b. Continuous Point Source

1) No mean wind

Let Q_2 = source emission rate (GM/T)

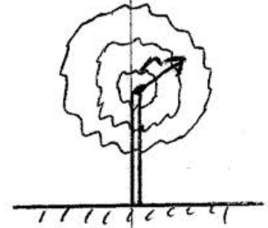
Thus $Q_2\,dt$ = mass emitted in time dt (like a burst or puff)

∴ Consider series of 3-D puffs emitted from source.

Integrate 3-D puff eq. over ~~time~~ all emission times.

Let: t_1 = time emissions started
t = current (observation) time
τ = time that any one puff was emitted.

$$c = \int_{t_1}^{t} \frac{Q_2}{\left[4\pi K(t-\tau)\right]^{3/2}} \exp-\left[\frac{r^2}{4K(t-\tau)}\right] d\tau$$



Upon integrating:

$$c = \frac{Q_2}{4\pi K r}\left[1 - \operatorname{erf}\left(\frac{r}{\left[4K(t-t_1)\right]^{1/2}}\right)\right]$$

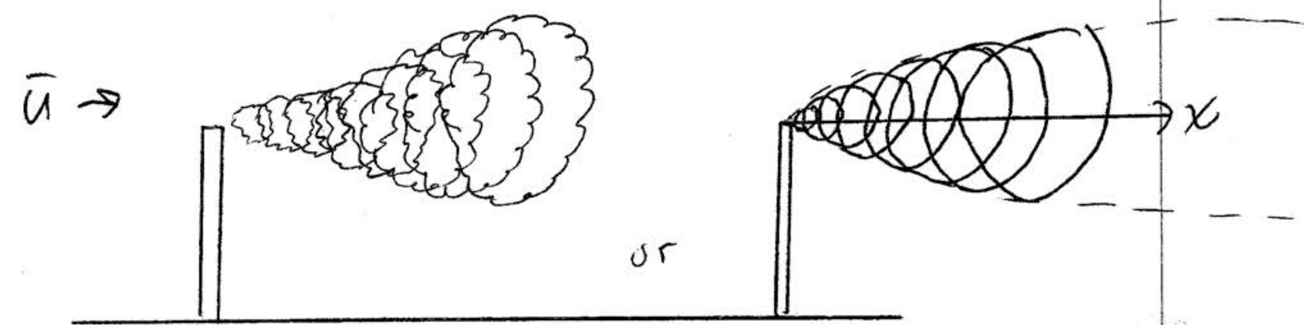
$$\boxed{c = \frac{Q_2}{4\pi K r}\operatorname{erfc}\left[\frac{r}{\{4K(t-t_1)\}^{1/2}}\right]} \qquad \text{IV.B.3.7}$$

Note

$$c \to \frac{Q_2}{4\pi K r} \qquad \text{as} \qquad t_1 \to -\infty$$

2) With Mean Wind

Theoretically, we could integrate over a series of puffs being advected downwind.

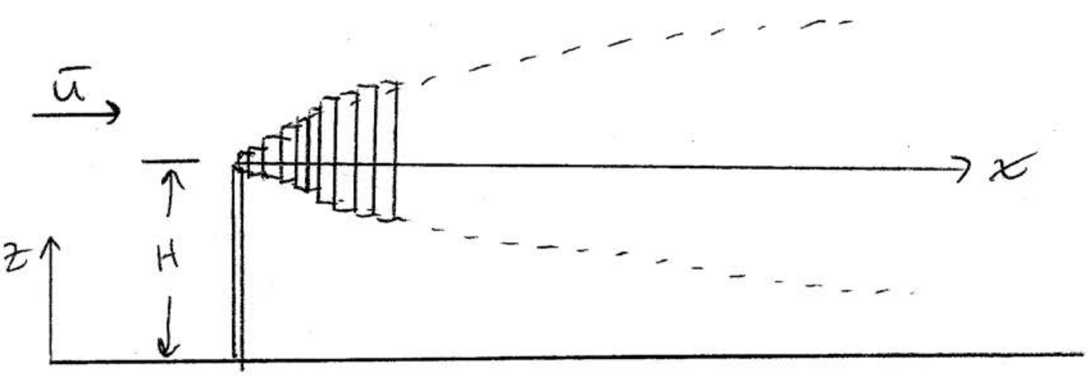


But very complicated to integrate.

Instead assume that the loss of pollutants diffusing in +x direction from a point are approx. balanced by pollutants diffusing in −x direction from previous puff.

> That is, assume <u>no</u> net diffusion in x-dir.
> Assume only advection in x-dir

This is the <u>"spreading disk"</u> diffusion model. Easier to integrate.



This is just 2−D diffusion! y & z

→ Works only if duration of emission is longer than travel time to the downwind position of interest.

Look at non-isotropic 2-D diffusion from H.

Source $= Q_1$ gm/m along x-axis

~~But wind is blowing ∴ Line concentration in x dir =~~

~~∴ $Q_1 = \frac{Q}{\bar{u}} \frac{gm/s}{m/s} = \left(\frac{gm}{m}\right)$~~

Spreading Disk model:

$$c = \frac{Q_1}{4\pi t (K_y K_z)^{1/2}} \exp -\left[\frac{y^2}{4K_y t} + \frac{(z-H)^2}{4K_z t}\right]$$

Let $\sigma_y \equiv \sqrt{2K_y t}$

$\sigma_z \equiv \sqrt{2K_z t}$

$Q_1 \frac{gm}{m} = Q\left(\frac{gm}{s}\right) / \bar{u}\left(\frac{m}{s}\right)$

$c = gm/m^3$

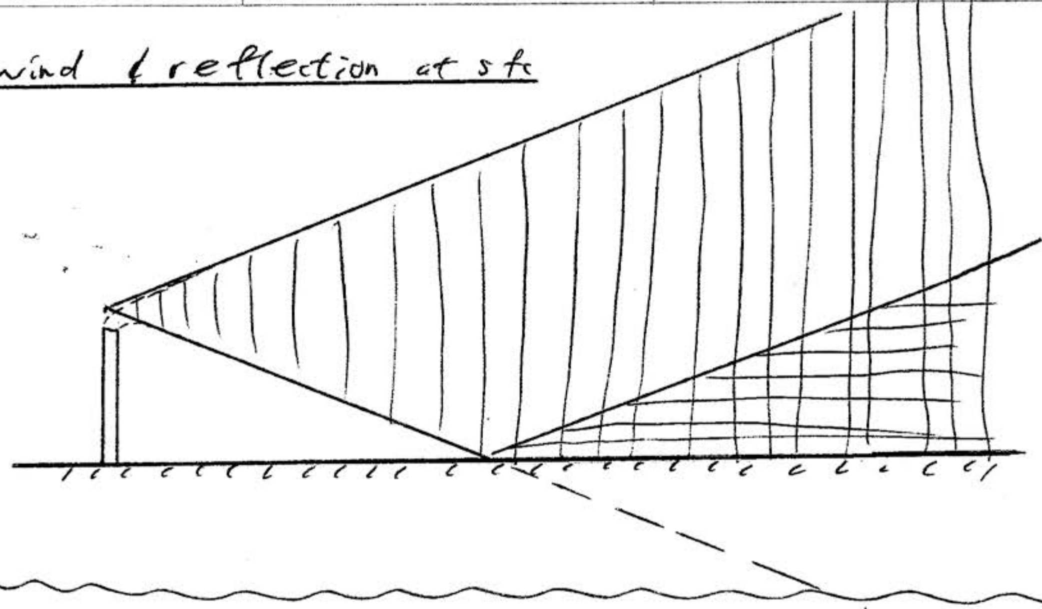
$$\boxed{c = \frac{Q}{2\pi \sigma_y \sigma_z \bar{u}} \exp -\left[\frac{y^2}{2\sigma_y^2} + \frac{(z-H)^2}{2\sigma_z^2}\right]}$$ IV.B.3.8

Note, the vertical spread may be different than horizontal spread.

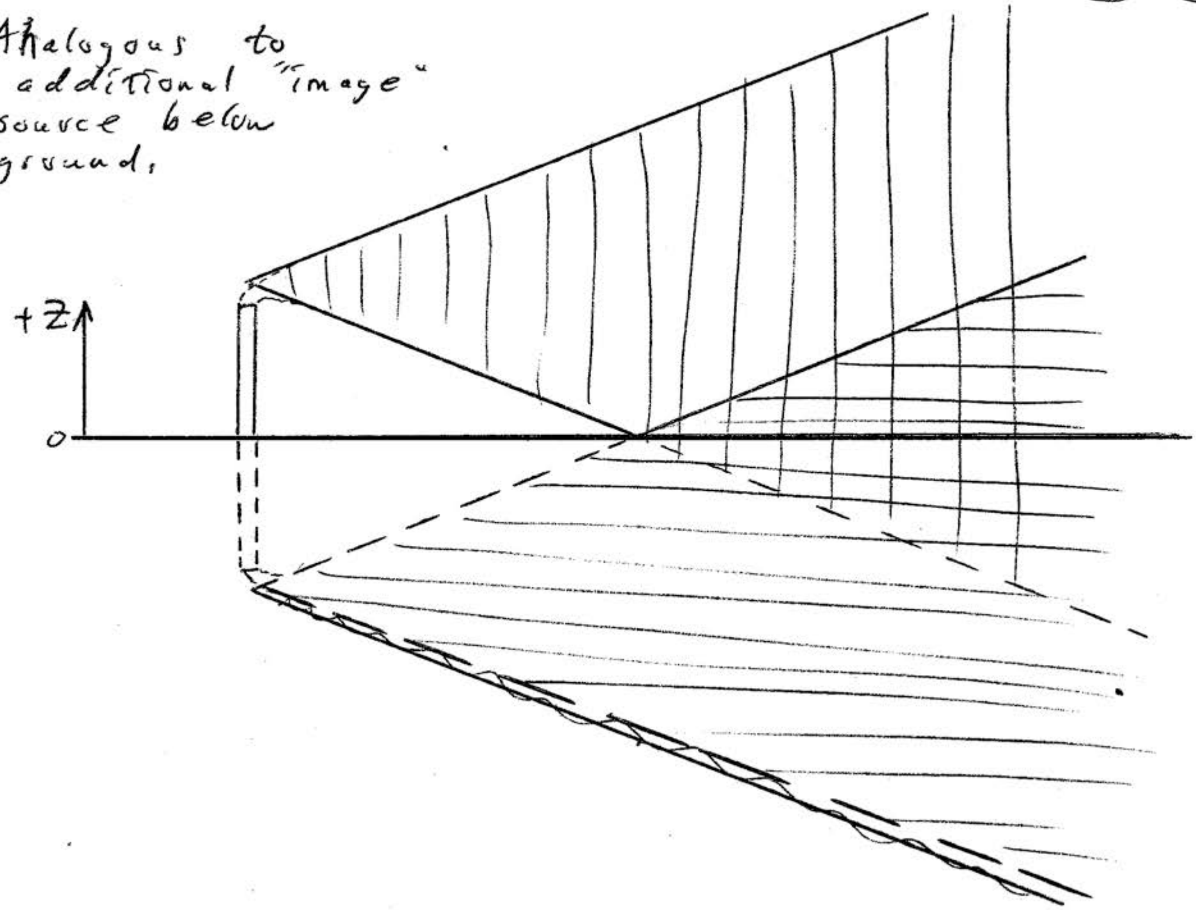
x is related to t by

$x = \bar{u} \cdot t$

3) Mean wind & reflection at sfc



Analogous to additional "image" source below ground.



This "image" has a plume concentration of

$$C_{im} = \frac{Q}{2\pi \sigma_y \sigma_z \bar{u}} \exp - \left[\frac{y^2}{2\sigma_y^2} + \frac{(z+H)^2}{2\sigma_z^2} \right]$$

(note z is neg.)

Summing the true & image concentrations:

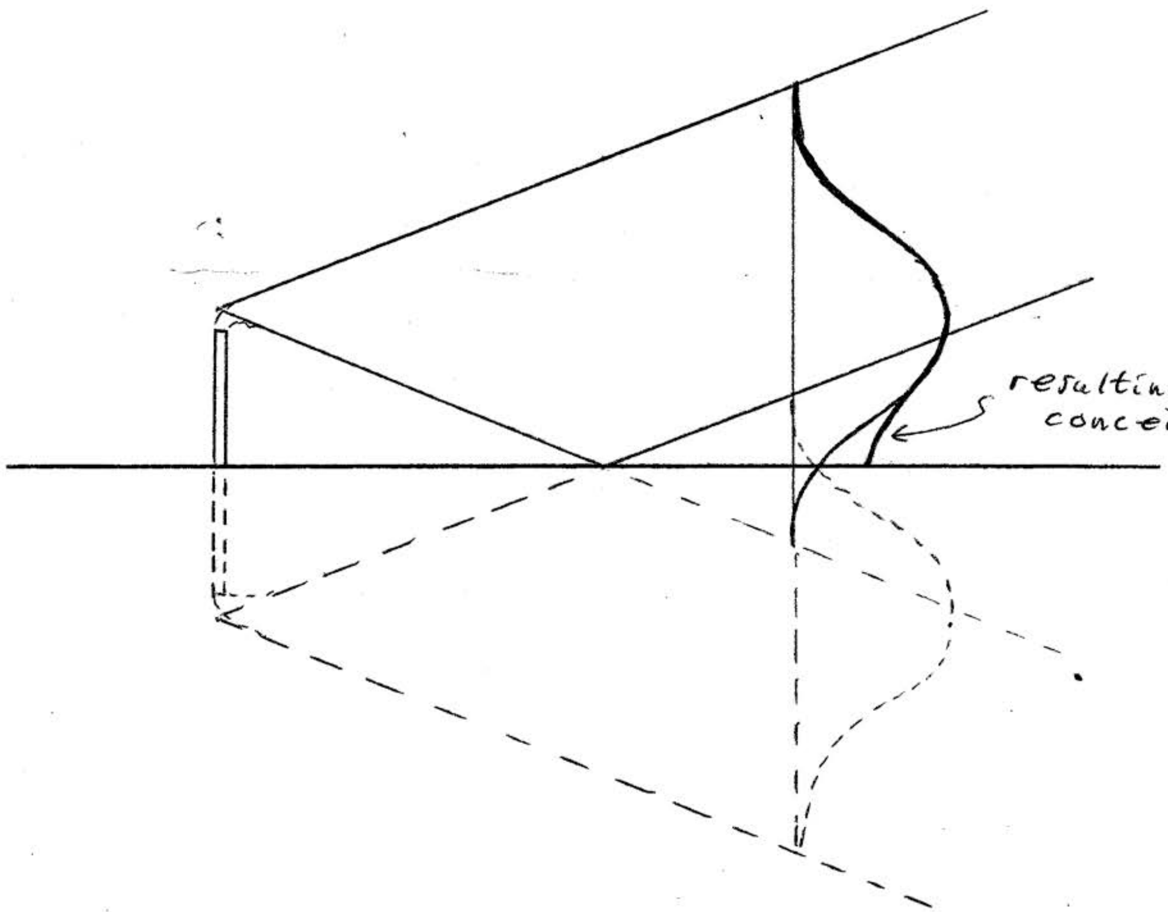
$$c = c_{true} + c_{imag.}$$

Assuming x-axis is aligned with mean wind

$c(x,y,z,H) =$

$$\boxed{c = \frac{Q}{2\pi\sigma_y\sigma_z\bar{u}} \exp -\left(\frac{y^2}{2\sigma_y^2}\right)\left\{\exp -\left[\frac{(z-H)^2}{2\sigma_z^2}\right] + \exp -\left[\frac{(z+H)^2}{2\sigma_z^2}\right]\right\}}$$

IV.B.3.9

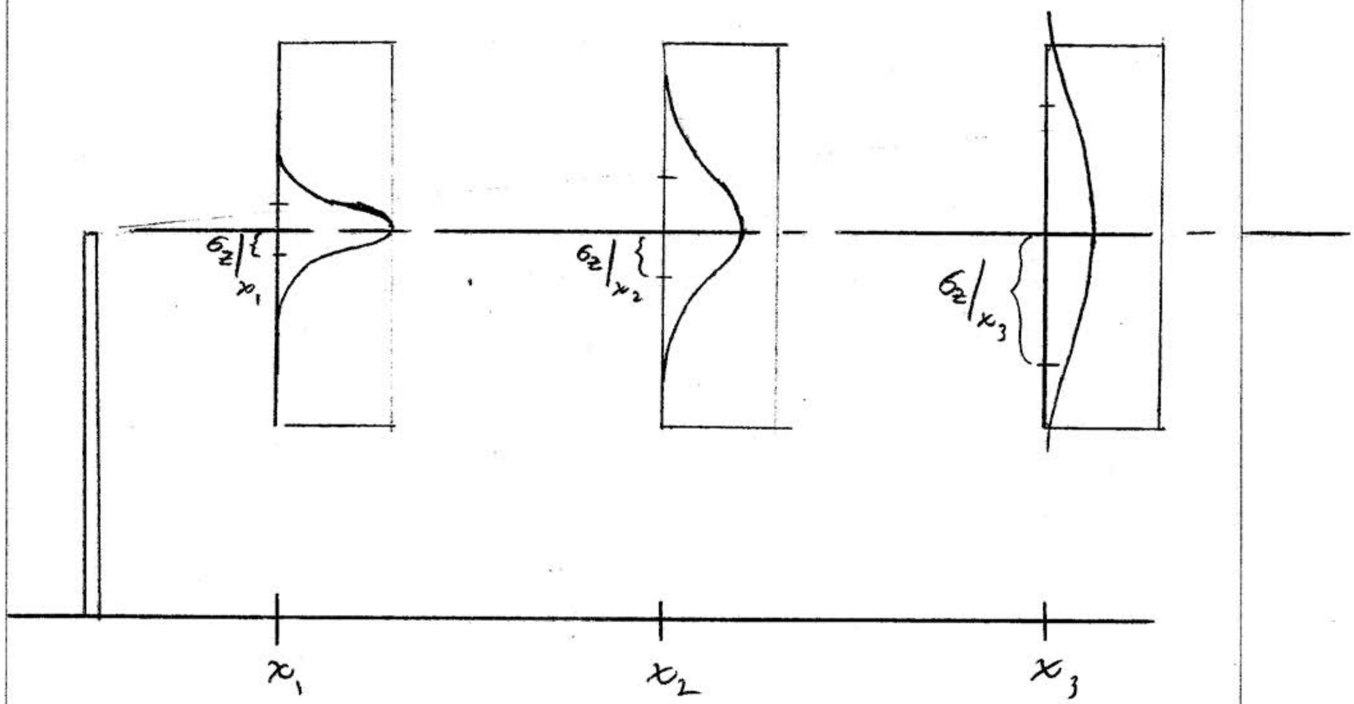


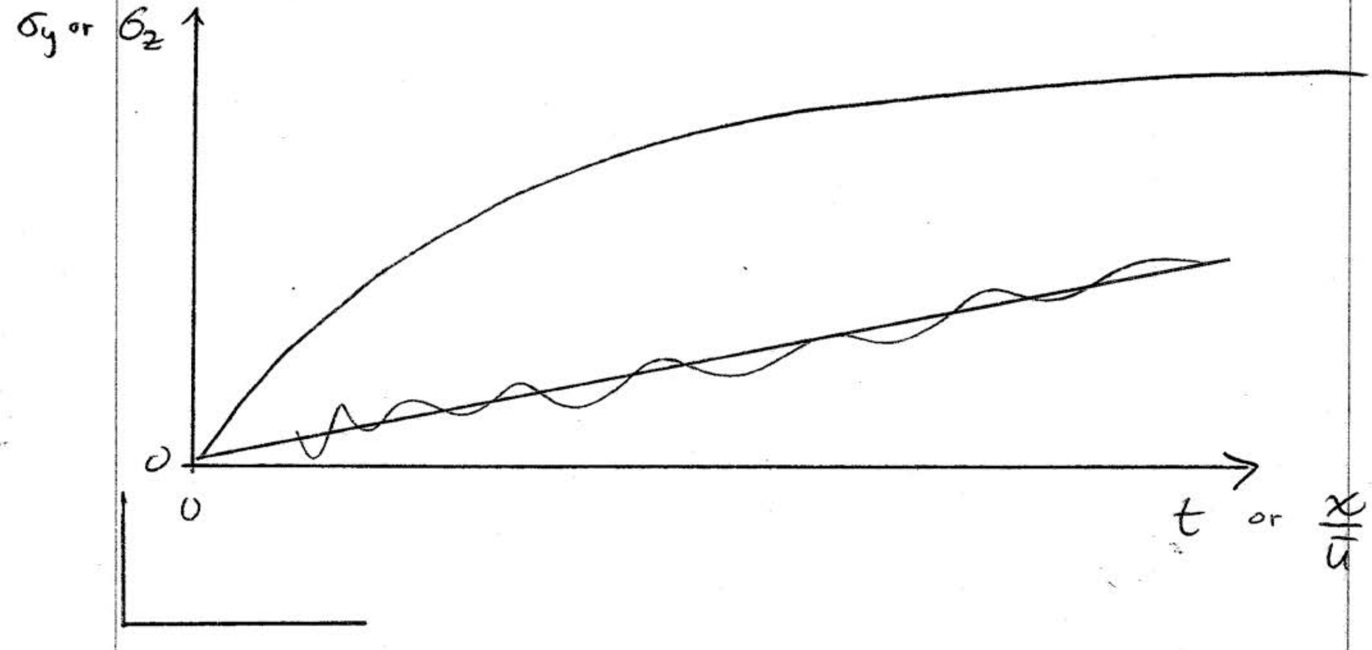
This assumes that there is <u>no</u> loss (or source) of pollutants at the surface (ie, total reflection)

Aside

$$\sigma_y \text{ or } \sigma_z = f(t) = f\left(\frac{x}{\bar{u}}\right)$$

For Neutral Stability





Fall 93

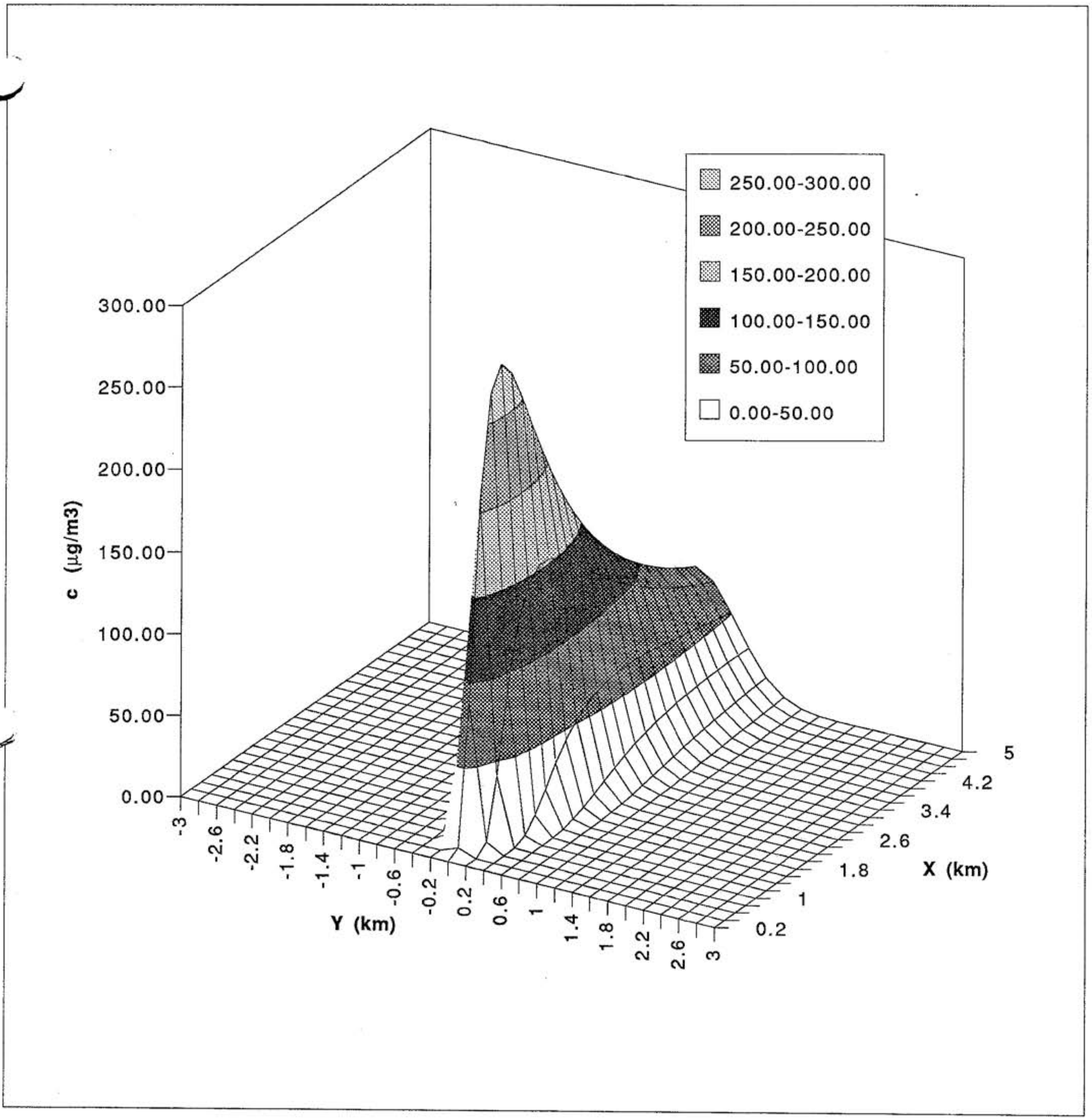
AOS/IES 535 - Homework: Gaussian Dispersion with Reflection Prof. Stull

Given: Emission rate of SO_2 from a point source is Q (g/s) = 263
Effective stack height of H (m) = 200
Mean wind from west constant with height of M = U(m/s) = 5
Assume D stability class. Use sigmas from HBH Table 4.5 for urban.

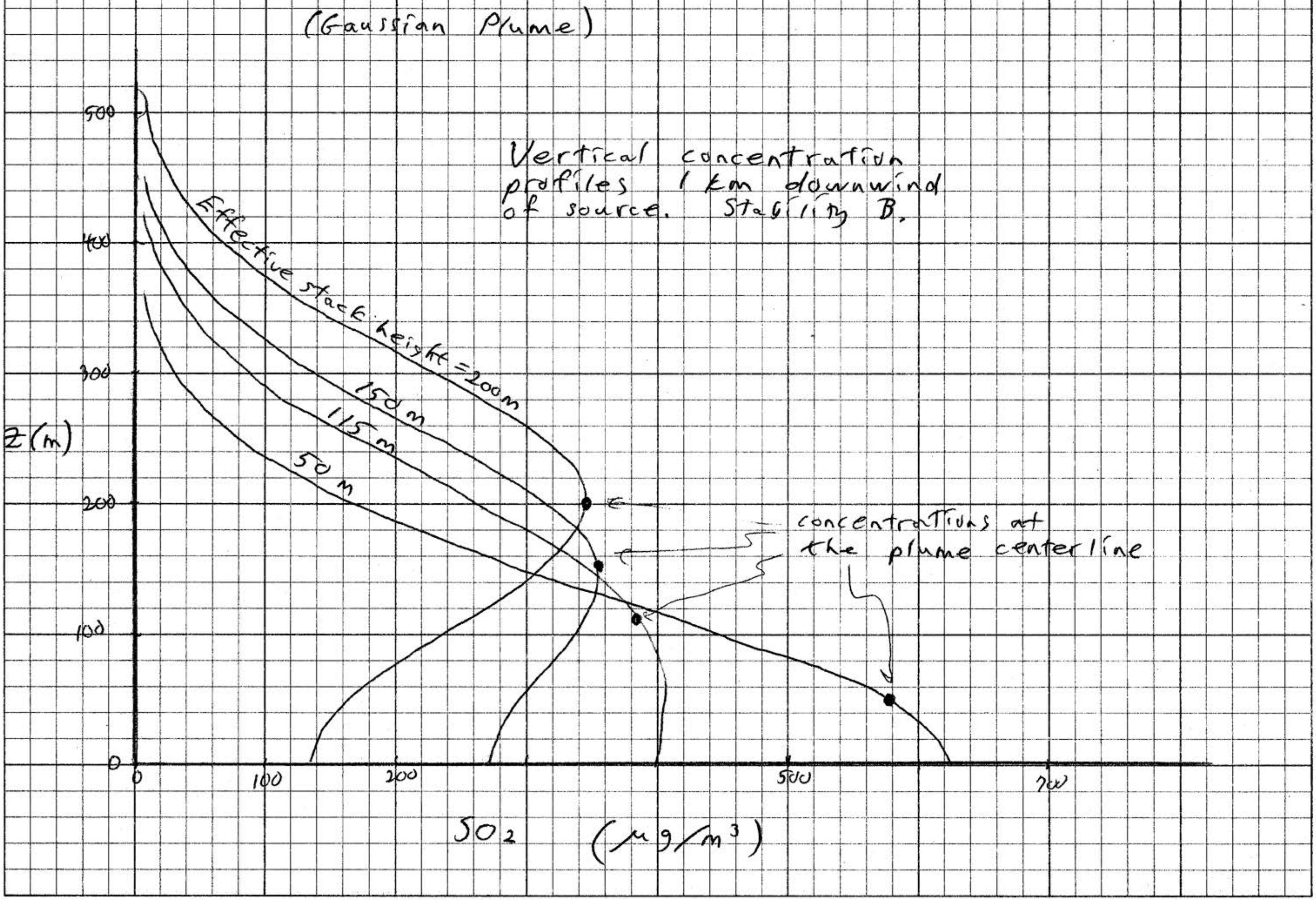
Prob: Calculate the concentration (μg/m3) at the surface for an array of grid points
(Hint, use eq 3.2 from Turner's Workbook, & be careful with units.)
From y (km)= -3 to y = 3 km, at increment Δy(km) = 0.2
From x (km)= 0.2 to x = 5 km, at increment Δx(km) = 0.2
Contour the resulting isopleths of equal concentration, for c(μg/m3) =
0.1, 10, 50, 100, 150, 200, 250, 300, etc.

Answer:

x(km) =	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	
sig y (m)=	30.792	59.423	86.211	111.41	135.22	157.82	179.34	199.9	219.6	238.51	256.72	274.29	
sig z (m)=	27.196	52.915	77.328	100.58	122.79	144.06	164.48	184.13	203.07	221.36	239.05	256.2	
	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	
-3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
-2.8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
-2.6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
-2.4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
-2.2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
-2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
-1.8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
-1.6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
-1.4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
-1.2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.1
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.10	0.23	
-0.8	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	0.30	0.76	1.50	2.50	10
-0.6	0.00	0.00	0.00	0.00	0.01	0.20	1.01	2.79	5.53	8.91	12.52	16.06	
-0.4	0.00	0.00	0.00	0.33	3.37	11.32	22.53	34.06	44.00	51.67	57.11	60.66	50
-0.2	0.00	0.01	6.01	41.30	89.64	125.86	145.52	152.88	152.69	148.35	141.94	134.67	100
0	0.00	4.21	88.59	206.91	267.62	280.92	271.00	252.17	231.17	210.84	192.26	175.68	150
0.2	0.00	0.01	6.01	41.30	89.64	125.86	145.52	152.88	152.69	148.35	141.94	134.67	
0.4	0.00	0.00	0.00	0.33	3.37	11.32	22.53	34.06	44.00	51.67	57.11	60.66	100
0.6	0.00	0.00	0.00	0.00	0.01	0.20	1.01	2.79	5.53	8.91	12.52	16.06	50
0.8	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.08	0.30	0.76	1.50	2.50	10
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.10	0.23	
1.2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.1
1.4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
1.6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
1.8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
2.2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
2.4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
2.6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
2.8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	


250.00-300.00
200.00-250.00
150.00-200.00
100.00-150.00
50.00-100.00
0.00-50.00
300.00
250.00
200.00
150.00
100.00
50.00
0.00
c (µg/m3)
-3
-2.6
-2.2
-1.8
-1.4
-1
-0.6
-0.2
0.2
0.6
1
1.4
1.8
2.2
2.6
3
Y (km)
5
4.2
3.4
2.6
1.8
1
0.2
X (km)

Example of Reflection from ground



Numerical Representation (Finite Difference)

Used where can't use simpler analytic solutions

$$\frac{\partial c}{\partial t} = K \frac{\partial^2 c}{\partial z^2} = K \frac{\partial}{\partial z}\left(\frac{\partial c}{\partial z}\right)$$

Let $t = i \Delta t$

$$\frac{\Delta c}{\Delta t} = K \frac{\partial}{\partial z}\left(\frac{\Delta c}{\Delta z}\right)$$

$$\frac{\Delta c}{\Delta t} = K \frac{\Delta}{\Delta z}\left(\frac{\Delta c}{\Delta z}\right) =$$

$$\begin{matrix} 0 & j+1 \\ & \\ 0 & j \\ & \\ 0 & j-1 \end{matrix} \quad \begin{matrix} \}\ \Delta z \\ \}\ \Delta z \end{matrix}$$

$$\frac{c_{i+1,j} - c_{i,j}}{\Delta t} = K \frac{\left[(\Delta c/\Delta z)^{+} - (\Delta c/\Delta z)^{-}\right)}{\Delta z}$$

$$= K \frac{\left[(c_{i,j+1} - c_{i,j})/\Delta z - (c_{i,j} - c_{i,j-1})/\Delta z\right]}{\Delta z}$$

$$= K \; \frac{c_{i,j+1} - 2c_{i,j} + c_{i,j-1}}{\Delta z^2}$$

$$\boxed{c_{i+1,j} = c_{i,j} + \left(K \frac{\Delta t}{\Delta z^2}\right)\left[c_{i,j+1} - 2c_{i,j} + c_{i,j-1}\right]}$$

Solve this eq. at every point, j, to make one step forward in time.

Repeat process, stepping from previous point to get forecast at any time t.

<u>Numerically unstable</u> if $\left(\frac{K \Delta t}{\Delta z^2}\right) > \frac{1}{2}$

$\therefore$ If K is determined by the physics of the flow, then

$$\boxed{\Delta t < \left(\frac{(\Delta z)^2}{2K}\right)}$$

To be <u>stable</u>

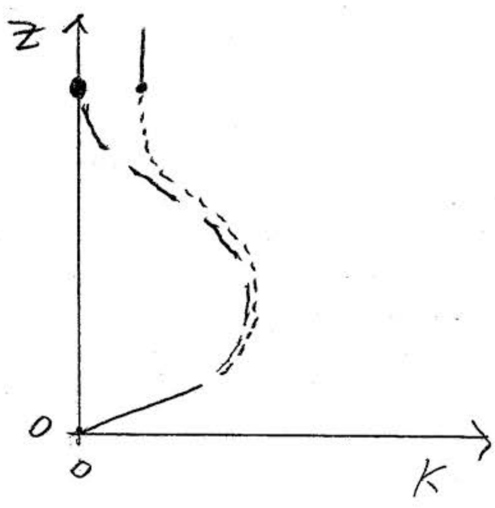
Parameterizing K

(Translates to parameterizing σ_y^2 @ σ_z^2 in air pollution studies)

1) $K=0$ at ground
2) $K=0$ where no turbulence (ie, above top of BL)
3) $K = k u_* z$ in surface layer $\left(= \overline{z'^2} \left| \frac{\partial \bar{u}}{\partial z} \right| \right.$ mixing length theory$\left. \right)$

   $\equiv \ell^2$ = mixing length $\propto k^2 z^2$ in sfc layer
4) Varies with turbulence intensity

   $K \uparrow$ as $TKE \uparrow$
5) $\therefore$ Also varies with static stability

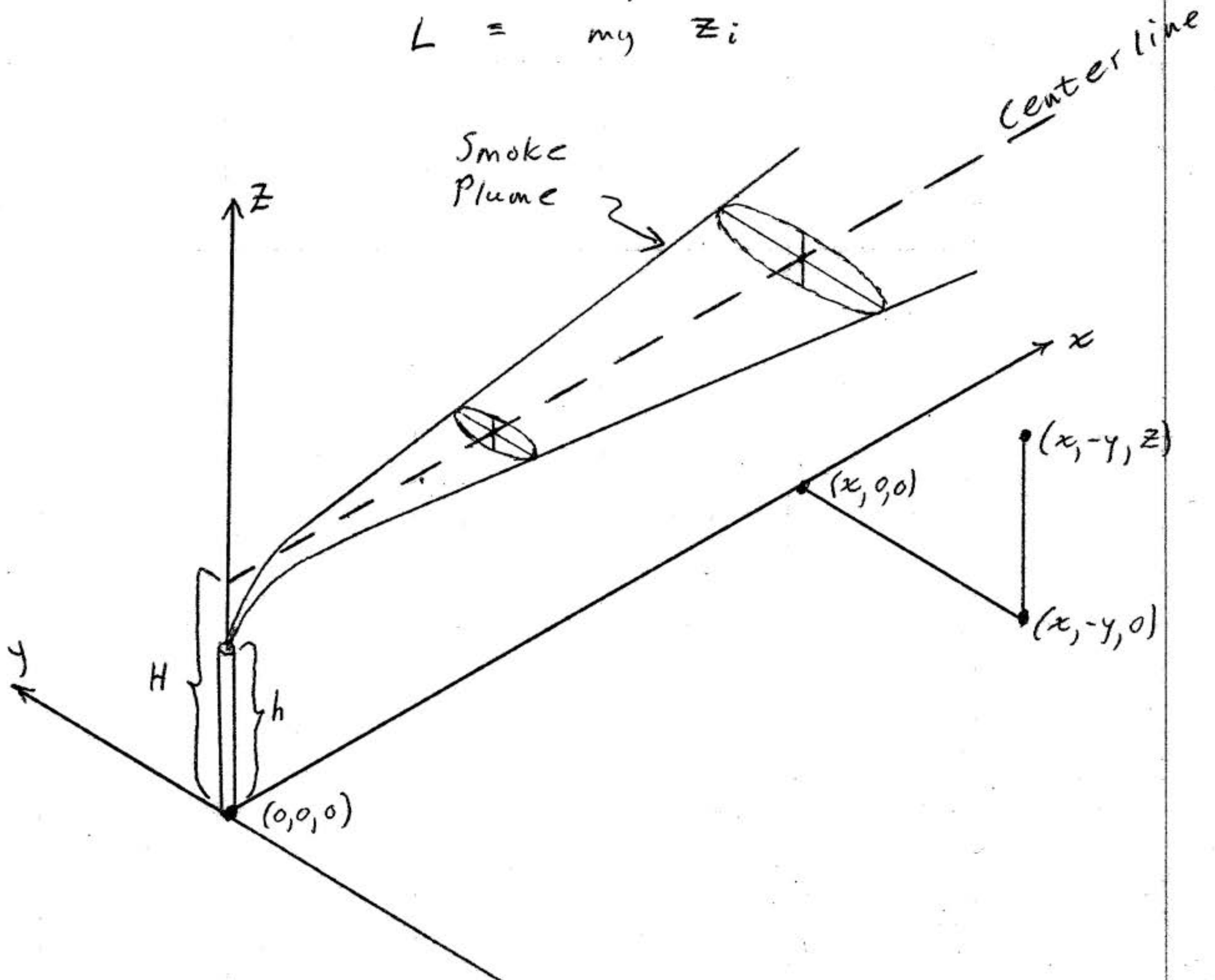


(K = const $\Rightarrow$ Ekman Spiral)

4. Practical Applications of Gaussian Diffusion

a. General Form

(Note: Turner's $\chi \equiv$ my c.
L = my z_i



$$\boxed{C(x,y,z,H) = \frac{Q}{2\pi\sigma_y\sigma_z\bar{u}} \cdot e^{-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2} \cdot \left[e^{-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2} + e^{-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2}\right]} \quad \text{IV.B.4.1}$$

Get σ_y & σ_z from tables, graphs, or formulas

$\sigma = \sigma(x, TKE, z_0)$

See Pasquill & Gifford's empirical curves: Turners Fig 3-2, 3-3

1) σ_y & σ_z vary with x, TKE

or t, $\bar{u}$, Ri, z_0

Get σ_z & σ_y from graphs, tables, or formulas.

Pasquill & Gifford have some empirical curves.

See Turner's Figs 3-2 (σ_y) & 3-3 (σ_z)

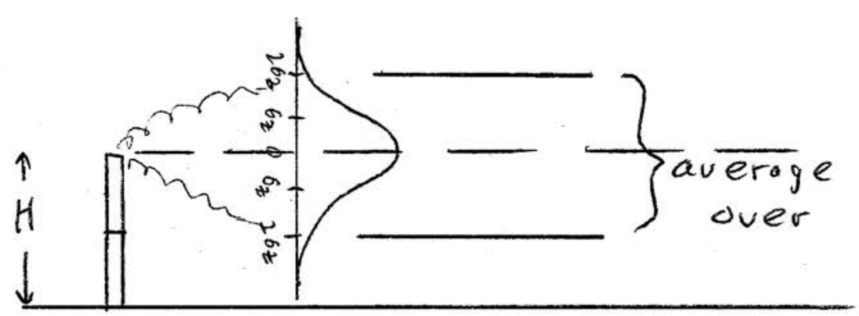
based on 10 min averages |

Will talk much more about σ_y & σ_z later!!

2) $\bar{u}$

Use an average $\bar{u}$ averaged over the plume

$$\bar{u}_{avg} = \frac{1}{4\sigma_z} \int_{H-2\sigma_z}^{H+2\sigma_z} \bar{u}\, dz$$

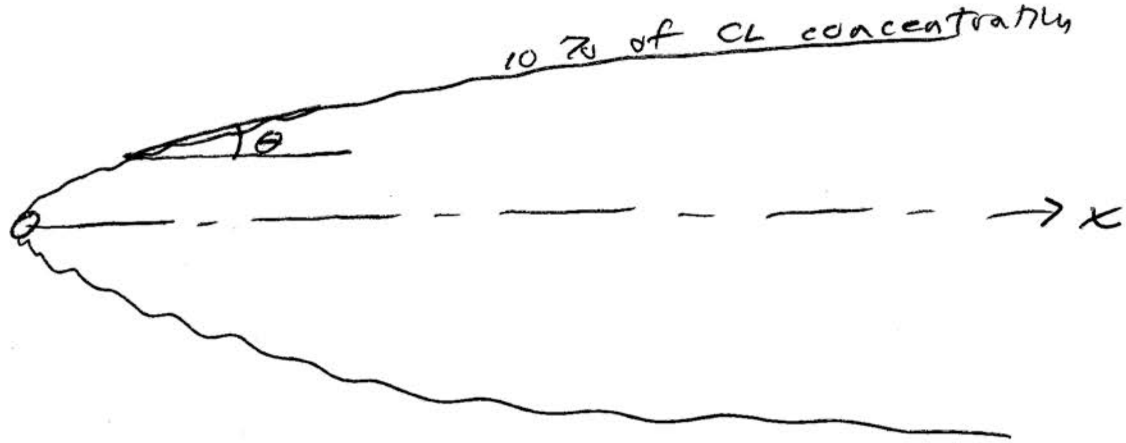


average the wind over this height.

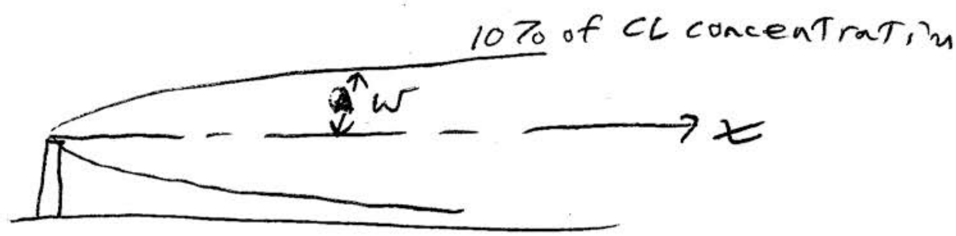
∴ "Surface" (10 m) wind is not always the best to use

~~Plume~~
(3) Photographs of plume spread

Top view:



Side view:



w = plume half width (or depth)

$\boxed{w = 2.15\ \sigma_z}$ for plume boundaries at $C = 10\%\ C_{CL}$

$$\tan\left(\frac{\theta}{2}\right) = \frac{2.15\ \sigma_y}{x}$$

Notes on the "Pasquill-Gifford Curves":

(Slade's stability curves don't represent any data very well.)

(~~Pasquill~~-Gifford's values of σ_y & σ_z are based on Mead's (1960) measurements of h & θ for open rural conditions, level terrain.)

best → (Note that Turner's workbook graphs of the stability curves best represent Pasquil's data.)

7.) σ_α = Standard Deviation of Horizontal Wind Direction, α

Prove: $\sigma_\alpha \approx \dfrac{\sigma_v}{\bar{u}}$

① Hint: Allign coordinate system such that x is along mean wind direction (ie, $\bar{v}=0$).

where $\sigma_\alpha \equiv \left(\overline{\alpha'^2}\right)^{1/2}$ = st. dev. of azimuthal wind direction

② Hint: $\bar{\alpha}=0$ α' = small

$\sigma_v \equiv \left(\overline{v'^2}\right)^{1/2}$ = st. dev. of wind in y-dir.

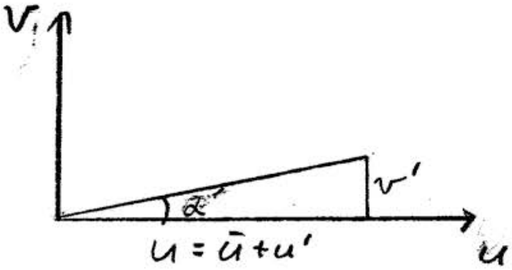
③ Hint $\bar{u}$ = mean wind in x dir. $\gg u', v'$

Proof

$u = \bar{u} + u'$

$v = \bar{v} + v'$

$= v'$ because $\bar{v}=0$.



$$\tan \alpha' = \frac{v}{u} = \frac{v'}{\bar{u}+u'}$$

But

$\tan \alpha' \approx \alpha'$ for small α'

Thus

$$\alpha' \approx \frac{v'}{\bar{u}+u'}$$

$$\alpha'^2 \approx \frac{v'^2}{(\bar{u}+u')^2} = \frac{v'^2}{\bar{u}^2}\frac{1}{\left(1+\frac{u'}{\bar{u}}\right)^2}$$

But using Taylor's Series

$$\left(1+\frac{u'}{\bar{u}}\right)^{-2} \approx 1 - 2\frac{u'}{\bar{u}} + 3\left(\frac{u'}{\bar{u}}\right)^2 - \cdots$$

$$\approx 1 \quad \text{for } u'/\bar{u} \ll 1$$

Thus: $\alpha'^2 \approx \dfrac{v'^2}{\bar{u}^2}$

Note, air poll. literature uses symbol θ instead of α. ∴ don't confuse it w pot. temp.

Upon averaging

$$\overline{\alpha'^2} = \frac{\overline{v'^2}}{\bar{u}^2} \xRightarrow{\sqrt{\ }} \boxed{\sigma_\alpha = \frac{\sigma_v}{\bar{u}}}$$

IV. B.5.2

IV. B.5.3

$\boxed{\sigma_\alpha = \dfrac{\sigma_y}{x}}$ using Taylors stat. theory. (only works close to the source).