

Fig. 11.19 Exaggerated idealization showing thermals with strong updrafts covering a relatively small fraction of the area, with weak downdrafts in between.

Although not plotted here, temperature histograms show negatively skewed frequency distributions in the ML, which change to symmetric distributions in the entrainment zone, similar to those of vertical velocity.

Joint probability distributions showing the relative frequency of vertical velocities and temperature fluctuations are shown in Fig 11.20 (Mahrt and Paumier, 1984; Deardorff and Willis, 1985). In the bottom 2/3 of the ML (Fig 11.20b) there is a predominance of cool downdrafts. At the top of the ML, however, we see a peak associated with cool updrafts (Fig 11.20c). In fact, there is a broad peak associated with both updrafts and downdrafts that are cool. These are the tops of thermals overshooting into the entrainment zone and then sinking back down.



various combinations of the two groups: $[z/z_i]$ and $[\overline{w'\theta_v'}/\overline{w'\theta_v'_s}]$. This greatly simplifies the design and conduct of our experiment.

Suppose the heat flux data from Fig 3.7 (reproduced as Fig 9.1a) represents the results of our experiment. The curves in this data set exhibit a common shape: there is a nearly-linear decrease of heat flux from the surface value to a small negative value near the top of the mixed layer. Above that, the flux reduces toward zero. As we shall soon learn in Chapter 11, the average depth of the mixed layer is frequently taken as the height where the heat flux is most negative. When each of the data curves is replotted in terms of the two dimensionless groups, as shown in Fig 9.1b, we happily find that all of the data is closely clustered around a single curve.

For step (4), an obvious choice of curve is a straight line between the surface and the top of the mixed layer. By definition we want the intercept of this line to equal 1, and by inspection it looks like the slope is roughly 1.2. This results in:

$$\frac{\overline{\mathbf{w}'\boldsymbol{\theta}_{v}}'}{\overline{\mathbf{w}'\boldsymbol{\theta}_{v}'}_{s}} = 1 - 1.2 \left[\frac{z}{z_{i}}\right] \quad \text{for } 0 \le (z / z_{i}) \le 1$$

which is also plotted in Fig 9.1b. As an independent test, the buoyancy flux data from Figs 3.1b, 3.2b, and 3.3b confirm the validity of our curve.



Fig. 9.1 Raw heat flux data from a simulation of Wangara Day 33 (a) replotted in a dimensionless framework (b). The empirical straight line estimate from similarity theory is also shown in (b).

Discussion. We hope to be able to use this equation to diagnose the value of the buoyancy flux at any height within the interior of a convective mixed layer on any other day at any other location, assuming we know the surface flux and the mixed layer depth. Even without this equation, we could use Fig 9.1b to determine the flux at any height.

$$\frac{\varepsilon_{q} \ z_{i}}{w_{*} \ q_{*}^{ML^{2}}} = 0.43 \left(\frac{z}{z_{i}}\right)^{-4/3}$$
(9.6.4r)

9.6.5 Example

Problem: Use similarity theory to develop an expression for vertical velocity variance, $\overline{w'}^2$, as a function of height given the (synthetic) measurements in Table 9-2.

Table 9-2. Synthetic vertical velocity variance	data.
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z	$\overline{w'^2}$ (m ² s ⁻²)					$\overline{w'^{2}}$ (m ² s ⁻²)		
(m)	Day 1	Day 2	Day 3	Day 4				
1500				0.4				
1400				0.6				
1300				0.8				
1200				1.0				
1100				1.1				
1000		0.10		1.3				
900		0.16		1.4				
800		0.22		1.4				
700	0.6	0.30		1.5				
600	0.9	0.37		1.6				
500	1.2	0.39		1.6				
400	1.5	0.40		1.5				
300	1.6	0.40	0.20	1.4				
200	1.5	0.36	0.36	1.2				
100	1.2	0.28	0.40	1.0				
0	0.8	0.20	0.20	0.8				
z _i (m)	750	1000	350	1500				
$\overline{w'\theta_{v's}}$ (K m/s)	0.33	0.03	0.09	0.16				

Solution: Although this data set exhibits a variety of magnitudes over a range of heights, each of the individual data curves has the same shape (see Fig 9.3a) — a clue that they are created by a common physical process that could possibly be described empirically.



The tabulated data includes mixed layer depth and surface heat flux. From these, we can calculate the length and velocity scales, z_i and w_* , assuming $g/\overline{\theta_v} = 0.0333 \text{ ms}^{-2} \text{K}^{-1}$. Using Buckingham Pi analysis, or by inspection for this simple case, we can create the following dimensionless groups: z/z_i , and $\overline{w'}^2/w_*^2$. When the original data is replotted in this dimensionless framework, lo and behold most of the data points collapse into a single curve, as plotted in Fig 9.3b. Thus, all of the data are similar, allowing us to use similarity theory.

By trial and error using simply power laws, we find that the following equation approximates the shape of the data, and is plotted as the curve in Fig 9.3b:

$$\frac{\overline{w'^2}}{w_*^2} = 1.7 \left(\frac{z}{z_i}\right)^{2/3} \cdot \left(1 - 0.8 \frac{z}{z_i}\right)^2$$

This equation is almost identical to (9.6.3c), except for the value of the regression coefficient (i.e., 1.7 vs. 1.8).

Discussion: The hope is that this equation, and the corresponding curve in Fig 9.3b, are "universal"; that is, they should work just as well for other free convection

situations. For example, on a different day with $z_i = 1200$ m and $\overline{w'\theta_{v_s}} = 0.2$ K m/s, we might wish to know the vertical velocity variance at z = 500 m without performing an experiment to measure it ourselves. At that height, $z/z_i = 0.42$, which can be used in the above equation or with Fig 9.3b directly to give us $\overline{w'}^2/w_*^2 = 0.42$. Since $w_* = 2$ m/s based on flux and z_i data given at the start of this example, we can easily solve for $\overline{w'}^2 = 0.42$.

1.68 $m^2 s^{-2}$.

The above example was more than just a contrived didactic case. If you look back at Fig 4.2a, you will see that vertical velocity variance measurements do indeed vary with height as described here.

9.7 The Log Wind Profile

One important application of similarity theory is to the mean wind profile in the surface layer. Since people spend most of their lives within the surface layer, the variation of wind speed with height affects their daily lives. The nature of this profile dictates the structure of buildings, bridges, snow fences, wind breaks, pollutant dispersion, and wind turbines, for example. Also, the surface layer wind profile has been studied extensively because of its accessibility to surface-based measurements.

As shown in Fig 9.4, the wind speed usually varies approximately logarithmically with height in the surface layer. Frictional drag causes the wind speed to become zero close to the ground, while the pressure gradient forces cause the wind to increase with height.



Figure 4.



Figure 5.



x