

ATSC 201 Fall 2023

Total mark out of 41

Chapter 10: A12e, A14e, A18e

Chapter 11: A14e, A17e, A18e, A19e

Chapter 14: A18a, A23, A28, A30

Chapter 10

A12e)

(3.5 marks)

Given the pressure gradient magnitude (kPa/1000km) below, find geostrophic wind speed for a location having $f_c = 1.1 \times 10^{-4} \text{ /s}$ and $\rho = 0.8 \text{ kg/m}^3$. e) 4

Given: $f_c =$ $M_{\tan} = 0.5 \cdot f_c \cdot R \cdot \left[-1 + \sqrt{1 + \frac{4 \cdot G}{f_c \cdot R}} \right]$ (10.34a)
 $\rho =$
 $\Delta P / \Delta d =$ 5 kPa/1000km

Find: $G =$? m/s
Geostrophic wind

Using equation 10.28:

$$G = \left| \frac{1}{\rho \cdot f_c} \cdot \frac{\Delta P}{\Delta d} \right|$$

Convert $\Delta P / \Delta d$ from kPa/1000km to Pa/m:

The 'kilo' (x1000) on top and bottom can cancel each other out, so
kPa / 1000km = Pa / 1000m. To get this into Pa / m, divide by 1000.

$$\Delta P / \Delta d = 0.005 \text{ Pa / m}$$

$$G = 56.82 \text{ m/s}$$

Check: Units ok. Physics ok.

Discussion: Note that G is proportional to the PGF. When Δd (spacing between isobars) is smaller, PGF is larger and so is G .

A14e)

(3 marks)

At the radius (km) given below from a low-pressure center, find the gradient wind speed given a geostrophic wind of 8 m/s and given $f_c = 1.1 \times 10^{-4} \text{ /s}$. e) 800.

Given: R = 900 km
 G = 8 m/s
 $f_c =$ $1.10 \times 10^{-4} \text{ /s}$

Find: $M_{tan} =$? m/s

Using eq. 10.34a:
$$M_{tan} = 0.5 \cdot f_c \cdot R \cdot \left[-1 + \sqrt{1 + \frac{4 \cdot G}{f_c \cdot R}} \right]$$

Convert R(km) into R(m):

R (m) = 900000

$M_{tan} =$ 7.44 m/s

Check: Units ok. Physics ok.

Discussion:

Gradient wind speed around a low is slower than the geostrophic wind because of the imbalance between the PGF and the Coriolis force caused by the curvature of the flow.

A18e)
 (3 marks)

Find the cyclostrophic wind at radius (m) given below, for a radial pressure gradient = 0.5 kPa/m: e) 18

Given: R = 18 m
 $\Delta P / \Delta R =$ 0.5 kPa/m

Find: $M_{cs} =$? m/s

Use eq. 10.46:

$$M_{cs} = \sqrt{\frac{R}{\rho} \cdot \frac{\Delta P}{\Delta R}}$$

assume $\rho =$ 1 kg / m^3

Convert $\Delta P / \Delta R$ (kPa/m) into $\Delta P / \Delta R$ (Pa/m):

$\Delta P / \Delta R =$ 500 Pa/m

$Mcs =$	94.87 m/s
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Check: Units ok. Physics ok.

Discussion: These are very strong winds because a PGF of 0.5 kPa/m is very large!

Chapter 11

A14e)

A14. Find the relative vorticity (s^{-1}) for the change of (U , V) wind speed ($m\ s^{-1}$), across distances of $\Delta x = 300\ km$ and $\Delta y = 600\ km$ respectively given below.
e) 50, -50

Given:	$\Delta U =$	50 m/s
	$\Delta V =$	-50 m/s
	$\Delta x =$	300 km
	$\Delta y =$	600 km

Find: $\zeta_r =$? /s

Use eq. 11.20:

$$\zeta_r = \frac{\Delta V}{\Delta x} - \frac{\Delta U}{\Delta y}$$

Convert $\Delta x(km)$ and $\Delta y(km)$ into $\Delta x(m)$ and $\Delta y(m)$:

$\Delta x\ (m) =$	300000 m
$\Delta y\ (m) =$	600000 m

$\zeta_r =$	-0.00025 /s
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Check: Units ok. Physics ok.

Discussion: Negative sign due to anticyclonic motion

A17e)
(4 marks)

If the relative vorticity is $5 \times 10^{-5}/s$, find the absolute vorticity at the following latitude: e) 70 deg.

Given: $\zeta_r = 5.00\text{E-}05 \text{ /s}$
 $\phi = 70 \text{ deg}$

Find: $\zeta_a = ? \text{ /s}$

Use eq. 11.23: $\zeta_a = \zeta_r + f_c$

where $f_c = 2 \cdot \Omega \cdot \sin \phi$:
 $2\Omega = 1.46\text{E-}04 \text{ /s}$

$f_c = 0.000137007 \text{ /s}$

$\zeta_a = 1.87\text{E-}04 \text{ /s}$

Check: Units ok. Physics ok.

Discussion: f_c increases with latitude and hence, absolute vorticity will be a maximum at the north pole. The 70th parallel passes through Greenland, and is in the Arctic Circle.

A18e)
 (3.5 marks)

If absolute vorticity is $5 \times 10^{-5} \text{ /s}$, find the potential vorticity ($\text{/m}^2\text{s}$) for a layer of thickness (km) of: e) 2.5

Given: $\zeta_a = 5.00\text{E-}05 \text{ /s}$
 $\Delta z = 2.5 \text{ km}$

Find: $\zeta_p = ? \text{ /m}^2\text{s}$

Use eq. 11.24: $\zeta_p = \frac{\zeta_r + f_c}{\Delta z} = \text{constant}$

where $\zeta_r + f_c = \zeta_a$ from eq. 11.23 or A17e.

Convert $\Delta z(\text{km})$ into $\Delta z(\text{m})$:
 $\Delta z (\text{m}) = 2500$

$\zeta_p = 2.00\text{E-}08 \text{ /m}^2\text{s}$

Check: Units ok. Physics ok.

Discussion: The potential vorticity is a useful definition in determining how a column of air would respond to stretching in order to conserve its potential vorticity in the absence of turbulent drag and heating. This reasoning is thought to influence storm development in the lee of the Rocky Mountains.

A19e)
(6 marks)

The potential vorticity is $1 \times 10^{-8} \text{ / (m}^2\text{s)}$ for a 10 km thick layer of air at latitude 48 degN. What is the change of relative vorticity (/s) if the thickness (km) of the rotating air changes to: e) 7.5?

Given: $\zeta_p = 1.00\text{E-}08 \text{ / (m}^2\text{s)}$
 $\Delta z_i = 10 \text{ km}$
 $\Delta z_f = 7.5 \text{ km}$
 $\phi = 48 \text{ deg}$

Find: $\Delta \zeta_r = ? \text{ /s}$

Use eq. 11.24:

$$\zeta_p = \frac{\zeta_r + f_c}{\Delta z} = \text{constant}$$

where $f_c = 2 \cdot \Omega \cdot \sin \phi$:

$$2\Omega = 1.46\text{E-}04 \text{ /s}$$

$$f_c = 1.08\text{E-}04 \text{ /s}$$

Convert $\Delta z_i(\text{km})$ and $\Delta z_f(\text{km})$ to $\Delta z_i(\text{m})$ and $\Delta z_f(\text{m})$:

$$\Delta z_i (\text{m}) = 10000$$

$$\Delta z_f (\text{m}) = 7500$$

$$\zeta_{ri} = -8.50\text{E-}06 \text{ /s}$$

Since we know ζ_p is constant, we can calculate new ζ_r with new Δz :

$$\zeta_{rf} = -3.35\text{E-}05 \text{ /s}$$

Therefore the change in relative vorticity is:

$$\Delta \zeta_r = \zeta_{rf} - \zeta_{ri} = -0.000025 \text{ /s}$$

Check: Units ok. Physics ok.
Discussion: For a fixed latitude, the planetary vorticity will not change. When the thickness decreases from 10km to 7.5km, the result is the generation of negative relative vorticity, causing the wind to spin faster in the clockwise direction (or slower in the counter-clockwise direction).

Chapter 14

A18a)
(2 marks)

Solution: See attached figure. Plotted in red
z: 0,1,2,3,4,5,6 (km)
a: (0,0),(120,5),(150,8),(180,12),(210,15),(240,25),(260,40)

Check: Curve looks reasonable, similar to Fig. 14.62c).
Discussion: The hodograph shows a view of the change in wind speed and direction with altitude. The sounding data given is showing an increase in the winds with height. These conditions favour multicell thunderstorms.

A23)
(3 marks)

Solution: See attached figure. Vector plotted in purple

Mean shear direction = imaginary line connecting point 0 to point 6

Mean shear magnitude (roughly)	5.8138645
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Check: Looks reasonable compared to textbook vectors
Discussion: The hodograph allows a very easy way to do the vector math to find the mean shear vector, even if the surface wind is not 0 m/s.

A28)

(3.5 marks)

Solution: See attached hodograph. X (&acceptable area) in black.

Method 1) Approximate by finding center of mass.

OR

Method 2) Vector sum method

dir (deg) 223.9047619
dir = 223.9 deg

speed (m/s) 15
speed = 15 m/s

Check: Looks to be near center of mass

Discussion: For a normal thunderstorm, under these environmental wind conditions, the general movement of the storm will move from the WSW at a speed of 15 m/s. This speed corresponds to roughly 54 km in one hour.

A30)

(6 marks)

Internal Dynamics method:

- 1) Approximate the 0.25 to 5.75 km layer shear vector using the 0 to 6 km mean shear vector
- 2) Draw line perpendicular to mean shear vector
- 3) R and L are long this line, 7.5 m/s from the center of mass

- 4) For right-moving supercell thunderstorms, estimated movement:

direction =	265 deg
speed =	12 m/s

For left-moving supercell thunderstorms, estimated movement:

direction =	212 deg
speed =	14 m/s

Given the hodograph shape, the right moving super cells would dominate.

(See Fig. 14.62)

Check: L and R points look similar to textbook hodographs

Discussion: Wind shear is only one of the main ingredients in the formation of a thunderstorm; others include the amount of available moisture, instability, and a trigger mechanism that will create uplift.

