ApP

A7g)
(7 marks)

> When air at latitude 60°N flows over a mountain
> range of height 2 km within a troposphere of depth
> 12 km, find the radius of curvature (km) at location "C" in Fig. 13.20
> given an average wind speed (m s–1)
> of: g) M=50m/s

Given: ϕ =   60 °   1.0471976 radians
           Δz_mtn =   2 km   2000 m
           Δz_T =   12 km   12000 m
           M =   50 m/s   0.05 km/s

Find: Rc =   ?   km

Can assume that the crest after location c is at the same latitude as loc. a
Therefore can use equation 13.4 to find the amplitude of the Rossby wave.

A=Δz_mtn/Δz_T*R_earth

where R_earth (km) =   6371
          (m) =   6371000

A =   1061.83 km   2A =   2123.6667 km
               1.06E+06 m

Now that we have A, we can find the change in latitude (and therefore the
change in fc) from point a to point c, i.e. over a north-south distance of 2A.

we know 111km = 1°lat   111
so 2A/111km = $\Delta\phi$ =   19.132132 ° Lat

so ϕ(c) = ϕ(a) - $\Delta\phi$
ϕ(c) =   40.87 °N   0.7132789 radians

use eqn 10.16
fc = 2*Ω*sin(ϕ)   where 2Ω =   1.46E-04 1/s

fc (at c) =   9.54E-05 1/s
fc (at a) =   1.26E-04 1/s

Use eqn 13.6 and 13.5

$$\zeta_p = \frac{(M/R) + f_c}{\Delta z} = \text{constant} \qquad \bullet(13.5)$$

$$\zeta_p = \frac{f_{c.a}}{\Delta z_{T.a}} \qquad (13.6)$$

Knowing that potential vorticity is conserved and that ΔzT(at a) = ΔzT(at c) we can rearrage these to be:

R (at c)= M/(fc(at a) - fc(at c))

                                                    1.62E+06

R (at c) =	1619847.06 m
	1619.85 km

Check: Units ok. Physics ok.

Discussion: The radius of curvature at location c must be positive because it is turning **cyclonically** to keep potential vorticity constant.

A14g)
(3.5 marks)

At an altitude where the ambient pressure is 85 kPa, convert the following vertical velocities (m s–1) into omega (Pa s–1): g) 50 m/s

Given: P =   85 kPa
            W =   50 m/s

Find: ω =   ?   Pa/s

Use eqn. 13.14:

$$\omega = -\rho \cdot |g| \cdot W \qquad \bullet(13.14)$$

|g| =   9.8 m/s^2

From Table 1- 5 estimate
ρ =   1.06 kg/m^3

ω =   -519.4 Pa/s

Check: Units ok. Physics ok.
Discussion: The term omega, by definition, is the change of pressure with time. So, when omega is negative, this means air is descending and that pressure is increasing as the air moves

A16g)
(3 marks)

> Find the vertical velocity (m s–1) at altitude 9 km in an 11 km thick troposphere, if the divergence (10–5 s–1) given below occurs within a 2 km thick layer within the top of the troposphere: g) D=5x10-5s-1

Given: D =   5.00E-05 /s

                Δz =   2 km   2000 m

Find: Wmid =   ?   m/s

Use eqn. 13.15:

$$W_{mid} = D \cdot \Delta z \qquad (13.15)$$

Wmid =	**0.10 m/s**

Check: Units ok. Physics ok.

Discussion: Due to the conservation of mass (continuity equation) if there is horizontal divergence/convergence, then there must be vertical motion to fill in the air that is leaving/entering horizontally.

Chapter 16

A3g)
(6 marks)

A3. Assume $\rho = 1$ kg m^{-3}, and latitude $20°$ Find the value of gradient wind (m s^{-1} and km h^{-1}) for:

Given: flow around a low-pressure center

$\rho =$	1 kg/m^3	
$\varphi =$	20 °	
R	50 km	50000 m
$\Delta P/\Delta R =$	20 kPa/100km	0.2 Pa/m

Find: Mtan =   ?   m/s or km/h

To find fc use eqn 10.16: fc = 2*Ω*sinφ
where Ω =   7.29E-05 /s

fc =   4.99E-05 /s

Use eq. 16.3:

$$\frac{1}{\rho} \cdot \frac{\Delta P}{\Delta R} = f_c \cdot M_{tan} + \frac{M_{tan}^2}{R}$$

if we re-arrange like this:

$$0 \quad = f_c \cdot M_{tan} + \frac{M_{tan}^2}{R} \quad - \quad \frac{1}{\rho} \cdot \frac{\Delta P}{\Delta R}$$

we can see that this is a quadtratic equation in Mtan.

Use quadratic formula:
when   ax^2 + bx + c = 0
then   x = (-b ± √(b^2-4ac)) / 2a

a =	1/R =	0.00002 1/m
b =	fc =	4.99E-05 1/s
c =	- 1/ρ * $\Delta P/\Delta R$ =	-0.2 m/s^2

x1 =	98.76 m/s	
x2 =	-101.25 m/s	Negative

Mtan =	98.76 m/s
	355.54 km/h

However, if a gradient wind is not possible for those conditions, explain why.

Recall from Ch.10 that solutions for the gradient wind around an anticyclone are physically realistic only for curvature Rossby numbers R0 ≤ 1/4. This question assumes flow around a low-pressure center (cyclone), for which stronger pressure gradients, smaller radii of curvature, and stronger winds are possible.

Check: Units ok. Physics ok.

Discussion: Plugging in Mtan in the gradient-wind equation (16.3) shows that the Centrifugal term is much larger than the Coriolis term in this case (approximately one order of magnitude).
Gradient winds are theoretical winds that follow curved isobars.

A5g)
(8.5 marks)

A5. Plot pressure vs. radial distance for the max pressure gradient that is admitted by gradient-wind theory at the top of a tropical cyclone for the latitudes (°) listed below. Use $z = 17$ km, $P_o = 8.8$ kPa.
  a. 5   b. 7   c. 9   d. 11   e. 13   f. 17   g. 19
  h. 21   i. 23   j. 25   k. 27   m. 29   n. 31   o. 33

Given: flow around a high
              ϕ =   19 °
              z =   17 km
              P0 =   8.8 kPa

Find: Plot P vs R

Use equation (a) from Higher Math box on page 619 (where R0=1/4):
              P = P0 - a*R^2

where   a = ρ * fc^2 / 8

find ρ using table 1-5:
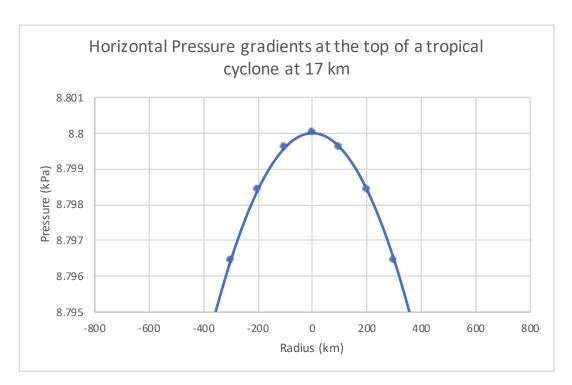 ρ =   0.14 kg/m^3

To find fc use eqn 10.16: fc = 2*Ω*sinϕ
where Ω =   7.29E-05 /s

fc =   4.75E-05 /s

a =	3.94E-11 kg/(m^3*s^2)
	3.94E-08 kPa/km^2

R (km)	P (kPa)
-800	8.77476421
-700	8.780678848
-600	8.785804868
-500	8.79014227
-400	8.793691052
-300	8.796451217
-200	8.798422763
-100	8.799605691
0	8.8
100	8.799605691
200	8.798422763
300	8.796451217
400	8.793691052
500	8.79014227
600	8.785804868
700	8.780678848
800	8.77476421

Horizontal Pressure gradients at the top of a tropical cyclone at 17 km

Check: Units ok. Physics ok.

Discussion: While the tropical cyclone has low pressure and convergence at the surface, there exists high pressure with diverging anticyclonic rotation aloft.

The maximum allowed pressure gradient for gradient winds around the upper-level high is very small. This is in part due to the small Coriolis force at low latitudes. In reality the pressure gradient can be larger and winds cross the isobars.

A6g)
(2.5 marks)

A6. At sea level, the pressure in the eye is 93 kPa and that outside is 100 kPa. Find the corresponding pressure difference (kPa) at the top of the tropical cyclone, assuming that the core (averaged over the tropical cyclone depth) is warmer than surroundings by (°C):

e. 1	f. 7	g. 10	h. 15

Given: P_B(eye) =   93 kPa
            P_B(∞)=   100 kPa
            ΔT =   10 °C

Find: ΔP_T =   ?   kPa

Use eqn. 16.5 $$\Delta P_T \approx a \cdot \Delta P_B - b \cdot \Delta T$$

where   ΔP_B =   7 kPa
            a ≈   0.15
            b ≈   0.7 kPa / K

ΔP_T =	-5.95 kPa

Check: Units ok. Physics ok.

Discussion:
This is an area of higher relative pressure than the surrounding environment

A8g)
(3 marks)

A8. Find the total entropy ($J \cdot kg^{-1} \cdot K^{-1}$) for:

Given: P =   90 kPa
            T =   30 °C   303 K
            r =   28 g/kg

Find: s =   ?   J/(kg*K)

Use eqn 16.7
$$s = C_p \cdot \ln\left(\frac{T}{T_o}\right) + \frac{L_v \cdot r}{T} - \Re \cdot \ln\left(\frac{P}{P_o}\right)$$

where   Cp =   1004 J/(kg*K)
            Lv =   2500 J/g
            R =   287 J/(kg*K)
            T0 =   273 K
            P0 =   100 kPa

s =	365.94 J/(kg*K)

Check: Units ok. Physics ok.

Discussion: The gain or loss of entropy can be related to mechanical energy, which can drive tropical cyclone-force winds.

A13g)
(3 marks)

A13. Use P_∞ = 100 kPa at the surface. What maximum tangential velocity (m s⁻¹ and km h⁻¹) is expected for an eye pressure (kPa) of:

a. 86	b. 88	c. 90	d. 92
e. 94	f. 96	g. 98	h. 100

Given: P_B(∞) =   100 kPa
        P_B(eye)=   98 kPa

Find: Mmax =   ?   m/s or km/h

Use eqn. 16.12

$$M_{max} = a \cdot (\Delta P_{max})^{1/2}$$

where   a =   20 m/s * kPa^(-1/2)

eqn 16.10

$$\Delta P_{max} = P_\infty - P_{eye}$$

         ΔPmax =   2 kPa

Mmax =	28.28 m/s
	101.82 km/h

Check: Units ok. Physics ok.

Discussion: According to the Saffir-Simpson Hurricane Wind Scale (Table 16-1) this would be a Category 5 Hurricane

A14vii)
(3.5 marks)

A14. For the previous problem, what are the peak velocity values (m s^{-1} and km h^{-1}) to the right and left of the storm track, if the tropical cyclone translates with speed (m s^{-1}):
  (i) 2   (ii) 4   (iii) 6   (iv) 8   (v) 10
  (vi) 12   (vii) 14   (viii) 16   (ix) 18   (x) 20

Given: Mtan =   28.28 m/s
              Mt =   14 m/s

Find: Mtot =   ?   m/s or km/h

Assume Northern Hemisphere

Right quadrant of storm (relative to direction of movement):
              Mtot =   Mtan + Mt

Mtot_right =	**42.28 m/s**
	152.22 km/h

Left quadarant of storm:
              Mtot =   Mtan - Mt

Mtot_left =	**14.28 m/s**
	51.42 km/h

Check: Units ok. Physics ok.

Discussion: The translation speed is the movement of the center of the storm and it adds to the rotation speed of the storm. It causes surface wind speeds to be stronger on the right(left) side in the Northern(Southern) hemisphere. In this example there are category-4 force wind speeds on the right side of the storm track, whereas the left side of the storm just experiences tropical storm force wind speeds