atsc507_fv3mpas

March 21, 2020

1. **(/10)** Consider a 1D domain, from $x = -1$ to $x = 1$. For $N = 3$ (i.e. 3 Voronoi regions), find the locations of the generating points z_i of a **centroidal** Voronoi tessellation for the density function $\rho = 1$. You should be able to do this by inspection, but explain how you got your answer regardless.

   Then, for the same domain, consider a sequence of points $x = -0.9, -0.8, -0.7, ..., 0.7, 0.8, 0.9$. Using the points in the sequence, approximate the cell centroids for $N = 3$. Does your answer match what you got by inspection? Why or why not?

   Hint 1: You should divide up the sequence of points into 3 regions between $x = -1$ and $x = 1$ for summation.

   Hint 2: Using a spreadsheet or a programming language may help with your computation.

2. **(/10)** Assume a constant, positive vertical velocity w blowing across MPAS vertical levels. Write down the **vertical flux divergence** (i.e. flux_{top} - flux_{bottom}) of a cell-averaged scalar ψ for cell k, expressed in terms of $\psi_{k-1}, \psi_k, \psi_{k+1}$, to **third-order accuracy**. Explain why your chosen scheme is an upwind-biased scheme.

   Hint: You may start from the expression of the flux across the top edge of the cell, i.e. the already-derived $F_{k+\frac{1}{2}}$. Unless you desperately want to, there is no need to start from the original FV definitions to derive the flux divergence.

3. **(/5)** For a cube with dimensions $2a \times 2a \times 2a$ inscribed within a sphere with radius R, show that $a = \frac{\sqrt{3}}{3}R$.

4. **(/20)** Using a programming language of your choice, construct and plot a **c12** cubed-sphere mesh using (a) an equidistant gnomonic projection, and (b) an equiangular gnomonic projection. Your plots should be similar to Fig. 1c in Putman and Lin (2007): "Finite-volume transport on various cubed-sphere grids." Obviously your meshes would be of a much coarser resolution. How do your two different projections compare? What happens to the two projections when you increase the resolution of your meshes, say to c24, c36, c48, etc.?