Similarity Relationship Handbook

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Tech Note Number BLRT-90-1

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Key

Symbol ***Variable Name*** ***Static Stability***

Range: [*lowest expected* to (*typical range*) to *maximum expected*] *units*

Equations:

symbol (*height range limits*) = (*best fit parameter* ± *95% of all data*) · terms (*eq #*)

Required input data: input variables are listed and defined

Comments: Special conditions, characteristics, or limitations are listed.

Sample Calculations with Typical Values: Example solution of one equation.

References Listed by Equation Number:

Common Symbols

0	denotes surface value when used as an argument
h	depth of the neutral boundary layer (base of capping inversion) or depth of the stable boundary layer (height where turbulence is near zero)
u_*	friction velocity = $[\ \overline{u'w'}(0)^2 + \overline{v'w'}(0)^2\ ]^{1/4}$
$\overline{u'w'}$ & $\overline{v'w'}$	kinematic vertical flux of U and V momentum, respectively.
w_*	convective velocity scale = $[\ (g/\overline{\theta})\ z_i\ \overline{w'\theta'}(0)\ ]^{1/3}$, where g is gravitational accel.
$\overline{w'\theta'}$	kinematic vertical heat flux
z	height above ground
z_i	depth of the convective mixed layer
θ	potential temperature

(For more details, see Stull, R.B., 1988: *An Intro. to Bound. Layer Meteor.*, Kluwer. 666 pp.)

σ_T Temperature Standard Deviation — Stable

Range: [0 to (0 to 0.5) to 1.0] °K

Equations:

$$\sigma_T (0 \le z \le h) = (3.4 \pm 1.0) \cdot \frac{[-\overline{w'\theta'}(z)]}{[\overline{u'w'}^2(z) + \overline{v'w'}^2(z)]^{1/4}} \tag{1}$$

$$\sigma_T (0 \le z \le h) = (2.45 \pm 2.45) \cdot \left[1 - (z/h)^{0.4}\right]^{0.75} \cdot \frac{[-\overline{w'\theta'}(0)]}{u_*} \tag{2}$$

$$\sigma_T (0 \le z \le h) = (1.7 \pm 1.7) \cdot [1 - (z/h)]^{1.25} \cdot \frac{[-\overline{w'\theta'}(0)]}{u_*} \tag{3}$$

$$\sigma_T (z > h) \approx 0 \tag{4}$$

Required input data for (2) and (3):

$\overline{w'\theta'}(0)$	turbulent kinematic heat flux near the surface (z=0)
u_*	friction velocity at the surface (z=0)
h	depth of the stable boundary layer

Comments:

a) σ_T is never negative.

b) σ_T is usually largest near the surface, and decreases with height to a value of 0 at z=h.

c) There is large scatter (±100%) of the observations about the curves in (2) and (3).

d) Eq (1) has less scatter, but is difficult to use because it requires as input the values of variables above the surface.

e) Gravity (buoyancy) waves can increase σ_T both below and above h.

Sample Calculations with Typical Values:

Define $\theta_*^{SL} \equiv \frac{[-\overline{w'\theta'}(z=0)]}{u_*}$

Let $\overline{w'\theta'}(0) = -0.02$ K m/s , $u_* = 0.1$ m/s, h = 300 m. Solve eq (3) at z = 10 m.

Thus $\theta_*^{SL} = 0.2$ °K , and $\sigma_T (z=10m) = 1.7 \cdot [1 - 10/300]^{1.25} \cdot (0.2°K) = 0.33°K$

References Listed by Equation Number (see detailed reference list attached):

(1) Nieuwstadt (84); Sorbjan (86, 87); Stull (88); Shao & Hacker (90)

(2) Caughey et al (79); Stull (88)

(3) Sorbjan (88)

σ_T Temperature Standard Deviation — Neutral

Range: [0 to (0) to 0] °K

Equations:

$$\sigma_T(z) = 0 \qquad (1)$$

Required input data:

(none)

Comments:

a) σ_T is always zero by definition in the neutral boundary layer, because "neutral" means there is no potential temperature change with height. Thus, although there is turbulence in the neutral case, there are no temperature variations associated with it.

b) The atmosphere is rarely perfectly neutral. For this "near-neutral" cases, use the appropriate stable or unstable equations.

Sample Calculations with Typical Values:

σ_T (z=10m) = 0°K

References Listed by Equation Number (see detailed reference list attached):

(1) Stull (88)

σ_T Temperature Standard Deviation — Unstable

Range: [0 to (0.1 to 1.5) to 2.25] °K

Equations:

$$\sigma_T (z > 1.5\, z_i) \approx 0 \tag{1}$$

$$\sigma_T (0.9\, z_i < z < 1.5\, z_i) = (4 \pm 4) \cdot \frac{\overline{w'\theta'}(0)}{w_*} \tag{2}$$

$$\sigma_T (0.03\, z_i \le z \le 0.9\, z_i) = (1.34 \pm 0.34) \cdot (z / z_i)^{-1/3} \cdot \frac{\overline{w'\theta'}(0)}{w_*} \tag{3}$$

$$\sigma_T (0 < z < 0.1 z_i) = (0.95 \pm 0.5) \cdot \left[\frac{0.4 \cdot z \cdot g \cdot \overline{w'\theta'}(0)}{\overline{\theta} \cdot u_*^3} \right]^{-1/3} \cdot \frac{\overline{w'\theta'}(0)}{u_*} \tag{4}$$

$$\sigma_T (0 < z \le 0.9\, z_i) = (1.4 \pm 0.3) \cdot (z / z_i)^{-1/3} \cdot (1 - 1.2\, z/z_i)^{2/3} \cdot \frac{\overline{w'\theta'}(0)}{w_*} \tag{5}$$

$$\sigma_T^2 (0.01 z_i \le z \le z_i) = \left[2 \cdot \frac{(1 - z / z_i)^{4/3}}{(z / z_i)^{2/3}} + 8 \cdot R^{4/3} \cdot \frac{(z / z_i)^{4/3}}{(1 - z/z_i + D)^{2/3}} \right] \cdot \left[\frac{\overline{w'\theta'}(0)}{w_*} \right]^2 \tag{6}$$

Required input data:

D	ratio of depth of capping inversion to mixed layer depth (only for eq 6)
R	ratio of heat flux at top of mixed layer to heat flux at surface (only eq 6)
$\overline{w'\theta'}(0)$	turbulent kinematic heat flux near the surface (z=0)
u_*	friction velocity at the surface (needed only for eq (4) in the surface layer)
w_*	convective velocity scale = $[(g / \overline{\theta}) \cdot z_i \cdot \overline{w'\theta'}(0)]^{1/3}$
z_i	depth of the unstable boundary layer (= height of first capping inversion)

Comments:

a) σ_T is never negative.

b) σ_T is usually large near z_i because of cool thermals overshooting into warmer air, and because of gravity (buoyancy) waves. There is 100% scatter in this region.

c) an additional class of equations involving local similarity was not included here because they requires knowledge of turbulent heat and momentum fluxes at height z. These are usually as unknown as the desired variable.

Sample Calculations with Typical Values:

Define $\theta_*^{ML} \equiv \dfrac{\overline{w'\theta'}(z=0)}{w_*}$

Let $\overline{w'\theta'}(0) = 0.2$ K m/s, $w_* = 1.88$ m/s, $z_i = 1000$ m.

Thus $\theta_*^{ML} = 0.106$ °K, and equation (3) can be solved at $z = 30$ m:

$$\sigma_T(z{=}30\text{m}) = 1.34 \cdot [\,30/1000]^{-1/3} \cdot (0.106°\text{K}) = 0.457\ °\text{K}$$

References Listed by Equation Number (see detailed reference list attached):

(1) Stull (88)

(2) Deardorff (74); André et al (78); Lenschow et al (80); Smedman & Högström (83)

(3) Wyngaard et al (71); Lenschow et al (80); Zhao et al (85); Chou & Zimmerman (89); Huynh (90)

(4) Wyngaard et al (71); Maitani & Ohtaki (87)

(5) Caughey et al (79); Sorbjan (86)

(6) Sorbjan (89)

Typical Ranges: Amiro (90); Andren (90)

σ_U Mean Wind Standard Deviation — Stable

Range: [0 to (0.01 to 0.5) to 1.5] m/s

Equations:

$$\sigma_U (0 \le z \le h) = (2.45 \pm 0.5) \cdot \left[1 - (z/h)^{1/2} \right]^{1/2} \cdot u_* \quad (1)$$

$$\sigma_U (0 \le z \le h) = (2.0 \pm 0.25) \cdot \left[1 - (z/h) \right]^{3/4} \cdot u_* \quad (2)$$

$$\sigma_U (0 \le z \le 0.1\,h) = (2.3 \pm 0.2) \cdot u_* \quad (3)$$

$$\sigma_U^2 + \sigma_V^2 \; (0 \le z \le 0.1\,h) = (8.5 \pm 2.) \cdot u_*^2 \quad (4)$$

$$\sigma_U^4 + \sigma_V^4 \; (0 \le z \le h) = (150. \pm 105.) \cdot \left[\overline{u'w'}^2(z) + \overline{v'w'}^2(z) \right] \quad (5)$$

$$\sigma_U (z > h) \approx \text{unknown (depends on shear \& Richardson number at z)} \quad (6)$$

Required input data:

u_* — friction velocity at the surface (z=0)

h — depth of the stable boundary layer

Comments:

a) σ_U is never negative.

b) σ_U is usually largest near the surface, and decreases with height to a small value at z=h.

c) Eq (5) is difficult to use because it requires as input the values of turbulent momentum fluxes above the surface.

Sample Calculations with Typical Values:

Let $u_* = 0.1$ m/s, $h = 300$ m. Solve eq (2) at $z = 10$ m.

Thus $\sigma_U (z=10m) = 2.0 \cdot [1 - 10/300]^{0.75} \cdot (0.1 \text{ m/s}) = 0.19$ m/s

References Listed by Equation Number (see detailed reference list attached):

(1) Caughey et al (79); Stull (88)

(2) Sorbjan (88)

(3) Smedman (88)

(4) Sorbjan (86)

(5) Nieuwstadt (84); Sorbjan (86, 87); Stull (88)

σ_U Mean Wind Standard Deviation — Neutral

Range: [0 to (0.5 to 2.0) to 3.2] m/s

Equations:

$$\sigma_U\ (0 \le z \le 0.1\ h) \;=\; (\,2.35 \pm 0.17\,) \cdot u_* \qquad (1)$$

$$\sigma_U^2 + \sigma_V^2\ (0 \le z \le 0.1\ h) \;=\; (\,8.5 \pm 0.6\,) \cdot u_*^2 \qquad (2)$$

$$\sigma_U\ (0 \le z \le 0.35\ h) \;=\; (\,2.4 \pm 0.5\,) \cdot (\,1 - z/h\,)^{5/2} \cdot u_* \qquad (3)$$

$$\sigma_U^2\ (0 \le z \le h) \;=\; (\,6.1 \pm 0.4\,) \cdot (\,1 - z/h\,)^2 \cdot u_*^2 \;+\; (\,z\,/\,h) \cdot \overline{u'^2}_{top} \qquad (4)$$

$$\sigma_U\ (0 \le z \le 2h) \;=\; (\,2.5 \pm 0.7\,) \cdot e^{-1.5\ z/h} \cdot u_* \qquad (5)$$

Required input data:

u_*	friction velocity at the surface (z=0)
h	depth of the neutral boundary layer
$\overline{u'^2}_{top}$	$= \sigma_U^2$ at top of the boundary layer (only for eq (4))

Comments:

a) σ_U is never negative.

b) σ_U is often larger than σ_V and σ_W, because the mean wind generates turbulence.

c) In most atmospheric boundary layers, the top of the neutral boundary layer (h) is marked by a capping inversion. In some theoretical/numerical studies such as was used to get eq (5), however, the neutral layer was assumed to be infinitely thick. For this latter case, h is defined as the lowest height where the wind perpendicular to the geostrophic wind first becomes zero (analogous to an Ekman layer height).

Sample Calculations with Typical Values:

Let: $u_* = 0.4$ m/s, h = 500 m. Solve (3) for z = 10 m:

σ_U (z=10m) = 2.4 · (1 - 10/500) · 0.4 m/s = 0.94 m/s

References Listed by Equation Number (see detailed reference list attached):

(1) Panofsky et al (77); Nichols & Readings (79); Smith (80); Panofsky & Dutton (84); Grant (86); Gunther & Lamb (89); Kristensen et al (89)

(2) Sorbjan (86)

(3) Brost et al (82); Grant (86); Stull (88); Andren (90)

(4) Nichols and Readings (79); Grant (86)

(5) Stull (88, based on data from Mason & Thomson (87))

Range: [0 to (0.2 to 1.5) to 3.0] m/s

Equations:

$$\sigma_U\,(0.1 \le z \le z_i) = (0.7 \pm 0.3)\cdot w_* \qquad (1)$$

$$\sigma_U^2\,(0 \le z \le z_i) = (1.44 \pm 0.4)\cdot (z/z_i)^{2/3}\cdot (1 - 0.7\, z/z_i)^2\cdot w_*^2 + (1.11 \pm ?)\cdot u_*^2 \qquad (2)$$

$$\sigma_U\,(z_i \le z \le 1.4\, z_i) = (0.48 \pm ?)\cdot (z/z_i)^{-1.8}\cdot w_* \qquad (3)$$

$$\sigma_U\,(0 \le z \le 0.1\, z_i) = \left[(8.0 \pm ?) - \frac{(0.64 \pm ?)\cdot z \cdot g \cdot \overline{w'\theta'}(z)}{\overline{T}\cdot\left(\overline{u'w'}^2(z) + \overline{v'w'}^2(z)\right)^{3/4}} \right]^{1/3} \qquad (4)$$

$$\sigma_U^2 + \sigma_V^2\,(0.1 \le z \le z_i) = (0.9 \pm 0.5)\cdot w_*^2 \qquad (5)$$

Required input data:

$\overline{w'\theta'}(0)$	turbulent kinematic heat flux near the surface (z=0)
u_*	friction velocity at the surface (needed only for eq (2))
w_*	convective velocity scale = $[(g/\overline{\theta})\cdot z_i \cdot \overline{w'\theta'}(0)]^{1/3}$
z_i	depth of the unstable boundary layer (= height of first capping inversion)

Comments:

a) σ_U is never negative.

b) In perfectly free convection there is no mean wind, making σ_U roughly constant with height above the surface layer. In more realistic cases with wind shear across the top of the boundary layer, Stull (88) shows examples where $\sigma_U = 1.7\, w_*$ near z_i. Although eq (3) above applies to the top of the mixed layer, it appears to be missing important physics such as the wind shear at z_i. Perhaps, an expression such as (4) from the σ_U Neutral summary, or (6) from the σ_T Unstable summary might work with different empirical constants.

c) Similarly, Stull (88) shows $\sigma_U = 2.4\, w_*$ near the surface in a windy mixed layer. Eq (2) might work for this case.

d) Eq (4) is difficult to use, because it requires knowledge of turbulent momentum and heat fluxes at height z.

Sample Calculations with Typical Values:

Let $w_* = 2$ m/s, $z_i = 1000$ m, and $u_* = 0.2$ m/s.

Use equation (2) for $z = 30$ m:

$\sigma_U^2 (z{=}30m) = 1.44 \cdot [30/1000]^{2/3} \cdot [1 - 0.7 \cdot (30/1000)]^2 \cdot 2^2 + 1.11 \cdot (0.2)^2 = 0.577\ m^2/s^2$. Thus, $\sigma_U (z{=}30m) = 0.76$ m/s .

References Listed by Equation Number (see detailed reference list attached):

(1) Wayland & Sethu (89); Huynh (90)
(2) Lenschow et al (88); Kristensen et al (89)
(3) Yasuda (88)
(4) Shao & Hacker (90)
(5) Andren (90)
Typical Ranges: Amiro (90); Andren (90)

σ_V Cross Wind Standard Deviation — Stable

Range: [0 to (0.01 to 0.5) to 1.5] m/s

Equations:

$$\sigma_V\,(0 \le z \le 0.1\,h) = (1.7 \pm 0.2) \cdot u_* \tag{1}$$

$$\sigma_V\,(0 \le z \le h) = (2.2 \pm 0.5) \cdot [1 - z/h]^{3/4} \cdot u_* \tag{2}$$

Required input data:

u_*	friction velocity at the surface (z=0)
h	depth of the stable boundary layer

Comments:

a) σ_V is never negative.

b) σ_V is usually largest near the surface, and decreases with height to a value of 0 at z=h.

c) Eq (2) for σ_V is similar to eq (2) for σ_U for the stable case. One would normally expect $\sigma_V < \sigma_U$ because the energy input is via the mean wind in the U direction. Although the data of Sorbjan shows the opposite, there is so much scatter as to leave the issue open.

d) Also see eqs (4) and (5) of σ_U stable.

Sample Calculations with Typical Values:

Let $u_* = 0.1$ m/s, h = 300 m. Solve eq (2) at z = 10 m.

Thus $\sigma_V\,(z{=}10m) = 2.2 \cdot [1 - 10/300]^{0.75} \cdot (0.1\ m/s) = 0.21\ m/s$

References Listed by Equation Number (see detailed reference list attached):

(1) Smedman (88); Hanna & Paine (89)

(2) Sorbjan (88)

σ_V Cross Wind Standard Deviation — Neutral

Range: [0 to (0.3 to 1.5) to 2.75] m/s

Equations:

$$\sigma_V \ (0 \le z \le 0.1\,h) = (2.05 \pm 0.42) \cdot u_* \quad (1)$$

$$\sigma_V \ (0 \le z \le 0.35\,h) = (1.7 \pm 0.3) \cdot (1 - z/h) \cdot u_* \quad (2)$$

$$\sigma_V^2 \ (0 \le z \le h) = (3.0 \pm 0.2) \cdot (1 - z/h)^2 \cdot u_*^2 + (z/h) \cdot \overline{v'^2}_{top} \quad (3)$$

$$\sigma_V \ (0 \le z \le 2h) = (1.6 \pm 0.1) \cdot (1 - 0.5\,z/h) \cdot u_* \quad (4)$$

Required input data:

u_*	friction velocity at the surface (z=0)
h	depth of the neutral boundary layer
$\overline{v'^2}_{top}$	$= \sigma_V^2$ at top of the boundary layer (only for eq (3))

Comments:

a) σ_V is never negative.

b) In most atmospheric boundary layers, the top of the neutral boundary layer (h) is marked by a capping inversion. In some theoretical/numerical studies such as was used to get eq (4), however, the neutral layer was assumed to be infinitely thick. For this latter case, h is defined as the lowest height where the wind perpendicular to the geostrophic wind first becomes zero (analogous to an Ekman layer height).

c) Also see eq (2) of the σ_U neutral section.

Sample Calculations with Typical Values:

Let: $u^* = 0.4$ m/s, $h = 500$ m. Solve (2) for $z = 10$ m:

$\sigma_V (z=10m) = 1.7 \cdot (1 - 10/500) \cdot 0.4$ m/s $= 0.67$ m/s

References Listed by Equation Number (see detailed reference list attached):

(1) Panofsky et al (77); Nichols & Readings (79); Smith (80); Panofsky & Dutton (84); Grant (86); Kristensen et al (89); Gunther & Lamb (90)

(2) Andren (90)

(3) Nichols and Readings (79); Grant (86)

(4) Stull (88, based on data from Mason & Thomson (87))

σ_V Cross Wind Standard Deviation — Unstable

Range: [0 to (0.2 to 2.0) to 3.0] m/s

Equations:

$$\sigma_V (0 \le z \le z_i) = [(0.4 \pm 0.2) \cdot (1 - z/z_i)^2 + (0.5 \pm 0.1)] \cdot w_* \qquad (1)$$

Required input data:

w_* — convective velocity scale = $[(g / \overline{\theta}) \cdot z_i \cdot \overline{w'\theta'}(0)]^{1/3}$

z_i — depth of the unstable boundary layer (= height of first capping inversion)

Comments:

a) σ_V is never negative.

b) If the mean wind is zero, then σ_V is expected to be approximately equal to σ_U.

c) In perfectly free convection there is no mean wind, making σ_V roughly constant with height above the surface layer. In more realistic cases one might expect larger values of σ_V near the surface and at the top of the mixed layer because of wind shear.

d) Also see eq (5) of the σ_U unstable section.

e) There is limited data for this section. Eq (1) seems to give numbers that are a bit too large.

Sample Calculations with Typical Values:

Let $w_* = 2$ m/s, $z_i = 1000$ m, and use equation (1) for $z = 30$ m:

$$\sigma_V (z{=}30m) = [0.4 \cdot (1 - 30/1000)^2 + 0.5] \cdot (0.2) = 1.75 \text{ m/s} .$$

References Listed by Equation Number (see detailed reference list attached):

(1) Wayland & Sethu (89)

Typical Ranges: Amiro (90)

σ_W Vertical Wind Standard Deviation — Stable

Range: [0 to (0.01 to 0.4) to 1.3] m/s

Equations:

$$\sigma_W \; (0 \le z \le 0.1\,h) = (1.4 \pm 0.2) \cdot u_* \tag{1}$$

$$\sigma_W |_{urban} \; (0 \le z \le z_i) = (2 \pm 1) \cdot u_* \tag{2}$$

$$\sigma_W |_{urban} \; (z_i \le z \le 5\,z_i) = (2 \pm 1) \cdot e^{-\frac{z - z_i}{2\,z_i}} \cdot u_* \tag{3}$$

$$\sigma_W \; (0 \le z \le h) = (1.58 \pm 0.25) \cdot \left[1 - (z/h)^{0.6} \right]^{1/2} \cdot u_* \tag{4}$$

$$\sigma_W \; (0 \le z \le h) = (1.73 \pm 0.5) \cdot [\, 1 - z/h \,]^{3/4} \cdot u_* \tag{5}$$

$$\sigma_W \; (0 \le z \le h) = (1.45 \pm 1.0) \cdot [\, \overline{u'w'}^2(z) + \overline{v'w'}^2(z) \,]^{1/4} \tag{6}$$

$$\sigma_W \; (0 \le z \le h) = \left[1 - \frac{0.8 \cdot z \cdot g \cdot \overline{w'\theta'}(z)}{T(z) \cdot [\, \overline{u'w'}^2(z) + \overline{v'w'}^2(z) \,]^{3/4}} \right]^{1/3} \tag{7}$$

Required input data:

u_*	friction velocity at the surface (z=0)
h	depth of the stable boundary layer
z_i	depth of the urban mixed layer under the stable boundary layer
$\overline{u'w'}(z)$	turbulent vertical momentum flux of u-wind at height z
$\overline{v'w'}(z)$	turbulent vertical momentum flux of v-wind at height z
$\overline{w'\theta'}(z)$	turbulent vertical heat flux at height z

Comments:

a) σ_W is never negative.

b) σ_W is usually largest near the surface, and decreases with height to a value of 0 at z=h.

c) Eqs (2) & (3) apply only to the urban stable boundary layer, where a shallow convective mixed layer caused by urban heating underlies a deeper capping stable layer.

d) Eqs (6) & (7) are difficult to use because they need values of turbulent fluxes at z. Of these two equations, (7) is a probably a bit more accurate.

e) σ_W is usually smaller than σ_U and σ_V, because the static stability suppresses vertical turbulent motions. However, gravity (buoyancy) waves in stable air can enhance σ_W.

Sample Calculations with Typical Values:

Let $u_* = 0.1$ m/s, $h = 300$ m. Solve eq (4) at $z = 10$ m.

Thus $\sigma_W (z{=}10\text{m}) = 1.58 \cdot [\, 1 - (10/300)^{0.6}\,]^{0.5} \cdot (0.1 \text{ m/s}) = 0.15$ m/s

References Listed by Equation Number (see detailed reference list attached):

(1) Hicks (81); Nieuwstadt (84); Panofsky & Dutton (84); Sorbjan (86, 87); Smedman (88); Wesely (88); Hanna & Paine (89); Leclerc & Thurtell (90); Rao & Schaub (90)

(2) Uno et al (89)

(3) Uno et al (89)

(4) Caughey et al (79); Nieuwstadt (84); Garratt & Ryan (89)

(5) Sorbjan (88)

(6) Nieuwstadt (84); Sorbjan (87)

(7) Shao & Hacker (90)

σ_W Vertical Wind Standard Deviation — Neutral

Range: [0 to (0.05 to 1.5) to 2.75] m/s

Equations:

$$\sigma_W \;(0 \le z \le 0.1\,h) = (1.22 \pm 0.08) \cdot u_* \qquad (1)$$

$$\sigma_W \;(0 \le z \le 0.35\,h) = (1.12 \pm 0.2) \cdot (1 - z/h)^{1/2} \cdot u_* \qquad (2)$$

$$\sigma_W \;(0 \le z \le 2h) = (1.25 \pm 0.15) \cdot (1 - 0.5\,z/h) \cdot u_* \qquad (3)$$

$$\sigma_W \;(0 \le z \le h) = (1.0 \pm ?) \cdot (1 - z/h)^{1/4} \cdot u_* \qquad (4)$$

Required input data:

u_*	friction velocity at the surface (z=0)
h	depth of the neutral boundary layer

Comments:

a) σ_W is never negative.

b) In most atmospheric boundary layers, the top of the neutral boundary layer (h) is marked by a capping inversion. In some theoretical/numerical studies such as was used to get eq (3), however, the neutral layer was assumed to be infinitely thick. For this latter case, h is defined as the lowest height where the wind perpendicular to the geostrophic wind first becomes zero (analogous to an Ekman layer height).

Sample Calculations with Typical Values:

Let: $u^* = 0.4$ m/s, $h = 500$ m. Solve (2) at $z = 10$ m:

$\sigma_W\,(z=10m) = 1.12 \cdot (1 - 10/500)^{1/2} \cdot 0.4$ m/s $= 0.44$ m/s

References Listed by Equation Number (see detailed reference list attached):

(1) Merry & Panofsky (76); Panofsky & Dutton (84); Grant (86); Sorbjan (86); McAneney et al (88); Kristensen et al (89); Gunther & Lamb (90)

(2) Brost (82); Grant (86); Andren (90)

(3) Stull (88, based on data from Mason & Thomson (87))

(4) Nichols and Readings (79); Grant (86)

σ_W Vertical Wind Standard Deviation — Unstable

Range: [0 to (0.02 to 2.5) to 4.0] m/s

Equations:

$$\sigma_W (z \approx 0) = (1.2 \pm ?) \cdot u_* \quad (1)$$

$$\sigma_W (0 \le z \le 0.1\, z_i) = (1 \pm 0.25) \cdot [u_{*L}^3(z) + (2.4 \cdot z \cdot g \cdot \overline{w'\theta'}(z) / \overline{\theta})]^{1/3} \quad (2)$$

$$\sigma_W (0 \le z \le 0.1\, z_i) = (1.25 \pm 0.5) \cdot [1 - (3 \pm 1)\cdot z/L]^{1/3} \cdot u_* \quad (3)$$

$$\sigma_W (0 \le z \le 0.1\, z_i) = (1.2 \pm 0.2) \cdot [g \cdot z \cdot \overline{w'\theta'}(0) / \overline{\theta}]^{1/3} = (1.6 \pm 0.3) \cdot (z / L)^{1/3} \cdot u_* \quad (4)$$

$$\sigma_W (0 \le z \le z_i) = (1.33 \pm 0.25) \cdot (z / z_i)^{1/3} \cdot (1 - 0.8\cdot z/z_i) \cdot w_* \quad (5)$$

$$\sigma_W (0 \le z \le z_i) = (1.26 \pm ?) \cdot (z / z_i)^{1/3} \cdot (1 - 1.2\cdot z/z_i)^{1/3} \cdot w_* \quad (6)$$

$$\sigma_W^2 (0 \le z \le z_i) = (1.44 \pm 0.4) \cdot [(z / z_i)^{2/3} \cdot (1 - 0.7\, z/z_i)^2 \cdot w_*^2 + (1.11 \pm ?) \cdot u_*^2] \quad (7)$$

$$\sigma_W (z_i \le z \le 1.4\, z_i) = (0.42 \pm ?) \cdot (z / z_i)^{-4.2} \cdot w_* \quad (8)$$

Required input data:

u_*	friction velocity at the surface (needed only for eq (7))
$u_{*L} = [\overline{u'w'}^2(z) + \overline{v'w'}^2(z)]^{1/4}$	is the local friction velocity at height z (only eq 2)
w_*	convective velocity scale = $[(g / \overline{\theta})\cdot z_i \cdot \overline{w'\theta'}(0)]^{1/3}$
z_i	depth of the unstable boundary layer (= height of first capping inversion)
$\overline{u'w'}(z)$	turbulent vertical momentum flux of u-wind at height z (only eq 2)
$\overline{v'w'}(z)$	turbulent vertical momentum flux of v-wind at height z (only eq 2)
$\overline{w'\theta'}(z)$	turbulent vertical heat flux at height z (only eq 2)
g	acceleration due to gravity = 9.8 m/s^2 (only eqs 2 & 4)
$\overline{\theta}$	mean potential temperature (°K) (only eqs 2 & 4)
L	Obukhov length (only eqs 3 & 4) = $- [\overline{u'w'}^2(0) + \overline{v'w'}^2(0)]^{3/4} / [0.4 \cdot (g/\overline{\theta}) \cdot \overline{w'\theta'}(0)]$

Comments:

a) σ_W is never negative.

b) σ_W is largest near the middle of the mixed layer because of the buoyant action of rising thermals. σ_W is usually larger than σ_U and σ_V in unstable conditions.

c) Eq (4) gives two identical forms of the same equation.

d) In humid environments, us the virtual potential temperature in place of every occurrence of the potential temperature. It gives a better estimate of buoyancy.

e) Eq (2) is difficult to use because it required knowledge of turbulent fluxes at z.

f) Wind shear at the top of the mixed layer can generate turbulence in excess of that indicated by eq (8).

Sample Calculations with Typical Values:

Let $w_* = 2$ m/s, $z_i = 1000$ m, and solve equation (5) at $z = 30$ m:

Thus $\sigma_W (z{=}30m) = 1.33 \cdot [\,30/1000]^{1/3} \cdot [\,1 - 0.8 \cdot (30/1000)] \cdot 2 = 0.81$ m/s .

References Listed by Equation Number (see detailed reference list attached):

(1) Maitani & Ohtaki (87)

(2) Caughey and Readings (71); Hicks (81); Wesely (88); Shao and Hacker (90)

(3) Hicks (81); Wesely (88); Leclerc & Thurtell (90)

(4) Wyngaard et al (71); Caughey & Palmer (79); Smith (80); Berkowicz & Prahm (84); Sorbjan (86)

(5) Deardorff (74); Willis & Deardorff (74); Andre et al (78); Caughey & Palmer (79); Lenschow et al (80); Smedman & Högström (83); Therry & Lecarrere (83); Zhou et al (85); Lenschow et al (88); Chou & Zimmerman (89); Wayland & Sethu (89); Andren (90); Greenhut & Mastrantonio (90); Huynh et al (90)

(6) Caughey & Palmer (79); Sorbjan (86)

(7) Kristensen et al (89)

(8) Yasuda (88)

Typical Ranges: Amiro (90)

σ_q Specific Humidity Standard Deviation — Stable

Range: [0 to (0 to 0.5) to 2.0] g/kg

Equations:

$$\sigma_q(z) = (3 \pm 3) \cdot [\, |\overline{w'q'}(z)| / u_{*L}(z) \,] \qquad (1)$$

Required input data:

$u_{*L} = [\, \overline{u'w'}^2(z) + \overline{v'w'}^2(z) \,]^{1/4}$ is the local friction velocity at height z

$\overline{u'w'}(z)$ turbulent vertical momentum flux of u-wind at height z

$\overline{v'w'}(z)$ turbulent vertical momentum flux of v-wind at height z

$\overline{w'q'}(z)$ turbulent vertical moisture flux at height z

Comments:

a) σ_q is never negative, but it is often very close to 0 in stable conditions.

b) Eq (1) is difficult to use, because it depends on turbulent fluxes at z. It also has 100% error bars.

c) There is virtually no data in the literature about this variable for stable conditions.

d) Moisture variance will be positive regardless of the sign of the moisture flux.

Sample Calculations with Typical Values:

Let $u_* = 0.1$ m/s, $h = 300$ m, $\overline{w'q'}(z) = -0.005$ (g/kg)·m/s at $z = 0$.

Higher, at $z = 150$ m, suppose $u_{*L} = 0.07$ m/s, $\overline{w'q'}(z) = -0.002$ (g/kg)·m/s.

Thus σ_q (z=150 m) = 3 · (0.002 (g/kg)·m/s) / (0.07 m/s) = 0.09 g/kg .

References Listed by Equation Number (see detailed reference list attached):

(1) Shao & Hacker (90)

σ_q Specific Humidity Standard Deviation — Neutral

Range: [0 to (0 to 0.75) to 3.0] g/kg

Equations:

$$\sigma_q \; (0 \le z \le h) = (2 \pm 2) \cdot (1 - z/h) \cdot \frac{|\overline{w'q'}(0)|}{u_*} \qquad (1)$$

Required input data:

u_*	friction velocity at the surface (z=0)
h	depth of the neutral boundary layer
$\overline{w'q'}(0)$	turbulent kinematic moisture flux at the surface (z = 0)

Comments:

a) σ_q is never negative.

b) No similarity relationships were found in the literature for this case. Eq (1) above was constructed for this study by Stull, based on a scientific guess of how the moisture variance should behave while obeying similarity scaling relationships. The parameter was chosen as an intermediate value between the stable and unstable values. Moisture variance can be nonzero for both positive and negative surface moisture fluxes. It is assumed that there is zero turbulence and zero moisture flux at the top of the boundary layer. Errors can easily exceed 100%.

c) If a similarity relationship for standard deviation of a pollutant or tracer can be found for neutral conditions, then moisture standard deviation probably obeys the same similarity scaling.

d) The range information stated above is also just a guess.

Sample Calculations with Typical Values:

Let: $u^* = 0.4$ m/s, $h = 500$ m, $\overline{w'q'}(0) = 0.01$ (g/kg)·m/s . Solve (1) at $z = 10$ m:

σ_q (z=10m) = 2 · (1 - 10/500) · (0.01 (g/kg)·m/s) / 0.4 m/s = 0.05 g/kg

References Listed by Equation Number (see detailed reference list attached):

(1) Stull (90, a scientific guess).

σ_q Specific Humidity Standard Deviation — Unstable

Range: [0 to (0.01 to 1) to 50] g/kg

Equations:

$$\sigma_q \, (0.01 \, z_i \leq z \leq 0.9 \, z_i) = (1.5 \pm 0.5) \cdot (z / z_i)^{-1/3} \cdot \frac{\overline{w'q'}(0)}{w_*} \qquad (1)$$

Required input data:

w_*	convective velocity scale = $[(g / \overline{\theta}) \cdot z_i \cdot \overline{w'\theta'}(0)]^{1/3}$
z_i	depth of the unstable boundary layer (= height of first capping inversion)
$\overline{w'q'}(0)$	turbulent kinematic moisture flux at the surface (z = 0)

Comments:

a) σ_q is never negative.

b) Near the top of the mixed layer there is often entrainment of drier air from aloft. As a result, there are moist thermals adjacent to dry entrained air at the top of the mixed layer. This can lead to extremely large standard deviations of moisture there.

Sample Calculations with Typical Values:

Let $w_* = 2$ m/s, $z_i = 1000$ m, and $\overline{w'q'}(0) = 0.056$ (g/kg)·m/s.

Solve equation (1) at z = 30 m:

$$\sigma_q \, (z{=}30\text{m}) = 1.5 \cdot [30/1000]^{-1/3} \cdot (0.056 \text{ (g/kg)·m/s}) / 2 \text{ m/s} = 0.13 \text{ g/kg} .$$

References Listed by Equation Number (see detailed reference list attached):

(1) Wyngaard et al (71); Smedman-Högström (73); Deardorff (74); Lenschow et al (80); Zhou et al (85); Huynh et al (90); Shao & Hacker (90)

TKE Turbulence Kinetic Energy Stable

Range: [0 to (0.0 to 0.5) to 2.5] m^2/s^2

Equations:

$$TKE\,(0 \le z \le h) = (5.5 \pm 2.5) \cdot u_{*L}^2 \quad (1)$$

$$TKE\,(0 \le z \le 0.1\,h) = (5.5 \pm 2.5)\, u_*^2 \quad (2)$$

$$TKE\,(0 \le z \le h) = (5.5 \pm 2.5) \cdot e^{-\frac{(3.5 \pm 0.5) \cdot z}{h}} \cdot u_*^2 \quad (3)$$

$$TKE\,(0 \le z \le h) = u_*^2 \cdot 10^{(1.67 \pm 0.2) \cdot (1 - z/h)^4} \quad (4)$$

Required input data:

- u_* — friction velocity at the surface (z=0)
- h — depth of the stable boundary layer
- $u_{*L} = [\overline{u'w'}^2(z) + \overline{v'w'}^2(z)]^{1/4}$ — local friction velocity at height z (only eq 1)
- $\overline{u'w'}(z)$ — turbulent vertical momentum flux of u-wind at height z (only eq 1)
- $\overline{v'w'}(z)$ — turbulent vertical momentum flux of v-wind at height z (only eq 1)

Comments:

a) TKE is never negative. It is often largest near the ground and decreases with height.

b) Other expressions for TKE can be constructed from the standard deviation equations using the definitions:

$$TKE = \frac{1}{2}\left[\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right] \quad \text{or} \quad TKE = \frac{1}{2}\left[\sigma_U^2 + \sigma_V^2 + \sigma_W^2\right]$$

c) TKE at night can increase and decrease sporadically or have "bursts". Also the TKE in the mid and upper boundary layer can be unrelated to TKE at the surface. The gradient Richardson number (see Stull (88)) is useful to diagnose whether turbulence occurs.

d) Gravity waves that propagate into the region from elsewhere can also create velocity fluctuations that appear like TKE.

e) Eq (1) is difficult to use because it requires knowledge of turbulent fluxes at z.

Sample Calculations with Typical Values:

Let $u_* = 0.1$ m/s, $h = 300$ m. Solve eq (3) at $z = 10$ m.

Thus TKE (z=10m) = $5.5 \cdot \exp[-3.5 \cdot 10 / 300] \cdot (0.1\ m/s)^2 = 0.049\ m^2/s^2$.

References Listed by Equation Number (see detailed reference list attached):

(1), (2), (3) Constructed from other equations in Stull (88)

(4) Constructed from TKE dissipation rates in Stull (88), Caughey et al (79), Andren (90)

TKE Turbulence Kinetic Energy — Neutral

Range: [0 to (0.1 to 4.0) to 6.0] m^2/s^2

Equations:

$$TKE\,(0 \le z \le h) = (5.5 \pm 2.5) \cdot u_{*L}^2 \qquad (1)$$

$$TKE\,(0 \le z \le 0.1\,h) = (5.5 \pm 2.5)\, u_*^2 \qquad (2)$$

$$TKE\,(0 \le z \le h) = (5.5 \pm 2.5) \cdot (1 - z/h) \cdot u_*^2 \qquad (3)$$

Required input data:

u_*	friction velocity at the surface (z=0)
h	depth of the neutral boundary layer
$u_{*L} = [\,\overline{u'w'}^2(z) + \overline{v'w'}^2(z)\,]^{1/4}$	local friction velocity at height z (only eq 1)
$\overline{u'w'}(z)$	turbulent vertical momentum flux of u-wind at height z (only eq 1)
$\overline{v'w'}(z)$	turbulent vertical momentum flux of v-wind at height z (only eq 1)

Comments:

a) TKE is never negative. It is often largest near the ground and decreases with height.

b) Other expressions for TKE can be constructed from the standard deviation equations using the definitions:

$$TKE = \frac{1}{2}\left[\,\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\,\right] \quad \text{or} \quad TKE = \frac{1}{2}\left[\,\sigma_U^2 + \sigma_V^2 + \sigma_W^2\,\right]$$

c) TKE can also be generated by wind shears across the top of the neutral boundary layer, particularly if it is capped by a stable layer (as is usually the case). Eq(3) should then be modified to linearly decrease to a nonzero value at the top of the boundary layer.

d) Eq (1) is difficult to use because it requires knowledge of turbulent fluxes at z.

Sample Calculations with Typical Values:

Let $u_* = 0.4$ m/s, $h = 500$ m. Solve eq (3) at z = 10 m.

Thus TKE (z=10m) = $5.5 \cdot [\,1 - 10/500] \cdot (0.4\text{ m/s})^2 = 0.86\ m^2/s^2$.

References Listed by Equation Number (see detailed reference list attached):

(1) Constructed from other equations in Stull (88)

(2) Constructed from other equations in Stull (88)

(3) Constructed from equations in Stull (88) and data from the BLX83 field program.

Range: [0 to (0.1 to 5.0) to 10.0] m^2/s^2

Equations:

$$TKE = 0.5 \cdot w_*^2 \cdot 10^{(0.67 \pm 0.4) \cdot [(1 - z/z_i)^2 - 1]} \qquad (1)$$

Required input data:

w_* convective scaling velocity at the surface (z=0)

z_i depth of the convective mixed layer

Comments:

a) TKE is never negative.

b) Such a differing variety of TKE data have been presented in the literature that it is difficult to get a single similarity expression. In some zero wind cases, the TKE appears to be largest at the middle of the mixed layer (Therry & Lacarrere, 83); for which the σ_W eq might be used. For cases with wind, the TKE appears largest at the surface, and quickly decreases to a nearly constant value over most of the interior of the mixed layer (Hechtel, 88). Eq (1) above describes the latter scenario. Comment (c) might be the best overall approach.

c) Other expressions for TKE can be constructed from the standard deviation equations using the definitions:

$$TKE = \frac{1}{2}\left[\overline{u'^2} + \overline{v'^2} + \overline{w'^2} \right] \quad \text{or} \quad TKE = \frac{1}{2}\left[\sigma_U^2 + \sigma_V^2 + \sigma_W^2 \right]$$

Sample Calculations with Typical Values:

Let $w_* = 2$ m/s, $z_i = 1000$ m. Solve eq (1) at z = 30 m.

Thus $TKE(z=30m) = 0.5 \cdot 2^2 \cdot 10^{0.67 \cdot [(1 - 30/1000)^2 - 1]} = 2 \cdot 10^{-0.04} = 1.83\ m^2/s^2$.

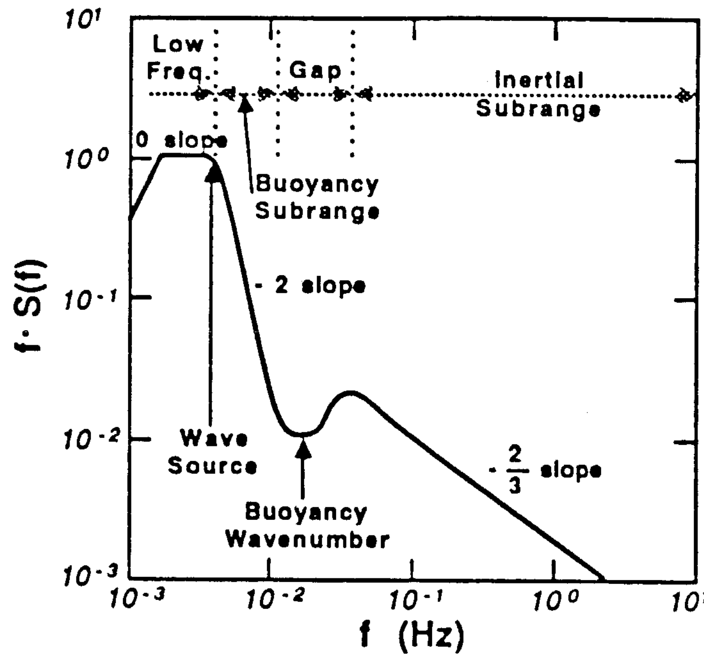
References Listed by Equation Number (see detailed reference list attached)**:**

(1) Constructed by Stull from TKE dissipation rate data of Caughey and Palmer (79); Andren (90); Huynh et al (90); and from TKE data of Hechtel (88).

f·S(f) Turbulence Energy Spectrum — Stable

Range: [0 to (0 to 1) to 5] m^2/s^2

Equations:



Required input data:

f   frequency (Hz), (or instead use wavelength (m) and wind speed (m/s))

Comments:

a) The turbulence state of the very stable boundary layer is so variable that no similarity equations have been presented in the literature. Part of the difficulty comes from the interactions between gravity (buoyancy) waves and turbulence. The largest spectral amplitude appears in the buoyancy subrange for waves, not for turbulence. Another difficulty is the intermittency and occasional absence of turbulence.

b) The graph above indicates the spectrum for one of the many possible turbulence states of the boundary layer. Its general applicability is not known.

c) For slightly stable (ie, near neutral) conditions, use the spectrum given in the Neutral section.

d) Wavelength instead of frequency can be input, using: f(Hz) = U / wavelength.

Sample Calculations with Typical Values:

Let wavelength = 20 m/cycle, U = 5 m/s. Thus, f = 5 / 20 = 1.57 Hz.

Thus, using the graph $f \cdot S_w(f) = 10^{-3}\ m^2/s^2$.

References Listed by Equation Number (see detailed reference list attached):

Graph   Stull (88, based on data from Nai-Ping et al 83, & Finnigan et al 84)

$f \cdot S(f)$ Turbulence Energy Spectrum — Near Neutral

Range: [0 to (0 to 0.1) to 0.5] m^2/s^2

Equations:

$$\pi_2 = f\, z / U \qquad (1)$$

$$\pi_3 = \frac{0.4 \cdot g \cdot z \cdot |\overline{w'\theta'}(0)|}{u_*^3 \cdot \theta} \qquad (2)$$

$$f \cdot S_U(f) = u_*^2 \cdot \pi_1 \cdot (1 + \pi_3^{2/3}) \qquad (3)$$

Required input data:

f	spectral frequency (s^{-1})
g	acceleration due to gravity = 9.8 m/s^2
$\overline{\theta}$	mean potential temperature (°K)
u_*	friction velocity at the surface (z=0)
U	mean horizontal wind speed at height (z)
$\overline{w'\theta'}(0)$	turbulent vertical heat flux at the surface
z	height of interest

Comments:

a) This spectrum is for the slightly stable to neutral surface layer.

b) The method is

(1) For any frequency of interest, f, calculate π_2 from eq (1)

(2) Next, calculate π_3 from eq (2), based on the surface heat flux. This flux is zero for neutral boundary layers.

(3) Using these π values, look on the graph to find the resulting value of π_1 .

(4) Finally use eq (3) to calculate $f \cdot S(f)$.

c) The integral of the spectrum $\int S(f)\, df$ over all frequencies is the velocity variance σ_U^2.

d) Wavelength instead of frequency can be input, using: $f = 2\pi\, U$ / wavelength.

Sample Calculations with Typical Values:

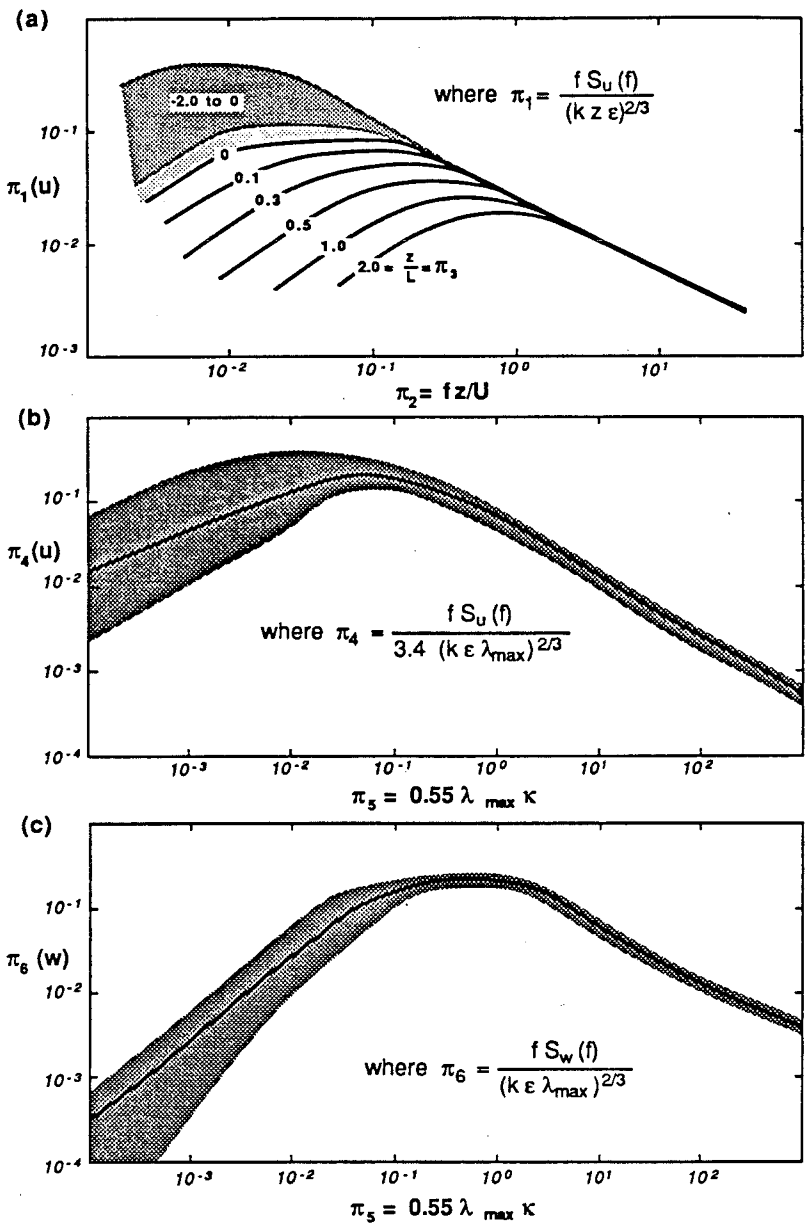
Let $u_* = 0.4$ m/s, $z = 10$ m, $\overline{w'\theta'}(0) = 0$, $U = 10$ m/s, wavelength = 20 m.

Thus $\pi_2 = 2\,\pi\,10\text{m} / 20\text{m} = 3.14$. $\pi_3 = 0$. $\pi_1 = 10^{-2}$ from graph.

$f \cdot S_U(f) = (0.4)^2 \cdot 10^{-2} = 0.0016\ m^2/s^2$.

References Listed by Equation Number (see detailed reference list attached):

(1), (2), (3) Kaimal et al (72); Moraes & Goedert (88); Skupniewicz et al (89)



(a)
-2.0 to 0
0
0.1
0.3
0.5
1.0
2.0 = z/L = π3
where π1 = f Su (f) / (k z ε)^2/3
π1 (u)
π2 = f z/U
(b)
π4 (u)
where π4 = f Su (f) / 3.4 (k ε λmax)^2/3
π5 = 0.55 λ max κ
(c)
π6 (w)
where π6 = f Sw (f) / (k ε λmax)^2/3
π5 = 0.55 λ max κ

f·S(f) Turbulence Energy Spectrum Unstable

Range: [0 to (0 to 1) to 5] m^2/s^2

Equations:

$$\pi_2 = f z_i / U \qquad (1)$$

$$\pi_3 = z / z_i \qquad (2)$$

$$f \cdot S(f) = \pi_1 \cdot w_*^2 \cdot 10^{(0.67 \pm 0.4) \cdot [(1 - \pi_3)^2 - 1]} \qquad (3)$$

Required input data:

f	spectral frequency (s^{-1})
w_*	convective velocity scale at the surface (z=0)
U	mean horizontal wind speed at height (z)
z	height of interest
z_i	depth of the unstable mixed layer

Comments:

a) The method is
   (1) For any frequency of interest, f, calculate π_2 from eq (1)
   (2) Next, calculate π_3 from eq (2).
   (3) Using these π values, look on the graph to find the resulting value of π_1. The various parts of these graphs can be used to give $f \cdot S_U(f)$, $f \cdot S_V(f)$, or $f \cdot S_W(f)$.
   (4) Finally use eq (3) to calculate f·S(f).

b) The integral of the spectrum $\int S(f)\,df$ over all frequencies is the velocity variance σ^2.

c) Wavelength instead of frequency can be input, using: $f = 2\pi U$ / wavelength.

d) Godowich (90) has an alternative graph normalized by σ^2_U, not shown here.

e) Busch et al (87) give spectra scaled by u_*, which may not be appropriate for a convective mixed layer. Therefore, it is not shown here.

Sample Calculations with Typical Values:

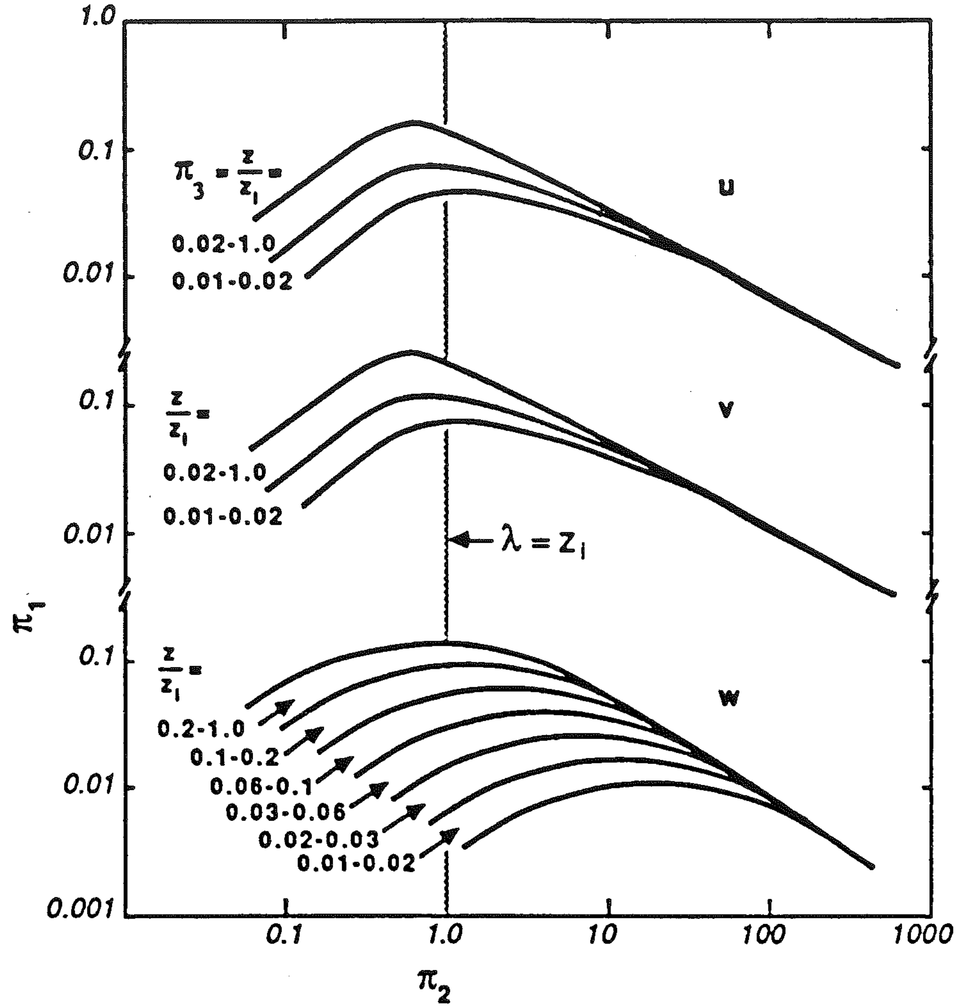
Let $w_* = 2$ m/s, $z = 30$ m, $z_i = 1000$ m, $U = 10$ m/s, wavelength = 20 m.

Thus $\pi_2 = 2\,\pi\,30\text{m} / 20\text{m} = 9.42$. $\pi_3 = 30 / 1000 = 0.03$. $\pi_1 = 0.03$ from graph for U.

$$f \cdot S_U(f) = 0.03 \cdot (2)^2 \cdot 10^{0.67 \cdot [(1 - 0.03)^2 - 1]} = 0.11\ m^2/s^2.$$

References Listed by Equation Number (see detailed reference list attached):

(1), (2), (3)	Constructed by Stull using data from Huynh et al (90), Andren (90)
Graph from	Kaimal et al (76)


1.0
0.1
0.01
0.1
0.01
0.1
0.01
0.001
$\pi_3 = \frac{z}{z_i} =$
0.02-1.0
0.01-0.02
u
$\frac{z}{z_i} =$
0.02-1.0
0.01-0.02
v
$\leftarrow \lambda = z_i$
$\frac{z}{z_i} =$
0.2-1.0
0.1-0.2
0.06-0.1
0.03-0.06
0.02-0.03
0.01-0.02
w
0.1
1.0
10
100
1000
π_1
π_2

Range: Vertical thickness of largest eddies [0 to (0.1 to 50) to 100] m
Horizontal width of largest eddies [0 to (1 to 200) to 1000] m

Equations:

$$\text{Vertical eddy thickness} \propto 4 \cdot \sigma_W / N \tag{1}$$

$$\lambda_x\ (0 \le z \le h) = (1.5 \pm 0.2) \cdot (z \cdot h)^{1/2} \tag{2}$$

$$\lambda_y\ (0 \le z \le h) = (0.7 \pm 0.1) \cdot (z \cdot h)^{1/2} \tag{3}$$

$$\lambda_z\ (0 \le z \le h) = (1 \pm 0.2) \cdot z^{0.8} \cdot h^{0.2} \tag{4}$$

$$\lambda_x\ (0 \le z \le 0.1\ h) = (2.77 \pm 0.2) \cdot z^{0.2} \cdot L^{0.8} \tag{5}$$

$$\lambda_y\ (0 \le z \le 0.1\ h) = (1.25 \pm 0.2) \cdot (z \cdot L)^{1/2} \tag{6}$$

Required input data:

h	depth of the stable boundary layer (where turbulence is near zero) (m)
N	Brunt-Vaisala frequency = $[(g/\theta)\ \partial\theta/\partial z]^{1/2}$ (1/s) (only eq 1)
σ_W	standard deviation of vertical velocity (m/s)
L	Obukhov length (m) (only eqs 5 & 6) = $- [\overline{u'w'}^2(0) + \overline{v'w'}^2(0)]^{3/4} / [0.4 \cdot (g/\overline{\theta}) \cdot \overline{w'\theta'}(0)]$

Comments:

a) Eddies in a stable boundary layer are "pancake" shaped; namely, vertically thin and horizontally wide. This is because static stability suppresses vertical motion.

b) The largest size eddies are usually the most energetic. They are formed by the mean flow, and thus scale to the flow size characteristics. A measure of the size of these eddies is the wavelength, $\lambda_{x,y,z}$, at the peak of the (U,V,W) velocity energy spectra. $\lambda_{x,y,z}$ is a measure of the eddy size in the x, y and z directions, respectively.

c) A wide range of smaller, weaker eddies also exist because of the inertial cascade of turbulent energy (see sections on the turbulence energy spectrum).

d) Horizontal coverage: If the gradient Richardson number (Ri) varies in the horizontal, then it is possible to have patches of turbulence where Ri < 0.25, with no turbulence elsewhere. Normally, either 100% or 0% of the area is covered by turbulence.

Sample Calculations with Typical Values:

Vertical: Let $g/\theta = 0.0333\ m\cdot s^{-2}\cdot K^{-1}$, $\partial\theta/\partial z = 0.02$ K/m. $\sigma_W = 0.15$ m/s. Solve eq (1).

Thus, $N = (0.0333 \cdot 0.02)^{1/2} = 0.026\ s^{-1}$. Vertical eddy size = $4 \cdot 0.15/0.026 = 23$ m.

Horizontal: Let h = 300 m. Solve eq (2) at z = 10 m.

Thus $\lambda_x = 1.5 \cdot (10\cdot 300)^{1/2} = 82$ m.

References Listed by Equation Number (see detailed reference list attached):

(1) Constructed from other equations in Stull (88)

(2), (3), (4) Constructed by Hanna et al (82) using data from Caughey et al (79)

(5), (6) Constructed by Stull from graphs in Kaimal et al (72), Busch (73)

Range: [0 to (1 to 500) to 2000] m

Equations:

$$\lambda_x(0 \le z \le 0.1\,h) = (20 \pm 5) \cdot z \quad (1)$$

$$\lambda_y(0 \le z \le 0.1\,h) = (5 \pm 0.8) \cdot z \quad (2)$$

$$\lambda_z(0 \le z \le 0.1\,h) = (3.0 \pm 0.3) \cdot z \quad (3)$$

Required input data:

z height above ground (m)

Comments:

a) Eddies in the neutral boundary layer are "egg" shaped, with the long axis parallel to the mean wind direction. In other words, the size in all three directions is approximately equal, but the along-wind direction is slightly greater. This is because it is the mean wind that generates this turbulence in the absence of large roughness elements such as buildings or trees.

b) There is little information in the literature about the dominant eddy size above the surface layer, although there is some suggestion that the size continues to increase with height.

c) λ is the wavelength of the peak of the energy spectrum. In other words, it is the size of the strongest eddy. Subscripts x,y, and z correspond to U, V, and W spectra.

d) Horizontal coverage: Eddies cover 100% of the horizontal area, and overlap in complex patterns.

e) In slightly unstable conditions, horizontal roll vortices form that are characterized by updrafts predominantly lined-up along the mean wind direction. If there is sufficient moisture in the air to create clouds, these are sometimes visible from the ground or satellite as parallel rows of clouds called cloud streets. The horizontal spacing between cloud streets is about 3 times the depth (h) of the boundary layer.

Sample Calculations with Typical Values:

Let h = 500 m. Solve eq (3) at z = 10 m.

Thus λ_x (z=10m) = 20 · 10 = 200 m. Similarly, λ_y (z=10m) = 50 m, and λ_z = 30 m.

References Listed by Equation Number (see detailed reference list attached):

(1), (2), (3) Constructed by Stull from graphs in Kaimal et al (72) & Busch (73)

Morphology of Thermals

Range: [50 to (100 to 2000) to 5000] m

Equations:

$$\Delta x, \Delta y, \Delta z \quad (0 \le z \le 0.1\, z_i) \;=\; (100 \pm 50 \text{ m}), (100 \pm 50 \text{ m}), (0.1 \cdot z_i) \qquad (1)$$

$$\Delta x = \Delta y = \Delta z \quad (0.1\, z_i \le z \le z_i) \;=\; (1.0 \pm 0.3) \cdot z_i \qquad (2)$$

$$\lambda_z (0 \le z \le 0.2\, z_i) \;=\; (5.9 \pm 1) \cdot z \qquad (3)$$

$$\lambda_z (0.2\, z_i \le z \le 0.8\, z_i) \;=\; (1.0 \pm 0.5) \cdot z_i \qquad (4)$$

$$\Delta x (0.05\, z_i \le z \le 0.5\, z_i) \;=\; (0.22 \pm 0.02) \cdot z^{1/3} \cdot z_i^{2/3} \qquad (5)$$

$$\text{fractional area coveragy by thermals} = (0.5 \pm 0.05) - 0.35 \cdot (z/z_i)^{3/2} \cdot (1.3 - z/z_i)^{9/4} \qquad (6)$$

Required input data:

z	height above the surface (m)
z_i	depth of the convective mixed layer (m)

Comments:

a) Near the surface there are updraft curtains that crisscross like a fishnet. The distance between curtains is about z_i. At the knots of this net are more intense warm updrafts called plumes that are shaped like fat "hot dogs", with the long axis vertical (eq 1).

b) These highly turbulent plumes cover roughly 20% of the ground, while broader slower downdrafts in between contain weaker turbulence. The most energetic turbulent eddies have a diameter roughly equal to their height above ground (eq 3).

c) These curtain structures merge to form broader updrafts called thermals (eq 2). Thermals look like short tree stumps, with diameter roughly equal to height, which are both equal to z_i . If sufficient moisture is present, cumulus clouds can form at the tops of thermals. At the top of the mixed layer, thermals overshoot vertically into warmer air aloft, and then sink back into the mixed layer. The dominant eddy is the thermal.

d) Well-defined thermal updrafts with strong turbulence cover roughly 15 to 43 % of the surface area. In between are weaker downdrafts of slightly cooler air.

e) Δx, Δy, & Δz are the dimensions of organized structures. Within these structures are various scales of turbulence. The wavelength of peak spectral intensity of W is λ_z.

Sample Calculations with Typical Values:

Let $z = 30$ m, and $z_i = 2000$ m. Use eq (1) to give: $\Delta x = \Delta y = 100$ m. $\Delta z = 200$ m.

References Listed by Equation Number (see detailed reference list attached):

(1), (2) Stull (88)

(3), (4) Kaimal et al (76); Huynh et al (90)

(5) Lenschow & Stephens (80); Greenhut & Khalsa (87)

(6) Young (88)

w' Vertical Velocity Excess within Thermals — Unstable

Range: [-1.0 to (0.1 to 5.0) to 10.0] m/s

Equations:

$$w' (0 \le z \le z_i) = (1.0 \pm 0.7) \cdot \left\{ 0.5 + \left[z/z_i - (z/z_i)^2 \right] \right\} \cdot w_* \qquad (1)$$

$$w'_{max} (0 \le z \le z_i) = (6.0 \pm 2.0) \cdot \left\{ 0.17 + \left[z/z_i - (z/z_i)^2 \right] \right\} \cdot w_* \qquad (2)$$

$$w'(0.05\, z_i \le z \le 1.2\, z_i) = (0.85 \pm 0.2) \cdot (z/z_i)^{1/3} \cdot (1.3 - z/z_i) \cdot w_* \qquad (3)$$

Required input data:

w_*	convective scaling velocity at the surface (z=0)
z_i	depth of the convective mixed layer

Comments:

a) If temperature or humidity is used to define a thermal, then sometimes $w' < 0$. However, usually w' is positive in thermals.

b) w' is the deviation of vertical velocity within a thermal from the average vertical velocity over a large area. w'_{max} is the extreme but rare value of w' .

Sample Calculations with Typical Values:

Let $w_* = 2$ m/s, $z_i = 1000$ m. Solve eq (1) at z = 30 m.

Thus w' (z=30m) = $\{0.5 + [\,30/1000 - (30/1000)^2\,]\}\cdot 2 = 1.06$ m/s.

References Listed by Equation Number (see detailed reference list attached):

(1) Constructed by Stull from data of Greenhut & Khalsa (87); Young (88).

(2) Constructed by Stull from data of Caughey et al (83); Deardorff & Willis (85)

(3) Young (88)

θ_V' Virtual Potential Temperature Excess within Thermals — Unstable

Range: [0 to (0.01 to 3.0) to 5.0] °K

Equations:

$$\theta_v'(0 \le z \le z_i) = [(2.5 \pm 1.5) - 3 \cdot z / z_i\ ] \cdot \frac{\overline{w'\theta_v'}(0)}{w_*} \quad (1)$$

$$\theta_{v\ max}'\ (0 \le z \le z_i) = [(10 \pm 3) - 3 \cdot z / z_i\ ] \cdot \frac{\overline{w'\theta_v'}(0)}{w_*} \quad (2)$$

$$\theta_v'(0 \le z \le 1.2\ z_i) = \left[-(0.5 \pm 0.2) \cdot \left\{ 1 - \frac{0.2}{\left[1 + 100 \cdot (0.9 - z/z_i)^2 \right]} \right\} + 2 \cdot | 0.9 - z/z_i | \right] \cdot \frac{\overline{w'\theta_v'}(0)}{w_*} \quad (3)$$

Required input data:

w_*	convective scaling velocity at the surface (m/s)
z_i	depth of the convective mixed layer (m)
$\overline{w'\theta_v'}(0)$	virtual potential temperature flux at the surface (K m/s)

Comments:

a) Virtual potential temperature, θ_v, is a measure of the buoyancy of the air that includes both temperature and humidity effects: $\theta_v = \theta \cdot (1 + 0.61\ r)$, where r is humidity mixing ratio in grams of water vapor per gram of air. In humid tropical environments a thermal might be everywhere cooler than its environment, but yet positively buoyant because of the high humidity excess. (Water vapor is less dense than air.)

b) θ_v' indicates how much warmer the thermal is than its environment. $\theta_{v\ max}$' is the corresponding peak value reported in the literature.

Sample Calculations with Typical Values:

Let $w_* = 2$ m/s, $z_i = 1000$ m, $\overline{w'\theta_v'}(0) = 0.2$ °K·m/s. Solve eq (1) at z = 30 m.

Thus $\theta_v' = [2.5 - 3 \cdot 30/1000] \cdot (0.2 / 2) = 0.241$°K .

References Listed by Equation Number (see detailed reference list attached):

(1) Constructed by Stull using data from Greenhut & Khalsa (87); Young (88)

(2) Constructed by Stull using data from Mahrt & Paumier (84); Deardorff & Willis (85)

(3) Young (88)

q' Specific Humidity Excess within Thermals Unstable

Range: [0 to (0.01 to 1.0) to 3.0] g/kg

Equations:

$$q'(0 \le z \le z_i) = [(4.0 \pm 2.0) - 2 \cdot z / z_i] \cdot \frac{\overline{w'q'}(0)}{w_*} \quad (1)$$

$$q'_{max}(0 \le z \le z_i) = [(15 \pm 5) - 2 \cdot z / z_i] \cdot \frac{\overline{w'q'}(0)}{w_*} \quad (2)$$

Required input data:

- w_* — convective scaling velocity at the surface (z=0)
- z_i — depth of the convective mixed layer
- $\overline{w'q'}(0)$ — kinematic moisture flux at the surface (g water / kg air)·(m/s)

Comments:

a) q'_{max} is the extreme specific humidity excess that happens only rarely. There is not much data on this variable.

Sample Calculations with Typical Values:

Let $w_* = 2$ m/s, $z_i = 1000$ m. $\overline{w'q'}(0) = 0.056$ (g/kg)·(m/s). Solve eq (1) at z = 30 m.
Thus $q'(z=30m) = [4 - 2(30/1000)] \cdot 0.056 / 2 = 0.11$ g/kg

References Listed by Equation Number (see detailed reference list attached):

(1) Constructed by Stull with data from Greenhut & Khalsa (87); Crum & Stull (87).

(2) This is a scientific guess by Stull, by analogy to the w'_{max} and θ'_{max} equations.

u' Mean Wind Excess within Thermals — Unstable

Range: [-5.0 to (-1.0 to -0.1) to +1.0] m/s

Equations:

$$u'(0 \le z \le z_i) = \left[-(1.5 \pm 0.5) + 0.5 \cdot (z / z_i)^{1/2} \right] \cdot u_* \quad (1)$$

Required input data:

- u_* — friction velocity at surface
- z_i — depth of the convective mixed layer

Comments:

a) There is large variability in u' values, especially where there is wind shear across the top of the mixed layer. Therefore, eq (1) might be significantly in error.

b) The negative sign for u' happens because thermals are usually slower than the mean wind.

Sample Calculations with Typical Values:

Let $u_* = 0.2$ m/s, $z_i = 1000$ m. Solve eq (1) at $z = 30$ m.

Thus $u'(z{=}30\text{m}) = [-1.5 + 0.5(30/1000)] \cdot 0.2 = -0.30$ m/s.

References Listed by Equation Number (see detailed reference list attached):

(1) Constructed by Stull using data from Greenhut & Khalsa (87)

v' Cross Wind Excess within Thermals — Unstable

Range: [-1.0 to (0.1 to 1.0) to +5.0] m/s

Equations:

Required input data:

Comments:

a) The range above is based on a scientific guess that thermals take air from the surface layer and move it upward. Friction near the surface causes the wind to blow in the positive y direction, assuming the x axis is aligned with the mean wind in the middle of the mixed layer. Thus, v' is probably positive.

Sample Calculations with Typical Values:

References Listed by Equation Number (see detailed reference list attached):

(There is not much data for v' in the literature.)

C_T2 Temperature Structure Parameter — Stable

Range: [0 to (0 to 0.5) to 5.0] $°K^2\ m^{-2/3}$

Equations:

$$C_T2\ (0.1h \le z \le 1.2h) = (0.025 \pm 0.005\ K^2\ m^{-2/3}) - 0.02 \cdot z/h \quad (1)$$

$$C_T2\ (0.2h \le z \le 0.8h) = (0.22 \pm 0.1) \cdot z^{-3} \cdot h^{7/3} \cdot \frac{(\overline{w'\theta'}(0))^2}{u_*^2} \quad (2)$$

$$C_T2\ (0.02h \le z \le 0.2h) = (10 \pm 5) \cdot z^{-1} \cdot h^{1/3} \cdot \frac{(\overline{w'\theta'}(0))^2}{u_*^2} \quad (3)$$

Required input data for (2) and (3):

$\overline{w'\theta'}(0)$	turbulent kinematic heat flux near the surface (z=0)
u_*	friction velocity at the surface (z=0)
h	depth of the stable boundary layer

Comments:

a) There is large scatter (±100%) of the observations about all the equations.

b) Gravity (buoyancy) waves and breaking K-H waves can increase C_T2 both below and above h.

Sample Calculations with Typical Values:

Define $\theta_*^{SL} \equiv \frac{[-\overline{w'\theta'}(z=0)]}{u_*}$

Let $\overline{w'\theta'}(0) = -0.02$ K m/s , $u_* = 0.1$ m/s, h = 300 m. Solve eq (3) at z = 10 m.

Thus $\theta_*^{SL} = 0.2$ °K, and $C_T2\ (z=10m) = 10 \cdot (10)^{-1} \cdot (300)^{1/3} \cdot (0.2°K)^2 = 0.27\ °K^2 \cdot m^{-2/3}$

References Listed by Equation Number (see detailed reference list attached):

(1) Coulter (90)

(2), (3) Constructed by Stull using data from Neff & Coulter (86)

C_{T^2} Temperature Structure Parameter — Neutral

Range: [0] $°K^2\ m^{-2/3}$

Equations:

$$C_{T^2}(z) = 0 \tag{1}$$

Required input data:

(none)

Comments:

a) C_{T^2} is always zero by definition in the neutral boundary layer, because "neutral" means there is no potential temperature change with height. Thus, although there is turbulence in the neutral case, there are no temperature variations associated with it.

b) The atmosphere is rarely perfectly neutral. For this "near-neutral" cases, use the appropriate stable or unstable equations.

Sample Calculations with Typical Values:

C_{T^2} (z=10m) = 0°K

References Listed by Equation Number (see detailed reference list attached):

(1) Stull (88)

$C_T{}^2$ Temperature Structure Parameter — Unstable

Range: $[\ 0 \text{ to } (\ 0.0002 \text{ to } 0.2\ ) \text{ to } 2.0\ ]\ ^\circ K^2\ m^{-2/3}$

Equations:

$$C_T{}^2\ (\ 0 \le z \le 0.1 z_i\ ) = (\ 2.68 \pm 0.5) \cdot (\ g / \theta\ )^{-2/3} \cdot [\ \overline{w'\theta'}(0) / z\ ]^{4/3} \qquad (1)$$

$$C_T{}^2\ (\ 0 \le z \le z_i\ ) = (\ 3.0 \pm 0.2\ ) \cdot \varepsilon_\theta \cdot \varepsilon^{-1/3} \qquad (2)$$

$$C_T{}^2\ (\ 0 \le z \le z_i\ ) = z_i^{5/6} \cdot z^{-3/2} \cdot [\ \overline{w'\theta'}(0) / w_*\ ]^2 \qquad (3)$$

$$C_T{}^2\ (\ 0 \le z \le 0.5\ z_i\ ) = (\ 2.67 \pm 0.5) \cdot z_i^{2/3} \cdot z^{-4/3} \cdot [\ \overline{w'\theta'}(0) / w_*\ ]^2 \qquad (4)$$

$$C_T{}^2\ (\ 0.5 z_i \le z \le 0.7 z_i\ ) = (6.72 \pm 1) \cdot z_i^{-2/3} \cdot [\ \overline{w'\theta'}(0) / w_*\ ]^2 \qquad (5)$$

$$C_T{}^2\ (\ 0.7 z_i \le z \le z_i\ ) = (\ 19.5 \pm 3) \cdot z_i^{-5/3} \cdot z \cdot [\ \overline{w'\theta'}(0) / w_*\ ]^2 \qquad (6)$$

$$C_T{}^2\ (\ 0 \le z \le 0.1\ z_i\ ) = (\ 1.45 \pm 0.3) \cdot |\ z/L|^{-2/3} \cdot (\ \overline{w'\theta'}(0) / u_*\ )^2 \cdot z^{-2/3} \qquad (7)$$

Required input data:

ε	dissipation rate of turbulence kinetic energy (only eq 2)
ε_θ	dissipation rate of temperature variance (only eq 2)
$\overline{w'\theta'}(0)$	turbulent kinematic heat flux near the surface (z=0)
u_*	friction velocity at the surface (needed only for eq (7) in the surface layer)
$\overline{u'w'}(z)$	turbulent vertical momentum flux of u-wind at height z (only eq 7)
$\overline{v'w'}(z)$	turbulent vertical momentum flux of v-wind at height z (only eq 7)
g	acceleration due to gravity = 9.8 m/s^2 (only eqs 1 & 7)
$\overline{\theta}$	mean potential temperature (°K) (only eqs 1 & 7)
L	Obukhov length (only eq 7) = $-\ [\ \overline{u'w'}^2(0) + \overline{v'w'}^2(0)\ ]^{3/4}\ /\ [\ 0.4 \cdot (g/\overline{\theta}) \cdot \overline{w'\theta'}(0)\ ]$
w_*	convective velocity scale = $[\ (g / \overline{\theta}\ ) \cdot z_i \cdot \overline{w'\theta'}(0)\ ]^{1/3}$
z_i	depth of the unstable boundary layer (= height of first capping inversion)

Comments:

a) Eq (2) is difficult to use because it contains values of turbulent dissipation rates.

b) Eq (7) might work when turbulence is generated by both heating and wind shear.

c) Above the top of the mixed layer, the turbulence decreases rapidly. However, the strong temperature gradient there can allow large values of $C_T{}^2$.

Sample Calculations with Typical Values:

Define $\theta_*^{ML} \equiv \frac{\overline{w'\theta'}(z=0)}{w_*}$

Let $\overline{w'\theta'}(0) = 0.2$ K m/s, $w_* = 1.88$ m/s, $z_i = 1000$ m.

Thus $\theta_*^{ML} = 0.106$ °K, and equation (4) can be solved at $z = 30$ m:

$$c_T{}^2 (z=30m) = 2.67 \cdot 1000^{2/3} \cdot 30^{-4/3} \cdot (0.106°K)^2 = 0.029 \ °K^2 \cdot m^{-2/3}$$

References Listed by Equation Number (see detailed reference list attached):

(1) Wyngaard et al (71)

(2) Stull (88)

(3) Constructed by Stull using data from Neff & Coulter (86)

(4), (5), (6) Hartman et al (90); Kaimal et al (76)

(7) Constructed by Stull using equation from Keder et al (89)

C_{V^2} Wind Structure Parameter — Stable

Range: [0 to (0 to 0.5) to 1.5] $(m/s)^2 \cdot m^{-2/3}$

Equations:

$$C_{V^2}(0 \le z \le 0.1h) = (25 \pm 10) \cdot (z \cdot h)^{-1/3} \cdot u_*^2 \quad (1)$$

$$C_{V^2}(0 \le z \le 0.1h) = (8.8 \pm 4) \cdot u_*^2 \cdot L^{-2/3} \quad (2)$$

$$C_{V^2}(0.1h \le z \le h) = (7 \pm 3) \cdot z^{-1} \cdot h^{1/3} \cdot u_*^2 \quad (3)$$

Required input data:

u_*	friction velocity at the surface (z=0)
h	depth of the stable boundary layer
$\overline{u'w'}(z)$	turbulent vertical momentum flux of u-wind at height z (only eq 3)
$\overline{v'w'}(z)$	turbulent vertical momentum flux of v-wind at height z (only eq 3)
g	acceleration due to gravity = 9.8 m/s^2 (only eq 3)
$\overline{\theta}$	mean potential temperature (°K) (only eq 3)
L	Obukhov length (only eq 3) = $-[\overline{u'w'}^2(0) + \overline{v'w'}^2(0)]^{3/4} / [0.4 \cdot (g/\overline{\theta}) \cdot \overline{w'\theta'}(0)]$

Comments:

a) In stable conditions, turbulence can be zero, intermittent and/or patchey. The relationships above assume a contiguous domain of turbulence coupled with surface forcings.

Sample Calculations with Typical Values:

Let $u_* = 0.1$ m/s, h = 300 m. Solve eq (1) at z = 10 m.

Thus $C_{V^2}(z=10m) = 25 \cdot [10 \cdot 300]^{-1/3} \cdot (0.1 \text{ m/s})^2 = 0.017 \; (m/s)^2 \cdot m^{-2/3}$

References Listed by Equation Number (see detailed reference list attached):

(1) Constructed by Stull using data from Neff & Coulter (86)
(2) Constructed by Stull using data from Stull (88) and Huynh et al (90)
(3) Constructed by Stull using data from Neff & Coulter (86)

C_{V^2} Wind Structure Parameter — Neutral

Range: [0 to (0 to 0.5) to 2] $(m/s)^2 \cdot m^{-2/3}$

Equations:

$$C_{V^2}(0 \le z \le 0.1h) = (5 \pm 1) \cdot u_*^2 \cdot z^{-2/3} \quad (1)$$

Required input data:

u_*	friction velocity at the surface (z=0)
h	depth of the neutral boundary layer (m)

Comments:

a) Although eq (1) is designed for the neutral surface layer, it might hold much higher in the boundary layer. There is little data for C_{V^2} higher in the neutral boundary layer.

Sample Calculations with Typical Values:

Let: $u_* = 0.4$ m/s, h = 500 m. Solve (3) for z = 10 m:

$C_{V^2}(z=10m) = 5 \cdot (0.4\ m/s)^2\ 10^{-2/3} = 0.17\ (m/s)^2 \cdot m^{-2/3}$

References Listed by Equation Number (see detailed reference list attached):

(1) Constructed by Stull from Stull (88) & Sorbjan (86)

C_{V^2} Wind Structure Parameter — Unstable

Range: [0 to (0.01 to 1.0) to 3.0] $(m/s)^2 \cdot m^{-2/3}$

Equations:

$$C_{V^2}\,(0.01z_i \le z \le 0.1z_i) = [\,(1.3 \pm 0.3) + 0.1 \cdot (z/z_i)^{-2/3}\,] \cdot w_*^2 \cdot z_i^{-2/3} \quad (1)$$

$$C_{V^2}\,(0.1z_i \le z \le z_i) = (2 \pm 1) \cdot w_*^2 \cdot z_i^{-2/3} \quad (2)$$

$$C_{V^2}\,(0 \le z \le z_i) = (2 \pm 1) \cdot w_*^2 \cdot z_i^{-2/3} \cdot 10^{(0.67 \pm 0.4)\cdot[\,(1 - z/z_i)^2 - 1\,]} \quad (3)$$

$$C_{V^2} = (2 \pm 1) \cdot \varepsilon^{2/3} \quad (4)$$

Required input data:

$\overline{w'\theta'}(0)$	turbulent kinematic heat flux near the surface (z=0)
w_*	convective velocity scale = $[\,(g/\overline{\theta})\cdot z_i \cdot \overline{w'\theta'}(0)\,]^{1/3}$
z_i	depth of the unstable boundary layer (= height of first capping inversion)
ε	dissipation rate of turbulence kinetic energy (only eq 4)

Comments:

a) Eq (4) is difficult to use because it contains a turbulence dissipation rate.

b) Above the top of the mixed layer, the turbulence decreases rapidly, and C_{V^2} probably tends toward zero.

Sample Calculations with Typical Values:

Let $w_* = 2$ m/s, $z_i = 1000$ m, and solve equation (1) for z = 30 m:

$$C_{V^2}\,(z{=}30m) = [\,1.3 + 0.1\cdot(30/1000)^{-2/3}] \cdot 2^2 \cdot 1000^{-2/3} = 0.093 \;(m/s)^2 \cdot m^{-2/3}.$$

References Listed by Equation Number (see detailed reference list attached):

(1) Kaimal et al (76)

(2) Kaimal et al (76); Neff & Coulter (86)

(3) Constructed by Stull using data from Kaimal et al (73); Huynh et al (90)

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Acknowledgements

Pete Pokrandt is gratefully acknowledged for his major effort in reviewing the current literature, without which this summary would not have been a success.