## A.4. ERRORS

**Error** is the difference between a measured (or estimated) value and the true (or reference) value.

### A.4.1. Systematic Error & Accuracy

If part of the error is **systematic** or **repeatable** (namely, you get the same error each time you make a measurement), then the difference between the average measurement and the true value is called the **bias**. Smaller bias magnitude (i.e., lower systematic errors) corresponds to greater **accuracy**. Namely, accuracy indicates how close your **average** observations are to truth (Fig. A4).

**Systematic errors** can be due to **errors in instrument calibration**, **personal errors** (such as parallax error in reading a dial), **erroneous experimental conditions** (such as not shielding a thermometer from sunlight), and **imperfect technique** (such as breathing on a thermometer before you read it).

If you can calculate or otherwise know the bias, then you can remove this bias from your observations to correct for systematic error. Namely, you can easily make your corrected observations more accurate.

### A.4.2. Random Error & Precision

After removing systematic errors, you might find that your observations still have some unexplained variability from measurement to measurement. These are called **random errors** (Fig. A.4). Experiments with smaller random errors are said to have higher **precision**; namely, they are more precise. The **standard deviation** (or spread) of your observations is a measure of the random error — greater standard deviation indicates greater random error and lower precision.

Random errors can be due to **errors in judgment** (such as by manually reading a dial with poor resolution), **fluctuating conditions** (such as trying to determine sea level on a wavy ocean), **small disturbances** (such mechanical vibrations of an instrumented tower in high winds), and **errors in definition** (such as measuring the dimension of a fractal-shaped cloud, which depends on the size of the measuring stick).

Unfortunately, the probabilistic nature of random errors makes them difficult to remove after the fact. Often, the only recourse is to repeat the experiment under better controlled conditions and with higher quality instruments, and be sure to take a large number of observations to improve the statistical robustness of your results.

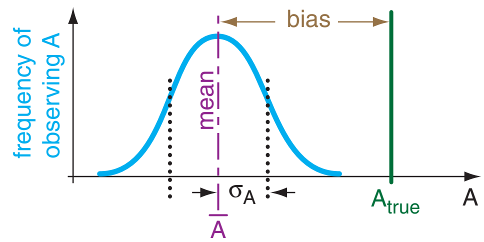


**Figure A.4**
*The dark curve is the frequency that different A values are observed. Accuracy is related to bias. Precision is related to spread of curve (described by standard deviation $\sigma_A$). $\overline{A}$ is the mean.*

### A.4.3. Reporting Observations

For any variable $A$ that you have measured $N$ times to yield a **data set** ($A_1, A_2, A_3, ..., A_N$), let $\overline{A}$ be the **mean value**, and $\sigma_A$ be the **standard deviation**. These are defined as:

$$\overline{A} = \frac{1}{N}\sum_{i=1}^{N} A_i \qquad \bullet\text{(A.8)}$$

and

$$\sigma_A = \left[\frac{1}{(N-1)}\sum_{i=1}^{N}(A-\overline{A})^2\right]^{1/2} \qquad \bullet\text{(A.9)}$$

where $i$ is a **dummy index** that points to a single **data element** $A_i$ in your data set.

After removing any known biases, the resulting observation is usually reported or written as a mean (average) value plus or minus (±) the standard deviation:

$$A = \overline{A} \pm \sigma_A \qquad \bullet\text{(A.10)}$$

where the precision or **standard uncertainty** is given by the standard deviation $\sigma_A$.

**Sample Application**
Given these $T$ observations: (15, 13, 20, 12, 10, 17, 18)°C. $T_{true}$ = 8°C. Find the mean, bias & std dev.

**Find the Answer**
Given: $T_{true}$ = 8°C, and the data set above.
Find: $\overline{T}$ = ? °C, $\sigma_T$ = ? °C, bias = ? °C.

Use eq. (A.8): $\overline{T}$ = (1/7)(15+13+20+12+10+17+18) = **<u>15°C</u>**
Define the deviation from mean as: $T' = T - \overline{T}$
Thus, our observation have $T'$ = (0, –2, 5, –3, –5, 2, 3)°C
Use eq. (A.9), rewritten as $\sigma_T = [(N-1)^{-1}\,\Sigma(A'^2)]^{1/2}$
Thus: $\sigma_T = [(1/6)\cdot(0+4+25+9+25+4+9)(°C)^2]^{1/2}$ = **<u>3.56°C</u>**
From the raw observations: **<u>T = 15 ± 3.56 °C</u>** .
*Bias* = $\overline{T} - T_{true}$ = 15 – 8 °C = **<u>7°C</u>** .

**Check**: Units OK. We seem to have a warm bias.
**Exposition**: Our observations are **<u>not accurate</u>** (large bias) and are **<u>not precise</u>** (large $\sigma_T$).

### HIGHER MATH • Error Propagation

Suppose $D$ is a function of $A$, $B$ and $C$, where $C$ is not a constant. Namely, $D(A, B, C)$.

If the error standard deviations $\sigma_A$, $\sigma_B$, and $\sigma_C$ for $A$, $B$ & $C$ are known, then the **propagation of errors** into the standard deviation $\sigma_D$ of variable $D$ is:

$$\sigma_D = \left[ \left(\frac{\partial D}{\partial A}\right)^2 \cdot \sigma_A^2 + \left(\frac{\partial D}{\partial B}\right)^2 \cdot \sigma_B^2 + \left(\frac{\partial D}{\partial C}\right)^2 \cdot \sigma_C^2 + 2 r_{AB} \cdot \left(\frac{\partial D}{\partial A}\right) \cdot \left(\frac{\partial D}{\partial B}\right) \cdot \sigma_A \cdot \sigma_B + 2 r_{AC} \cdot \left(\frac{\partial D}{\partial A}\right) \cdot \left(\frac{\partial D}{\partial C}\right) \cdot \sigma_A \cdot \sigma_C + 2 r_{BC} \cdot \left(\frac{\partial D}{\partial B}\right) \cdot \left(\frac{\partial D}{\partial C}\right) \cdot \sigma_B \cdot \sigma_C \right]^{1/2} \quad \text{(A.a)}$$

The correlation coefficients $r$ are defined as

$$r_{AB} = \frac{1}{(N-1) \cdot \sigma_A \cdot \sigma_B} \cdot \sum_{i=1}^{N} \left[ (A_i - \overline{A}) \cdot (B_i - \overline{B}) \right]$$

and correlations $r_{BC}$ and $r_{AC}$ are defined similarly. If $A$ and $B$ are independent, then $r_{AB} = 0$. Correlations between the other variables could also be zero.

For example, suppose you measure air density ($\rho = \overline{\rho} \pm \sigma_\rho$) and temperature ($T = \overline{T} \pm \sigma_T$), and calculate pressure ($P$) using the ideal gas law $P = \rho \cdot \Re \cdot T$, where $\Re$ is a constant. Thus, from calculus: $\partial P / \partial \rho = \Re \cdot T$, and $\partial P / \partial T = \rho \cdot \Re$. Assume $\rho$ and $T$ are independent, thus the correlation coefficient $r_{\rho T} = 0$.

Our best estimate of pressure is

$$\overline{P} = \overline{\rho} \cdot \Re \cdot \overline{T} \; .$$

To estimate the pressure error $\sigma_P$, use eq. (A.a) to propagate the other errors into the pressure error:

$$\sigma_P = \left[ \left(\frac{\partial P}{\partial \rho}\right)^2 \cdot \sigma_\rho^2 + \left(\frac{\partial P}{\partial T}\right)^2 \cdot \sigma_T^2 \right]^{1/2}$$

$$\sigma_P = \left[ (\Re \cdot T)^2 \cdot \sigma_\rho^2 + (\Re \cdot \rho)^2 \cdot \sigma_T^2 \right]^{1/2}$$

Multiply the right side by 1 in the form of $\overline{P} / \left(\overline{\rho} \cdot \Re \cdot \overline{T}\right)$

$$\sigma_P = \overline{P} \cdot \left[ \left(\frac{1}{\overline{\rho}}\right)^2 \cdot \sigma_\rho^2 + \left(\frac{1}{\overline{T}}\right)^2 \cdot \sigma_T^2 \right]^{1/2}$$

or

$$\sigma_P = \overline{P} \cdot \left[ (\sigma_\rho / \overline{\rho})^2 + (\sigma_T / \overline{T})^2 \right]^{1/2}$$

where we use the averages as our best estimates of $P$, $\rho$, and $T$. This last result looks like eq. (A.14). In fact, we could have used eq. (A.14) directly and avoided all the calculus.

Thus, we would report our calculated pressure as:

$$P = \overline{P} \pm \sigma_P$$

Similarly, the mean and standard deviation of some other variable $B$ would be $\overline{B}$ and $\sigma_B$.

For example, Newton's constant of gravitation $G$ is reported (CODATA 2006) as:

$$G = 6.67428\text{x}10^{-11} \pm 0.00067\text{x}10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \; .$$

## A.4.4. Error Propagation

**Error propagation** tells us how the errors in $A$ and $B$ affect the error of $D$, where $D$ depends on $A$ and $B$ according to some equation. Namely, how can we estimate $\sigma_D$ knowing $\sigma_A$ and $\sigma_B$ ? Assume the errors in $A$ and $B$ are independent of each other.

For a simple sum or difference (e.g., $D = A + B$, or $D = A - B$), then

$$\sigma_D = [\sigma_A{}^2 + \sigma_B{}^2]^{1/2} \quad \text{(A.11)}$$

For $D = c \cdot A$ where $c$ is a constant, then

$$\sigma_D = c \cdot \sigma_A \quad \text{(A.12)}$$

Similarly, if $D = c_A \cdot A \pm c_B \cdot B$ where $c_A$ and $c_B$ are different constants, then

$$\sigma_D = [c_A{}^2 \cdot \sigma_A{}^2 + c_B{}^2 \cdot \sigma_B{}^2]^{1/2} \quad \text{(A.13)}$$

For a simple product $D = c \cdot A \cdot B$ or quotient $D = c \cdot A / B$, then

$$\sigma_D = \overline{D} \cdot [\, (\sigma_A / \overline{A}\,)^2 + (\sigma_B / \overline{B}\,)^2 \,]^{1/2} \quad \text{(A.14)}$$

where $\overline{A}$ is the average of $A$, $\overline{B}$ is the average of $B$, and $\overline{D}$ is the average of $D$ (*i.e.*, $\overline{D} = c\,\overline{A}\,\overline{B}$, *or* $\overline{D} = c\,\overline{A}\,/\overline{B}$ ).

For a simple power relationship $D = c \cdot A^m$ where $m$ is a fixed constant, then

$$\sigma_D = \overline{D} \cdot m \cdot (\sigma_A / \overline{A}\,) \quad \text{(A.15)}$$

For the general case of a product of factors raised to various fixed (errorless) powers $D = c \cdot A^m \cdot B^q$, then

$$\sigma_D = \overline{D} \cdot [\, m^2 \cdot (\sigma_A / \overline{A}\,)^2 + q^2 \cdot (\sigma_B / \overline{B}\,)^2 \,]^{1/2} \quad \text{(A.16)}$$

For a logarithm such as $D = \ln(c \cdot A)$, where $c$ is a constant, then

$$\sigma_D = (\sigma_A / \overline{A}\,) \quad \text{(A.17)}$$

For an exponential such as $D = e^{c \cdot A}$ where $c$ is a constant, then

$$\sigma_D = c \cdot \overline{D} \cdot \sigma_A \quad \text{(A.18)}$$

For more complicated relationships, the rules above can be combined or used sequentially (or see the HIGHER MATH box).

**Sample Application**

Observations give $P_1 = 100 \pm 0.1$ kPa, $P_2 = 50 \pm 0.5$ kPa, and $\overline{T_v} = 260 \pm 5$ K. Use hypsometric eq. to find $\Delta z$.

**Find the Answer**

Given: $\overline{P}_1 = 100$ kPa, $\sigma_{P1} = 0.1$ kPa, $\overline{P}_2 = 50$ kPa, $\sigma_{P2} = 0.5$ kPa, $\overline{T_v} = 260$ K, $\sigma_T = 5$ K.

Hyp. eq.(1.26a): $\Delta z = a \cdot \overline{T_v} \cdot \ln(P_1/P_2)$, $a$=29.3m K$^{-1}$.

Find: $\Delta z$ = ? ± ? m. Namely, find $\overline{\Delta z}$ = ?m, $\sigma_{\Delta Z}$ = ? m

Method: Use error propagation rules sequentially.

For $(P_1/P_2)$: *Average(P/P)* = (100kPa)/(50kPa) = 2

Use eq. (A.14): $\sigma_{P/P} = 2 \cdot [(0.1/100)^2 + (0.5/50)^2]^{1/2} = 0.02$

For $a \cdot \ln(P_1/P_2)$: *Average* =(29.3m K$^{-1}$)·ln(2) = 20.31 m K$^{-1}$

Use eq. (A.17): $\sigma_{a \cdot ln}$ =(29.3m K$^{-1}$)·(0.02/2) = 0.293 m K$^{-1}$

For $\overline{T_v} \cdot a \cdot \ln(P_1/P_2)$:*Average*=(20.31m K$^{-1}$)·(260K)= **5281m**

Use eq. (A.14): $\sigma_{\Delta Z}$=(5281m)·[(5/260)$^2$+(0.293/20.31)$^2$]$^{1/2}$ = **127 m**

Thus: **Δz = 5281 ± 127 m**

**Exposition**: Notice that error-propagation eqs. (A.11 - A.18) are dimensionally consistent. A good check.

## A SCIENTIFIC PERSPECTIVE • Have Passion

The best scientists and engineers need more than the good habits of diligence and meticulousness. They need passion for their field, and they need creativity. In this regard, they are kindred spirits to artists, composers, musicians, authors, and poets.

While an observation is something that can usually be quantified, the explanation or theory for it comes from the minds of people. For example, does light consist of particles (photons) or waves? Probably it consists of neither, but those are two theories from the creative imagination of scientists that have proved useful in explaining the observations.

The joy that a scientist feels after successfully explaining an observation, and pride that an engineer feels for making something work within the constraints of physics and economics, are no less intense than the joy and pride felt by an artist who has just completed his or her masterpiece.

Approach your work with passion, evaluate your result objectively, and enjoy your travel through life as you help society.

## A.5. A SCIENTIFIC PERSPECTIVE

Science is a philosophy. It is faith in a set of principles that guide the actions of scientists. It is a faith based on observation. Scientists try to explain what they observe. Theories not verified by observations are discarded. This philosophy applies to **atmospheric science**, also known as **meteorology**.

A good theory is one that works anywhere, anytime. Such a theory is said to be **universal**. Engineers utilize universal theories with the expectation they will continue working in the future. The structures, machines, circuits, and chemicals designed by engineers that we use in every-day life are evidence of the success of this philosophy.

But we scientists and engineers are people, and share the same virtues and foibles as others. Those of you planning to become scientists or engineers might appreciate learning some of the pitfalls so that you can avoid them, and learning some of the tools so that you can use them to good advantage.

For this reason, scattered throughout the book are boxes called "A SCIENTIFIC PERSPECTIVE", summarized in Table A–5. These go beyond the mathematical preciseness and objective coldness that is the stereotype of scientists. These boxes cover issues and ideas that form the fabric of the philosophy of science. As such, many are subjective. While they give you one scientist's (my) perspectives, I encourage you to discuss and debate these issues with other scientists, colleagues, and teachers.

**Table A-5**. Guidelines & issues for scientists. Chapters & topics of the "A SCIENTIFIC PERSPECTIVE" boxes.