HW 4 Answer Key

ATSC 201 Fall 2025

Total mark out of 38

Chapter 10: **A12g, A14g**

Chapter 11: **A14g, A17g, A18g, A19g**

Chapter 14: **A18c, A23, A28, A30**

Chapter 10

A12g)

> **Given the pressure gradient magnitude (kPa/1000km) below, find geostrophic wind speed for a location having fc = 1.1x10^-4 /s and rho = 0.8kg/m^3.**
> **g) 7**

fc = 1.10E-04 /s

rho = 0.8 kg/m^3

delta_P / delta_d = 7 kPa/1000km

Find: G = ? m /s

Geostrophic wind

Using equation 10.28:

$$G = \left| \frac{1}{\rho \cdot f_c} \cdot \frac{\Delta P}{\Delta d} \right|$$

Convert ΔP / Δd from kPa/1000km to Pa/m:
The 'kilo' (x1000) on top and bottom can cancel each other out, so kPa / 1000km = Pa / 1000m. To get this into Pa / m, divide by 1000.

ΔP / Δd = 0.007 Pa / m

G = 79.55 m/s

Check: Units ok. Physics ok.

Discussion: Note that G is proportional to the PGF. When Δd (spacing between isobars) is smaller, PGF is larger and so is G.

A14g) **At the radius (km) given below from a low-pressure center, find the gradient wind speed given a geostrophic wind of 8 m/s and given fc = 1.1x10^-4 /s.**
g) 1200.

Given:	R =	1200	km
	G =	8	m/s
	fc =	1.10E-04	/s
Find:	Mtan =	?	m/s

Using eq. 10.34a:

$$M_{\tan} = 0.5 \cdot f_c \cdot R \cdot \left[-1 + \sqrt{1 + \frac{4 \cdot G}{f_c \cdot R}} \right]$$

Convert R(km) into R(m):

R (m) = 1200000

Mtan = 7.57 m/s

Check: Units ok. Physics ok.

Discussion: Gradient wind speed around a low is slower than the geostrophic wind because of the imbalance between the PGF and the Coriolis force caused by the curvature of the flow.

Chapter 11

A14g)

A14. Find the relative vorticity (s–1) for the change of (U , V) wind speed (m s–1), across distances of Δx = 300 km and Δy = 600 km respectively given below.
g) 20, 20

Given:	ΔU =	20	m/s
	ΔV =	20	m/s
	Δx =	300	km
	Δy =	600	km

Find: ζr = ? /s

Use eq. 11.20:

$$\zeta_r = \frac{\Delta V}{\Delta x} - \frac{\Delta U}{\Delta y}$$

Convert Δx(km) and Δy(km) into Δx(m) and Δy(m):

Δx (m) =	300000	m
Δy (m) =	600000	m

$$\boxed{\zeta r = 3.3333E\text{-}05 \ /s}$$

Check: Units ok. Physics ok.
Discussion: Positive sign due to cyclonic motion

A17g) **If the relative vorticity is 5x10^-5/s, find the absolute vorticity at the following latitude: g) 60 deg.**

Given:	ζr =	5.00E-05	/s
	ϕ =	60	deg
Find:	ζa =	?	/s

Use eq. 11.23:

$$\zeta_a = \zeta_r + f_c$$

where fc = 2*Ω*sinϕ:

2Ω =	1.46E-04	/s

fc = 0.000126267 /s

ζa = 1.76E-04 /s

Check: Units ok. Physics ok.

Discussion: Fc increases with latitude and hence, absolute vorticity will be a maximum at the north pole.

A18g) **If absolute vorticity is 5x10^-5 /s, find the potential vorticity (/m*s) for a layer of thickness (km) of: g) 3.5**

Given:	ζa =	5.00E-05	/s
	Δz =	3.5	km
Find:	ζp =	?	/(m*s)

Use eq. 11.24:

$$\zeta_p = \frac{\zeta_r + f_c}{\Delta z} = \text{constant}$$

where ζr + fc = ζa from eq. 11.23 or A17e.

Convert Δz(km) into Δz(m):

Δz (m) = 3500

ζp = 1.43E-08 /(m*s)

Check: Units ok. Physics ok.

Discussion: The potential vorticity is a useful definition in determining how a column of air would respond to stretching in order to conserve its potential vorticity in the absence of turbulent drag and heating. This reasoning is thought to influence storm development in the lee of the Rocky Mountains.

A19g) **The potential vorticity is 1x10^-8 /(m*s) for a 10 km thick layer of air at latitude 48 degN. What is the change of relative vorticity (/s) if the thickness (km) of the rotating air changes to: g) 6.5?**

Given:	ζp =	1.00E-08	/(m*s)
	Δzi =	10	km
	Δzf=	6.5	km
	ϕ =	48	deg

Find: $\Delta\zeta r$ = ? /s

Use eq. 11.24:

$$\zeta_p = \frac{\zeta_r + f_c}{\Delta z} = \text{constant}$$

where fc = $2*\Omega*\sin\phi$:

2Ω = 1.46E-04 /s

fc = 1.08E-04 /s

Convert Δzi(km) and Δzf(km) to Δzi(m) and Δzf(m):

Δzi (m) = 10000

Δzf (m) = 6500

ζri = -8.50E-06 /s

Since we know ζp is constant, we can calculate new ζr with new Δz:

ζrf = -4.35E-05 /s

Therefore the change in relative vorticity is:

$\Delta\zeta r$ = $\zeta rf - \zeta ri$ = -3.50E-05 /s

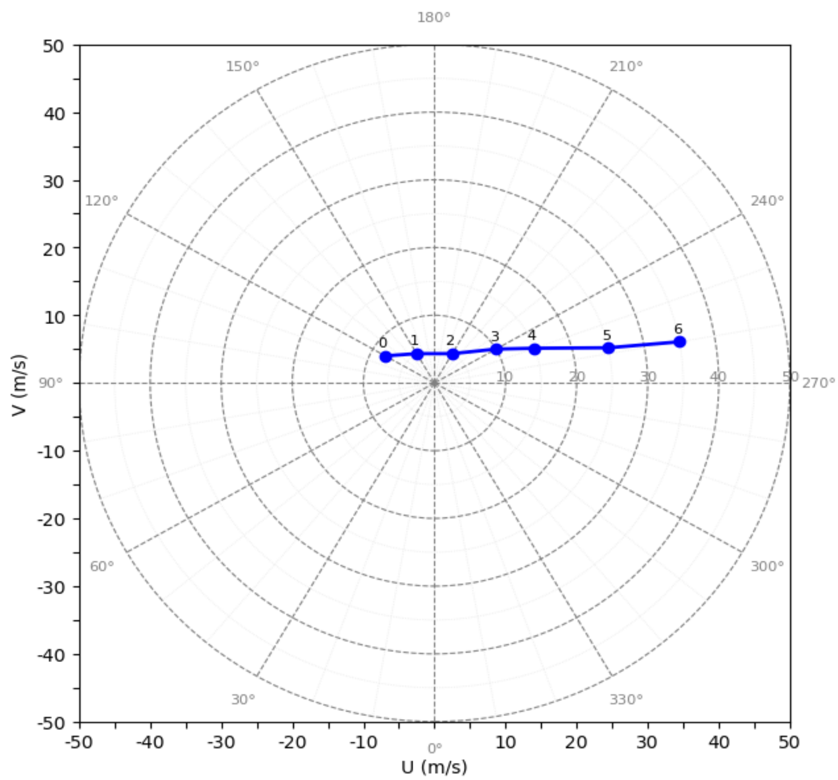
Check: Units ok. Physics ok.

Discussion: For a fixed latitude, the planetary vorticity will not change. When the thickness decreases from 10km to 6.5km, the result is the generation of negative relative vorticity, causing the wind to spin faster in the clockwise direction (or slower in the counter-clockwise direction).

Chapter 14

A18c)

Solution: See attached figure. Ch14_18c
z: 0,1,2,3,4,5,6 (km)
c. (120,8),(150,5),(210,5),(240,10),(250,15),(258,25),(260,35)

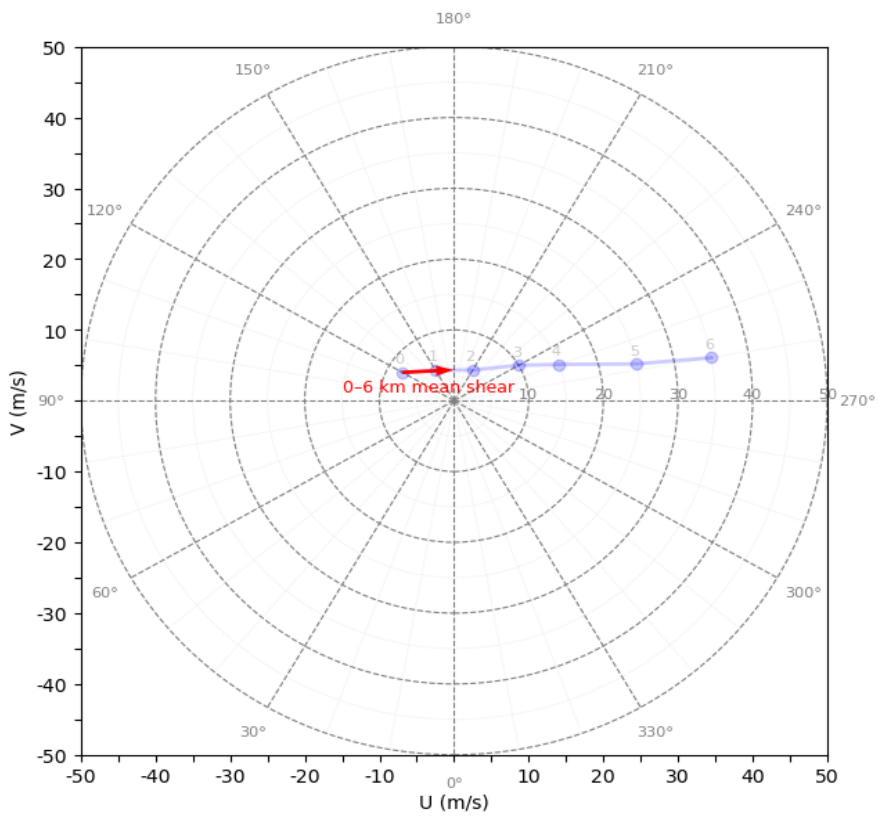


Check: Curve looks reasonable, similar to Fig. 14.62a).
Discussion: The hodograph shows how wind speed and direction change with altitude.

A23)

Graphically, using your hodograph plot from exercise A18, plot the 0 to 6 km mean shear vector (m/s).

Solution:



Mean shear direction = imaginary line connecting point 0 to point 6	
Mean shear magnitude (roughly)	6.9

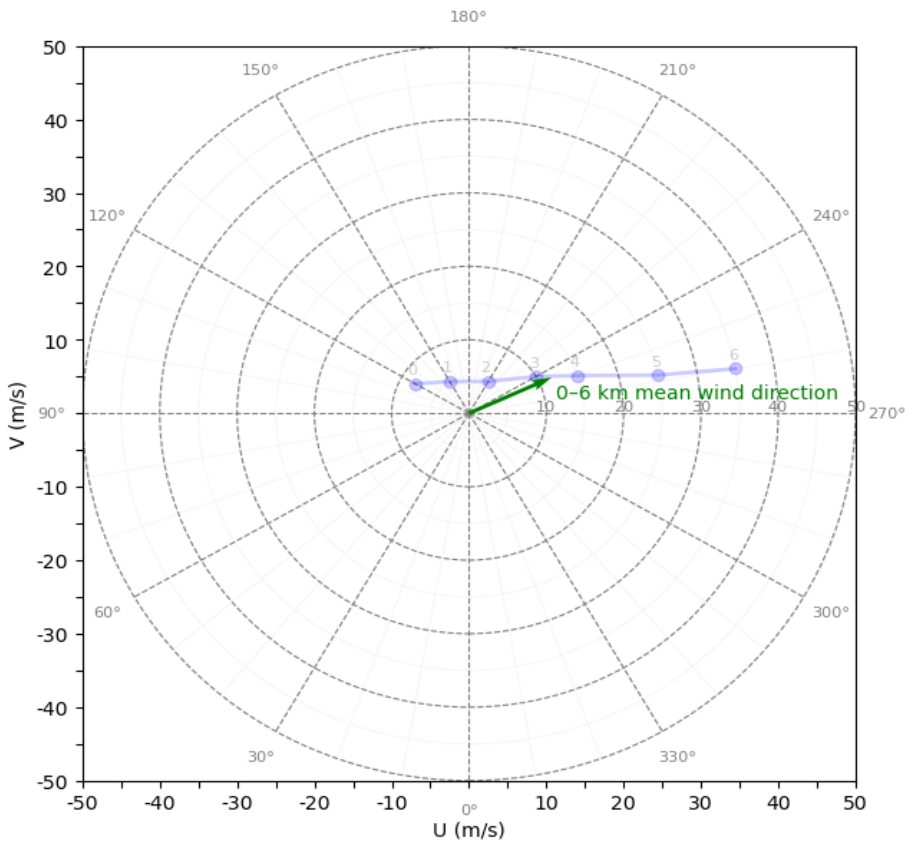
Check: Looks reasonable compared to textbook vectors

Discussion: The hodograph allows a very easy way to do the vector math to find the mean shear vector, even if the surface wind is not 0 m/s.

A28)

Graphically, using your hodograph plot from exercise A18, find the mean environmental wind direction (deg) and speed (m/s), for normal storm motion.

Solution:



Method 1) Approximate by finding center of mass.
OR
Method 2) Vector sum method

U=	10.68	m/s	V=	4.87 m/s
dir (deg)	245.487369			
dir =	**245.5**	deg		
speed (m/s)	11.7379427			
speed =	**11.7 m/s**			

Check: Looks to be near center of mass

Discussion: For a normal thunderstorm, under these environmental wind conditions, the general movement of the storm will move from the SW at a speed of 11.7 m/s. This speed corresponds to roughly 42 km/h.

A30)

Given the hodograph shape from exercise A18, indicate whether right or left-moving supercells would dominate. Also, starting with the "normal storm motion" from exercise A28 (based on hodograph from A18), use Internal Dynamics method on your hodograph to graphically estimate the movement (i.e. direction and speed) of Right-and Left-moving supercell thunderstorms.

<u>Internal Dynamics method:</u>

1) Approximate the 0.25 to 5.75 km layer shear vector using the 0 to 6 km mean shear vector
2) Draw line perpendicular to mean shear vector
3) R and L are long this line, 7.5 m/s from the center of mass
4) For right-moving supercell thunderstorms, estimated movement:

direction =	**283.4**	**deg**
speed =	**11.4**	**m/s**

For left-moving supercell thunderstorms, estimated movement:

direction =	**219.8**	**deg**
speed =	**16.1**	**m/s**

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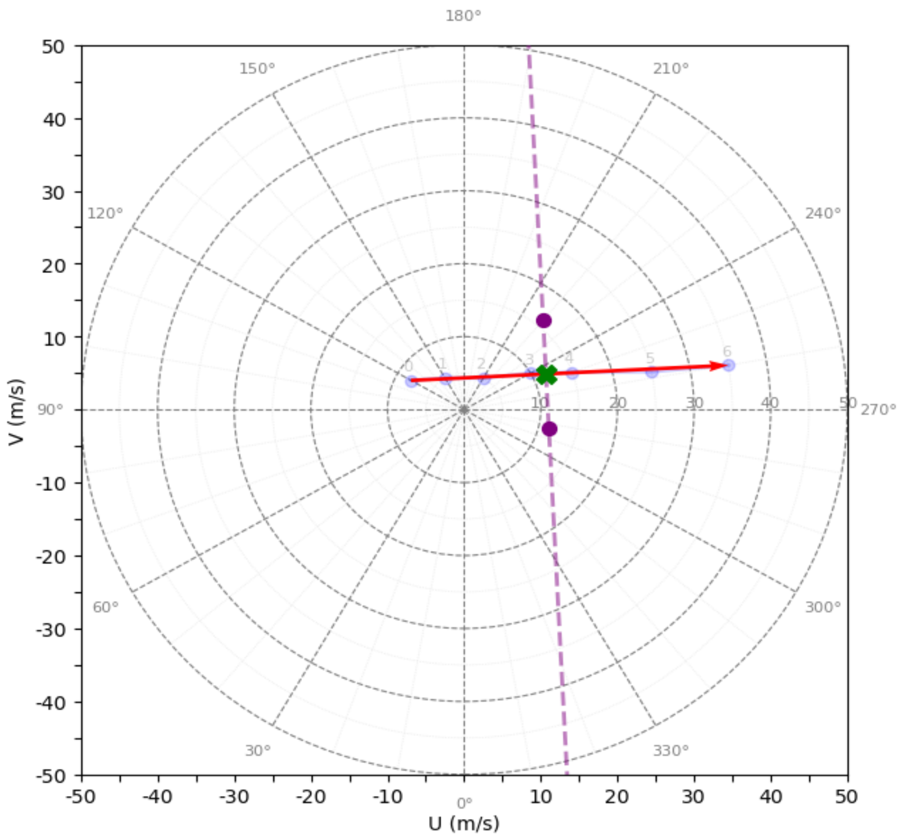

Because the hodograph is relatively straight, both are expected to exist with roughly equal strength

(See Fig. 14.62)

Check: L and R points look similar to textbook hodographs

Discussion: Wind shear is only one of the main ingredients in the formation of a thunderstorm; others include the amount of available moisture, instability, and a trigger mechanism that will create uplift.