ATSC 201 Fall 2023
Total marks out of 42
Chapter 15: A15e, A16e, A25e, A27e, A29e, A33e, A36e

Chapter 15

A15e)
(5.5 marks)

Given a lightning discharge current (kA) below and a voltage difference between the beginning to end of the lightning channel of 10^10V, find (1) the resistance of the ionized lightning channel and (2) the amount of charge (C) transferred between the cloud and the ground during the 20 micro second lifetime of the lightning stroke. e) 10

Given:	Δt =	20	µs
	V =	10000000000	V
	I =	10	kA
Convert	Δt =	2.00E-05	s
	I =	10000	A

1) Use eqn from "INFO: Electricity in a Channel"

$$V = I \cdot R$$

Rearrange to R = V / I

R = 1.00E+06 ohms

2) Use eqn from "INFO: Electricity in a Channel"

$$I = \Delta Q \ / \ \Delta t$$

Rearrange to ΔQ = I * Δt

ΔQ = 0.20 C

Check: Units ok. Physics ok.
Discussion: Lightning strikes are one of the most fatal weather hazards in North America. 0.20 C is a huge charge.

A16e)
(3.5 marks)

To create lightning in (1) dry air, and (2) cloudy air, what voltage difference is required, given a lightning stroke length (km) of: e) 1

Given: Δz = 1 km

Find:	ΔV_dry =	?	V
	ΔV_cloudy=	?	V

Use eqn. 15.16: $\Delta V_{lightning} = B \cdot \Delta z$

where	Bdry =	3.00E+09 V/km
	Bcloudy =	1.00E+09 V/km

ΔV_dry =	**3.00E+09 V**
ΔV_cloudy =	**1.00E+09 V**

Check: Units ok. Physics ok.
Discussion: Lightning can be deadly, regardless of the type of air
we are looking at. The longer the stroke, the more dangerous.

A25e)
(3 marks)

What is the minimum inaudibility distance for hearing thunder from a sound source 7km high in an environment of T = 20 deg C with no wind. Given a lapse rate (degC/km) of: e) 7.5

Given:	z =	7.00	km
	T =	20.00	deg C
	γ =	7.50	degC/km

Find:	xmax =	?	km

Use eqn. 15.38:

$$x_{\max} \cong 2 \cdot \sqrt{T \cdot z / \gamma}$$

xmax =	**8.64 km**

Check: Units ok. Physics ok.
Discussion: The position of T in the equation indicates
that warmer air propogates sound better.

A27e)

For a Rankine Combined Vortex model of a tornado, plot the pressure (kPa) and tangential wind speed (m/s) vs radial distance (m) out to 125m, for a tornado of core radius 25m and core pressure deficit (kPa) of: e) 0.5

(13 marks split into 7 for the answers, 5 for the plot, 1 for check+disc)

Given:	Ro =	25 m	
	ΔP (at core) =	0.5 kPa	500 Pa

Find: P (kPa) and Mtan (m/s) for R = 0 to 125 m

Inner region:

$$\frac{M_{\tan}}{M_{\tan\max}} = \frac{R}{R_o}$$

$$\frac{\Delta P}{\Delta P_{\max}} = 1 - \frac{1}{2}\left(\frac{R}{R_o}\right)^2$$

Outer region:

Outer Region ($R > R_o$):

$$\frac{M_{\tan}}{M_{\tan\max}} = \frac{R_o}{R} \qquad \bullet(15.42)$$

$$\frac{\Delta P}{\Delta P_{\max}} = \frac{1}{2}\left(\frac{R_o}{R}\right)^2 \qquad \bullet(15.43)$$

It is evident from eqns 15.40-15.43 that we need to find ΔPmax and Mtanmax.
First, we can see in eqn 15.41 that at the core

ΔP = ΔPmax = 500 Pa

We can then use eqn 15.44 to calculate Mtanmax

$$\Delta P_{\max} = \rho\cdot(M_{\tan\max})^2 \qquad \bullet(15.44)$$

where rho = 1 kg/m^3

Mtanmax = 22.36 m/s
(at R = Ro = 25m)

We can now calculate our Mtan values for our plot:
Use eqn. 15.40 for R < Ro:

Mtan = Mtanmax*(R/Ro)

Use eqn. 15. 42 for R > Ro:

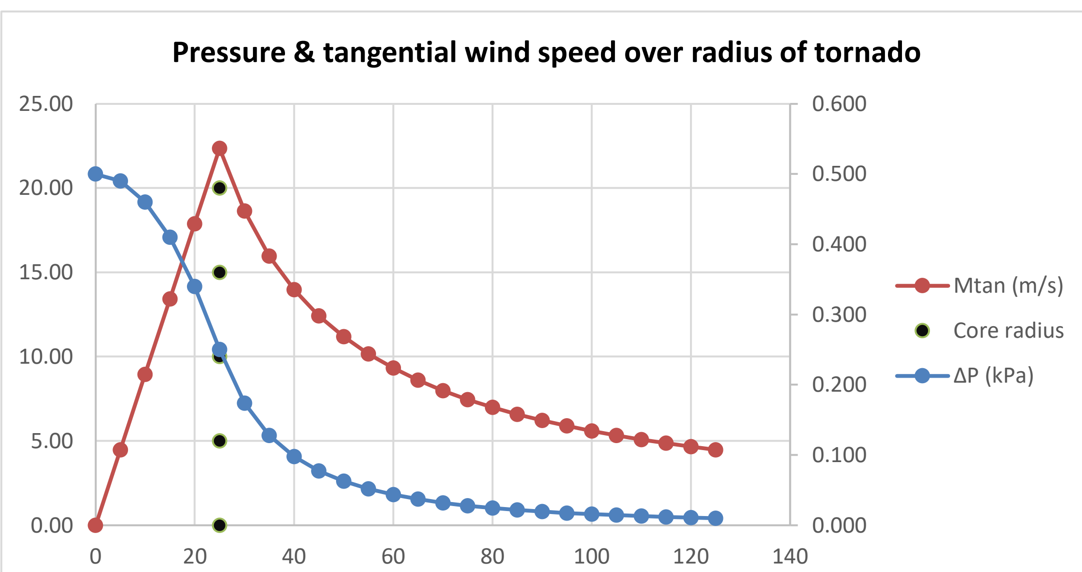
$$\frac{M_{\tan}}{M_{\tan\max}} = \frac{R_o}{R}$$

And our ΔP values:
Use eqn. 15.41 for R<Ro:

ΔP = ΔPmax*(1 - 0.5*(R/Ro)^2)

Use eqn. 15.43 for R>Ro: (see above screenshot)

	R (m)	Mtan (m/s)	ΔP (Pa)	ΔP (kPa)	
Core region	0	**0.00**	500	**0.500**	25
	5	**4.47**	490	**0.490**	25
	10	**8.94**	460	**0.460**	25
	15	**13.42**	410	**0.410**	25
	20	**17.89**	340	**0.340**	25
	25	**22.36**	250	**0.250**	25
Outer region	30	**18.63**	173.61	**0.174**	
	35	**15.97**	127.55	**0.128**	
	40	**13.98**	97.66	**0.098**	
	45	**12.42**	77.16	**0.077**	
	50	**11.18**	62.50	**0.063**	
	55	**10.16**	51.65	**0.052**	
	60	**9.32**	43.40	**0.043**	
	65	**8.60**	36.98	**0.037**	
	70	**7.99**	31.89	**0.032**	
	75	**7.45**	27.78	**0.028**	
	80	**6.99**	24.41	**0.024**	
	85	**6.58**	21.63	**0.022**	
	90	**6.21**	19.29	**0.019**	
	95	**5.88**	17.31	**0.017**	
	100	**5.59**	15.63	**0.016**	
	105	**5.32**	14.17	**0.014**	
	110	**5.08**	12.91	**0.013**	
	115	**4.86**	11.81	**0.012**	
	120	**4.66**	10.85	**0.011**	
	125	**4.47**	10.00	**0.010**	



Check: Units ok. Physics ok. Looks like Fig. 15.33

Discussion: Those core winds are not very strong compared to a tornado with a larger core pressure deficit.

A29e)
(2.5 marks)

What are the Enhanced Fujita and TORRO intensity indices for a tornado of max wind speed (m/s) of: e) 60

Given: Mmax = 60 m/s

Find: EF and TORRO ratings.

Use Tables 15-3 and 15-4:

A tornado with max wind speed of 60 m/s would be rated an EF2.
A tornado with max wind speed of 60 m/s would be rated a T4.

Discussion: A tornado of EF2 and T4 can be expected to produce considerabledamage. Roofs could be ruined, walls could collapse, windows would break, and cars and trucks could be blown over. For Torro scale T4, cars can be lifted off the ground.

A33e)
(4 marks)

A mesocyclone at 38N is in an environment where the vertical stretching ($\Delta W/\Delta z$) is (20 m/s)/(2km). Find the rate of voticity spin-up due to stretching only, given an initial relative voriticity (/s) of: e) 0.0010.

Given:	ΔW =	20 m/s		
	Δz =	2 km	2000 m	
	ϕ =	38 deg		
	ζ_r =	0.001 /s		

Find: $\Delta\zeta_r/\Delta t$ (/s^2) due to stretching only.

Use stretching portion of eqn 15.51:

$$\Delta\zeta_r/\Delta t = (\zeta_r + f_c)*(\Delta W/\Delta z)$$

where $f_c = 2*\Omega*\sin\phi$ Ω = 7.29E-05 /s

f_c = 8.98E-05 /s

$\Delta\zeta_r/\Delta t$ =	**1.09E-05 /s^2**

Check: Units ok. Physics ok.

Discussion: Stretching is only part of the ingredient for tornadic rotation. Stretching results in an increase in the amount of spin in the atmosphere.

A36e)
(11 marks)

Given the hodograph of winds in Fig.15.40a. Assume W=0 everywhere. Calculate helicity H based on the wind-vectors for the following pairs of heights (km): e) 4,5

Given:	W =	0	m/s		
	zf =	5	km	5000	m
	zi =	4	km	4000	m

Find: H = ? m/s^2

W = 0 everywhere simplifies eqn 15.52 to 15.53:

$$H = -U_{avg}*(\Delta V/\Delta z) + V_{avg}*(\Delta U/\Delta z)$$

where $U_{avg} = 0.5*(U_f + U_i)$ and $V_{avg} = 0.5*(V_f + V_i)$

From Fig. 15.40a:

Mi (@z=3km) =	15	m/s
Mf (@z=4km) =	20	m/s
αi (@z=3km)=	190	deg
αf (@z=4km)=	220	deg

From eqns. 1.3 and 1.4:

$$U = -M*\sin(\alpha)$$

$V = -M*\cos(\alpha)$

Ui =	2.60	m/s
Vi =	14.77	m/s
Uf =	12.86	m/s
Vf =	15.32	m/s
Uavg =	7.73	m/s
Vavg =	15.05	m/s
ΔV = Vf - Vi =	0.55	m/s
ΔU = Uf - Ui =	10.25	m/s
Δz = zf - zi =	1000	m
H =	**0.15**	**m/s^2**

Check: Units ok. Physics ok.

Discussion: Since there are no vertical winds, this value represents the streamwise-vorticity contribution to the total helicity.