| a. 2 | b. 5 | c. 10 | d. 20 | e. 30 | f. 40 | g. 50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| h. -0.2 i. -0.5 j. -1.0 k. -3 $1 .-5$ m. -0.03 |  |  |  |  |  |  |.

Given:

$$
\begin{aligned}
& P= \\
& W=
\end{aligned}
$$

85 kPa
$30 \mathrm{~m} / \mathrm{s}$
Find:
$\omega=$
?
$\mathrm{Pa} / \mathrm{s}$

Use eqn. 13.14:

$$
\omega=-\rho \cdot|g| \cdot W
$$

$$
|\mathrm{g}|=\quad 9.8 \mathrm{~m} / \mathrm{s}^{\wedge} 2
$$

From Table 1-5 estimate

$$
\rho=\quad 1.06 \mathrm{~kg} / \mathrm{m}^{\wedge} 3
$$

$\omega=\quad-311.64 \mathrm{~Pa} / \mathrm{s}$

Check: Units ok. Physics ok.
Discussion: The term omega, by definition, is the change of pressure with time. So, when omega is negative, this means air is descending and that pressure is increasing as the air moves

## A16e)

(3 marks)
A16. Find the vertical velocity ( $\mathrm{m} \mathrm{s}^{-1}$ ) at altitude 9 km in an 11 km thick troposphere, if the divergence $\left(10^{-5} \mathrm{~s}^{-1}\right)$ given below occurs within a 2 km thick layer within the top of the troposphere.
$\begin{array}{llllllll}\text { a. } 0.2 & \text { b. } 1 & \text { c. } 1.5 & \text { d. } 2 & \text { e. } 3 & \text { f. } 4 & \text { g. } 5 & \text { h. } 6\end{array}$
$\begin{array}{llllll}\text { i. }-0.3 & \text { j. }-0.7 & \text { k. }-1.8 & \text { l. }-2.2 & \text { m. }-3.5 & \text { n. }-5\end{array}$

$$
? z=\quad 3 \mathrm{~km}
$$

3000 m

Find: $\quad$ Wmid $=\quad ? \quad \mathrm{~m} / \mathrm{s}$

Use eqn. 13.15:

$$
\begin{equation*}
W_{\text {mid }}=D \cdot \Delta z \tag{13.15}
\end{equation*}
$$

Wmid $=\quad 0.06 \mathrm{~m} / \mathrm{s}$

Check: Units ok. Physics ok.

Discussion: Due to the conservation of mass (continuity equation) if there is horizontal divergence/convergence, then there must be vertical motion to fill in the air that is leaving/entering horizontally.

