MPAS NUMERICS, DYNAMICS, AND RESEARCH

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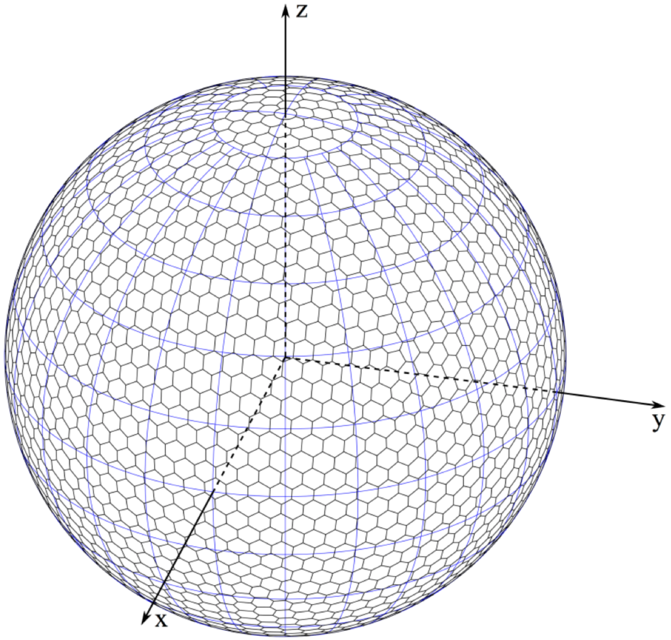
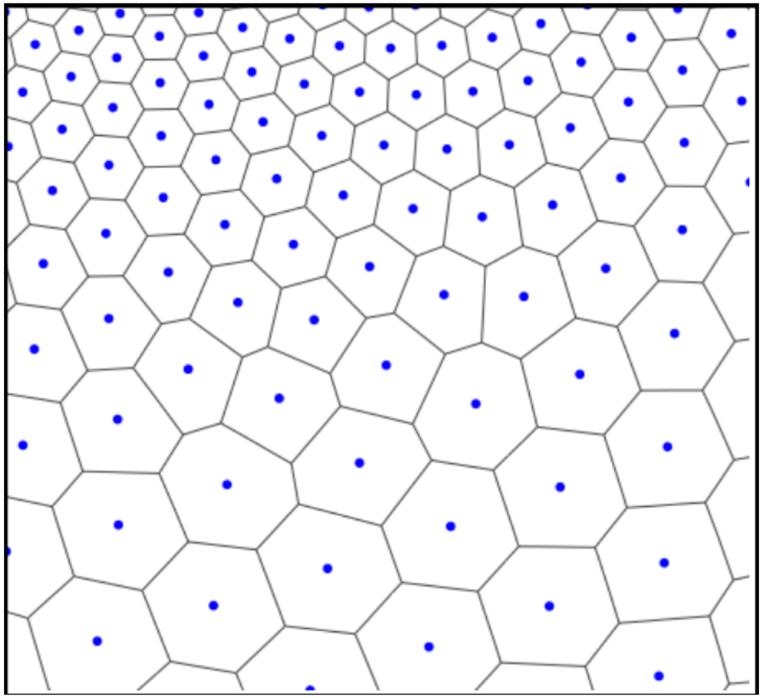
Mar. 21, 2019



http://mpas-dev.github.io/atmosphere/tutorial.html

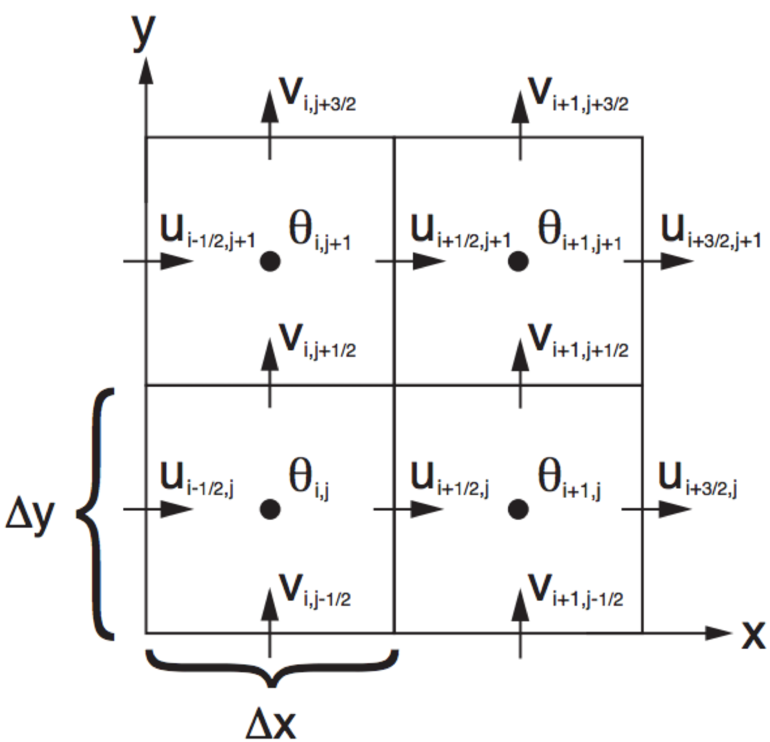
Overview of seminar series

- Introduction to MPAS
- The MPAS Mesh
- Introduction to Finite-Volume Discretization
- **MPAS Numerics, Dynamic, and Research**

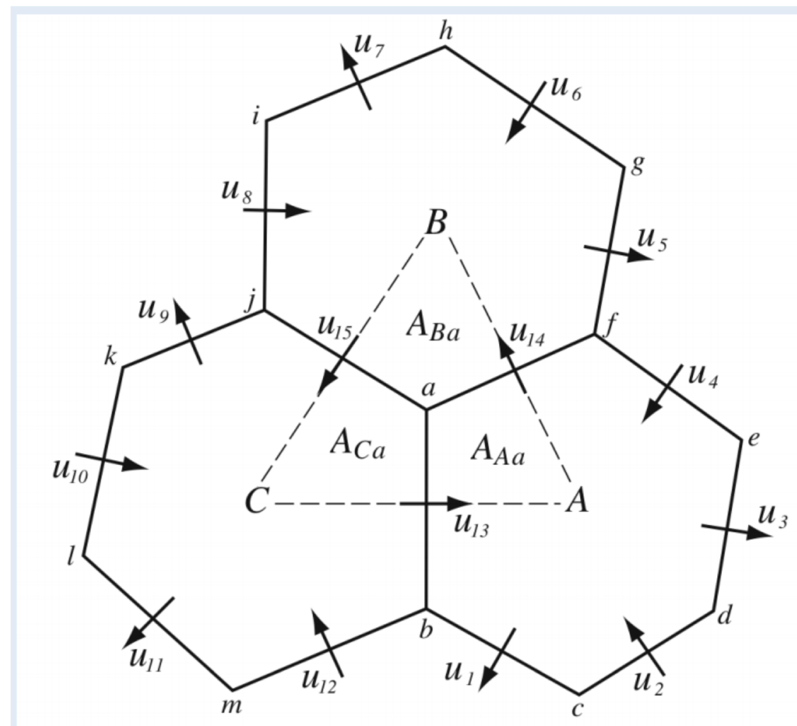


http://mpas-dev.github.io/atmosphere/tutorial.html

Seminar 1: WRF vs. MPAS



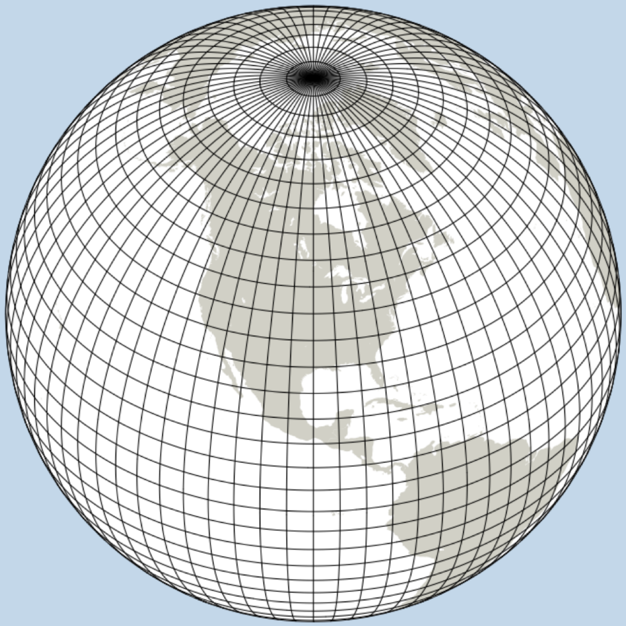
horizontal grid



http://mpas-dev.github.io/atmosphere/tutorial.html

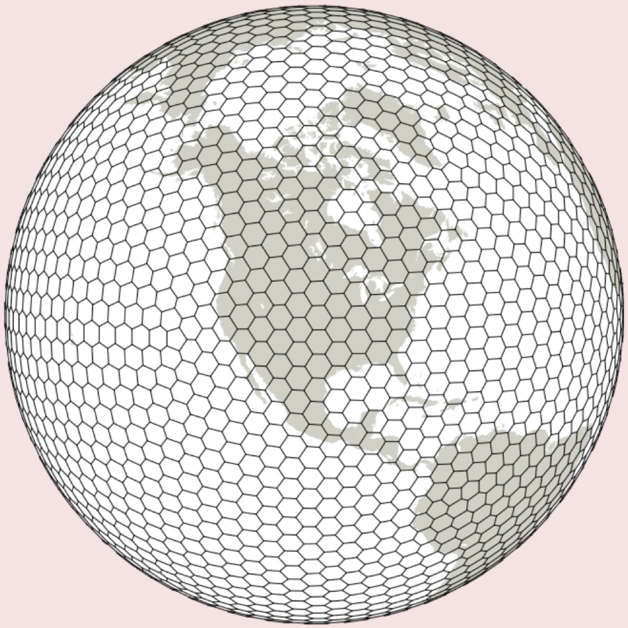
- Eulerian, finite-difference, **structured**
- Eulerian, finite-volume, **unstructured**

Seminar 1: WRF vs. MPAS



WRF
Lat-Lon global grid

- Anisotropic grid cells
- Polar filtering required
- Poor scaling on massively parallel computers

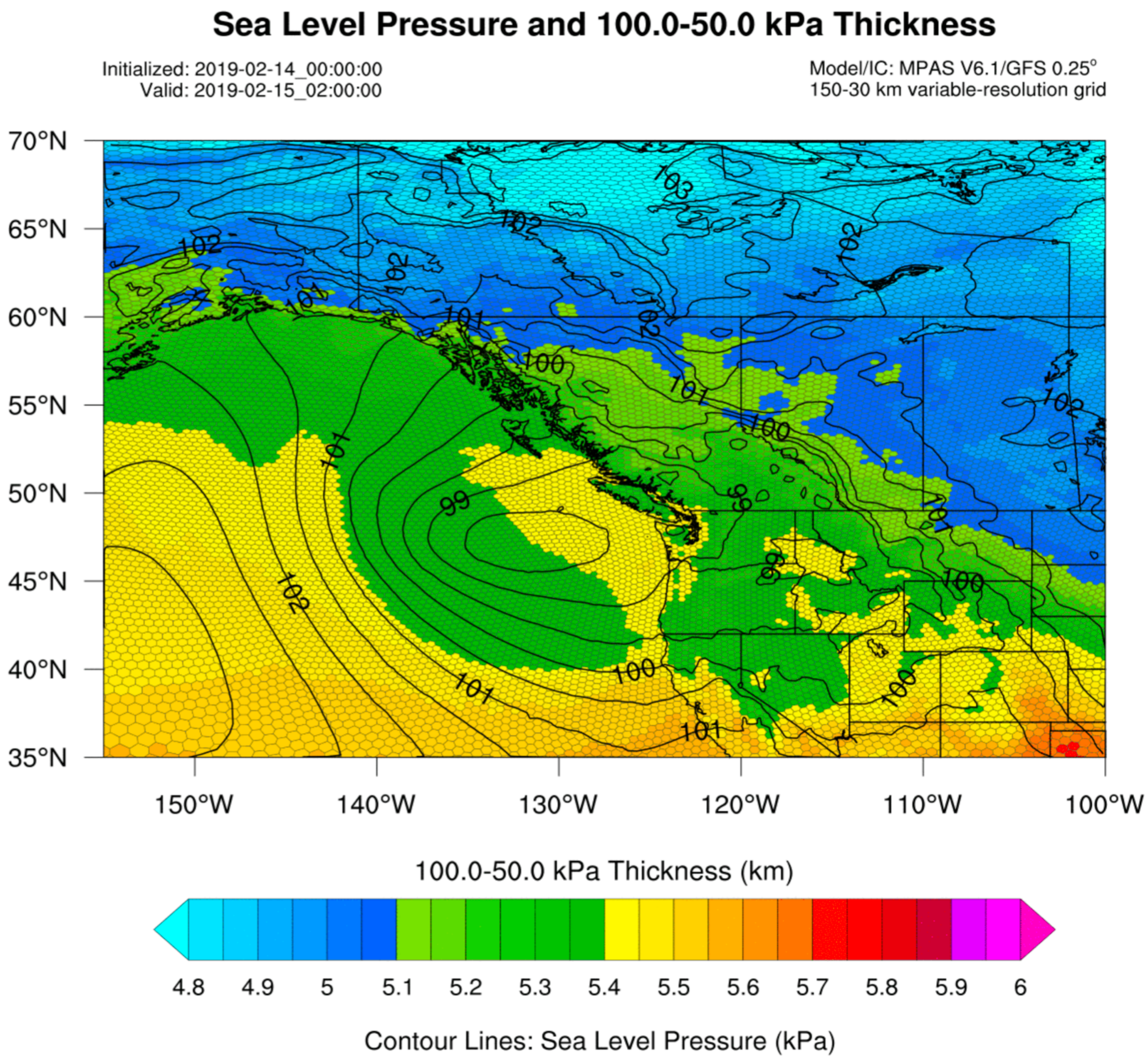


MPAS
Unstructured Voronoi (hexagonal) grid

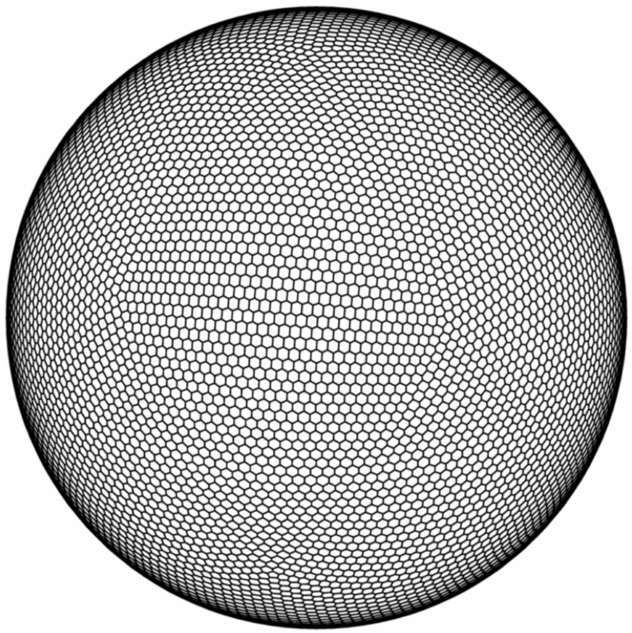
- Good scaling on massively parallel computers
- No pole problems

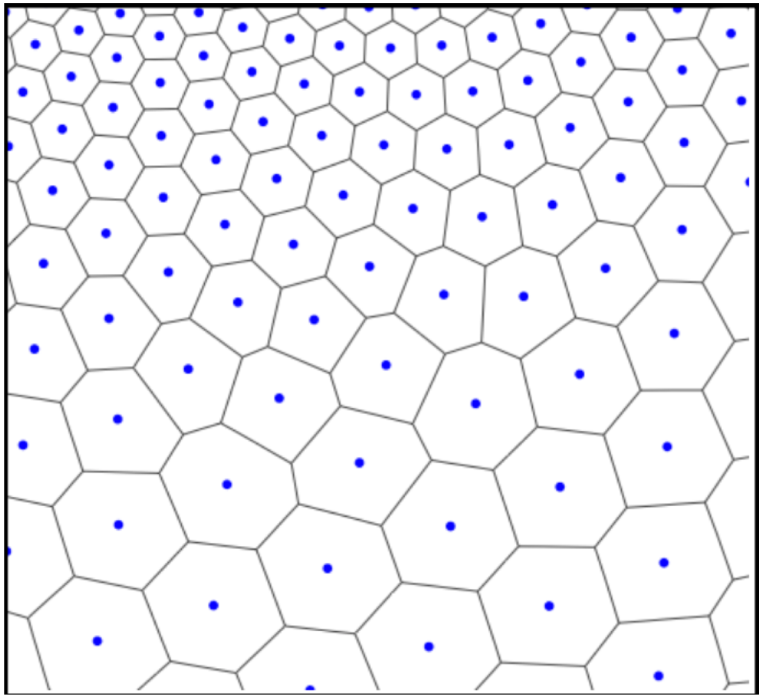
http://mpas-dev.github.io/atmosphere/tutorial.html

Seminar 1: MPAS Forecasts



Seminar 2: Spherical Centroidal Voronoi Tessellations





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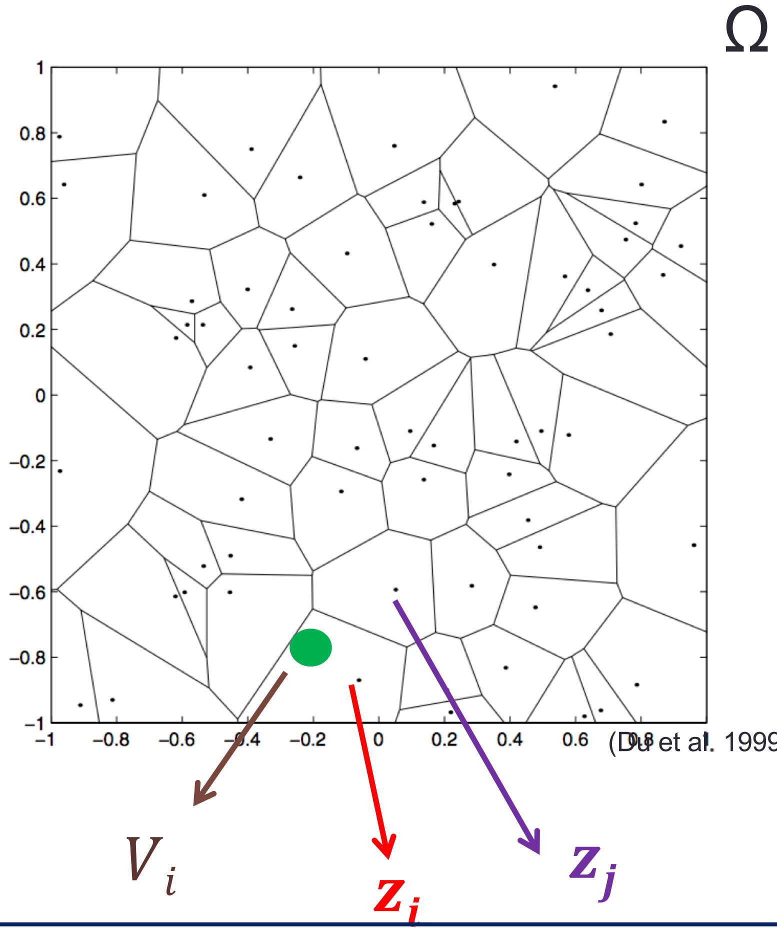
Seminar 2: Spherical Centroidal Voronoi Tessellations

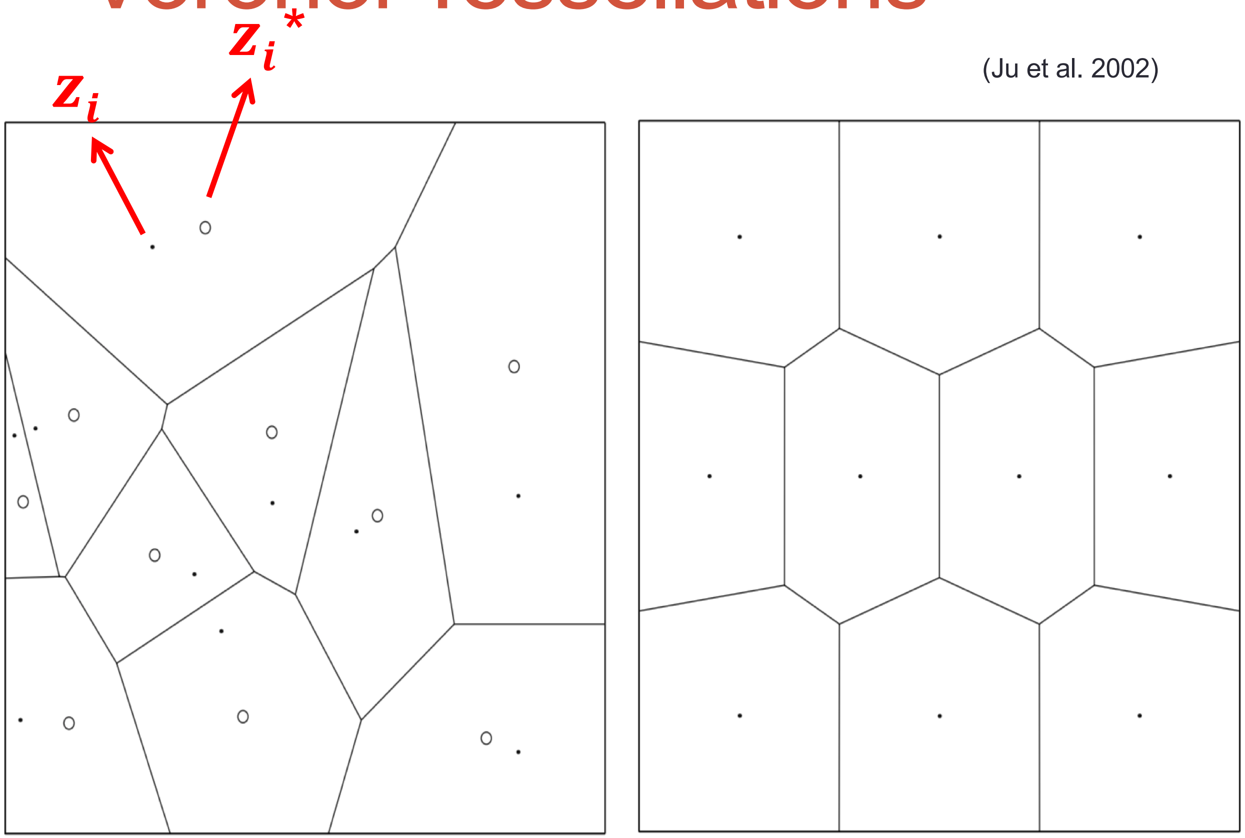


(Du et al. 1999)

- Points associated with a Voronoi region V_i make up a Voronoi set $\hat{V}_i$

$$\hat{V}_i = \{\boldsymbol{x} \in \Omega \mid |\boldsymbol{x} - \boldsymbol{z_i}| < |\boldsymbol{x} - \boldsymbol{z_j}| \text{ for } j = 1, \dots, k, j \neq i\}$$

Seminar 2: Spherical Centroidal Voronoi Tessellations



(Ju et al. 2002)

$\mathbf{z}_i \neq \mathbf{z}_i^*$, $i = 1, \ldots, k$ $\quad$ $\mathbf{z}_i = \mathbf{z}_i^*$, $i = 1, \ldots, k$

- In general, the generating point z_i of each Voronoi region is not the same as the mass centroid z_i^* of the region

Seminar 3: Finite-Volume Discretization

$$\frac{d\bar{U}}{dt} = -\frac{1}{V}\oiint_{\partial CV} (\vec{F}\cdot\hat{n})\,dA + \bar{S}$$

- Change in the control volume average $\bar{U}$ is due to stuff leaving and entering control volume, and stuff $\bar{S}$ being created or destroyed on the inside

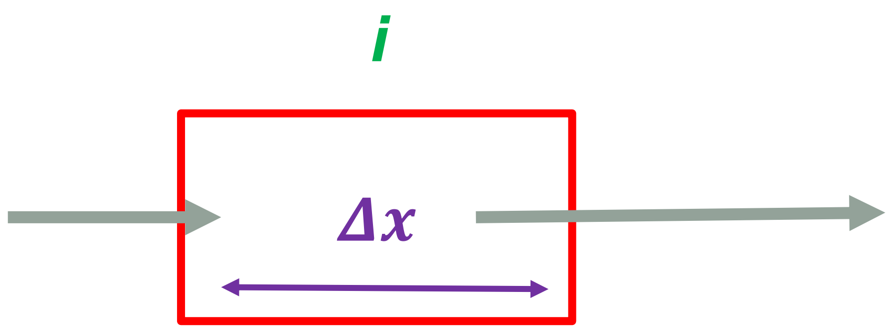
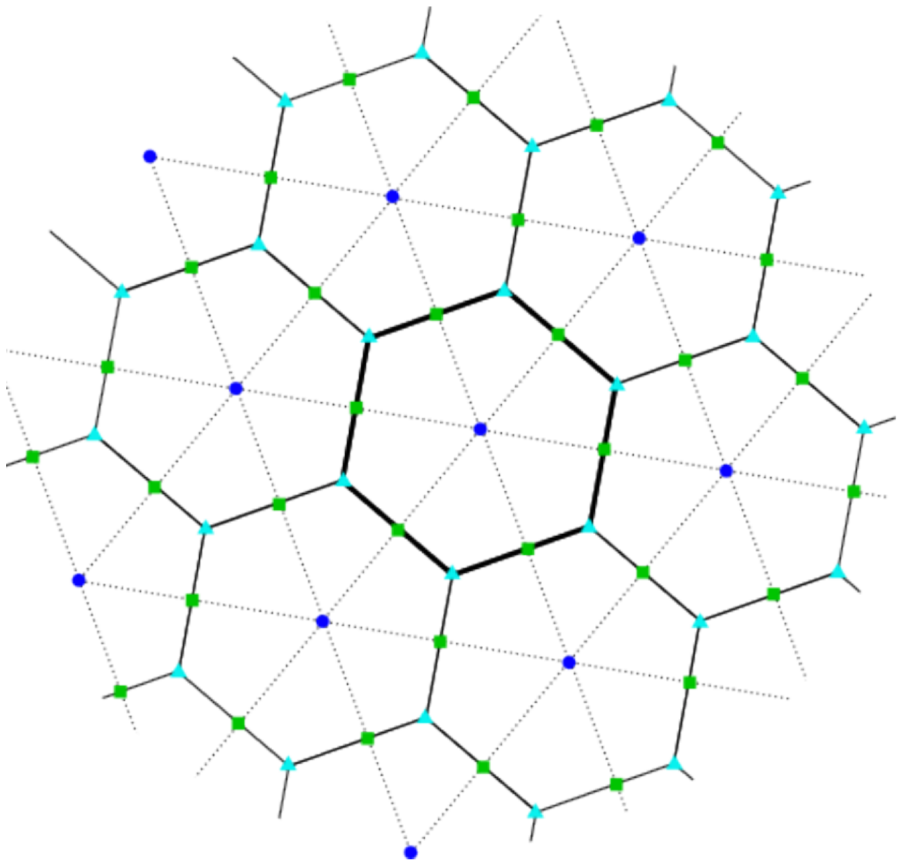


Table of Contents

- Vertical Coordinate System
- Horizontal Coordinate System
- MPAS Dynamical Equations
- Vertical Discretization
- Horizontal Discretization
- Summary



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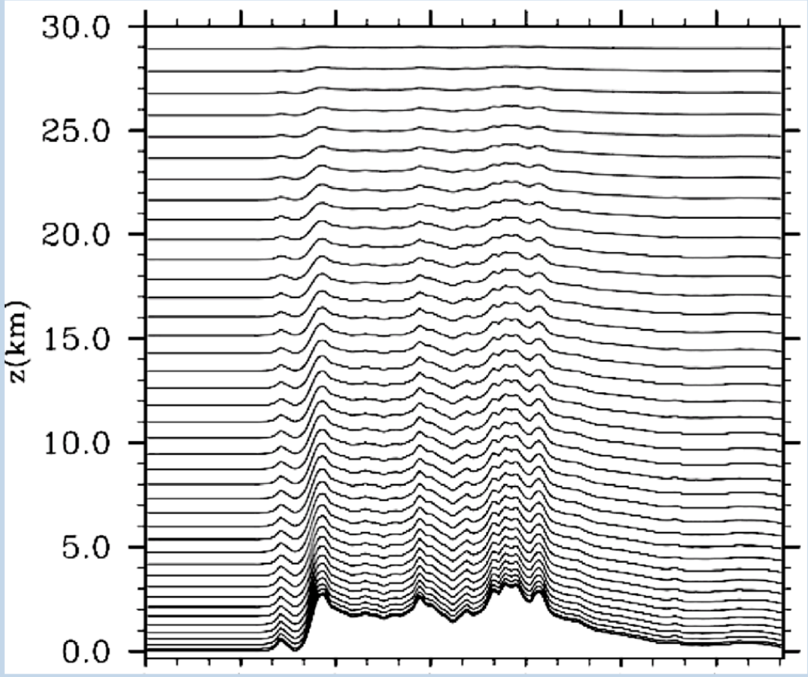
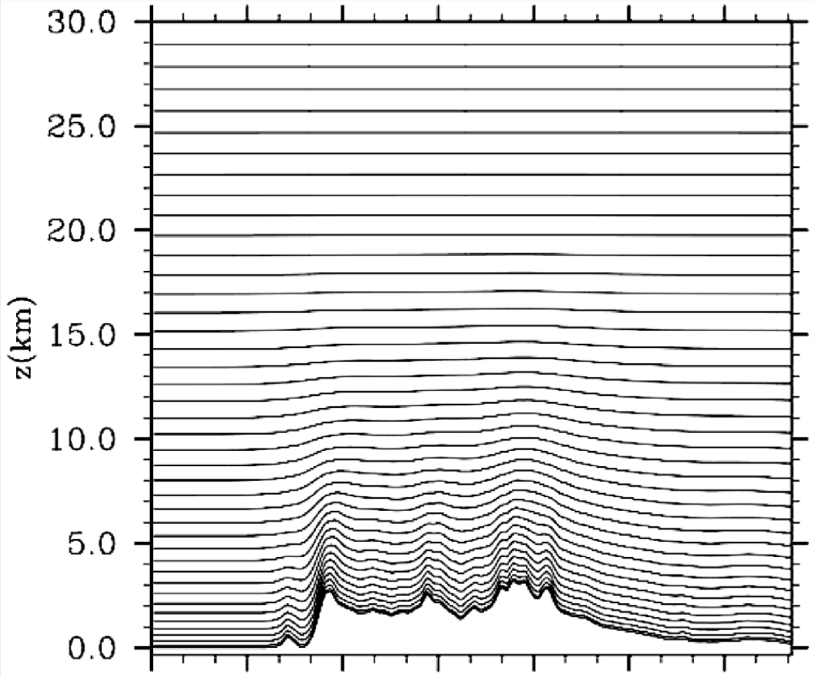
Vertical Coordinate System



WRF
Pressure-based
terrain-following sigma
vertical coordinate



MPAS
Height-based hybrid smoothed
terrain-following vertical coordinate

- Improved numerical accuracy

http://www2.mmm.ucar.edu/projects/mpas/tutorial/UK2015/slides/MPAS-solver_physics.pdf

Vertical Coordinate System

Basic terrain-following (**BTF**; Gal-Chen and Somerville 1975):

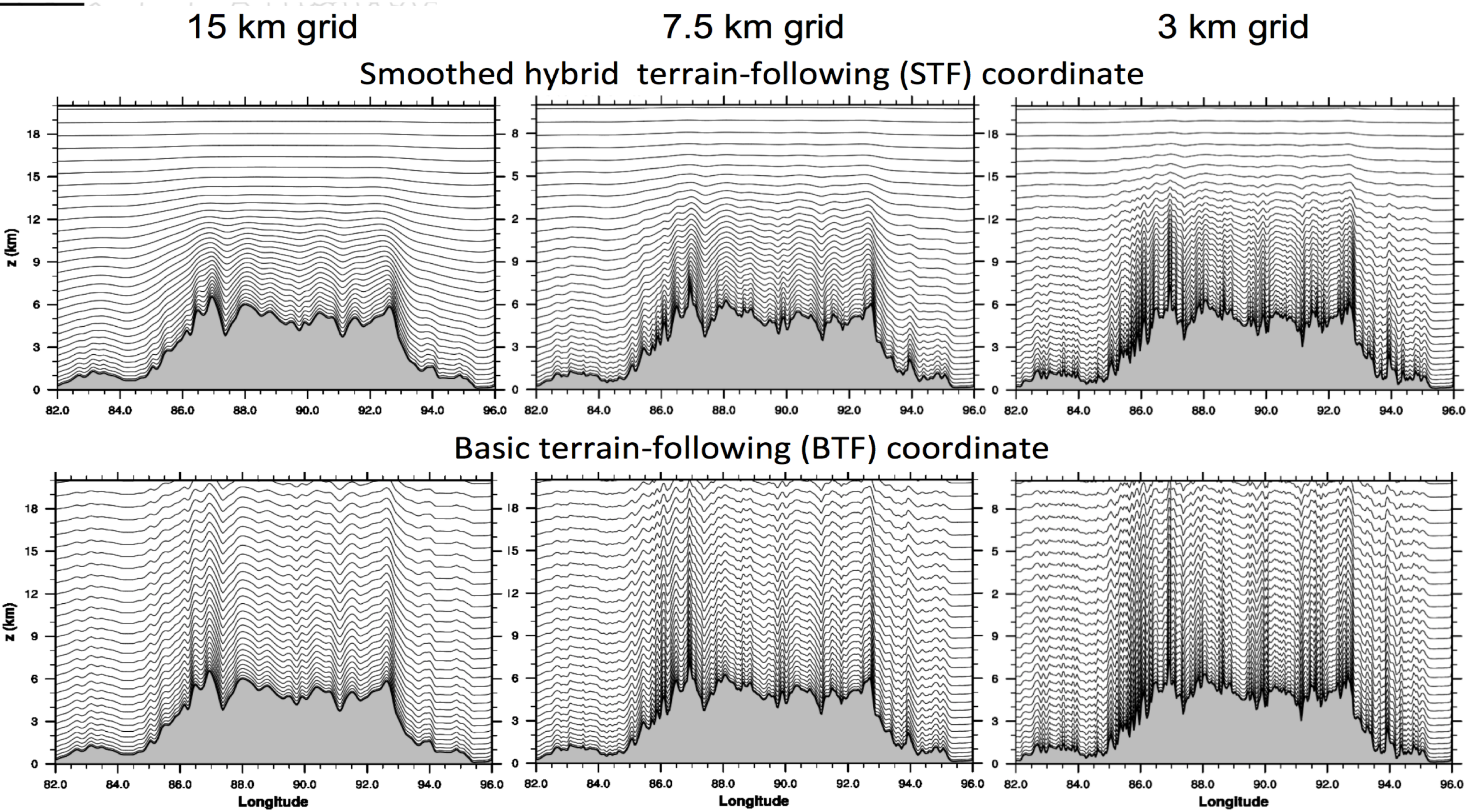
$$z(x, y, \zeta) = \zeta + \left[1 - \frac{\zeta}{z_t}\right] h(x, y)$$

Smoothed terrain-following (**STF**; Klemp 2011)

$$\boxed{z(x, y, \zeta) = \zeta + A(\zeta) h_s(x, y, \zeta)}$$

$z(x, y, \zeta) =$ geometric height [m]
$\zeta =$ terrain-following constant; for BTF $\zeta = z$ at z_t (model lid)
$h(x, y) =$ terrain height [m]
$h_s(x, y, \zeta) =$ terrain influence in STF; $h_s(x, y, 0) = h$
$A(\zeta) =$ controls how quickly terrain-following -> constant height

Vertical Coordinate System



(Model top is at 30 km)

http://www2.mmm.ucar.edu/projects/mpas/tutorial/UK2015/slides/MPAS-solver_physics.pdf

Vertical Coordinate System - Init

- **Terrain**
  - 30 arcsec data averaged over grid-cell areas
  - Single pass of smoothing using 4th-order Laplacian
- **Vertical Coordinate**
  - Choice non-smoothed (BTF) or *n*-times smoothed (STF); iterate upwards from $h_s^n(x, y, 0) = h$

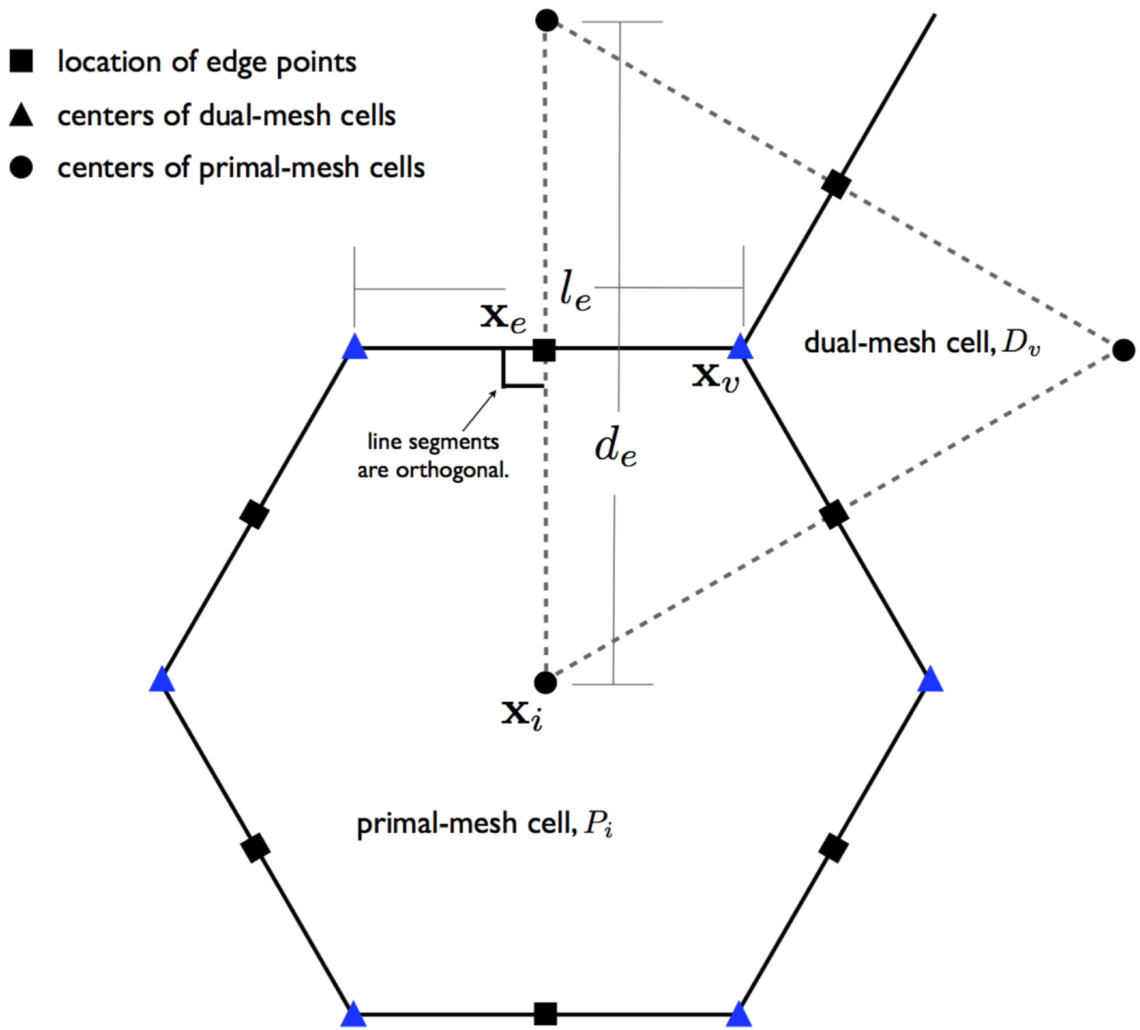
$\beta(\zeta) =$ smoothing coefficient
$d =$ grid-cell length scale
$z_H =$ height where $\zeta = z$
$\nabla_\zeta^2 h_s^n =$ Laplacian

$$h_s^{n+1} = h_s^n + \beta(\zeta) d^2 \nabla_\zeta^2 h_s^n$$

$$A(\zeta) = \cos^6\left(\frac{\pi}{2}\frac{\zeta}{z_H}\right), \qquad \zeta < z_H$$

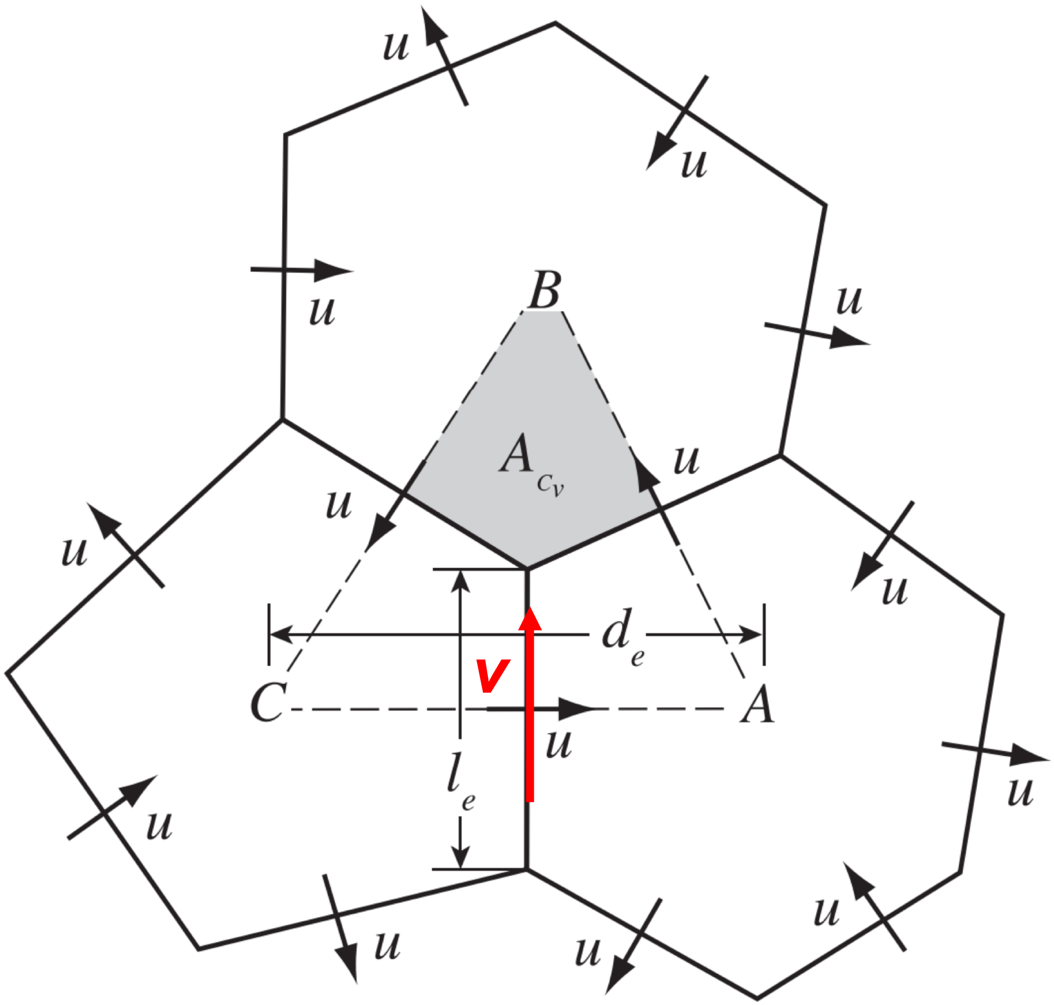
$$A(\zeta) = 0, \qquad \zeta \geq z_H$$

Horizontal Coordinate System



http://www2.mmm.ucar.edu/projects/mpas/mpas_atmosphere_users_guide_6.0.pdf

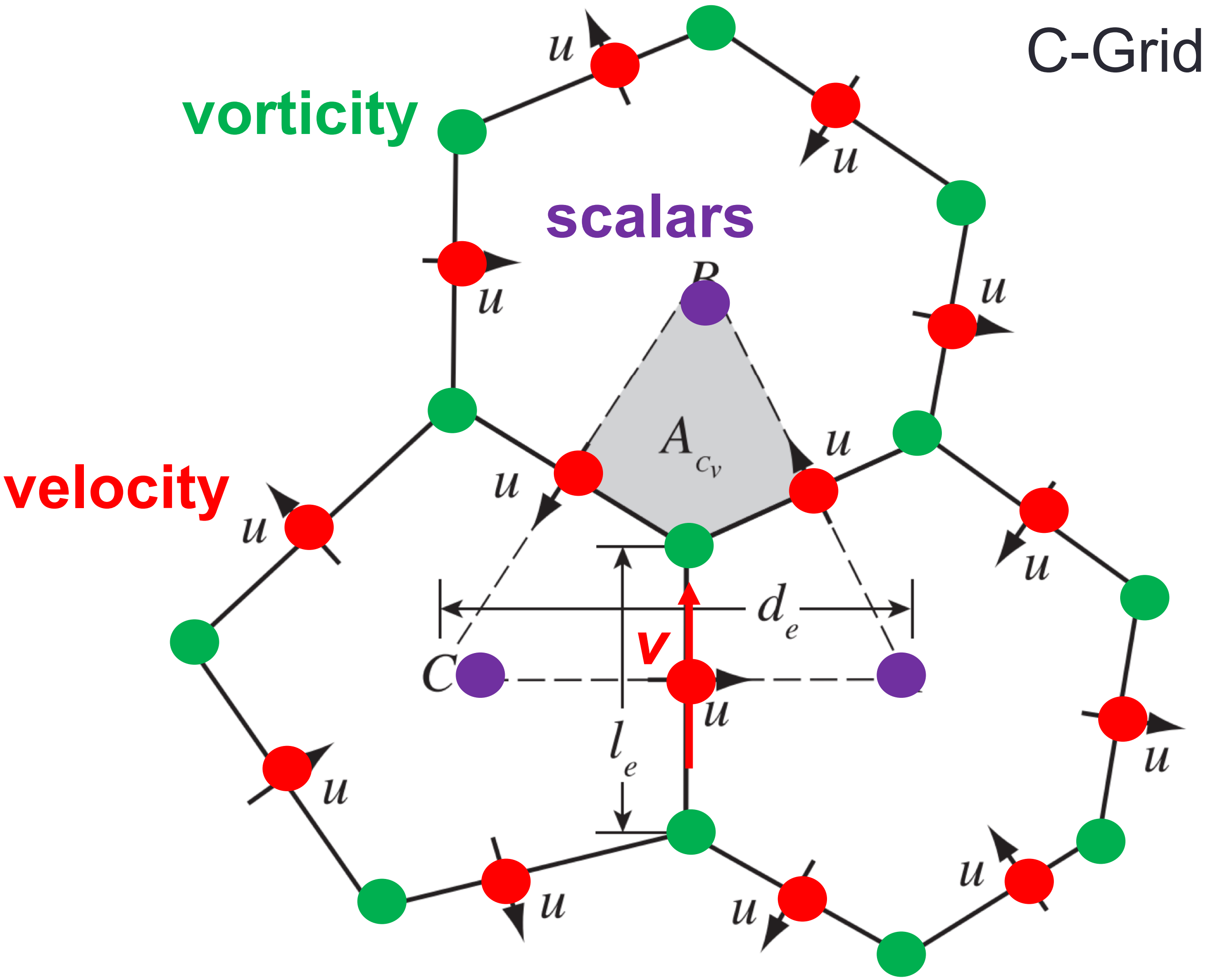
Horizontal Coordinate System



Skamarock et al. (2012)

Horizontal Coordinate System

C-Grid Staggering



Skamarock et al. (2012)

MPAS Dynamical Equations

- Geostrophic flow
- Dry
- Incompressible
- Pythagorean

$$u = -\frac{1}{\rho f}\frac{\partial p}{\partial y}$$

$$v = \frac{1}{\rho f}\frac{\partial p}{\partial x}$$

$$a^2 + b^2 = c^2$$

Skamarock et al. (2012)

MPAS Dynamical Equations

- Coupled variables in conservative form
- Hybrid-height coordinate
- Nonhydrostatic
- Fully compressible
- Coupled (moist) potential temperature

$$\theta_m = \theta[1 + (R_v/R_d)q_v]$$

$$p = p_0\left(\frac{R_d\zeta_z\Theta_m}{p_0}\right)^{\gamma}$$

$$\frac{\rho_m}{\rho_d} = 1 + q_v + q_c + q_r + \cdots$$

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\nabla_\zeta\left(\frac{p}{\zeta_z}\right) - \frac{\partial \mathbf{z}_H p}{\partial \zeta}\right] - \eta \mathbf{k} \times \mathbf{V}_H - \boldsymbol{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \boldsymbol{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K - eW\cos\alpha_r - \frac{\boldsymbol{v}_H W}{r_e} + \mathbf{F}_{\mathbf{V}_H},$$

$$\frac{\partial W}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\frac{\partial p}{\partial \zeta} + g\tilde{\rho}_m\right] - (\nabla \cdot \boldsymbol{v}W)_\zeta + \frac{uU + vV}{r_e} + e(U\cos\alpha_r - V\sin\alpha_r) + F_W,$$

$$\frac{\partial \Theta_m}{\partial t} = -(\nabla \cdot \mathbf{V}\theta_m)_\zeta + F_{\Theta_m},$$

$$\frac{\partial \tilde{\rho}_d}{\partial t} = -(\nabla \cdot \mathbf{V})_\zeta, \quad \text{and}$$

$$\frac{\partial Q_j}{\partial t} = -(\nabla \cdot \mathbf{V}q_j)_\zeta + F_{Q_j}.$$

Skamarock et al. (2012)

MPAS Dynamical Equations

- Coupled variables in conservative form
- Hybrid-height coordinate
- Nonhydrostatic
- Fully compressible
- Coupled (moist) potential temperature

Moist (virtual) θ

$$\theta_m = \theta[1 + (R_v/R_d)q_v]$$

$$p = p_0\left(\frac{R_d \zeta_z \Theta_m}{p_0}\right)^{\gamma}$$

Eqn of state

$$\frac{\rho_m}{\rho_d} = 1 + q_v + q_c + q_r + \cdots$$

Moist air

Skamarock et al. (2012)

Hor. momentum

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\nabla_\zeta\left(\frac{p}{\zeta_z}\right) - \frac{\partial \mathbf{z}_H p}{\partial \zeta}\right] - \eta \mathbf{k} \times \mathbf{V}_H$$
$$- \boldsymbol{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \boldsymbol{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K$$
$$- eW\cos\alpha_r - \frac{\boldsymbol{v}_H W}{r_e} + \mathbf{F}_{\mathbf{V}_H},$$

$$\frac{\partial W}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\frac{\partial p}{\partial \zeta} + g\tilde{\rho}_m\right] - (\nabla \cdot \boldsymbol{v}W)_\zeta + \frac{uU + vV}{r_e}$$
$$+ e(U\cos\alpha_r - V\sin\alpha_r) + F_W,$$

Vert. momentum

$$\frac{\partial \Theta_m}{\partial t} = -(\nabla \cdot \mathbf{V}\theta_m)_\zeta + F_{\Theta_m},$$

Temperature

$$\frac{\partial \tilde{\rho}_d}{\partial t} = -(\nabla \cdot \mathbf{V})_\zeta, \quad \text{and}$$

Continuity

$$\frac{\partial Q_j}{\partial t} = -(\nabla \cdot \mathbf{V}q_j)_\zeta + F_{Q_j}.$$

Moisture

MPAS Dynamical Equations

- Flux-form prognostic equations

$$\left(U, V, W, \Theta_m, Q_j\right) = \widetilde{\rho_d} \cdot (u, v, w, \theta_m, q_j)$$

$$j = v \text{ (vapour)}, c \text{ (cloud)}, r \text{ (rainwater)}, \text{etc.}$$

$$\widetilde{\rho_d} = \rho_d \left(\frac{1}{\frac{d\zeta}{dz}} \right) = \rho_d \zeta_z$$

$$\boldsymbol{v} = (u, v, w), \boldsymbol{V} = (U, V, W)$$

Skamarock et al. (2012)

MPAS Dynamical Equations

- Vertical momentum

$$\frac{\partial W}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\frac{\partial p}{\partial \zeta} + g\tilde{\rho}_m\right] - (\nabla \cdot \boldsymbol{v}W)_\zeta + \frac{uU + vV}{r_e} + e(U\cos\alpha_r - V\sin\alpha_r) + F_W,$$

Skamarock et al. (2012)

MPAS Dynamical Equations

- Vertical momentum

Pressure gradient | Gravity | Advection | Curvature

$$\frac{\partial W}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\frac{\partial p}{\partial \zeta} + g\tilde{\rho}_m\right] - (\nabla \cdot \boldsymbol{v}W)_\zeta + \frac{uU + vV}{r_e} + e(U\cos\alpha_r - V\sin\alpha_r) + F_W,$$

Coriolis | Physics

Skamarock et al. (2012)

MPAS Dynamical Equations

- Horizontal momentum

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\nabla_\zeta\left(\frac{p}{\zeta_z}\right) - \frac{\partial \mathbf{z}_H p}{\partial \zeta}\right] - \eta \mathbf{k} \times \mathbf{V}_H$$

$$- \boldsymbol{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \boldsymbol{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K$$

$$- eW \cos\alpha_r - \frac{\boldsymbol{v}_H W}{r_e} + \mathbf{F}_{\mathbf{V}_H},$$

Skamarock et al. (2012)

MPAS Dynamical Equations

- Horizontal momentum

Pressure gradient

Nonlinear Coriolis (Vorticity)

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\nabla_\zeta\left(\frac{p}{\zeta_z}\right) - \frac{\partial \mathbf{z}_H p}{\partial \zeta}\right] - \eta \mathbf{k} \times \mathbf{V}_H$$

Advection

$$- \boldsymbol{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \boldsymbol{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K$$

Kinetic energy gradient

$$- eW \cos\alpha_r - \frac{\boldsymbol{v}_H W}{r_e} + \mathbf{F}_{\mathbf{V}_H},$$

Coriolis

Curvature

Physics

Skamarock et al. (2012)

Transport (Advection)

- ψ = some quantity

$$\frac{\partial \rho \psi}{\partial t} = -\nabla_{\zeta} \cdot \mathbf{v_H}(\rho \psi) - \frac{\partial W \rho \psi}{\partial z}$$

Skamarock et al. (2012)

Scalar Transport

- FV formulation over, integrate over cell

$$\int_V \frac{\partial \rho\psi}{\partial t} dV = -\int_V \big(\nabla \cdot \mathbf{v}(\rho\psi)\big) dV$$

Skamarock et al. (2012)

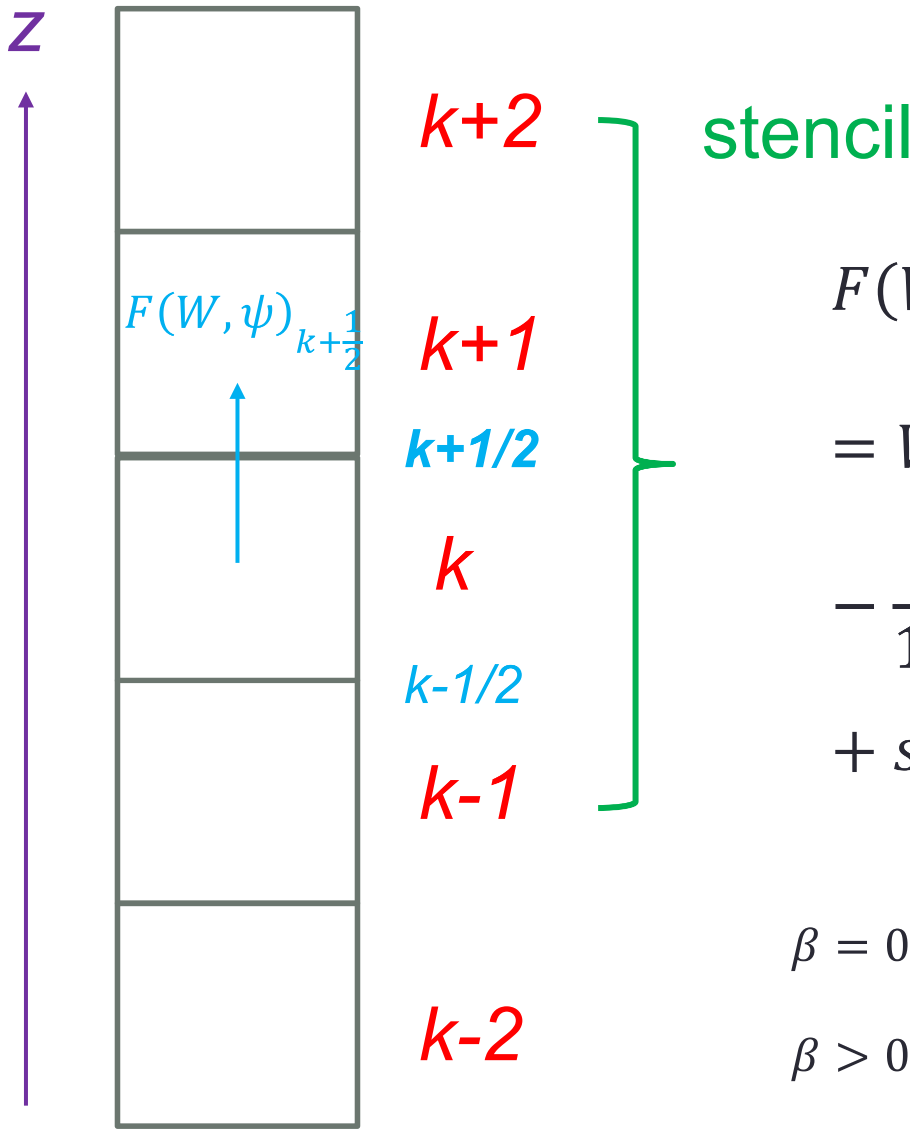
Scalar Transport

- Apply divergence theorem; control-volume average definition
- Σ = surface of polyhedron; $d\sigma =$ surface element

$$\frac{\partial \overline{\rho\psi}}{\partial t} = -\frac{1}{V}\int_{\Sigma} (\rho\psi)\mathbf{v}\cdot\mathbf{n}\,d\sigma$$

Skamarock et al. (2012)

Vertical Discretization



$$W = \rho w$$

$$\delta_z^2 \psi_k = \psi_{k-1} - 2\psi_k + \psi_{k+1}$$

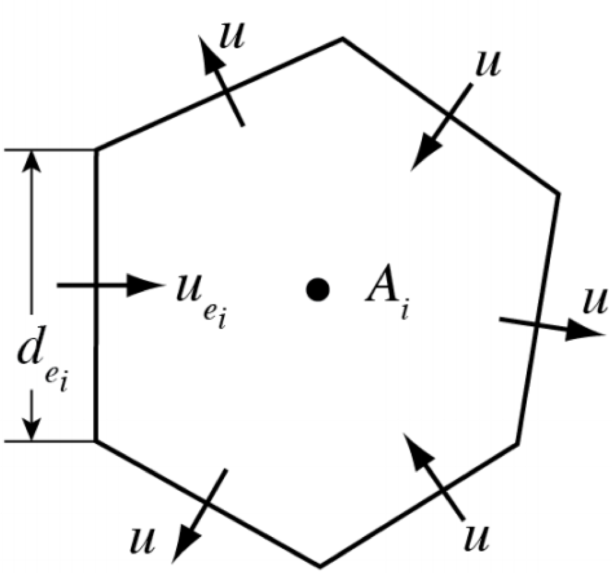
$$F(W,\psi)_{k+\frac{1}{2}} = W_{k+\frac{1}{2}} \left[\frac{1}{2}(\psi_{k+1} + \psi_k) - \frac{1}{12}(\delta_z^2 \psi_{k+1} + \delta_z^2 \psi_k) + sign(W)\frac{\beta}{12}(\delta_z^2 \psi_{k+1} - \delta_z^2 \psi_k) \right]$$

$\beta = 0$: 4th-order scheme, neutral

$\beta > 0$: 3rd -order scheme, damping

Horizontal Discretization

- Sum counter-clockwise around hexagon edges

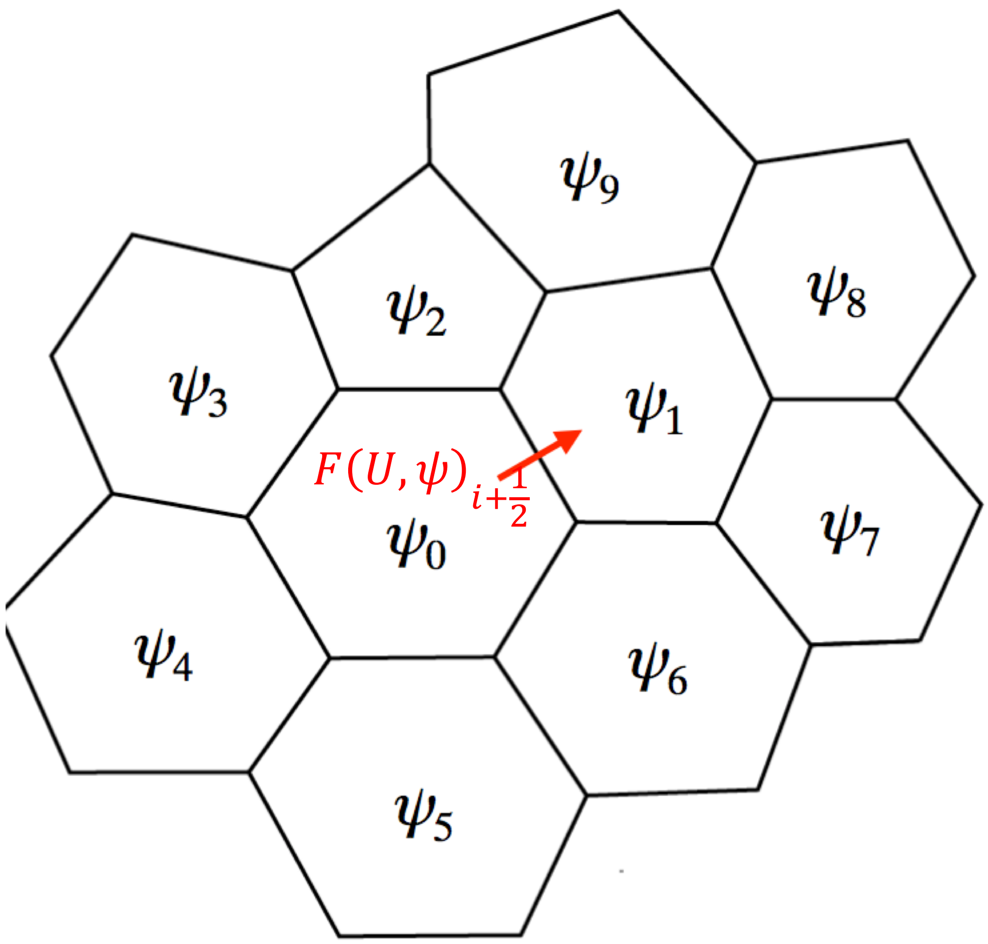


$$-\frac{1}{V}\int_{\Sigma}(\rho\psi)\mathbf{v}_{\mathbf{H}}\cdot\mathbf{n}\,d\sigma$$

$$\approx -\frac{1}{A_i}\sum_{n_{e_i}} d_{e_i}\overline{(\rho\mathbf{v}_{\mathbf{H}}\cdot\mathbf{n}_{\mathrm{e_i}})\psi_i}$$

https://www.earthsystemcog.org/site_media/projects/dycore_test_group/20160122_MPAS_configuration_overview.pdf

Horizontal Discretization



Horizontal Discretization

$$F(U,\psi)_{i+\frac{1}{2}} = U_{i+\frac{1}{2}}\left[\frac{1}{2}(\psi_{i+1}+\psi_i) - \frac{1}{12}(\delta_x^2\psi_{i+1}+\delta_x^2\psi_i) + sign(U)\frac{\beta}{12}(\delta_x^2\psi_{i+1}-\delta_x^2\psi_i)\right]$$

- x = direction normal to cell edges
- Coordinates aren't next to each other in the horizontal; can't do straight-forward computation of $\delta_x^2\psi_i$

$$\delta_x^2\psi_i = \Delta x^2\frac{\partial^2\psi}{\partial x^2} + \mathcal{O}(\Delta x^4)$$

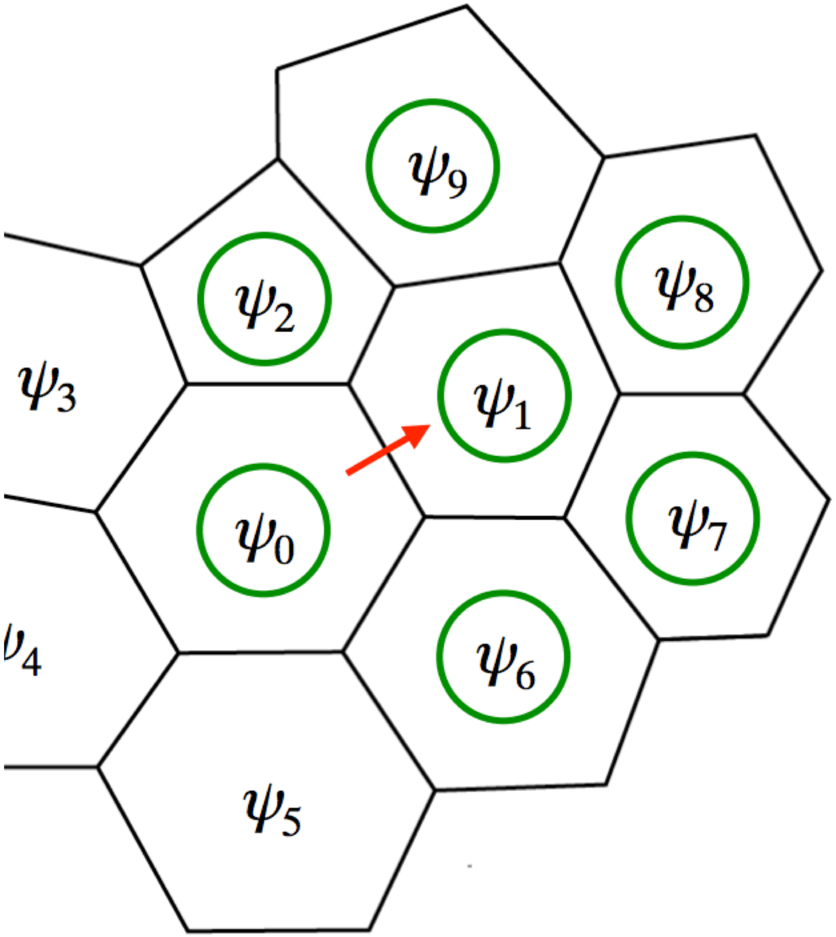
Horizontal Discretization

$$F(U,\psi)_{i+\frac{1}{2}}$$
$$= U_{i+\frac{1}{2}}\left[\frac{1}{2}(\psi_{i+1}+\psi_i)\right.$$
$$-\Delta x_e^2\frac{1}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1}+\left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\}$$
$$\left.+\,sign(U)\Delta x_e^2\frac{\beta}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1}-\left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\}\right]$$

Horizontal Discretization

https://www.earthsystemcog.org/site_media/projects/dycore_test_group/20160122_MPAS_configuration_overview.pdf

$$F(U,\psi)_{i+\frac{1}{2}} = U_{i+\frac{1}{2}}\left[\frac{1}{2}(\psi_{i+1}+\psi_i) - \Delta x_e^2 \frac{1}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1} + \left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\} + sign(U)\Delta x_e^2 \frac{\beta}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1} - \left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\}\right]$$

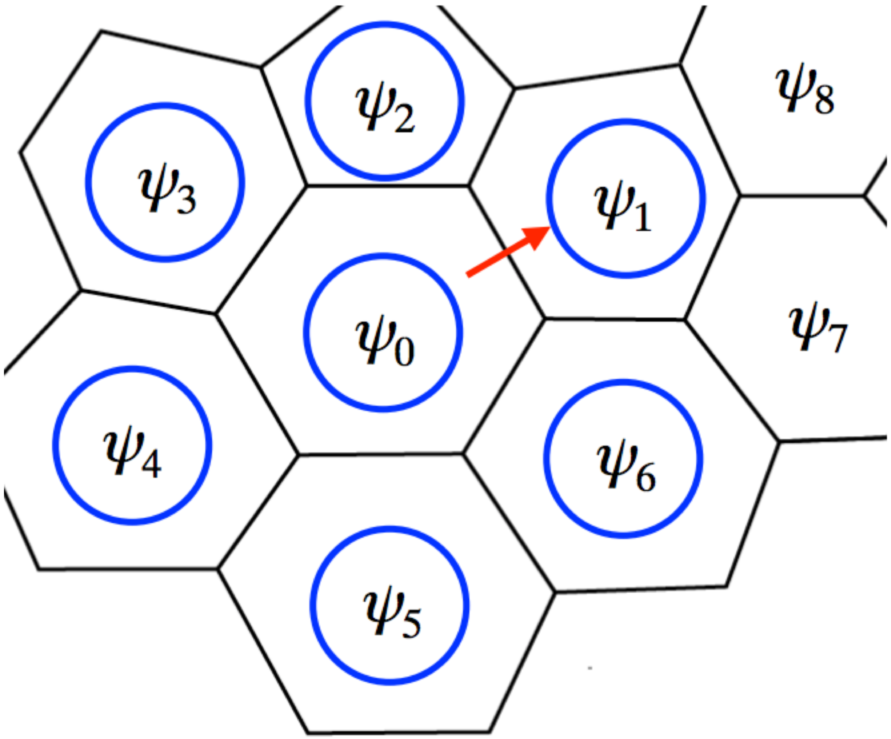


Fit polynomial to approximate second derivatives (Skamarock and Gassmann 2011)

Horizontal Discretization

https://www.earthsystemcog.org/site_media/projects/dycore_test_group/20160122_MPAS_configuration_overview.pdf

$$F(U,\psi)_{i+\frac{1}{2}}$$
$$= U_{i+\frac{1}{2}}\left[\frac{1}{2}(\psi_{i+1}+\psi_i)\right.$$
$$-\Delta x_e^2\frac{1}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1}+\left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\}$$
$$+\,sign(U)\Delta x_e^2\frac{\beta}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1}\right.$$
$$\left.\left.-\left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\}\right]$$



Fit polynomial to approximate second derivatives (Skamarock and Gassmann 2011)

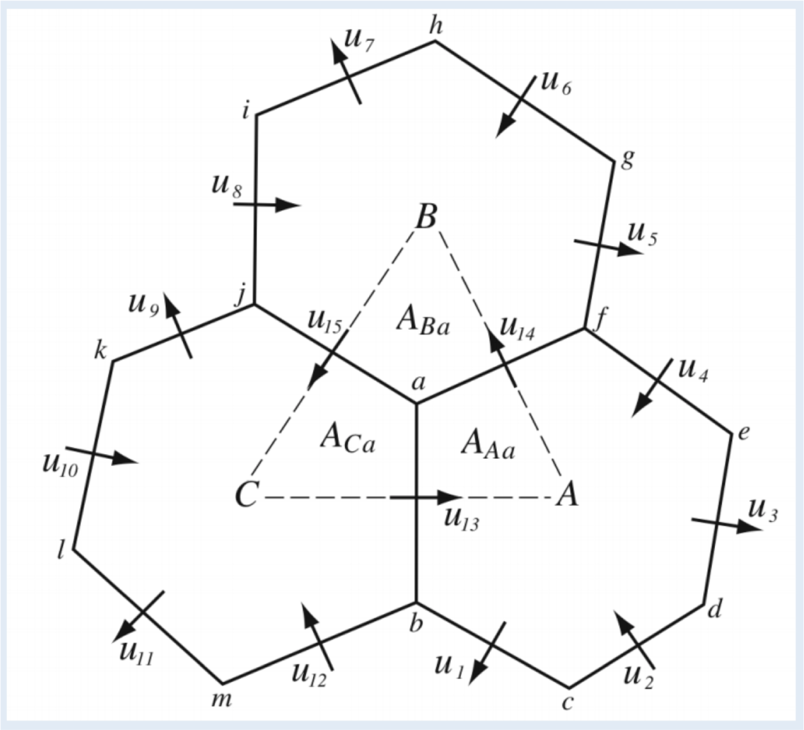
Not discussed

https://www.convertwithcontent.com/web-marketing-in-a-nutshell/

- Derivations
- Types of hybrid coordinates
- Nonlinear Coriolis term
- Kinetic energy gradient
- Explicit spatial filters
- Time discretization (split-explicit RK3); treatment of acoustic modes
- Top-of-model Rayleigh damping

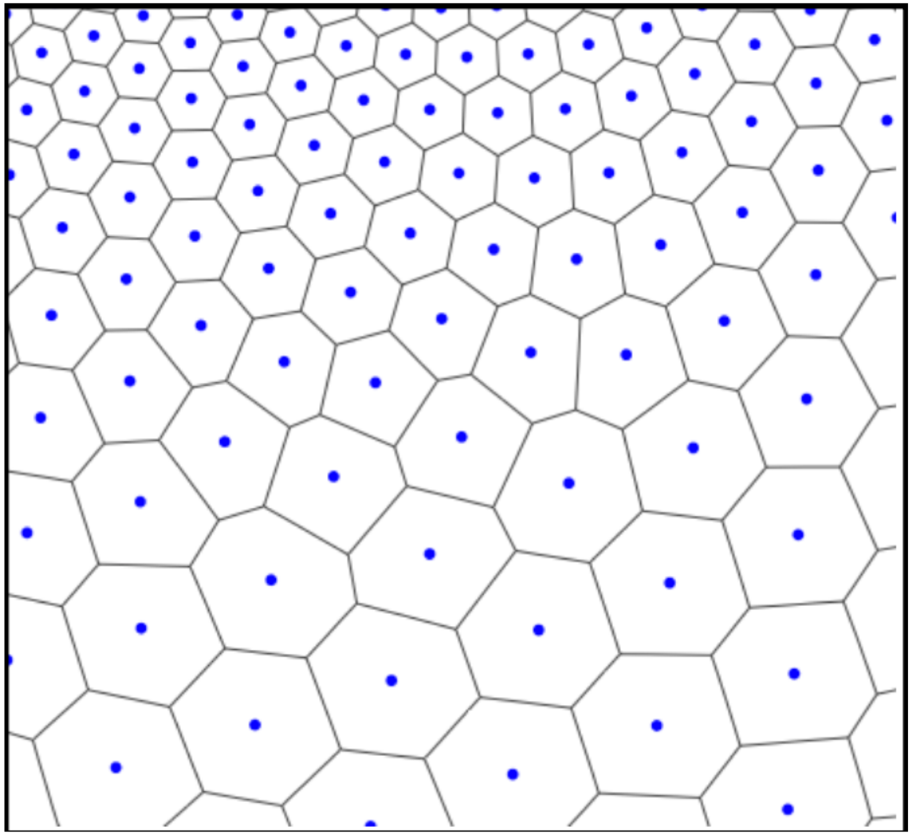
Summary

- MPAS is a cool model
- Hybrid-height, terrain-influenced vertical coordinate
- C-Grid staggering on a Voronoi mesh
- Nonhydrostatic, conservative equations
- Sophisticated FV discretization on an unstructured mesh



http://mpas-dev.github.io/atmosphere/tutorial.html

Thank you (???)



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