ATSC 201 Fall 2023
Chapter 4: A1e, A4e, A13e, A17e, A25e, E17
Chapter 15: A1e, A3e
Total marks out of 48.5

Chapter 4

A1e) (5 marks)

Compare the saturation vapor pressures (with respect to liquid water) calculated with the Clausius-Clapeyron equation and with Tetens' formula, for T(°C): e) 25

Given:	T =	25 °C		298.15 K

Use eqn 4.1a (Clausius Clapeyron eqn)

$$e_s = e_o \cdot \exp\left[\frac{L}{\Re_v} \cdot \left(\frac{1}{T_o} - \frac{1}{T}\right)\right]$$

where Rv =	461	J/(K*kg)
To =	273.15	K
eo =	0.611	kPa
L = Lv (for liquid water) =	2.50E+06	J/kg

Use eqn 4.2 (Teten' formula)

$$e_s = e_o \cdot \exp\left[\frac{b \cdot (T - T_1)}{T - T_2}\right]$$

where b =	17.2694	
eo =	0.611	kPa
T1 =	273.15	K
T2 =	35.86	K

Clausius-Clapeyron:

es = 3.229 kPa

Teten's:

es = 3.169 kPa

Clausius-Clapeyron's saturation vapor pressure is slightly larger than Teten's.

Check: Units ok. Physics ok.

Discussion: The Clausius-Clapeyron gives a saturation vapor pressure slightly larger to Teten's formula for a temperature of 25°C. This is consistent with the graph on p.89 from the sample application. Teten's formula accounts for L varying slightly, but it appears the difference is usually negligible.

A4e)
(18 marks)

Calculate the values of es (kPa), r (g/kg), q (g/kg), ρv (g/m^3), RH (%), Td (°C), LCL (km), Tw (°C), rs(g/kg), qs (g/kg), and ρvs (g/m^3), given the following atmospheric state: e) P = 80kPa, T = 5°C, e = 0.5kPa.

Given:	P =	80 kPa		
	T =	5 degC	278.15 K	
	e =	0.5 kPa		

Find:	es =	?	kPa
	r =	?	g/kg
	q =	?	g/kg
	ρv =	?	g/m^3
	RH =	?	%
	Td =	?	degC
	LCL =	?	km
	Tw =	?	degC
	rs =	?	g/kg
	qs =	?	g/kg
	ρvs =	?	g/m^3

Use eqn. 4.1a:

$$e_s = e_o \cdot \exp\left[\frac{L}{\Re_v} \cdot \left(\frac{1}{T_o} - \frac{1}{T}\right)\right]$$

where eo = 0.611 kPa
To = 273.15 K
Lv/Rv = 5423 K

es =	**0.87 kPa**

Use eqn 4.4: r = ε*e/ (P - e)
where ε = 622 g/kg   0.622 g/g

r =	**3.91 g/kg**

HW 7 Answer Key

Use eqn. 4.7: q = ε*e/P

q =	3.89	g/kg

Use eqn. 4.10: ρv = e/(Rv * T)
where Rv = 4.61E-04 kPa*m^3/(K*g)

ρv =	3.90	g/m^3

Use eqn. 4.14a: RH =(e/ es) * 100

RH =	57.27	%

Use eqn. 4.15a:

$$T_d = \left[\frac{1}{T_o} - \frac{\Re_v}{L} \cdot \ln\left(\frac{e}{e_o} \right) \right]^{-1}$$

where eo =	0.611	kPa
To =	273	K
Rv/Lv =	1.84E-04	/K
Td =	270.27	K
Td =	**-2.88**	**degC**

Use eqn. 4.16a: LCL = a*(T-Td)
where a = 0.125 km/degC

LCL =	0.98	km

To find wet bulb temperature using Normand's Rule:
Use eqn 4.21: Tw = T_LcL + Γs*LCL

Find Γs with eqn 4.37a:

$$\Gamma_s = \frac{|g|}{C_p} \cdot \frac{\left(1 + \frac{r_s \cdot L_v}{\Re_d \cdot T}\right)}{\left(1 + \frac{L_v{}^2 \cdot r_s \cdot \varepsilon}{C_p \cdot \Re_d \cdot T^2}\right)}$$

Use eqn. 3.3: cp = cpd*(1+1.84*r)

where cpd =	1004	J/(kg*K)
r (g/g) =	0.00391	
cp =	1011.2268	J/(kg*K)

g/cp =	9.69	K/km	(I calculated some intermediate
Lv/cp =	2472.24	K	values to make the huge eqn nicer
Lv/Rd =	8707.77	K	to input)

Γs = 5.38 K/km

Find T_LcL with eq 4.20: T_LcL = T - Γd*LCL
where Γd = 9.8 K/km

T_LcL = 268.50 K

Tw = 273.80 K

Tw =	**0.65 degC**

There are other methods to get here, too

Use eqn 4.5: rs = (ε*es)/(P-es)

rs =	**6.86 g/kg**

rs = 0.0068628 g/g

Use eqn. 4.8: qs = (ε*es)/P

qs =	**6.79 g/kg**

Use eqn. 4.12: ρvs = es/(Rv*T)

ρvs =	**6.81 g/m^3**

Check: Units ok. Physics ok.
Discussion:
Because the air at this level is unsaturated, any liquid droplets in the air will tend to evaporate. Imagine leaving a wet towel in a room; it will eventually dry up due to evaporation. This evaporation causes cooling in the wet towel. The wet-bulb temperature represents the temperature of the wet towel after the evaporative cooling. Therefore, you would always expect the wet-bulb temperature to be cooler than the dry-bulb temperature in unsaturated air.

A13e) (3.5 marks)

For air at sea level, find the total water mixing ratio for a situation where:
e) T=8°C, rL = 2 g/kg.

Given:	r =		8 g/kg
	rL =		2 g/kg
Find:	rT =	?	g/kg

Use eqn 4.32a:

rT = rs + rL + ri

Need rs and ri:
Assume ri = 0 g/kg for 8deg air at sea level.

rT =	**10 g/kg**

Check: Units ok. Physics ok.

Discussion: Since liquid water exists in our parcel, we can assume that the air is saturated, and that the amount of water vapor in the air is the maximum amount of water vapor that the air can hold (as determined by its temperature).

A17e)
(3 marks)

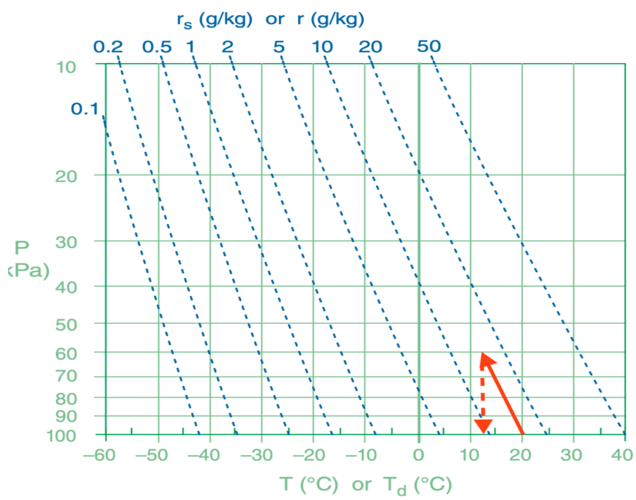
Given an air parcel starting at 100kPa with dewpoint (degC) given below, use Fig. 4.7 to find the parcel's final dewpoint (degC) if it rises to a height where P = 60kPa. e) 20

Given:

Pi =	100	kPa
Tdi =	20	degC
Pf =	60	kPa

Find:
Tdf = ?

See fig.



Tdf =	**13 degC**

Check: Units ok. Physics ok.

Discussion: The dew point temperature decreases with height because air expands and cools adiabatically - just like with regular temperature.

A25e)
(3 marks)

Given an air parcel that starts at a height where P = 100kPa with T = 25degC and r = 12 g/kg (ie. it is initially unsaturated). After rising to its final height it has an rL (g/kg) value listed below. Assuming no precipitation falls out, find the value for r (g/kg) for this now-saturated air parcel. e) 2.5

Given:	P_initial =	100	kPa
	T_initial =	25	degC
	r_initial =	12	g/kg
	rL_final =	2.5	g/kg
	rL_initial =	0	g/kg
Find:	r_final = ?		g/kg

Use eqn 4.35b:

$$(r + r_L)_{initial} = (r + r_L)_{final} \qquad \bullet(4.35b)$$

Re-arrange: r_final = r_initial + rL_initial - rL_final

r_final = 9.5 g/kg

Check: Units ok. Physics ok.

Discussion: In this situation, we are assuming that the water vapor condenses into liquid form, and the liquid droplets are suspended in the air. This way, we can assume that the total mixing ratio remains constant, as water is not lost through precipitation.

E17)
(8 marks)

Create a thermo diagram using a spreadsheet to calculate isohumes (for r = 1, 3, 7, 10, 30 g/kg) and dry adiabats (for θ = -30, -10, 10, 30 °C)**, all plotted on the same graph vs P on an inverted log-scale similar to Figs. 3.3 and 4.7.**

Find: Plot isohumes on a thermo diagram.
Note, only isohumes required as stated in the assignment page

$$T = \left[\frac{1}{\quad} - \frac{\Re_v}{\quad} \cdot \ln\left(\frac{r \cdot P}{\quad} \right)\right]^{-1}$$

HW 7 Answer Key

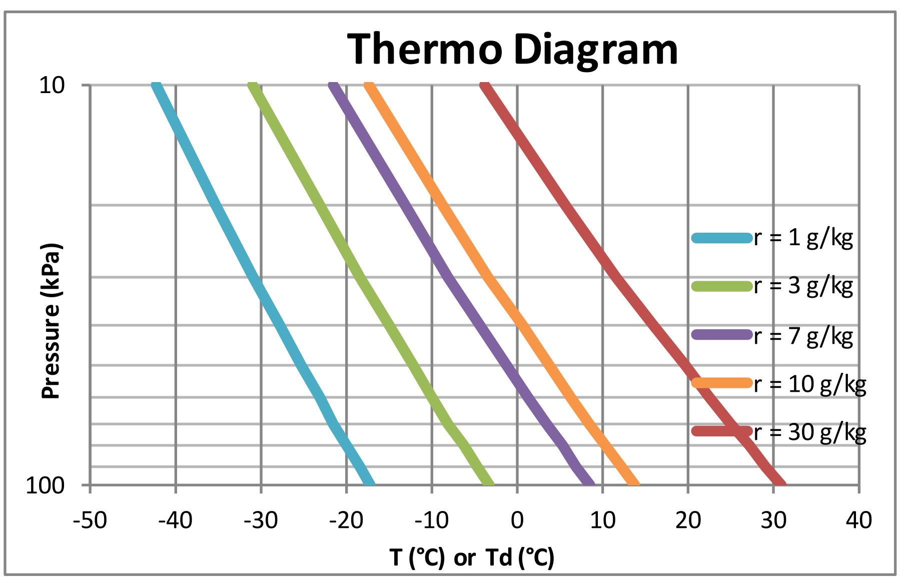
Use: $$T_d = \left[\frac{\;}{T_o} - \frac{\;}{L}\ln\left(\frac{\;}{e_o\cdot(r+\varepsilon)}\right)\right]$$ qn 4.15b

Or: $$T = \left[\frac{1}{T_o} - \frac{\Re_v}{L_v}\cdot\ln\left\{\frac{r_s\cdot P}{e_o\cdot(r_s+\varepsilon)}\right\}\right]^{-1}$$ eqn 4.36

eo =	0.611	kPa
Rv/Lv =	1.84E-04	1/K
To =	273	K
ε =	0.622	g/g

P (kPa)	T for r=1	T for r=3	T for r=7	T for r=10	T for r=30
10	-42.27	-30.99	-21.56	-17.39	-3.84
20	-35.26	-23.27	-13.21	-8.75	5.75
30	-30.96	-18.52	-8.06	-3.43	11.68
40	-27.81	-15.03	-4.28	0.49	16.05
50	-25.31	-12.27	-1.28	3.60	19.52
60	-23.23	-9.96	1.23	6.19	22.43
70	-21.44	-7.98	3.38	8.43	24.93
80	-19.88	-6.24	5.27	10.39	27.13
90	-18.48	-4.68	6.97	12.15	29.10
100	-17.21	-3.28	8.50	13.73	30.89

r (g/g)
0.001
0.003
0.007
0.01
0.03



Check: Units ok. Physics ok. Looks like fig 4.7.

Discussion: When we are given the air temperature and the dew point temperature of a parcel, we can use the thermodynamic diagram to find its lifting condensation level by following the dry adiabat from the temperature point, and the isohume from the dew-point temperature point.

Chapter 15

A1e)
(5 marks)

If a thunderstorm cell rains for 0.5 h at the precipitation rate (mm/hr) below, calculate both the net latent heat released into the atmosphere, and the average warming rate within the troposphere. e) 150

Given:	Δt =	0.5	hr	1800	s
	RR =	150	mm/hr		

Find: HRR = ? J/(m^2 * s)

Use eqn 15.2:

$$H_{RR} = a \cdot RR \qquad \bullet(15.2)$$

where

a = 694 (J/s*m^2)/(mm/hr)

HRR = 104100.00 J/(s*m^2)

Net latent heat released = HRR * Δt = 1.87E+08 J/m^2

Use eqn 15.3:

$$\Delta T/\Delta t = b \cdot RR \qquad (15.3)$$

where

b = 0.33 K/(mm of rain)

ΔT/Δt = 49.5 K/hr

Check: Units ok. Physics ok.

Discussion: As raindrops form in the troposphere in a thunderstorm, they release latent heat due to condensation. The amount or rain received at the surface is the amount of rain that did not evaporate and cool the atmosphere; thus, we can use this rain rate to estimate the amount of latent heat released and the warming caused by it. Note, however, that the temperature increase calculated is as if the air in the thunderstorm is trapped in the air column. In reality, there would be mixing with the environment, and so we would see a smaller effective temperature increase.

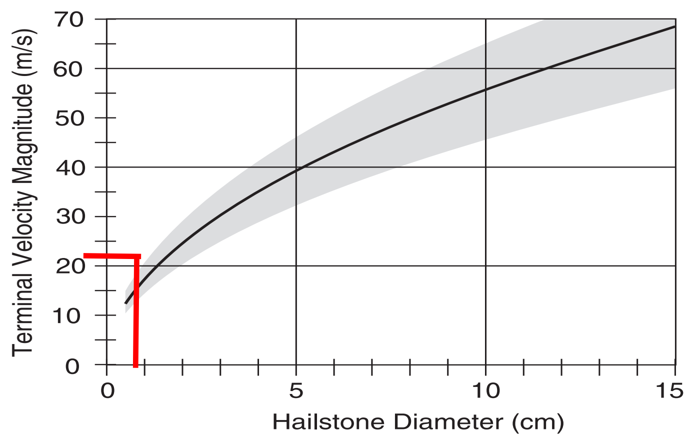
Graphically estimate the terminal fall velocity of hail of diameter (cm): e) 1.7

A3e)

(3 marks) Given: hail diameter = 1.7 cm

Find: Terminal fall velocity of hailstone.

Using Fig. 15.4



Approximate terminal fall velocity of hailstone:	**22 m/s**

accept values between 20 and 25 m/s

Check: Physics ok.

Discussion: A hailstone of diameter 1.7cm belongs to size code 2 in the TORRO hailstone size classification.