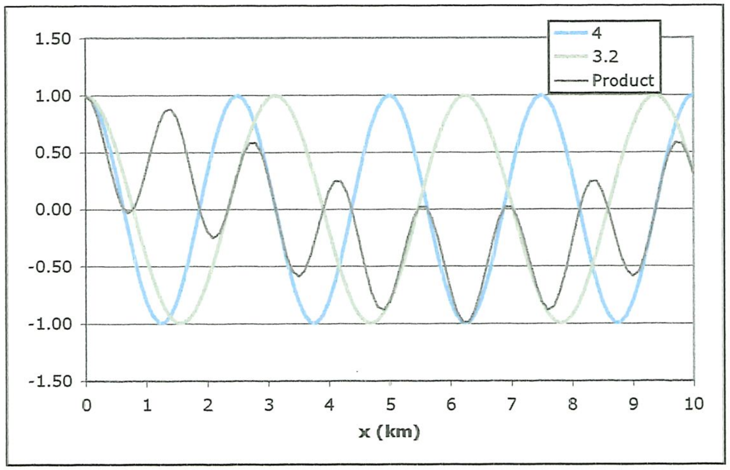
1) Product of 2 waves ... Stull Feb 2013



Amplitude (°C)= 1

Domain (km) = 10

Wavenumber (waves/domain)

4	3.2
n	m

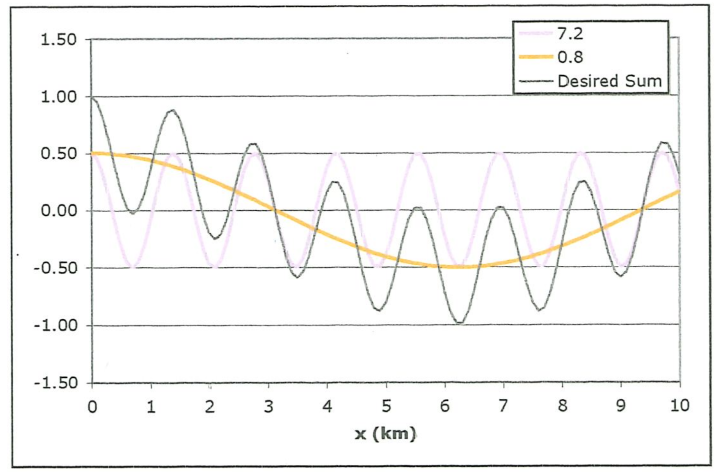
<= **PLAY WITH THESE**

2.50	3.13	= wavelength (km)
2.50	3.13	= wavelength/Δx

Actual Waves:

x (km)	Wavelength (km) 2.50	3.13	Product
0	1.00	1.00	1.00
0.1	0.97	0.98	0.95
0.2	0.88	0.92	0.81
0.3	0.73	0.82	0.60
0.4	0.54	0.69	0.37
0.5	0.31	0.54	0.17
0.6	0.06	0.36	0.02
0.7	-0.19	0.16	-0.03
0.8	-0.43	-0.04	0.02
0.9	-0.64	-0.24	0.15
1	-0.81	-0.43	0.34
1.1	-0.93	-0.60	0.56
1.2	-0.99	-0.75	0.74
1.3	-0.99	-0.86	0.86
1.4	-0.93	-0.95	0.88

2) ... acts like sum of 2 different waves ...



Amplitude (°C)= 0.5 = Original amplitude / 2

Domain (km) = 10

Wavenumber (waves/domain)

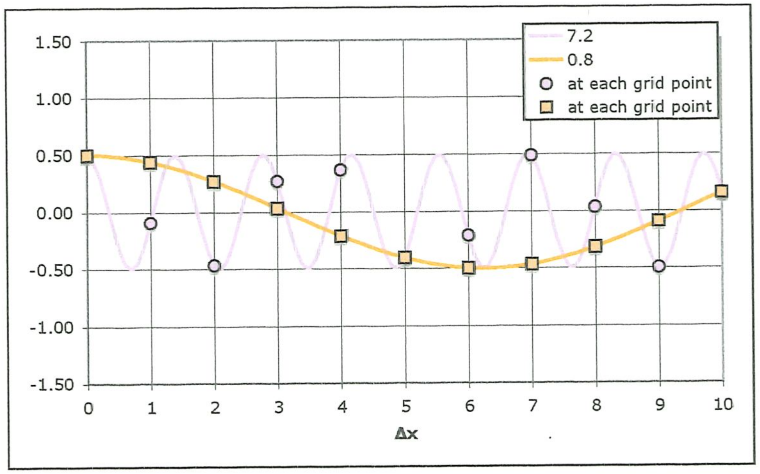
7.2	0.8
= n + m	= n – m

1.39	12.50	= wavelength (km)
1.39	12.50	= wavelength/Δx

Actual Waves:

x (km)	Wavelength (km) 1.39	12.50	Desired Sum
0	0.50	0.50	1.00
0.1	0.45	0.50	0.95
0.2	0.31	0.50	0.81
0.3	0.11	0.49	0.60
0.4	-0.12	0.49	0.37
0.5	-0.32	0.48	0.17
0.6	-0.46	0.48	0.02
0.7	-0.50	0.47	-0.03
0.8	-0.44	0.46	0.02
0.9	-0.30	0.45	0.15
1	-0.09	0.44	0.34
1.1	0.13	0.43	0.56
1.2	0.33	0.41	0.74
1.3	0.46	0.40	0.86
1.4	0.50	0.38	0.88

3) … but when discretized (sampled only at grid points) …



Amplitude (°C)= 0.5

Δx (km) = 1

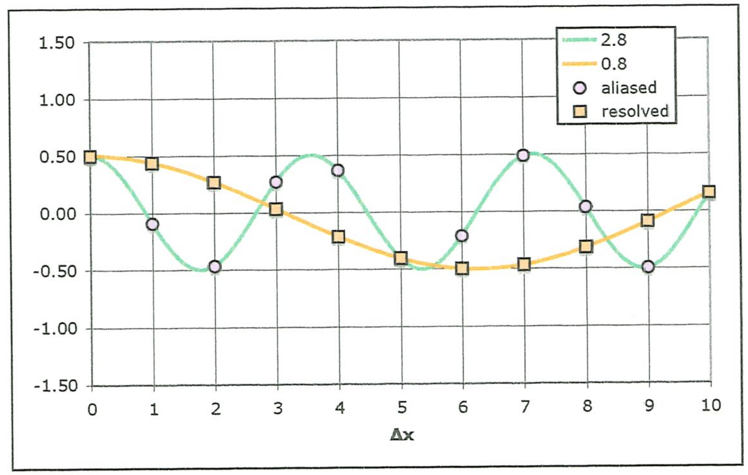
Domain (km) = 10 (km)

Wavenumber (waves/domain)		
7.2	0.8	
= n + m	= n − m	
1.39	12.50	= wavelength (km)
1.39	12.50	= wavelength/Δx

Discretized Waves:

grid point	x at each grid point	Wavelength (km) 1.389	12.5
0	0	0.50	0.50
1	1	-0.09	0.44
2	2	-0.46	0.27
3	3	0.27	0.03
4	4	0.36	-0.21
5	5	-0.40	-0.40
6	6	-0.21	-0.50
7	7	0.48	-0.46
8	8	0.03	-0.32
9	9	-0.50	-0.09
10	10	0.15	0.15
11			
12			
13			
14			

4) … causes the unresolved short wave to be aliased into a longer resolved wave …



Amplitude= 0.5

Domain (km) = 10

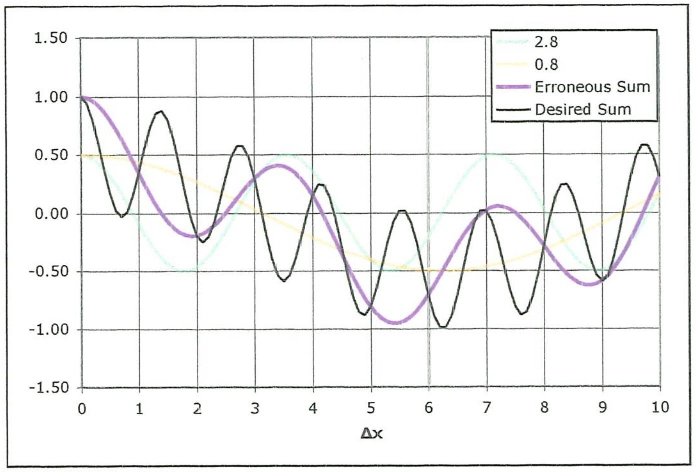
Jmax = Domain / Δx = 10

Wavenumber (waves/domain)		
2.8	0.8	
=Jmax−(m+n)	= n − m	
3.57	12.50	= wavelength (km)
3.57	12.50	= wavelength/Δx

Erroneous Result:

Δx	**aliased** Wavelength (km) 3.571	**resolved** 12.500	Erroneous Sum
0	0.50	0.50	1.00
0.1	0.49	0.50	0.99
0.2	0.47	0.50	0.97
0.3	0.43	0.49	0.93
0.4	0.38	0.49	0.87
0.5	0.32	0.48	0.80
0.6	0.25	0.48	0.72
0.7	0.17	0.47	0.64
0.8	0.08	0.46	0.54
0.9	-0.01	0.45	0.44
1	-0.09	0.44	0.34
1.1	-0.18	0.43	0.25
1.2	-0.26	0.41	0.15
1.3	-0.33	0.40	0.07
1.4	-0.39	0.38	-0.01

5) ... causing the sum of resolved waves to NOT equal the desired signal.



	For our case
The highest wavenumber that can be resolved is called the **Nyquist wavenumber** = Jmax / 2 =	5
This Nyquist wavenumber corresponds to a **smallest resolvable wavelength** = 2 Δx = Domain / Nyquist =	2
	(km)

Wavenumbers greater than Nyquist cannot be resolved.			
Our original wave of unresolved wavenumber =			7.2
is	2.2	greater than ...	
		...the Nyquist wavenumber of	5
Hence, the wave is folded to a new wavenumber			
that is	2.2	smaller than Nyquist	2.8

...which is the same as the [Jmax -(m+n)] wavenumber from the previousl page.

Unresolved waves having wavenumber mount **s** above the Nyquist wavenumber are **folded** into erroneous resolved waves having wavenumber distance **s** below Nyquist.

See illustration in Warner Fig. 3.22 .