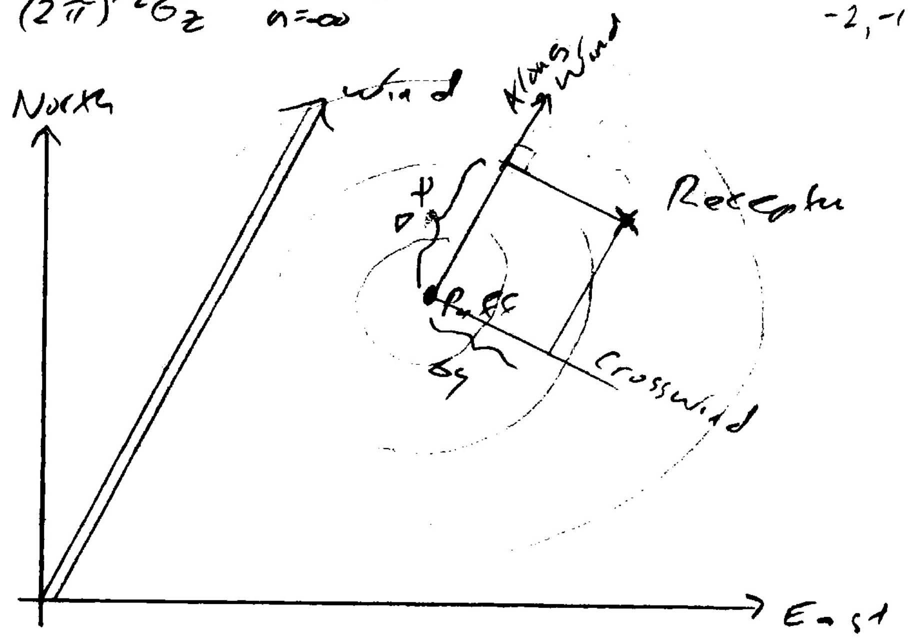
2.1.1 Integrated Puff Sampling Function. p 2-4 of Tech Guide

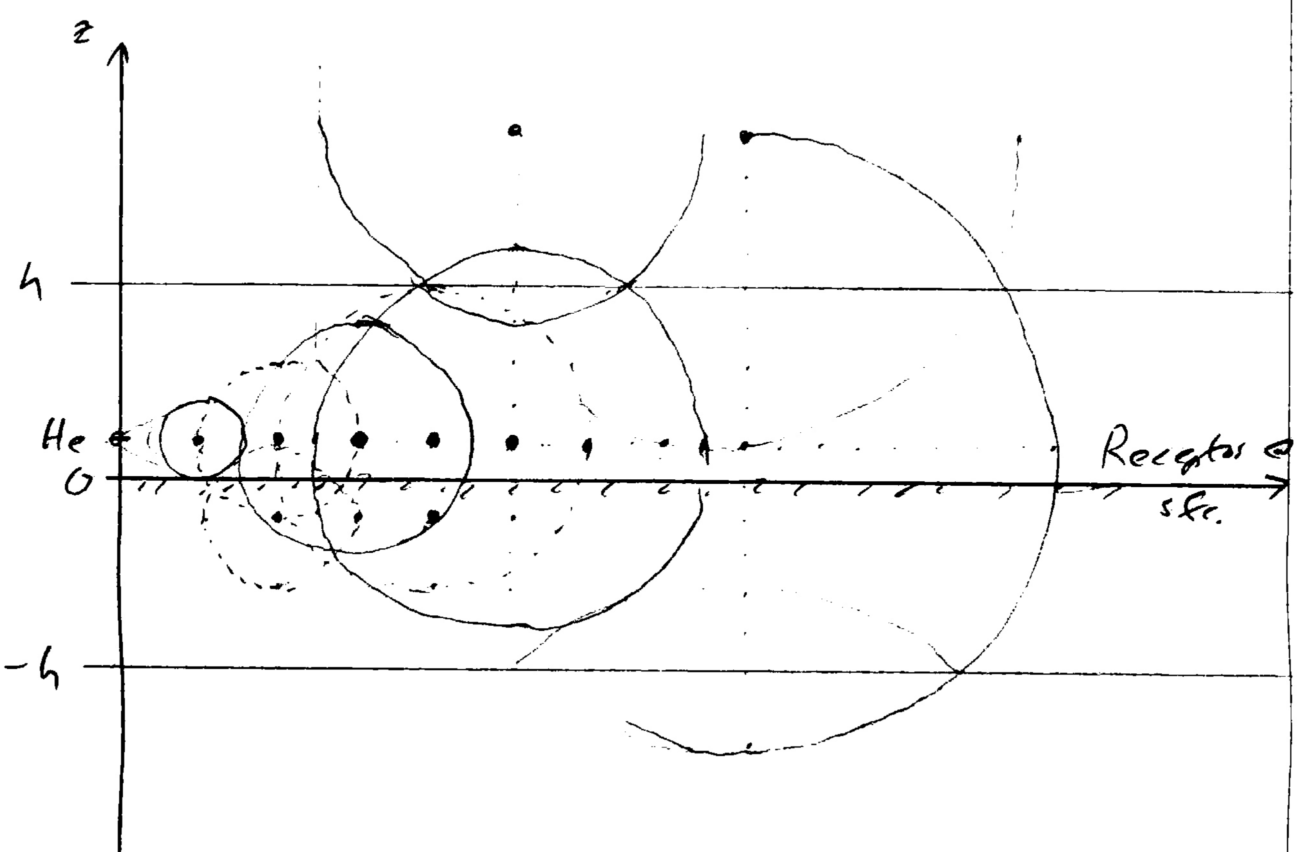
Instantaneous:

$$C = \frac{Q}{2\pi \sigma_x \sigma_y} g\, e^{-\frac{1}{2}\left(\frac{\Delta x}{\sigma_x}\right)^2} e^{-\frac{1}{2}\left(\frac{\Delta y}{\sigma_y}\right)^2} \qquad \text{Eq (2-1)}$$

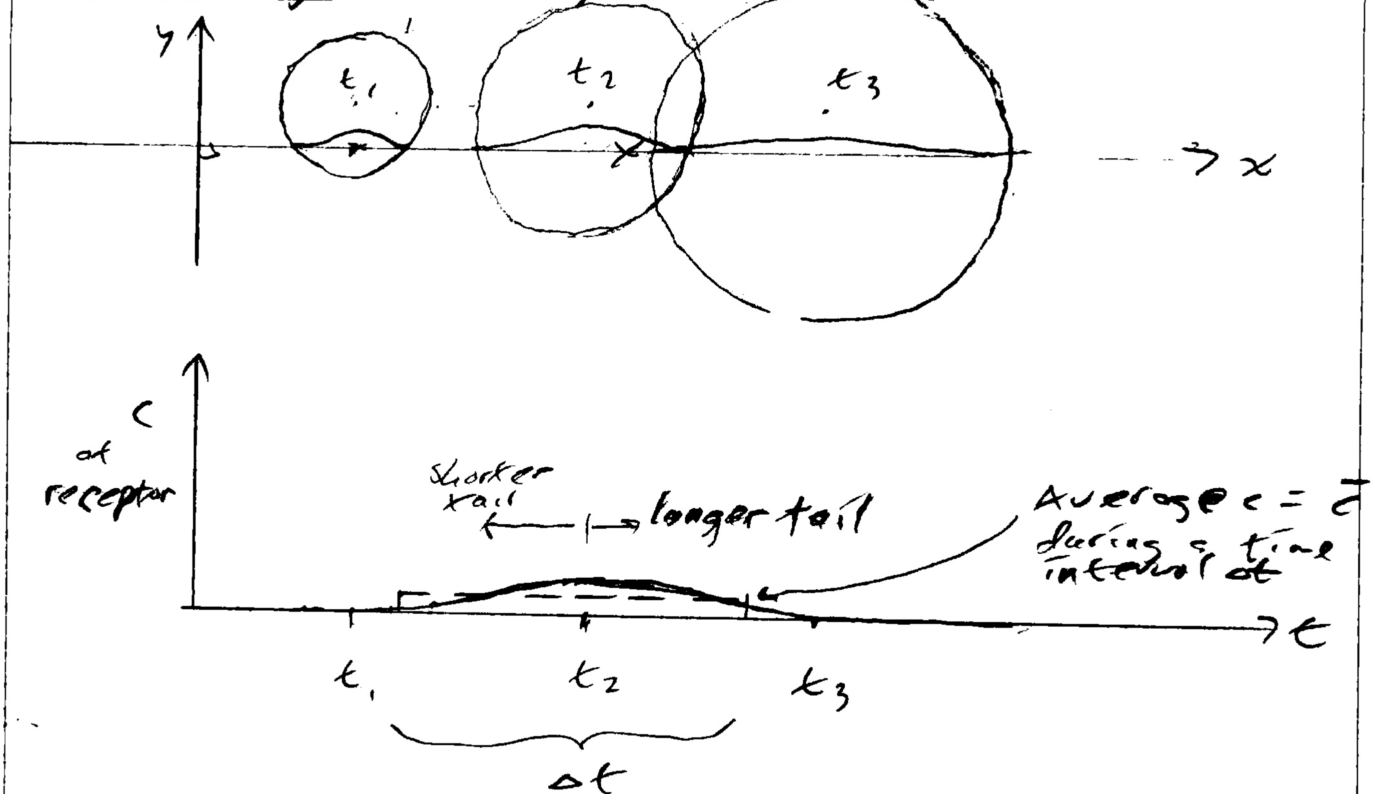
$$\text{where } g = \frac{2}{(2\pi)^{1/2}\sigma_z} \sum_{n=-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{H_e + 2nh}{\sigma_z}\right)^2} \qquad \text{Eq (2-2)}$$

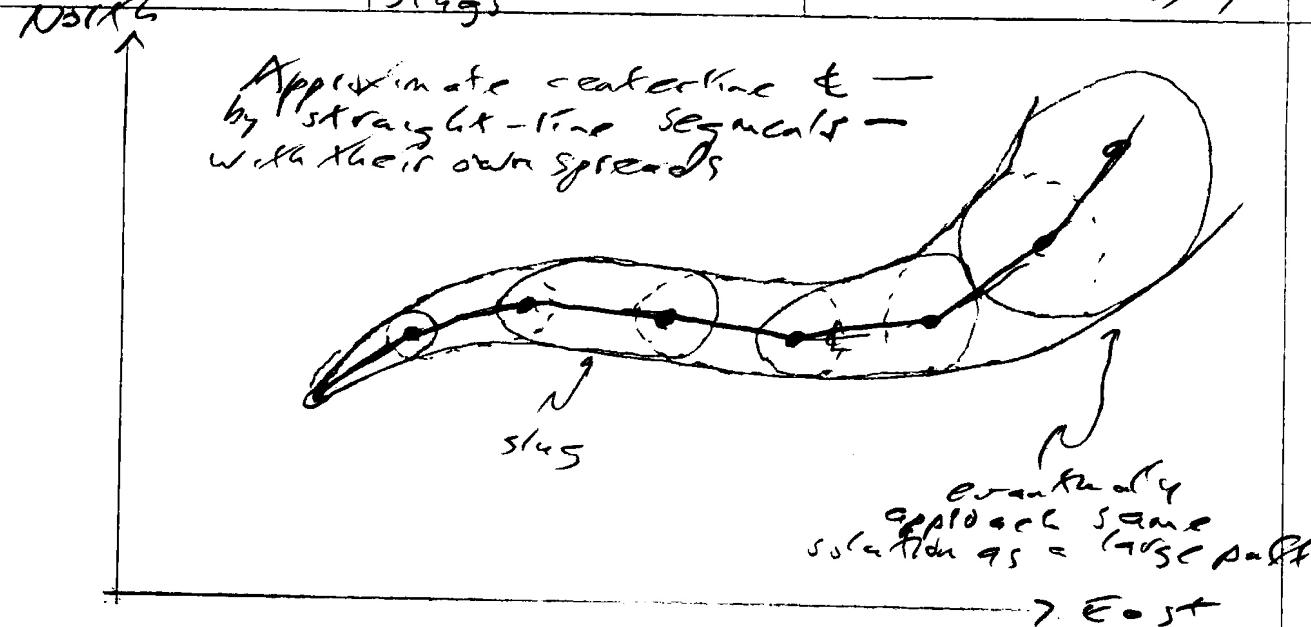
for n = integer -2, -1, 0, 1, 2





Time Averaged as one puff blows

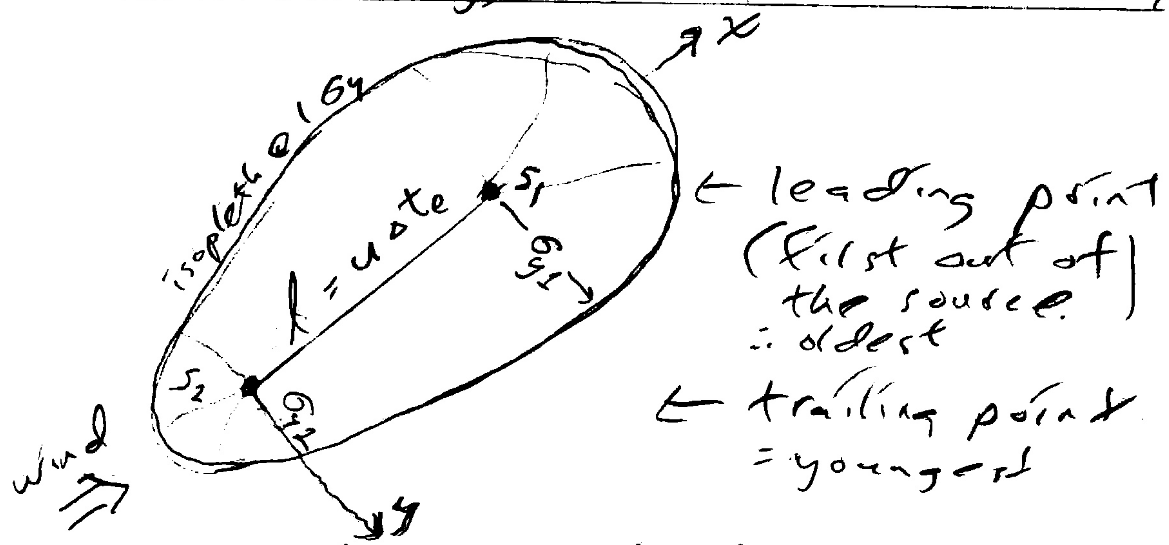




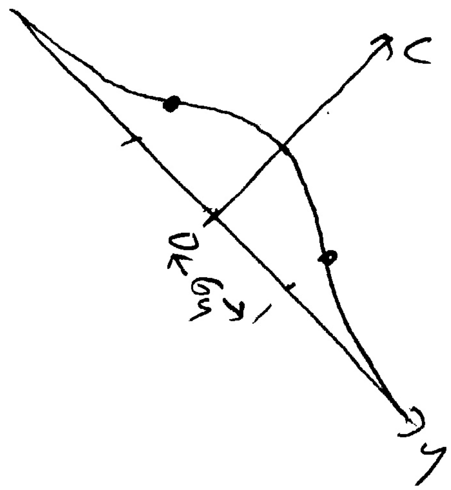
Advantage: - avoids need for excessively large number of puffs for initial plume.

- ∴ faster computation. (need to Lagrangian advect only the 2 slug end points), But since each point is shared w̄ 2 slugs,

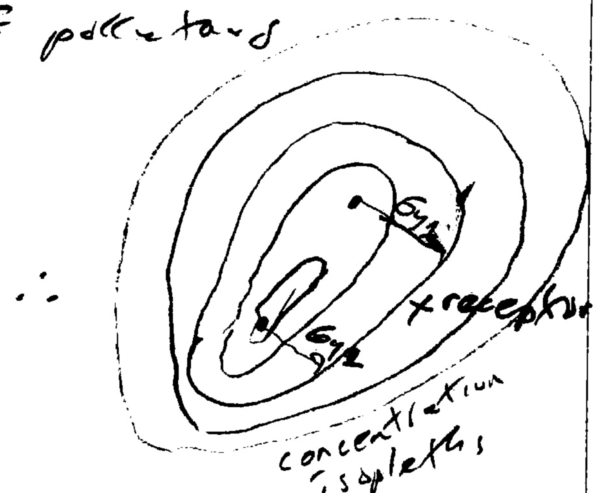
∴ need only 1 Lagrangian prediction / slug.



Let:
u = mean wind speed (m/s)
Δt_e = emission duration for this slug (s)
σ_y = cross-wind spread of pollutant



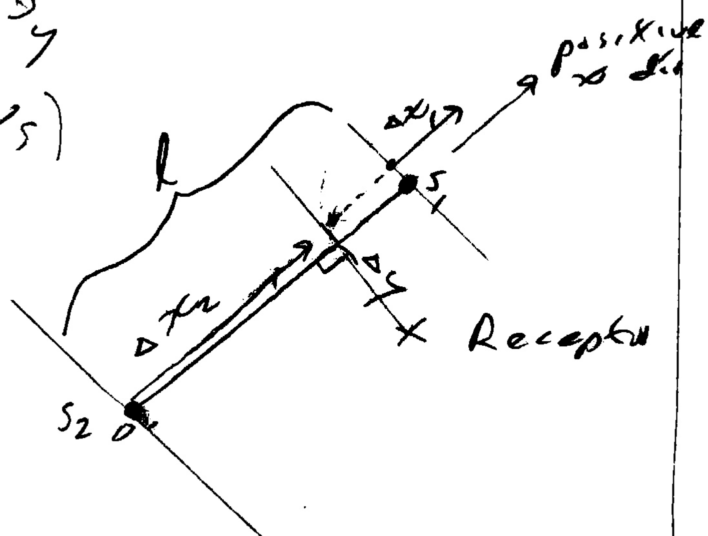
∴



q = emission rate (g/s)
Δx_2 = distance to receptor from S_2 in direction toward S_1

l = length of slug in horizontal $= u \cdot \Delta t_e$

Δx_1 = dist. to receptor negative in this example with magnitude $|l - \Delta x_2|$
$\therefore \Delta x_1 = -(l - \Delta x_2)$
$= \Delta x_2 - l$



$$c(t) = \frac{F \cdot q}{(2\pi)^{1/2} u' \sigma_y} \cdot g \cdot e^{-\frac{1}{2}\left(\frac{\Delta y}{\sigma_y}\right)^2 \left(\frac{u}{u'}\right)^2} \qquad (2\text{-}14)$$

$$F = \frac{1}{2}\left\{ \operatorname{erf}\left(\frac{\Delta x_2}{2^{1/2}\sigma_{y_2}}\right) - \operatorname{erf}\left(\frac{\Delta x_1}{2^{1/2}\sigma_{y_1}}\right)\right\} \qquad (2\text{-}15)$$

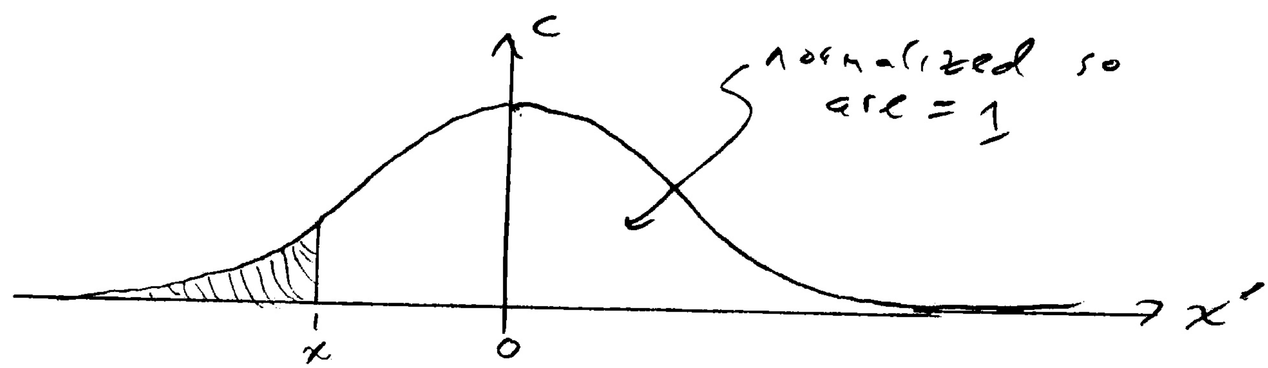
= "traversal fy fnt"

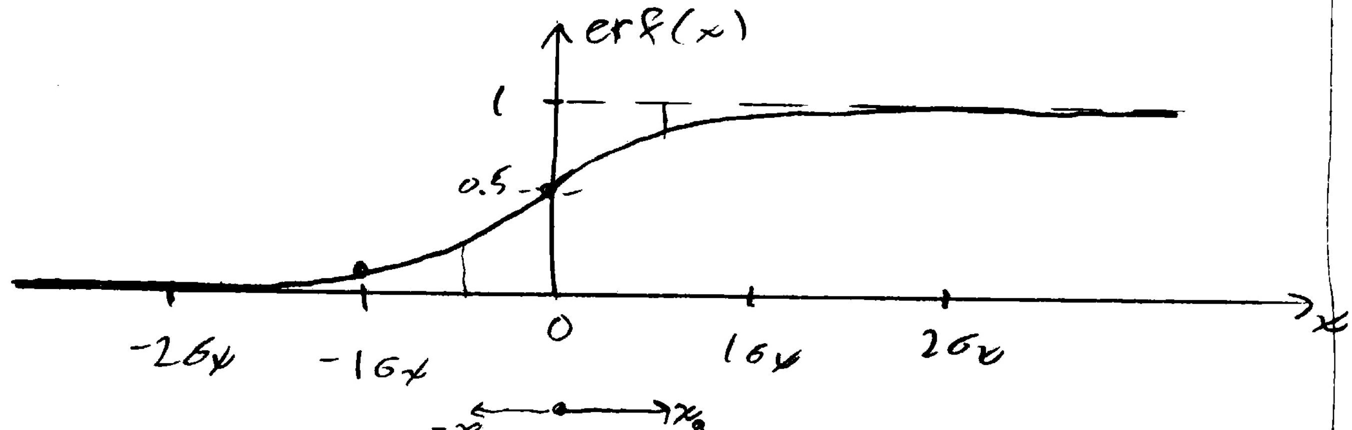
$$g = \frac{2}{(2\pi)^{1/2}\sigma_z} \sum_{n=-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{H+2nh}{\sigma_z}\right)^2}$$

where $u' \equiv (u^2 + \sigma_u^2)^{1/2}$

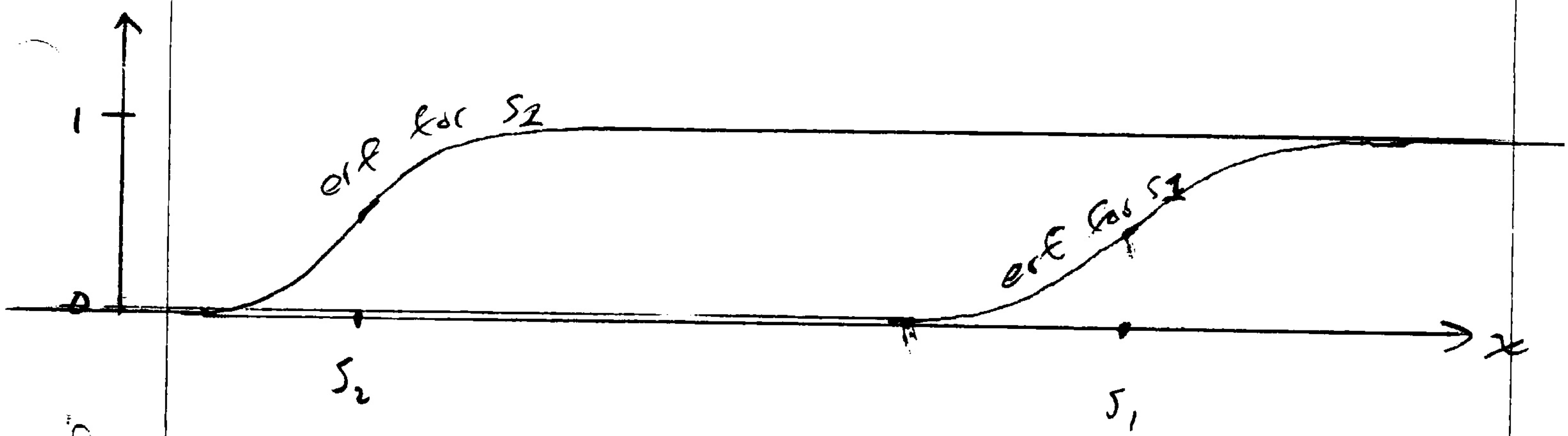
fudge to prevent divide-by-zero for calm winds $u=0$.

Recall $\operatorname{erf}(x) = \int_{-\infty}^{x} G(x')\,dx'$

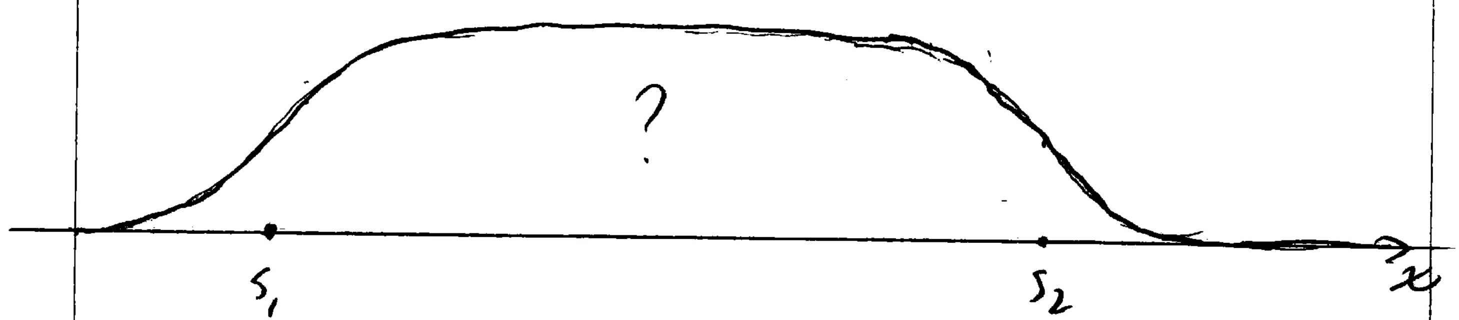




Note: $\operatorname{erf}(-x_a) = 1 - \operatorname{erf}(x_a)$ by inspection



What do you get when you subtract erf for S_1 from erf for S_2?



Answer: you get a "slug".

What happens when $u \to 0$

- Look at crosswind (by) term: ? $\to 1$

- Look at initial factor $\frac{F \cdot q}{\dots}$ = ?

Ans:

are the units reasonable?

- What happens to ℓ = ? Answer: $\ell \to 0$
- What happens to Δx_1 = ? Answer: $\Delta x_1 = \Delta x_2 \equiv 0$
- What happens to the slug term F ?

detour

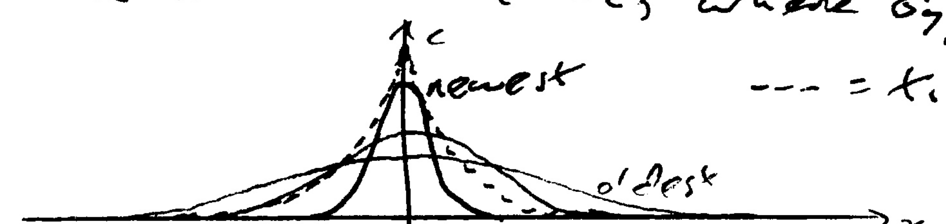
- But the Tech Note (p2-8, last ¶) says

$$F \to -\mathrm{erf}\left(\frac{\Delta x}{2^{1/2}\sigma_y}\right)$$

Why?

Answer: emissions are still happening over a finite time Δt, where $\sigma_y \propto \Delta t$. or $\propto \Delta t^{1/2}$

--- = time avg.



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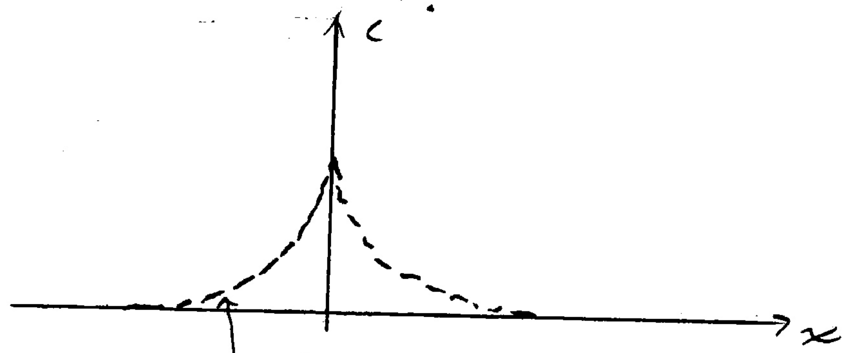
Answer continued:



is the $-x$ portion of erf.

Is this true? Can check w HW

where $\bar{\sigma}_y = \text{avg}(\sigma_y)$

– end of this detour –

Recall from Taylor's stat. theory

Near source (small times)
for $t < t_L$ $\quad \sigma_y \approx \sigma_v (t)$

Far from source (longer time)
for $t > t_L$

$$\sigma_y \approx \sigma_v (2 \cdot t \cdot t_L)^{1/2}$$

where t_L = Lagrangian time scale ≈ 60 s.

But for air quality, need $\bar{c}$ avg over time

p 2-9, eq 2-16 thru 2-20 is for the special case of the newest slug where the S_2 point is still at the emission source.

But can ignore this eq.

Because the last sentence on p 2-9 says that a more general solution is found by numerically integrating eq 2-14 over time.

Your will do this in your HW (R, matlab, Python)