

ATSC 201 Fall 2025

Chapter 4: A1g, A4g, A13g, A17g, A25g, E17

Total marks out of 40.5

Chapter 4

A1g)

(5 marks)

Compare the saturation vapor pressures (with respect to liquid water) calculated with the Clausius-Clapeyron equation and with Tetens' formula, for T(°C): g) 15

Given: T = 15 °C 288.15 K

Use eqn 4.1a (Clausius Clapeyron eqn)

$$e_s = e_o \cdot \exp \left[\frac{L}{R_v} \cdot \left(\frac{1}{T_o} - \frac{1}{T} \right) \right]$$

where $R_v = 461 \text{ J/(K*kg)}$
 $T_o = 273.15 \text{ K}$
 $e_o = 0.611 \text{ kPa}$
 $L = L_v \text{ (for liquid water) } = 2.50\text{E}+06 \text{ J/kg}$

Use eqn 4.2 (Teten' formula)

$$e_s = e_o \cdot \exp \left[\frac{b \cdot (T - T_1)}{T - T_2} \right]$$

where $b = 17.2694$
 $e_o = 0.611 \text{ kPa}$
 $T_1 = 273.15 \text{ K}$
 $T_2 = 35.86 \text{ K}$

Clausius-Clapeyron:

$e_s = 1.717 \text{ kPa}$

Teten's:

$e_s = 1.706 \text{ kPa}$

Clausius-Clapeyron's saturation vapor pressure is slightly larger than Teten's.

HW 7 Answer Key

Check: Units ok. Physics ok.

0.5 for check 0.5 for discussion

Discussion: The Clausius-Clapeyron gives a saturation vapor pressure slightly larger to Tetten's formula for a temperature of 15°C . This is consistent with the graph on p.89 from the sample application. Tetten's formula accounts for L varying slightly, but it appears the difference is usually negligible.

HW 7 Answer Key

A4g)

(18 marks)

Calculate the values of e_s (kPa), r (g/kg), q (g/kg), ρ_v (g/m³), RH (%), T_d (°C), LCL (km), T_w (°C), r_s (g/kg), q_s (g/kg), and ρ_{vs} (g/m³), given the following atmospheric state: $P = 80$ kPa, $T = 15^\circ\text{C}$, $e = 1$ kPa.

Given:

$P =$	80 kPa	
$T =$	15 degC	288.15 K
$e =$	1 kPa	

Find:

$e_s =$	?	kPa
$r =$	?	g/kg
$q =$	?	g/kg
$\rho_v =$	?	g/m ³
RH =	?	%
$T_d =$	?	degC
LCL =	?	km
$T_w =$	?	degC
$r_s =$	?	g/kg
$q_s =$	?	g/kg
$\rho_{vs} =$	?	g/m ³

Use eqn. 4.1a:

$$e_s = e_o \cdot \exp \left[\frac{L}{R_v} \cdot \left(\frac{1}{T_o} - \frac{1}{T} \right) \right]$$

where $e_o =$ 0.611 kPa
 $T_o =$ 273.15 K
 $L/R_v =$ 5423 K

$e_s =$ 1.72 kPa

Use eqn 4.4: $r = \epsilon \cdot e / (P - e)$

where $\epsilon =$ 622 g/kg 0.622 g/g

$r =$ 7.87 g/kg

Use eqn. 4.7: $q = \epsilon \cdot e / P$

$q =$ 7.78 g/kg

Use eqn. 4.10: $\rho_v = e / (R_v \cdot T)$

where $R_v =$ 4.61E-04 kPa*m³/(K*g)

HW 7 Answer Key

$$\rho_v = 7.53 \text{ g/m}^3$$

Use eqn. 4.14a: $RH = (e / e_s) * 100$

$$RH = 58.23 \%$$

Use eqn. 4.15a:

$$T_d = \left[\frac{1}{T_o} - \frac{R_v}{L} \cdot \ln \left(\frac{e}{e_o} \right) \right]^{-1}$$

where $e_o = 0.611 \text{ kPa}$

$T_o = 273 \text{ K}$

$R_v/L_v = 1.84E-04 \text{ /K}$

$T_d = 279.94 \text{ K}$

$$T_d = 6.79 \text{ degC}$$

Use eqn. 4.16a: $LCL = a * (T - T_d)$

where $a = 0.125 \text{ km/degC}$

$$LCL = 1.03 \text{ km}$$

To find wet bulb temperature using Normand's Rule:

Use eqn 4.21: $T_w = T_{LCL} + \Gamma_s * LCL$

Find Γ_s with eqn 4.37a:

$$\Gamma_s = \frac{|g|}{C_p} \cdot \frac{\left(1 + \frac{r_s \cdot L_v}{R_d \cdot T} \right)}{\left(1 + \frac{L_v^2 \cdot r_s \cdot \epsilon}{C_p \cdot R_d \cdot T^2} \right)}$$

Use eqn. 3.3: $cp = cp_d * (1 + 1.84 * r)$

where $cp_d = 1004 \text{ J/(kg*K)}$

$r \text{ (g/g)} = 0.00787$

$cp = 1018.545 \text{ J/(kg*K)}$

$g/cp = 9.62 \text{ K/km}$

$Lv/cp = 2454.48 \text{ K}$

$Lv/Rd = 8707.77 \text{ K}$

(intermediate values to make the huge eqn look nicer to input)

$\Gamma_s = 4.27 \text{ K/km}$

Find T_{LCL} with eq 4.20: $T_{LCL} = T - \Gamma_d * LCL$

where $\Gamma_d = 9.8 \text{ K/km}$

HW 7 Answer Key

$$T_{LCL} = 278.10 \text{ K}$$

$$T_w = 282.47 \text{ K}$$

$$T_w = 9.32 \text{ degC}$$

Use eqn 4.5: $r_s = (\epsilon^* e_s) / (P - e_s)$

$$r_s = 13.65 \text{ g/kg}$$

$$r_s = 0.0136462 \text{ g/g}$$

Use eqn. 4.8: $q_s = (\epsilon^* e_s) / P$

$$q_s = 13.35 \text{ g/kg}$$

Use eqn. 4.12: $p_{vs} = e_s / (R_v^* T)$

$$p_{vs} = 12.93 \text{ g/m}^3$$

Check: Units ok. Physics ok.

Discussion:

Because the air at this level is unsaturated, any liquid droplets in the air will tend to evaporate. Imagine leaving a wet towel in a room; it will eventually dry up due to evaporation. This evaporation causes cooling in the wet towel. The wet-bulb temperature represents the temperature of the wet towel after the evaporative cooling. Therefore, you would always expect the wet-bulb temperature to be cooler than the dry-bulb temperature in unsaturated air.

HW 7 Answer Key

A13g)

(3.5 marks)

For air at sea level, find the total water mixing ratio for a situation where:
g) $T=7^{\circ}\text{C}$, $r_L = 4 \text{ g/kg}$.

Given: $T = 7^{\circ}\text{C}$
 $r_L = 4 \text{ g/kg}$

Find: $r_T = ? \text{ g/kg}$

Use eqn 4.32a:

$$r_T = r_s + r_L + r_i$$

Need r_s and r_i :

Assume $r_i = 0 \text{ g/kg}$ for 12deg air at sea level.

Can estimate r_s from table 4-1:

$r_s = 6.3 \text{ g/kg}$

Or can calculate using eq 4.5 in Table 4-2a:

$$r_s = \epsilon * e_s / (P - e_s)$$

Using Table 4-1 to estimate e_s :

$e_s = 1 \text{ kPa}$

$P = 101.325 \text{ kPa}$

Assume $\epsilon = 622 \text{ g/kg}$

$r_s = 6.1998505 \text{ g/kg}$

$r_T = 10.19985 \text{ g/kg}$

Check: Units ok. Physics ok.

Discussion: Since liquid water exists in our parcel, we can assume that the air is saturated, and that the amount of water vapor in the air is the maximum amount of water vapor that the air can hold (as determined by its temperature).

HW 7 Answer Key

A17g)
(3 marks)

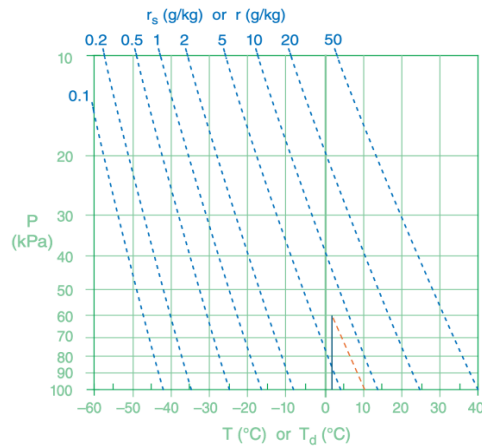
Given an air parcel starting at 100kPa with dewpoint (degC) given below, use Fig. 4.7 to find the parcel's final dewpoint (degC) if it rises to a height where $P = 60\text{kPa}$. g) 10

Given:

$P_i = 100 \text{ kPa}$
 $T_{di} = 10 \text{ degC}$
 $P_f = 60 \text{ kPa}$

Find:

$T_{df} = ? \text{ degC}$



See attached Fig. 4.7.

$T_{df} = 2 \text{ degC}$

Check: Units ok. Physics ok.

Discussion: The dew point temperature decreases with height because air expands and cools adiabatically - just like with regular temperature.

HW 7 Answer Key

A25g)
(3 marks)

Given an air parcel that starts at a height where $P = 100\text{kPa}$ with $T = 25\text{degC}$ and $r = 12\text{ g/kg}$ (ie. it is initially unsaturated). After rising to its final height it has an $r_L\text{ (g/kg)}$ value listed below. Assuming no precipitation falls out, find the value for $r\text{ (g/kg)}$ for this now-saturated air parcel. g) 3.5

Given: $P_{\text{initial}} =$ 100 kPa
 $T_{\text{initial}} =$ 25 degC
 $r_{\text{initial}} =$ 12 g/kg
 $r_{L_{\text{final}}} =$ 3.5 g/kg
 $r_{L_{\text{initial}}} =$ 0 g/kg

Find: $r_{\text{final}} =$? g/kg

Use eqn 4.35b:

$$(r + r_L)_{\text{initial}} = (r + r_L)_{\text{final}} \quad \bullet(4.35b)$$

Re-arrange: $r_{\text{final}} = r_{\text{initial}} + r_{L_{\text{initial}}} - r_{L_{\text{final}}}$

$r_{\text{final}} =$ 8.5 g/kg

Check: Units ok. Physics ok.

Discussion: In this situation, we are assuming that the water vapor condenses into liquid form, and the liquid droplets are suspended in the air. This way, we can assume that the total mixing ratio remains constant, as water is not lost through precipitation.

HW 7 Answer Key

E17)

(8 marks)

Create a thermo diagram using a spreadsheet to calculate isohumes (for $r = 1, 3, 7, 10, 30$ g/kg) and dry adiabats (for $\theta = -30, -10, 10, 30$ °C), all plotted on the same graph vs P on an inverted log-scale similar to Figs. 3.3 and 4.7.

Find: Plot isohumes on a thermo diagram.

Use:
$$T_d = \left[\frac{1}{T_o} - \frac{\mathfrak{R}_v}{L} \ln \left(\frac{r \cdot P}{e_o \cdot (r + \epsilon)} \right) \right]^{-1} \quad \text{eqn 4.15b}$$

Or:
$$T = \left[\frac{1}{T_o} - \frac{\mathfrak{R}_v}{L_v} \cdot \ln \left\{ \frac{r_s \cdot P}{e_o \cdot (r_s + \epsilon)} \right\} \right]^{-1} \quad \text{eqn 4.36}$$

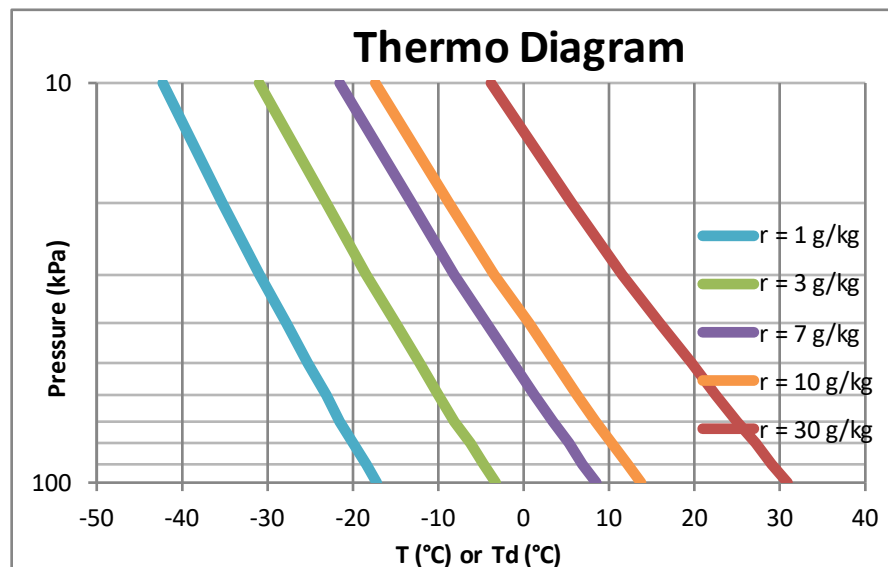
HW 7 Answer Key

$e_o =$ 0.611 kPa
 $R_v/L_v =$ 1.84E-04 1/K
 $T_o =$ 273 K
 $\epsilon =$ 0.622 g/g

P (kPa)	T for r=1	T for r=3	T for r=7	T for r=10	T for r=30
10	-42.27	-30.99	-21.56	-17.39	-3.84
20	-35.26	-23.27	-13.21	-8.75	5.75
30	-30.96	-18.52	-8.06	-3.43	11.68
40	-27.81	-15.03	-4.28	0.49	16.05
50	-25.31	-12.27	-1.28	3.60	19.52
60	-23.23	-9.96	1.23	6.19	22.43
70	-21.44	-7.98	3.38	8.43	24.93
80	-19.88	-6.24	5.27	10.39	27.13
90	-18.48	-4.68	6.97	12.15	29.10
100	-17.21	-3.28	8.50	13.73	30.89

r (g/g)

0.001
 0.003
 0.007
 0.01
 0.03



Check: Units ok. Physics ok. Looks like fig 4.7.

Discussion: When we are given the air temperature and the dew point temperature of a parcel, we can use the thermodynamic diagram to find its lifting condensation level by following the dry adiabat from the temperature point, and the isohume from the dew-point temperature point.