Atmospheric Boundary Layers, Turbulence & Dispersion

(A Review for the BC Ministry of Environment)

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Topics:
1. Boundary Layer Basics
2. Static Stability
3. BL Evolution
4. Turbulence Generation
5. Turb. Kinetic Energy
6. TKE & Dispersion
7. Pile-burn Case Study
8. Clarifications of Your Issues

If requested, if time:
9. Dispersion in the Convective ML
Survey Results

1) PBL expertise: 1 high, 7 medium, 0 low
2) Turbulence: 1 high, 5 medium, 2 low
3) Plume dispersion: 2 high, 6 medium, 0 low
4) Thermo diagram: 2 tephi, 3 skew-T, 4 not use
5) Potential temperature: 2 high, 4 medium, 2 low
6) Virtual temperature: 1 high, 2 medium, 5 low
7) AERMOD, 3 Hysplit, 7 CALPUFF, 1 CMAQ, 1 GEM-MACH, 1 CAMx
8) Complex terrain: 6 high, 1 medium, 1 low
Survey Results - Clarification Desired

- PBL formation and breakdown
- winter PBLs
- PBL in valleys. Also valley clouds vs. inversions
- PBL vs smoke dispersion
- dispersion prediction days in advance
- using soundings to anticipate dispersion
- how to understand Weinstein’s Air Qual. Advisory fcsts.
- surface roughness
- venting: roles of shear and buoyancy
- how fine a resolution in NWP is numerically stable
- using weather forecasts to anticipate episodes of poor air quality
1. Atmospheric Boundary Layer (ABL)

Some Definitions:

**ABL** = the portion of the atmosphere that feels the effects of the Earth’s surface, on a timescale of roughly an hour ——————————>

**PBL** = planetary boundary layer = ABL

**ML** = mixed layer (or mixing layer), when the ABL is so strongly mixed by turbulence that conserved properties (humidity, pollutant concentration, potential temperature) are roughly uniform with height.
Evolution of Thoughts about the Atmospheric Boundary Layer (ABL)

(a) No height limit. Doesn’t apply in general to the whole ABL, but works for initial growth of atmos. internal boundary layer (IBLs).

(b) Based on theoretical Ekman spiral, assuming molecular diffusion on a rotating planet. Doesn’t work in real atmos. (not even for neutral ABLs)

(c) Based on thermodynamics.

where:

- \( z \) = height
- \( h \) = ABL height
- \( x \) = downwind distance
- \( M \) = wind speed
- \( u^* \) = friction velocity (a measure of surface stress)
- \( f_c \) = Coriolis parameter (1/s) (related to Earth’s rotation)
- \( \theta \) = potential temperature
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- $\theta$ = potential temperature
- Turbulence Creates the ABL.
- The ABL Traps Turbulence. Thus: feedback.

\[ \theta = T + \Gamma z = \text{potential temperature}, \]
where \( \Gamma = 9.8 \, ^\circ\text{C}/\text{km} = \text{dry adiabatic lapse rate} \]
There is **always** a capping inversion at the top of the ABL. Let $z_i$ be the inversion height.
2. Static Stability

FLOW STABILITY

Stability is a characteristic of how a system reacts to small disturbances. If the disturbance is damped, the system is said to be stable. If the disturbance causes an amplifying response (irregular motions or regular oscillations), the system is unstable.

For fluid-flow stability we will focus on turbulent responses spanning the smallest eddies to deep thunderstorms. The stability characteristics are:

- **Unstable** air becomes, or is, turbulent (irregular, gusty, stormy).
- **Stable** air becomes, or is, laminar (non-turbulent, smooth, non-stormy).
- **Neutral** air has no tendency to change (disturbances neither amplify or dampen).

Flow stability is controlled by ALL the processes (buoyancy, inertia, wind shear, rotation, etc.) acting on the flow. However, to simplify our understanding of flow, we sometimes focus on just a subset of processes. If you ignore all processes except buoyancy, then you are studying static stability. If you include buoyancy and wind-shear processes, then you are studying dynamic stability.
2. Static Stability

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---

Little or no pollutant dispersion if the flow is not turbulent.

- Vigorous pollutant dispersion
- No pollutant dispersion
- Modest pollutant dispersion

<- use thermo diagram to determine static stability
Thermo Diagrams

(a) EMAGRAM
(b) STÜVE or pseudoadiabatic
(c) SKEW-T LOG-P
(d) TEPHIGRAM
For remote viewers you can get these diagrams by:

1) Do a Google Search on “Practical Meteorology Stull”
2) Click on the “Thermo diagrams” link next to Chapter 5.
3) Select whichever thermo diagram you are comfortable with.
4) Also, you might want to download: Skew-T-atmos.boundary.layer
MORE ON THE SKEW-T

Because of the popularity of the Skew-T, we dissect it in Fig. 5.4, to aid in its interpretation. Tephigram users should find this equally useful, because the only noticeable difference is that the isobars are slightly curved, and increase in slope higher in the tephigram.

The Skew-T diagram is trivial to create, once you have already created an Eogram. The easiest way is to start with a graphic image of an Eogram, and skew it into a parallelogram (Fig. 5.5) using a graphics or drawing program.
Skew-T Log-P Diagram (ABL)

Zoomed into the bottom 3 km of the atmosphere.

Perfect for ABL static-stability analysis.

Free downloads: search on “Practical Meteorology Stull”. See end of Chapter 5.
Next: step by step instructions on how to determine static stability from a sounding (using a contrived example)
Step 1
find kinks in the sounding (including top and bottom end points)

Figure 5.14a
Static stability determination. Step 1: Plot the sounding.
Step 1
find kinks in the sounding (including top and bottom end points)

Figure 5.14a
Static stability determination. Step 1: Plot the sounding.
Step 2

from each kink, conceptually lift an air parcel following a dry adiabat.

If buoyancy force is in the same direction as your lifting, then label with “U” for unstable.

If buoyancy would return it to its starting point, then label as “S” for stable.

Otherwise, “N” for neutral.

**Figure 5.14b**

Static stability determination. Step 2: Upward displacements.
Step 3
from each kink, conceptually lower an air parcel following a dry adiabat.

If buoyancy force is in the same direction as your initial motion, then label with “U” for unstable.

If buoyancy would return it to its starting point, then label as “S” for stable.

Otherwise, “N” for neutral.
Step 4

for all the motions labeled with “U”, continue moving the partial dry adiabatically until it hits the sounding or the ground.

The superposition of these “U” domains identifies all the Unstable layers in the atmosphere.

Unstable always wins.

Figure 5.14c & d
Static stability determination. (c) Step 3: Downward displacement...
Step 4

Unstable always wins.

The superposition of these “U” domains identifies all the Unstable layers in the atmos.

Figure 5.14c & d
Static stability determination. (c) Step 3: Downward displace-
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Unstable always wins.

Step 4 for all the motions labeled with "U", continue moving the partial dry adiabatically until it hits the sounding or the ground.

The superposition of these “U” domains identifies all the Unstable layers in the atmos.

Remember: Unstable regions cause rapid dispersion.

Unstable always wins.

Figure 5.14c & d
Static stability determination. (c) Step 3: Downward displace-
Step 5
all remaining portions of “S” regions that are not unstable are STABLE,
and all remaining portions of “N” regions that are not unstable are NEUTRAL.
Q: The approx. static stability of the shaded layer is

A) stable
B) neutral
C) conditionally unstable
D) unstable
E) not enough info
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A) stable
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D) unstable  
E) not enough info
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Scenario: **nighttime** over a forest. Calm winds. Clear skies.

Work in teams to plot this sounding, & identify the altitudes (top and bottom) of stable, neutral, and unstable layers.

IR radiation upward from top of forest

Sounding

<table>
<thead>
<tr>
<th>P (kPa)</th>
<th>T (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>8</td>
</tr>
<tr>
<td>84</td>
<td>6</td>
</tr>
<tr>
<td>90</td>
<td>11</td>
</tr>
<tr>
<td>94</td>
<td>13</td>
</tr>
<tr>
<td>98</td>
<td>13</td>
</tr>
<tr>
<td>100</td>
<td>17</td>
</tr>
</tbody>
</table>
3. Boundary Layer Evolution

(a) Day (3 PM)
- Free atmosphere
- Entrainment zone
- Mixed layer

(b) Night (3 AM)
- Free atmosphere
- Capping inversion
- Residual layer
- Stable boundary layer
Boundary Layer Evolution

additional variables
Boundary Layer Evolution - Summer, Fair Wx

(a) Summer

Free Atmosphere (FA)
Cap. Inversion
Entr. Zone (EZ)
CI
Mixed Layer (ML)
Residual Layer (RL)
SBL

(b) Night

06 12 18
03 06 18
04 08 12 16 20 24

Summer: (a) Day

Z (km)
θ

0 1
0 1

0 6 12 15
0 9 18

0 6
Boundary Layer Evolution - Winter, Fair Wx
Boundary Layer Evolution - Inversions & Clouds

- Free Atmosphere
- Capping Inversion
- Mixed Layer
- Residual Layer
- Stable BL

Height, z

Day 1
- E.Z.
- Mixed Layer
- Stable BL

Night 1
- Capping Inversion
- Residual Layer

Day 2
Boundary Layer Evolution - Inversions & Clouds

Phase 1
Nocturnal Inversion Burn-off

Phase 2
Rapid Rise

Phase 3
Quasi-steady

Height, $z$

- LCL
- $z_i$

~1 km

sunrise mid morning mid afternoon
Boundary Layer Evolution - Synoptic Variations

poor air quality

Venting by thunderstorms

Venting by fronts
Take a 5 minute “stretch” break
4. Turbulence Generation in ABL

three mechanisms:

1) **Buoyancy** (warm air rising or cold air sinking as thermals) => thermal generation of turbulence in UNSTABLE ABL.
Turbulence Generation in ABL

three mechanisms:

2. **Wind shear** (change of wind speed or direction with $z$) => mechanical generation of turb. in neutral & stable ABL.

**boundary shear**
- at the ground

**free shear**
- across a temperature inversion

$M$ is wind speed

$Z$ is altitude

20 m

2 km

$T$ is temperature

inversion layer
2. **Wind shear** (change of wind speed or direction with \( z \)) => mechanical generation of turb. in neutral & stable ABL.

**Turbulence Generation in ABL**

three mechanisms:

- **boundary shear** at the ground
  - \( M \) is wind speed
  - 20 m

- **free shear** across a temperature inversion
  - 2 km
  - KH wave cloud
  - May Wong 2016
3) **Obstacle wakes** (behind trees, bldgs, mountains, cars) => mechanical generation of turbulence
Buoyantly generated turbulence behaves in a special way. Thus, pollutant dispersion also behaves in a special way in convective mixed layers.
Turbulence = gustiness

Figure 1.1
Local Cartesian coordinates and velocity components.

\[ u'(t) = U(t) - \bar{U} \]
\[ v'(t) = V(t) - \bar{V} \]
\[ w'(t) = W(t) - \bar{W} \]
\[ T'(t) = T(t) - \bar{T} \]

Figure 18.23
The instantaneous wind speed \( U \) shown by the zigzag line. The average wind speed \( \bar{U} \) is shown by the thin horizontal dashed line. A gust velocity \( u' \) is the instantaneous deviation of the instantaneous wind from the average.
Variance and Standard Deviation

The variance $\sigma^2$ of vertical velocity is an overall statistic of gustiness:

$$\sigma_w^2 = \frac{1}{N} \sum_{k=1}^{N} (W_k - \bar{W})^2$$

$$= \frac{1}{N} \sum_{k=1}^{N} (w_k')^2$$

$$= w'^2 \quad \text{(18.22)}$$

Similar definitions can be made for $\sigma_u^2$, $\sigma_v^2$, $\sigma_\theta^2$, etc.

Standard Deviation:

$$\sigma_w = \sqrt{\sigma_w^2} = (w')^{1/2}$$
Isotropy

If turbulence has nearly the same variance in all three directions, then turbulence is said to be isotropic. Namely:

\[ \sigma_u^2 = \sigma_v^2 = \sigma_w^2 \]

(18.27)

Anisotropic (not isotropic) examples:

- \( \sigma_w > \sigma_u \)
- \( \sigma_u > \sigma_w \)
Why do we care about sigma-velocity?

**Dispersion Near & Far from the Source**

Close to the source (at small times after the start of dispersion), eq. (19.13a) reduces to

\[ \sigma_y \approx \sigma_v \cdot \left( \frac{x}{M} \right) \]  \hspace{1cm} (19.14)

while far from the source it can be approximated by:

\[ \sigma_y \approx \sigma_v \cdot \left( 2 \cdot t_L \cdot \frac{x}{M} \right)^{1/2} \]  \hspace{1cm} (19.15)

There are similar equations for \( \sigma_z \) as a function of \( \sigma_w \).

Greater \( \sigma_w \) causes faster spread \( \sigma_z \).

Greater \( \sigma_v \) causes faster spread \( \sigma_y \).

Greater \( \sigma_u \) causes faster spread \( \sigma_x \).
Anisotropy and Dispersion

Statically: stable neutral unstable

Isotropy:
- anisotropic
- isotropic

Behavior:
- fanning: $\sigma_Z < \sigma_Y$
- coning: $\sigma_Z = \sigma_Y$
- looping: $\sigma_Z > \sigma_Y$

Std Deviations:
- $\sigma_W < \sigma_V$
- $\sigma_W = \sigma_V$
- $\sigma_W > \sigma_V$
5. Turbulence Kinetic Energy

An overall measure of the intensity of turbulence is the **turbulence kinetic energy** per unit mass \((TKE)\):

\[
TKE = 0.5 \cdot \left[ (u')^2 + (v')^2 + (w')^2 \right] \quad \text{(18.28a)}
\]

\[
TKE = 0.5 \cdot \left[ \sigma_u^2 + \sigma_v^2 + \sigma_w^2 \right] \quad \text{(18.28b)}
\]

\(TKE\) is usually produced at the scale of the boundary-layer depth. The production is made mechanically by wind shear and buoyantly by thermals.

A statistic tells us about turbulent energy.
Turbulent energy cascades through the inertial subrange, where the large-size eddies drive medium ones, which in turn drive smaller eddies. Molecular viscosity continuously damps the tiniest (microscale) eddies, dissipating TKE into heat. TKE is not conserved.

The tendency of TKE to increase or decrease is given by the following TKE budget equation:

$$\frac{\Delta \text{TKE}}{\Delta t} = A + S + B + Tr - \varepsilon$$

where $A$ is advection of TKE by the mean wind, $S$ is shear generation, $B$ is buoyant production or consumption, $Tr$ is transport by turbulent motions and pressure, and $\varepsilon$ is viscous dissipation rate. For stationary (steady-state) turbulence, the tendency term on the left side of eq. (18.29) is zero.
**In a nutshell:**

- **S** Increases as wind shear increases
- **B** positive when warm thermals are rising or when cold “thermals” are sinking. (this happens in statically unstable air)
- **B** negative when warm air tries to sink, or cold air tries to rise. (this happens in statically stable air)
- **ε** always causes TKE to decrease.

Turbulence is NOT conserved. TKE will decay exponentially toward zero. Turbulence can be maintained ONLY if it is continually generated.
6. TKE and Dispersion

- Shear Generation Rate (m^2/s^3)
- Buoyant Generation Rate (m^2/s^3)
Static Stability & Flow Type

Shear Generation Rate (m^2/s^3)

Buoyant Generation Rate (m^2/s^3)

0.002

0.004

0.006

-0.002

-0.004

-0.006

Flow Type

Static Stability

forced convection

neutral

free convection

waves

stable

SST

Turbulence

NO

unstable

B

Buoyant Generation Rate (m^2/s^3)
Richardson Number

Shear Generation Rate (m²/s³)

Buoyant Generation Rate (m²/s³)

\[ R_f = \frac{-B}{S} \]
Dynamics Stability

\[ R_f = \frac{-B}{S} \]

Shear Generation Rate (m²/s³)

Buoyant Generation Rate (m²/s³)

Rf = 0

Rf = 1

Rf = -1

Rf = ∞

Dynamically Stable

NO Turbulence

Dynamically Unstable

& Turbulent

Buoyant Generation Rate (m²/s³)
### Table 19-2a. Pasquill-Gifford turbulence types for Daytime. $M$ is wind speed at $z = 10$ m.

<table>
<thead>
<tr>
<th>$M$ (m s$^{-1}$)</th>
<th>Insolation (incoming solar radiation)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strong</td>
<td>Moderate</td>
</tr>
<tr>
<td>&lt; 2</td>
<td>A</td>
<td>A to B</td>
</tr>
<tr>
<td>2 to 3</td>
<td>A to B</td>
<td>B</td>
</tr>
<tr>
<td>3 to 4</td>
<td>B</td>
<td>B to C</td>
</tr>
<tr>
<td>4 to 6</td>
<td>C</td>
<td>C to D</td>
</tr>
<tr>
<td>&gt; 6</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

### Table 19-2b. Pasquill-Gifford turbulence types for Nighttime. $M$ is wind speed at $z = 10$ m.

<table>
<thead>
<tr>
<th>$M$ (m s$^{-1}$)</th>
<th>Cloud Coverage</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>≥ 4/8 low cloud or thin overcast</td>
<td>≤ 3/8</td>
</tr>
<tr>
<td>&lt; 2</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>2 to 3</td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td>3 to 4</td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td>4 to 6</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>&gt; 6</td>
<td>D</td>
<td>D</td>
</tr>
</tbody>
</table>
Take a 5 minute “stretch” break
7. Pile-burn Case Study. Sep 2016 near Smithers

thanks to Ben Weinstein

“A forest company burned 400+ industrial slash piles < 20 km from Smithers. The venting forecast was ‘good’, however the plume was trapped aloft and had a defined top and bottom (see pictures). It ended up descending into town in the evening and led to high exposure for a short time, causing me much grief and extra workload.”

Morning or Mid-day

photos provided by Ben Weinstein
Morning or Mid-day

photos provided by
Ben Weinstein
Evening. Photos provided by Ben Weinstein
One possible explanation.

Discussion? Thoughts?
8. Clarifications of Some of Your Issues

- PBL formation and breakdown
- winter PBLs
- PBL in valleys. Also valley clouds vs. inversions
- PBL vs smoke dispersion
- dispersion prediction days in advance
- using soundings to anticipate dispersion
- how to understand Weinstein’s Air Qual. Advisory fcsts.
- surface roughness
- venting: roles of shear and buoyancy
- how fine a resolution in NWP is numerically stable
- using weather forecasts to anticipate episodes of poor air quality
Clarifications

• how fine a resolution in NWP is numerically stable?

• We run NWP models at $\Delta x = 1.3$ km every day, & 0.44 km for case studies.
• “Resolution” = 7 $\Delta x$ in most NWP
• Our “nowcasting” map combines NWP, station data, and very-high res. DEM.

Nowcast created by Nadya Moisseeva
Clarifications
• Valley PBLs, inversions & dispersion

See Chapter 17 of Stull 2015: Practical Meteorology.

Anabatic & katabatic winds during fair weather.
Clarifications

- Valley PBLs, inversions & dispersion

See Chapter 17 of Stull 2015: Practical Meteorology.

Valley inversions in **fair weather**.

Gap winds.

needs **cold** air under **warm**
Clarifications

- Valley PBLs, inversions & dispersion

See Chapter 17 of Stull 2015: Practical Meteorology.

For synoptically windy conditions.

Mountain waves

\[ Fr = \frac{\lambda}{2W} \propto \frac{M}{W \cdot N_{BV}} = \text{Froude Number} \]
Clarifications

- Valley PBLs, inversions & dispersion

See Chapter 17 of Stull 2015: Practical Meteorology.

Downslope winds

Bora

Foehn = Chinook

Channeled flow.
Survey Results - Clarification Presented

✓ PBL formation and breakdown
✓ winter PBLs
✓ PBL in valleys. Also valley clouds vs. inversions
✓ PBL vs smoke dispersion
  • dispersion prediction days in advance (use NWP)
✓ using soundings to anticipate dispersion
  • how to understand Weinstein’s Air Qual. Advisory fcsts.
  • surface roughness (rougher terrain causes slower winds but greater turbulence intensity near the ground)
✓ venting: roles of shear and buoyancy (greater shear & buoyancy increase dispersion rate, but other factors for venting include inversion height and wind speed)
✓ how fine a resolution in NWP is numerically stable
✓ using weather forecasts to anticipate episodes of poor air quality
Atmospheric Boundary Layers, Turbulence & Dispersion

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For free book online: https://www.eoas.ubc.ca/books/Practical_Meteorology/

the end
Strangely, for a convective ML, the centreline can descend.

(show Deardorff tank movie if time)
Time-averaged Centreline Height \( (Z_{CL}) \) vs. Downwind Distance \( (x) \)

\[
Z_{CL} = \frac{z_{CL}}{z_i}
\]

**Figure 19.7**

Height of the averaged pollutant centerline \( z_{CL} \) with downwind distance \( x \), normalized by mixed-layer scales. Dimensionless source heights are \( Z_s = z_s/z_i \): 0.025 (thick solid line); 0.25 (dashed); 0.5 (dotted); and 0.75 (thin solid). The plume is neutrally buoyant.

\[
\omega_* = \left[ \frac{|g| z_i F_H}{T_v} \right]^{1/3} = \text{Deardorff velocity} \quad \text{(m s}^{-1})
\]  

\( F_H = \text{surface heat flux, } z_i = \text{mixed layer depth, } M = \text{wind speed} \)
Cross-wind Integrated Concentration, $C_y$
Cy vs. z and x, for various effect stack heights

Isopleths of dimensionless crosswind-integrated concentration $C_y = c_y z_i M / Q$ in a convective mixed layer, where $c_y$ is crosswind integrated concentration, $z_i$ is depth of the mixed layer, $M$ is mean wind speed, and $Q$ is emission rate of pollutants. Source heights are $Z_s = z_s / z_i = (a) 0.75$, (b) 0.5, (c) 0.25, (d) 0.025, and are plotted as the large black dot at left.