HW 13 Answer Key

ATSC 201 Fall 2024

Chapter 13: A3f, A7f, A14f, A16f

Chapter 16: A3f, A5f, A6f, A8f, A13f, A14f(x).

Total marks out of 44

Chapter 13

A3f)
(4 marks)

Given a tropospheric depth of 12 km at latitude 45°N, what is the meridional (north-south) amplitude (km) of upper-atmosphere (Rossby) waves triggered by mountains, given an average mountainrange height (km) of: f) Δz_mtn = 1.4km

Given:	Δz_mtn =	1.4	km
	Δz_T =	12	km
	ϕ =	45	deg
Find:	A =	?	km

Use eqn 13.3: A=fc*Δz_mtn/(β*Δz_T)

where fc/β = Rearth*tan(ϕ) from page 444 below eqn 13.3

Rearth = 6371 km

fc/β = 6371 km

A = 743.28 km

Check: Units ok. Physics ok.

Discussion: The higher the mountains, the shorter the column of air will be, and with this greater change, relative vorticity will need to decrease more to conserve potential vorticity. This decreasing relative vorticity is what initiates the Rossby wave.

A7f)
(7 marks)

When air at latitude 60°N flows over a mountain range of height 2 km within a troposphere of depth 12 km, find the radius of curvature (km) at location "C" in Fig. 13.20 given an average wind speed (m s–1) of: f) M=45m/s

Given:	ϕ =	60 °	1.04719755 radians
	Δz_mtn =	2 km	2000 m
	Δz_T =	12 km	12000 m
	M =	45 m/s	0.045 km/s

Find: Rc = ? km

Can assume that the crest after location c is at the same latitude as loc. a
Therefore can use equation 13.4 to find the amplitude of the Rossby wave.

A=Δz_mtn/Δz_T*R_earth

where R_earth (km) = 6371
(m) = 6371000

A = 1061.83 km 2A = 2123.66667 km
1.06E+06 m

Now that we have A, we can find the change in latitude (and therefore the change in fc) from point a to point c, i.e. over a north-south distance of 2A.

we know 111km = 1°lat 111
so 2A/111km = Δϕ = 19.1321321 ° Lat

so ϕ(c) = ϕ(a) - Δϕ
ϕ(c) = 40.87 °N 0.71327885 radians

use eqn 10.16

fc = 2*Ω*sin(ϕ)   where 2Ω =   1.46E-04 1/s

fc (at c) =   9.54E-05 1/s
fc (at a) =   1.26E-04 1/s

Use eqn 13.6 and 13.5

$$\zeta_p = \frac{(M/R) + f_c}{\Delta z} = \text{constant} \qquad \bullet(13.5)$$

$$\zeta_p = \frac{f_{c.a}}{\Delta z_{T.a}} \qquad (13.6)$$

Knowing that potential vorticity is conserved and that ΔzT(at a) = ΔzT(at c) we can rearrage these to be:

R (at c)= M/(fc(at a) - fc(at c))

R (at c) =	**1457862.35**	**m**
	1457.86	**km**

Check: Units ok. Physics ok.

Discussion: The radius of curvature at location c must be positive because it is turning **cyclonically** to keep potential vorticity constant.

A14f)
(3.5 marks)

At an altitude where the ambient pressure is 85 kPa, convert the following vertical velocities (m s–1) into omega (Pa s–1): f) 40 m/s

Given:	P =	85	kPa
	W =	40	m/s

Find: ω = ? Pa/s

Use eqn. 13.14:

$$\omega = -\rho \cdot |g| \cdot W \qquad (13.14)$$

|g| = 9.8 m/s^2

From Table 1- 5 estimate

ρ = 1.06 kg/m^3

ω =	**-415.52 Pa/s**

Check: Units ok. Physics ok.

Discussion: The term omega, by definition, is the change of pressure with time. So, when omega is negative, this means air is descending and that pressure is increasing as the air moves

A16f)
(3 marks)

> Find the vertical velocity (m s–1) at altitude 9 km in an 11 km thick troposphere, if the divergence (10–5 s–1) given below occurs within a 2 km thick layer within the top of the troposphere: f) D=4x10-5s-1

Given:	D =	4.00E-05 /s	
	Δz =	2 km	2000 m
Find:	Wmid =	? m/s	

Use eqn. 13.15:

$$W_{mid} = D \cdot \Delta z \qquad (13.15)$$

Wmid =	**0.08 m/s**

Check: Units ok. Physics ok.

Discussion: Due to the conservation of mass (continuity equation) if there is horizontal divergence/convergence, then there must be vertical motion to fill in the air that is leaving/entering horizontally.

Chapter 16

A3f)
(6 marks)

A3. Assume $\rho = 1$ kg m^{-3}, and latitude 20° Find the value of gradient wind (m s^{-1} and km h^{-1}) for:

Given:	flow around a low-pressure center			
	ρ =	1 kg/m^3		
	ϕ =	20 °		
	R	75 km	75000 m	
	ΔP/ΔR =	10 kPa/100km	0.1 Pa/m	

Find: Mtan = ? m/s or km/h

To find fc use eqn 10.16: fc = 2*Ω*sinϕ
where Ω = 7.29E-05 /s

fc = 4.99E-05 /s

Use eq. 16.3:

$$\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta R} = f_c \cdot M_{tan} + \frac{M_{tan}^2}{R}$$

if we re-arrange like this:

$$0 = f_c \cdot M_{tan} + \frac{M_{tan}^2}{R} - \frac{1}{\rho}\cdot\frac{\Delta P}{\Delta R}$$

we can see that this is a quadtratic equation in Mtan.

Use quadratic formula:
when ax^2 + bx + c = 0
then x = (-b ± √(b^2-4ac)) / 2a

a =	1/R =	1.3333E-05	1/m
b =	fc =	4.99E-05	1/s
c =	- 1/ρ * ΔP/ΔR =	-0.1	m/s^2

x1 =	84.75	m/s	
x2 =	-88.49	m/s	wind speed

Mtan =	**84.75**	**m/s**
	305.11	**km/h**

However, if a gradient wind is not possible for those conditions, explain why.

Ch.10 that

Check: Units ok. Physics ok.

Discussion: Plugging in Mtan in the gradient-wind equation (16.3) shows that the Centrifugal term is much larger than the Coriolis term in this case (approximately one order of magnitude).
Gradient winds are theoretical winds that follow curved isobars.

A5f)
(8.5 marks)

A5. Plot pressure vs. radial distance for the max pressure gradient that is admitted by gradient-wind theory at the top of a tropical cyclone for the latitudes (°) listed below. Use z = 17 km, P_0 = 8.8 kPa.

a. 5	b. 7	c. 9	d. 11	e. 13	f. 17	g. 19
h. 21	i. 23	j. 25	k. 27	m. 29	n. 31	o. 33

Given: flow around a high

ϕ =	17	°
z =	17	km
P0 =	8.8	kPa

Find: Plot P vs R

Use equation (a) from Higher Math box on page 619 (where R0=1/4):

P = P0 - a*R^2

where a = ρ * fc^2 / 8

find ρ using table 1-5:

ρ = 0.14 kg/m^3

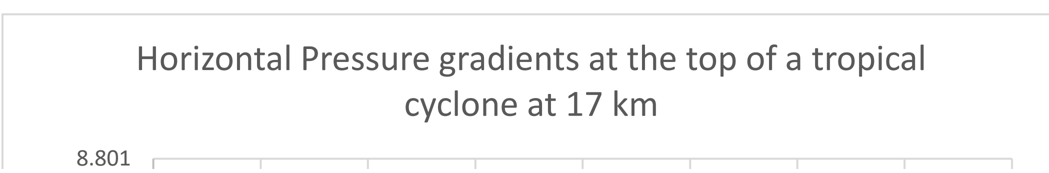
To find fc use eqn 10.16: fc = 2*Ω*sinϕ

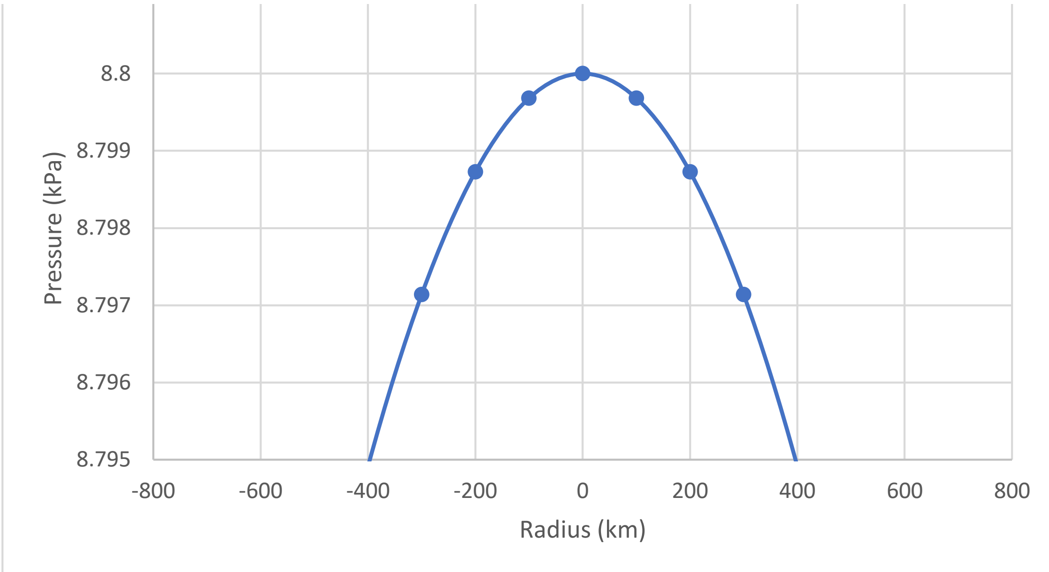
where Ω = 7.29E-05 /s

fc = 4.26E-05 /s

a =	**3.18E-11**	**kg/(m^3*s^2)**
	3.18E-08	**kPa/km^2**

R (km)	P (kPa)
-800	8.77964816
-700	8.78441812
-600	8.78855209
-500	8.79205006
-400	8.79491204
-300	8.79713802
-200	8.79872801
-100	8.799682
0	8.8
100	8.799682
200	8.79872801
300	8.79713802
400	8.79491204
500	8.79205006
600	8.78855209
700	8.78441812
800	8.77964816





Check: Units ok. Physics ok.

Discussion: While the tropical cyclone has low pressure and convergence at the surface, there exists high pressure with diverging anticyclonic rotation aloft.
The maximum allowed pressure gradient for gradient winds around the upper-level high is very small. This is in part due to the small Coriolis force at low latitudes. In reality the pressure gradient can be larger and winds cross the isobars.

A6f)
(2.5 marks)

A6. At sea level, the pressure in the eye is 93 kPa and that outside is 100 kPa. Find the corresponding pressure difference (kPa) at the top of the tropical cyclone, assuming that the core (averaged over the tropical cyclone depth) is warmer than surroundings by (°C):

e. 1 f. 7 g. 10 h. 15

Given:	P_B(eye) =	93	kPa
	P_B(∞)=	100	kPa
	ΔT =	7	°C

Find:	ΔP_T =	?	kPa

Use eqn. 16.5 $$\Delta P_T \approx a \cdot \Delta P_B - b \cdot \Delta T$$

where	ΔP_B =	7	kPa
	a ≈	0.15	
	b ≈	0.7	kPa / K

ΔP_T = -3.85 kPa

Check: Units ok. Physics ok.

Discussion:
This is an area of higher relative pressure than the surrounding environment

A8f)
(3 marks)

A8. Find the total entropy ($J \cdot kg^{-1} \cdot K^{-1}$) for:

Given:	P =	100	kPa		
	T =	30	°C	303	K
	r =	2	g/kg		

Find:	s =	?	J/(kg*K)

Use eqn 16.7 $$s = C_p \cdot \ln\left(\frac{T}{T_o}\right) + \frac{L_v \cdot r}{T} - \Re \cdot \ln\left(\frac{P}{P_o}\right)$$

where	Cp =	1004	J/(kg*K)
	Lv =	2500	J/g
	R =	287	J/(kg*K)
	T0 =	273	K
	P0 =	100	kPa

s = 121.18 J/(kg*K)

Check: Units ok. Physics ok.

Discussion: The gain or loss of entropy can be related to mechanical energy, which can drive tropical cyclone-force winds.

A13f)
(3 marks)

A13. Use P_∞ = 100 kPa at the surface. What maximum tangential velocity (m s^{-1} and km h^{-1}) is expected for an eye pressure (kPa) of:

a. 86	b. 88	c. 90	d. 92
e. 94	f. 96	g. 98	h. 100

Given: P_B(∞) = 100 kPa
P_B(eye)= 96 kPa

Find: Mmax = ? m/s or km/h

Use eqn. 16.12

$$M_{max} = a \cdot (\Delta P_{max})^{1/2}$$

where a = 20 m/s * kPa^(-1/2)

eqn 16.10

$$\Delta P_{max} = P_\infty - P_{eye}$$

ΔPmax = 4 kPa

Mmax = 40.00 m/s
144.00 km/h

Check: Units ok. Physics ok.

Discussion: According to the Saffir-Simpson Hurricane Wind Scale (Table 16-1) this would be a Category 5 Hurricane

A14x)
(3.5 marks)

> A14. For the previous problem, what are the peak velocity values (m s^{-1} and km h^{-1}) to the right and left of the storm track, if the tropical cyclone translates with speed (m s^{-1}):
> (i) 2 (ii) 4 (iii) 6 (iv) 8 (v) 10
> (vi) 12 (vii) 14 (viii) 16 (ix) 18 (x) 20

Given:	Mtan =	40.00	m/s
	Mt =	20	m/s
Find:	Mtot = ?		m/s or km/h

Assume Northern Hemisphere

Right quadrant of storm (relative to direction of movement):

Mtot = Mtan + Mt

Mtot_right =	**60.00**	**m/s**
	216.00	**km/h**

Left quadarant of storm:

Mtot = Mtan - Mt

Mtot_left =	**20.00**	**m/s**
	72.00	**km/h**

Check: Units ok. Physics ok.

Discussion: The translation speed is the movement of the center of the storm and it adds to the rotation speed of the storm. It causes surface wind speeds to be stronger on the right(left) side in the Northern(Southern) hemisphere. In this example there are category-4

Northern(Southern) hemisphere. In this example there are category 4 force wind speeds on the right side of the storm track, whereas the left side of the storm just experiences tropical storm force wind speeds (based on Saffir-Simpson Hurricane Wind scale).