Binary

ATSC 212

What is it?

Everyday numbers are represented using bases. Namely any number can be represented as

$$N = \Sigma b^i q_i$$

where b is the value of the base taken to the power of rank i, and q is the remainder at that rank. Our decimal number system uses base 10, for example

$$155 = 10^2 * 1 + 10^1 * 5 + 10^0 * 5$$

Binary is simply base 2, that is to say $N = \Sigma 2^i q_i$. Instead of using powers of 10, we use powers of 2.

How do we represent binary numbers?

In base 10, we have ten different digits to represent numbers (0...9). You can think of it in terms of counting. We have to count through ten different numbers to reach the base, at which point we start again, noting that we have a value at the next rank (ie counting to 11 gets us to 10 and then one more).

In binary (base 2), we only need two digits to reach the base (0, 1). So all numbers will be represented with 0's and 1's. (ie 11 in binary is 1011...$2^3 + 2^1 + 1$)

As you might guess, in other bases you only need enough digits to reach the base.

Converting to binary: long division

There are many ways to convert from decimal to binary, but the easiest is long division. The idea is to repeatedly divide a number by 2 writing out the remainders in order from least significant to most significant. Once you get down to 1, simply add an extra 1 for the most significant digit. For example, convert 143...

143 / 2 = 71	remainder 1 (LSD)	
71 / 2 = 35	remainder 1	
35 / 2 = 17	remainder 1	143 = 10001111
17 / 2 = 8	remainder 1	
8 / 2 = 4	remainder 0	
4 / 2 = 2	remainder 0	
2 / 2 = 1	remainder 0	
1	add an extra 1 (MSD)	

Another example: 255

255 / 2 = 127	remainder 1
127 / 2 = 63	remainder 1
63 / 2 = 31	remainder 1
31 / 2 = 15	remainder 1
15 / 2 = 7	remainder 1
7 / 2 = 3	remainder 1
3 / 2 = 1	remainder 1
1	MSD 1

So 255 = 11111111

Converting from binary to decimal

The easiest approach is to use the formula from the first slide, sum up all the powers of two where the digit is 1. For example,

$10110011 = 2^7 + 2^5 + 2^4 + 2^1 + 2^0$

$= 128 + 32 + 16 + 2 + 1$

$= 179$

The key is to remember that the least significant digit is the 0^{th} power.