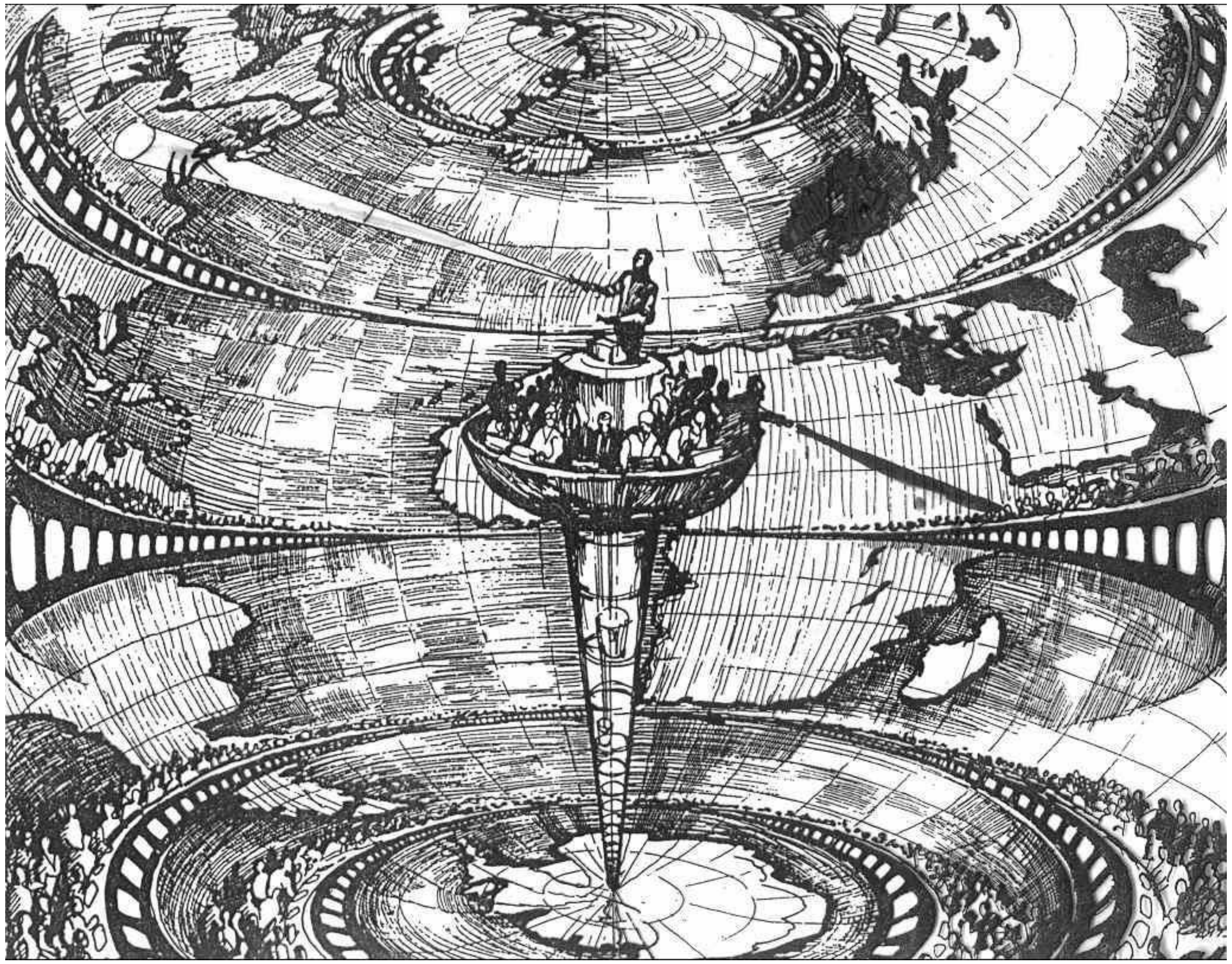
ATSC 507 Numerical Weather Prediction
(NWP)



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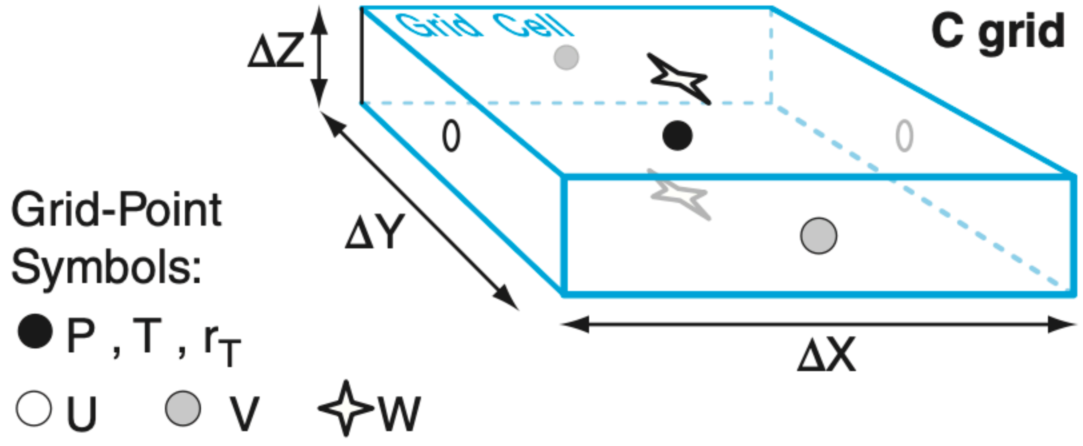
2025

Learning Goals

By the end of this course, you should be able to:
explain the scientific basis for NWP
utilize vertical and horiz. coordinate transformations
make finite diff. approximations to time & space derivatives
analyze data, including error propagation
use the von Neumann method to find errors & num. stability
correct for systematic errors with postprocessing
reduce random errors via ensemble forecasts
produce probabilistic forecasts from ensembles
verify forecast skill statistically
describe semi-Lagrangian advection methods
describe data assimilation methods
explain the basis for finite-volume NWP models: MPAS, FV3
install, compile, and run the WRF-ARW model
explain how physics parameterizations are used for subgrid phenomena

Factors and Terms in WRF eqs.

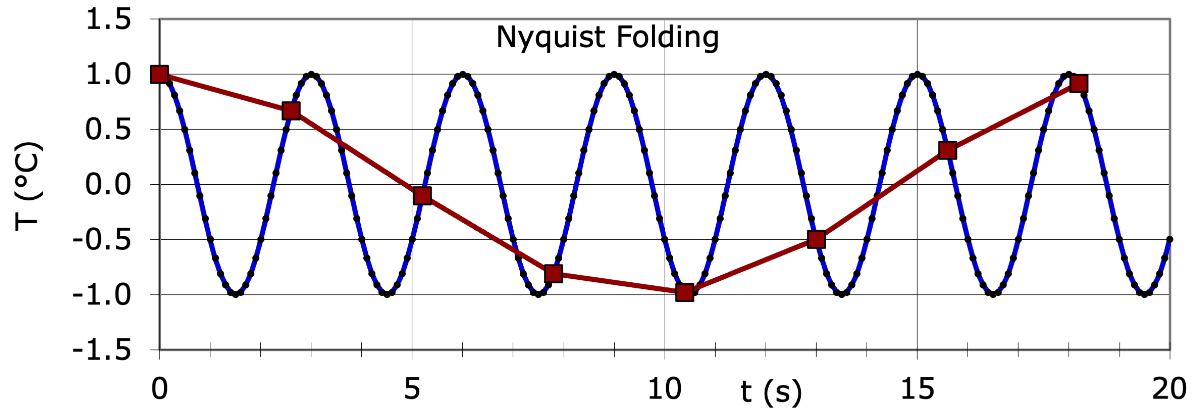
- curvature terms (due to inertia on spherical earth
- metric terms (due to terrain following vert. coord.)
- map scaling & rotating factors (for horiz. coord.)
- normalizing factors -> anomaly eqs.



- staggered grids (Arakawa grids)
- stencils & grid computation rules

Errors

- truncation errors (in the Taylor's series)
- amplitude errors (linear; damping or blowing up)
- phase speed errors (different wavelengths move at different speeds. Issues: numerical dispersion; ghost modes)
- group speed (energy flow) errors. 3ptCTCS has $2\Delta x$ waves that move in the wrong direction
- aliasing (short wavelengths folded into longer wavelengths. Thus, need to smooth the terrain to match the horiz. resolution)



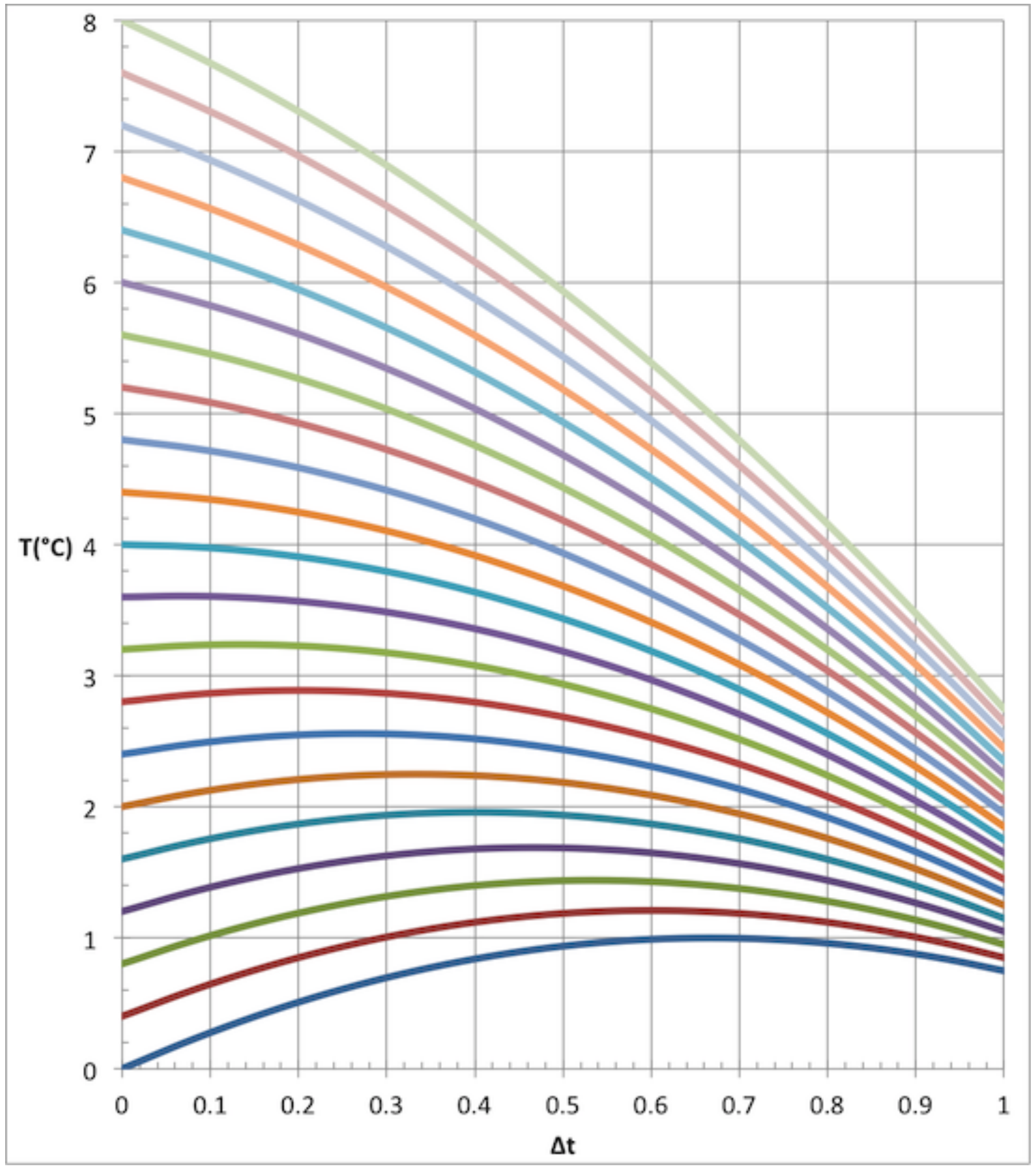
- nonlinear errors
- programming errors & bugs
- round-off errors (due to digital approx. of numbers)
- dynamic instability (chaos; butterfly effect)

Finite-diff. approx. to horizontal derivatives

- centered & one-sided (upwind) differences
- analogous to **interpolation**
- different orders of accuracy (in Taylor's series)
- higher derivatives (second, third, etc.)

Finite-diff. approx. to time derivatives

- explicit & implicit methods
- analogous to **extrapolation**
- Euler forward (unstable)
- leapfrog, etc.
- 3rd order Runge Kutta (used in WRF-ARW)
- Adams Bashforth (off-centered verision used in WRF-NMM)
- Lax Wendroff
- Matsuno



Key Points from Error Analysis: NWP models have "7 Δx" resolution

- results from truncation error of Taylor series
- found by comparing finite-difference solution for a 3-pt centered difference to the analytical solution
- see Warner Fig. 3.22

- **Significance**: need Δx to be 1/7 of the smallest scale you want to resolve.

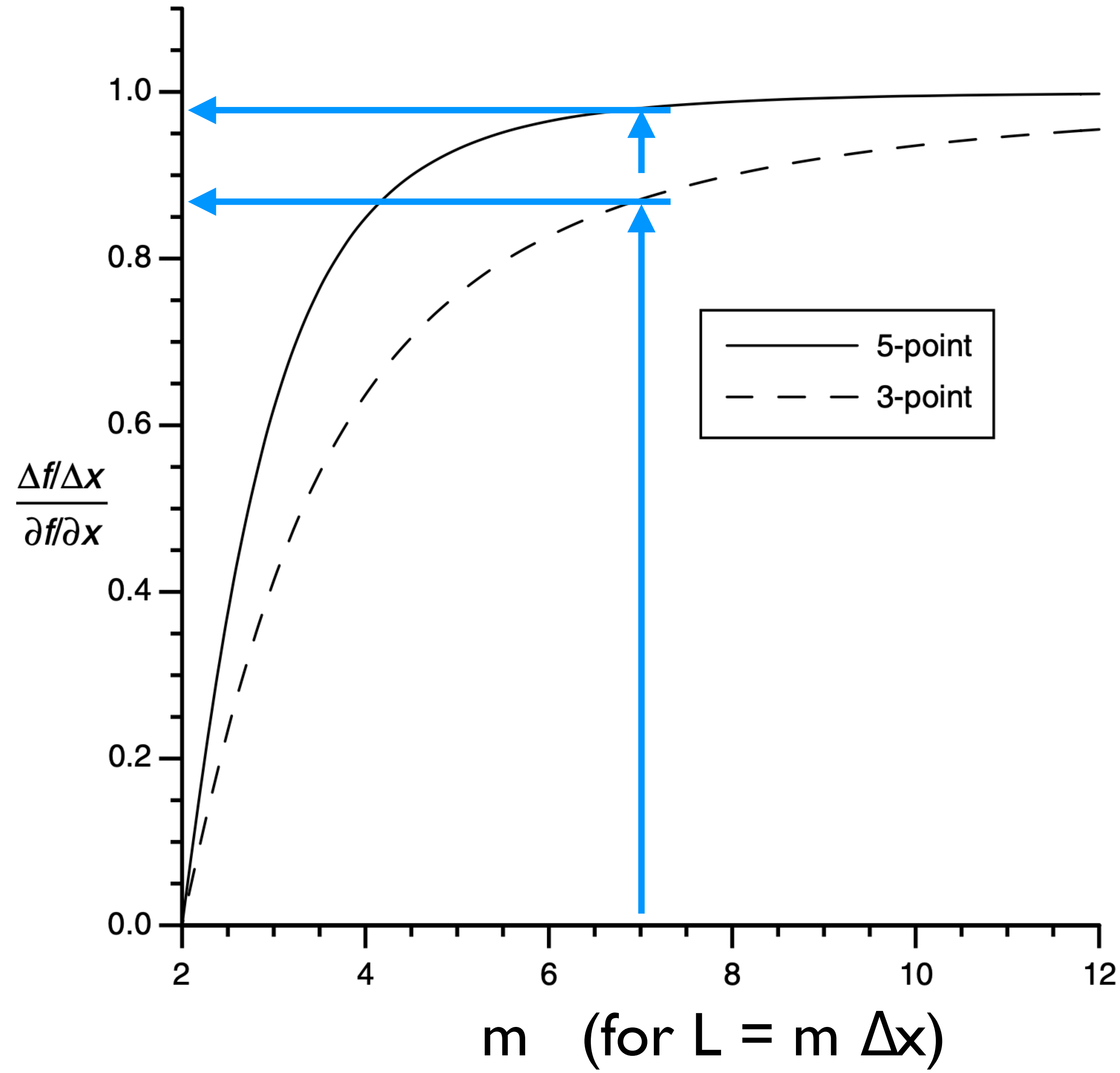


Fig. 3.22 The ratio of the value of the numerical approximation to the derivative of the cosine function and the value of the true derivative, for different numbers of grid increments per wavelength (how well the wave is resolved), for the five-point (fourth-order) and three-point (second-order) approximations.

Key Points from Error Analysis: von Neumann Linear Stability Analysis

Von Neumann Method for Determining Numerical Stability of a Finite-Difference Scheme

1. Write the finite difference eq. for 1-D linear advection of temperature T

   $\partial T/\partial t = -U_o \cdot \partial T/\partial x$

   where U_o is constant wind speed. Use j as grid index, n as time index.

2. Assume the following eigenmode solution, & plug into the finite diff. eq.

   $T = [A(k)]^n \cdot \exp(i\,k\,j\,\Delta x)$

   where A is amplitude, k is wavenumber, i is sqrt(–1), Δx is grid spacing

3. Divide the resulting eq. by

   $[A(k)]^n \cdot \exp(i\,k\,j\,\Delta x)$

4. Solve the resulting equation for amplitude A(k).

5. Use Euler's notation to convert into sines and/or cosines

   $e^{iy} - e^{-iy} = 2\,i\,\sin(y)$ and $e^{iy} + e^{-iy} = 2\cos(y)$

   $e^{iy} = \cos(y) + i\sin(y)$ and $e^{-iy} = \cos(y) - i\sin(y)$

6. Replace $U_o\,\Delta t / \Delta x$ with the Courant Number C_R.

7. Simplify the result where possible, and collect the real terms together and the imaginary terms together. These are the real and imaginary parts of the Amplitude, where $A(k) = A_R(k) + i\,A_I(k)$.

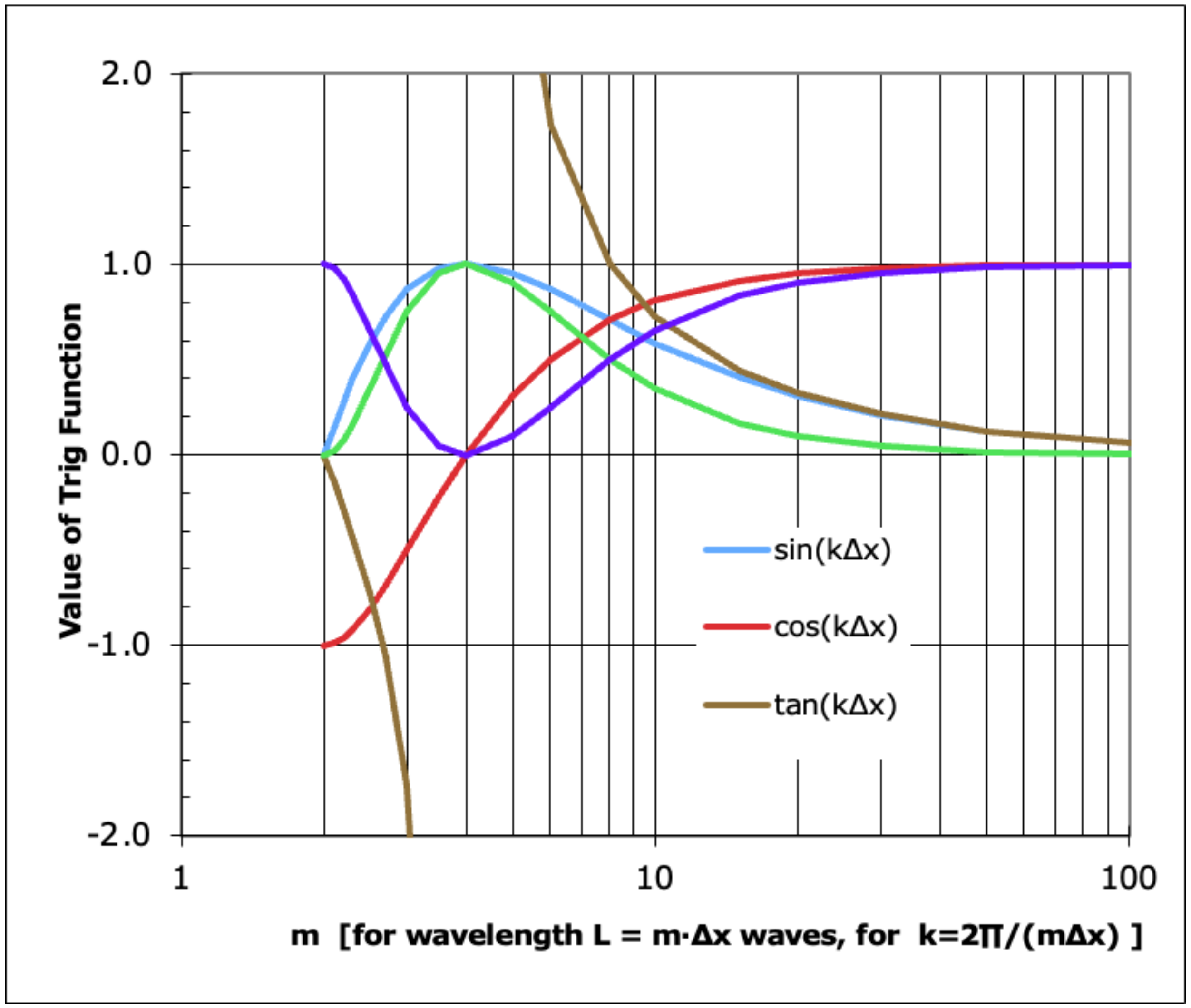
8. Find the amplitude modulus: $|A(k)| = \text{sqrt}\{[A_R(k)]^2 + [A_I(k)]^2\}$.

9. Replace k with m, using: $k = 2\pi / L = 2\pi / (m\,\Delta x)$

10. Find the worst-case m that gives the worst (most unstable) amplitude modulus, and assume that this wave exists in the atmosphere. (Utilize the handout of sine and cosine values for various m values.)

11. Make stability conclusions:
    Stable, but damped, if $|A(m)| < 1$.
    Stable if $|A(m)| = 1$.
    Unstable (blows up) if $|A(m)| > 1$.

See Press et al, 2007: Numerical Recipes, 3rd Ed., Cambridge

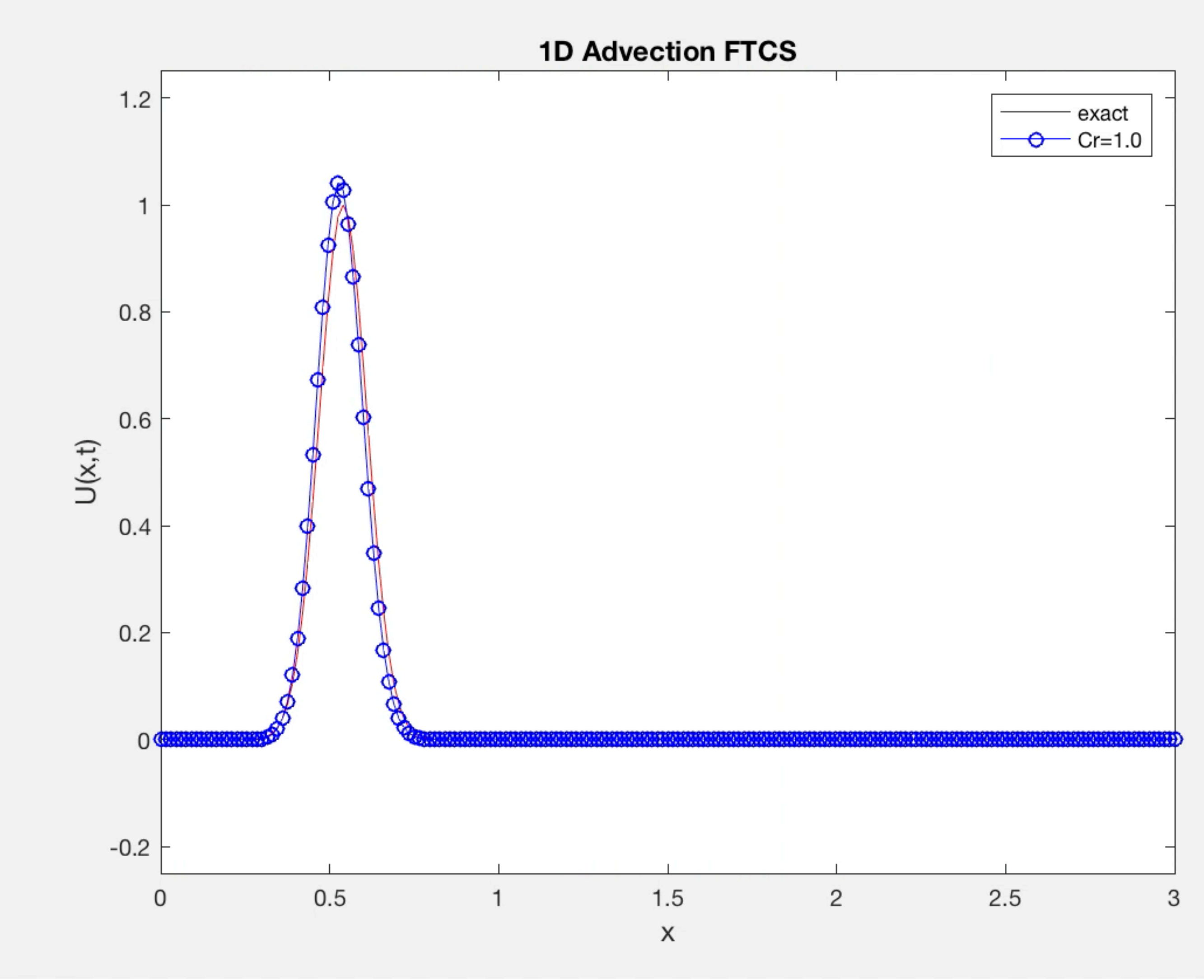


- **Significance**: the smallest wavelengths (2Δx or 4Δx) are often the most problematic (i.e., errors grow fastest)

Key Points from Error Analysis: CFL Stability Criterion

(CFL = Courant-Friedrichs-Lewy)

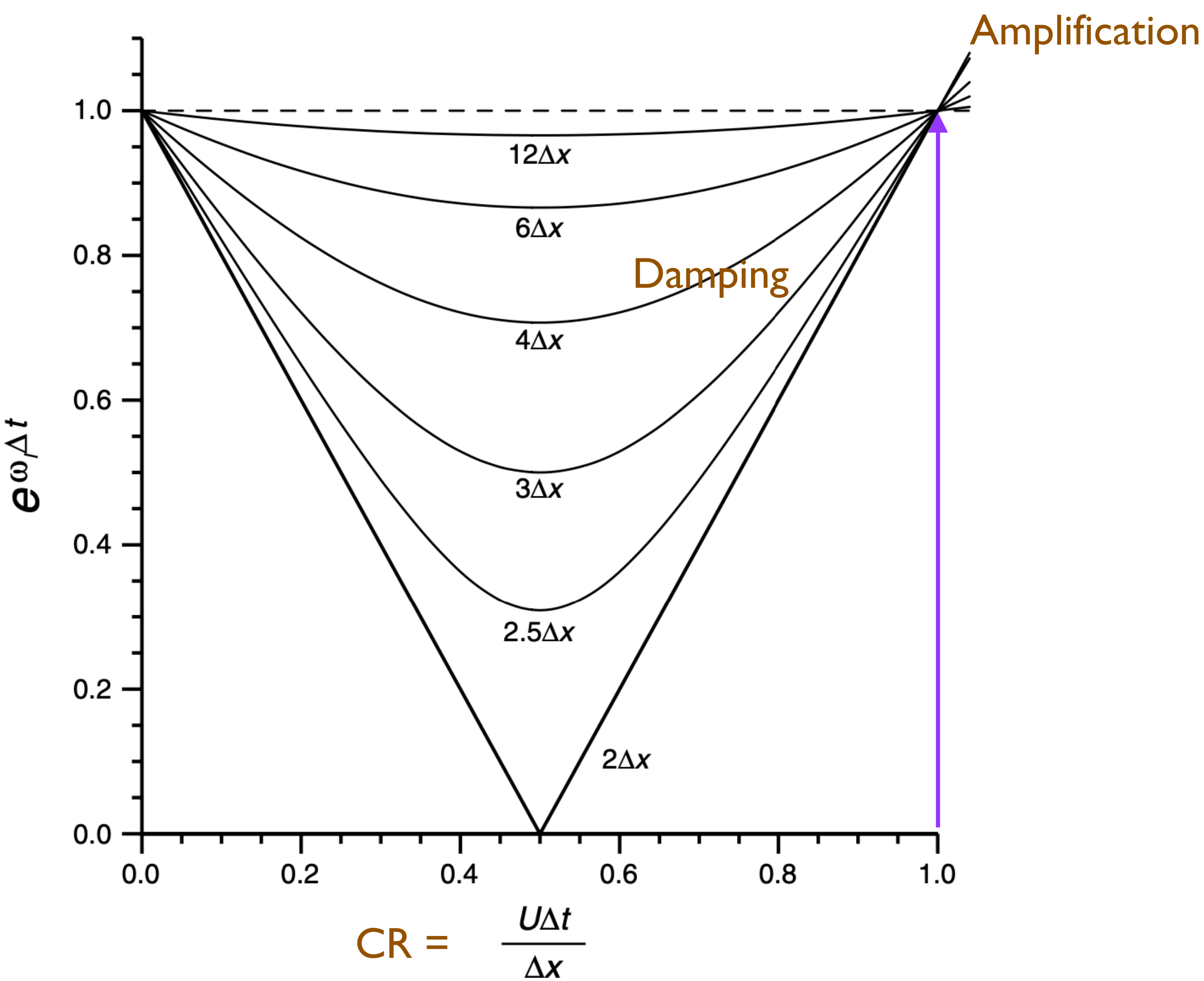
- Courant Number:
  $CR = U\,\Delta t / \Delta x$
- Numerically unstable if
  CR > a threshold (≈ 1)
- first appeared in our linear stability analysis for Forward in Time, Centered in Space (FTCS).



- **Significance**:
  smaller Δx requires smaller Δt.
- **Significance**: halving Δx requires 8x the number of computations

Key Points from Error Analysis: Damping

- some schemes either cause major damping of short wavelengths for CR< 1, or cause amplification for CR > 1.
- found for finite-difference solution for a forward-in-time, backward in space.
- see Warner Fig. 3.23
- **Significance**: This scheme won't work for most weather modeling.



Key Points from Error Analysis: NWP obeys different physics than real atm

- found by with Lax method (doing spatial averaging on the advection term)

$$\partial T/\partial t = -U_o \, \partial T/\partial x$$

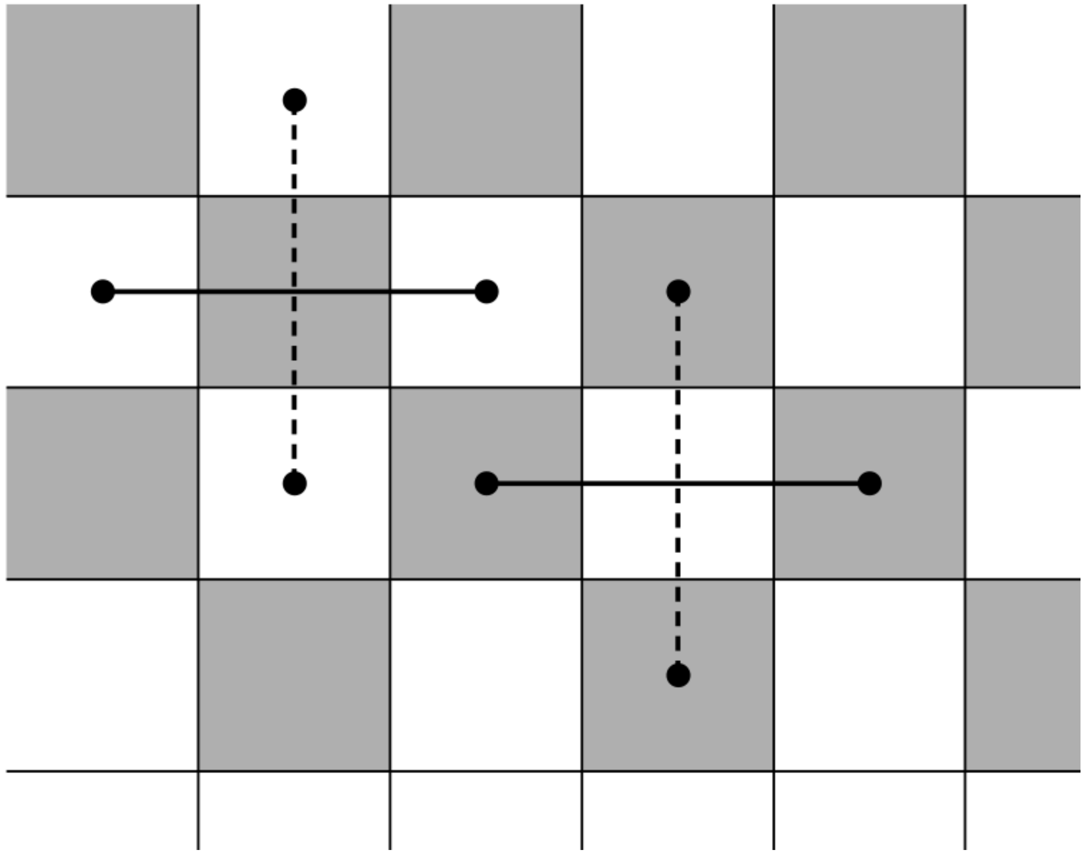
- increased the numerical stability

- **Significance**: acted like adding an unintended physical Diffusion term, in addition to the desired advection term.

$$\partial T/\partial t = -U_o \, \partial T/\partial x + [(\Delta x)^2/(2\Delta t]\, \partial^2 T/\partial x^2$$

Key Points from Error Analysis: Leapfrog

- centered in time, centered in space
- absolutely numerically stable when CR ≤ 1.
- no damping, but …
- … checkerboard issue. Solution for black cells diverge from the solution for white cells.



- **Implication**: Need to add diffusion between white and black cells.

Key Points from Error Analysis: CFL theory for 1-D, 2-D, and 3-D

- found by comparing finite-difference solutions Lax single-step solutions for multiple dimensions
- Let CR_{theory} be the theoretical max Courant number that is numerically stable in **1-D** for linear advection.

$$\Delta t_{max} < CR_{theory} \cdot (\Delta x / U_{max})$$

- **Significance**: Smaller time step is needed for a full **3-D** numerical forecast.

$$\Delta t_{max} < [CR_{theory} / (3^{1/2})] \cdot (\Delta x / U_{max})$$

Key Points from Error Analysis: Runge-Kutta 3rd Order (RK3)

- For linear advection in 1-D, different combinations of time and spatial differencing have different stability requirements. Some are never stable.

Time Scheme	Spatial order			
	3rd	4th	5th	6th
Leapfrog	*Unstable*	0.72	*Unstable*	0.62
RK2	0.88	*Unstable*	0.30	*Unstable*
RK3	1.61	1.26	1.42	1.08

WRF Tech note table 3.1. Max stable Courant number.

- **Significance**: RK3 with 3rd order spacial differencing in WRF has much larger CR_{theory}, enabling you to take larger Δt .

Key Points from Error Analysis: Finite Difference for Diffusion

$$\partial T/\partial t = D \cdot \partial^2 T/\partial x^2$$

- forward in time, centered in space is OK.

  Has num. stab. criterion: $D \, \Delta t / (\Delta x)^2 \leq 0.5$

- centered in time, centered in space is unstable.

- **Caution**: Pro: causes damping. Con: can cause phase change (+ , -) of waves at each time step.

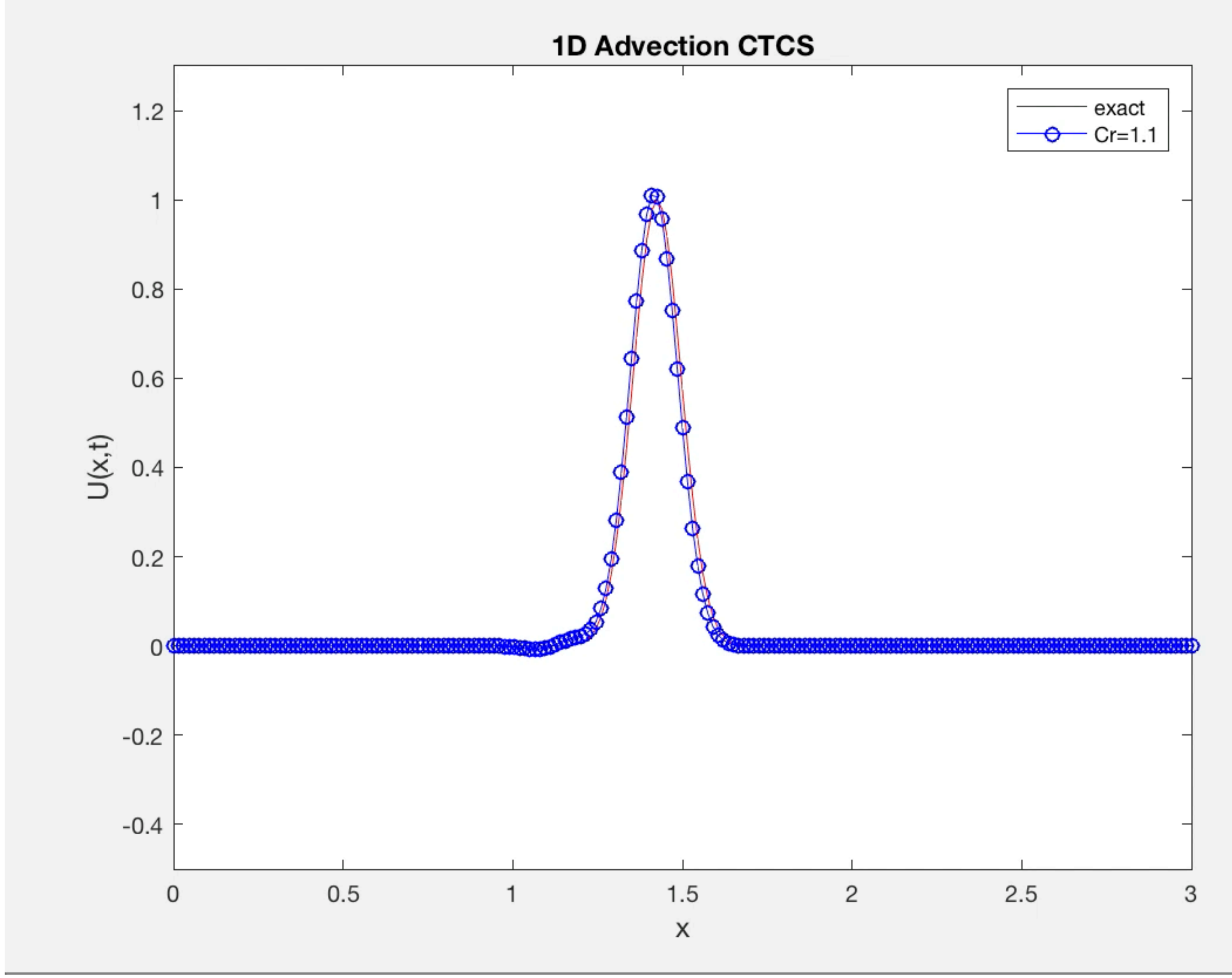
Key Points from Error Analysis: Implicit Methods

- absolutely numerically stable for any time step and grid spacing.
- stable for linear advection and for diffusion
- Crank-Nicholson methods

- **Note**: Requires iteration within each time step. (i.e., more costly)

Key Points from Error Analysis: Numerical artifacts / ghost modes / parasitic modes

- some methods create a physical solution AND an unwanted computational (parasitic / ghost) mode.
- also, wave energy (group velocity) moves in the wrong direction for 3pt CTCS.
- **Significance**: Even if the NWP approximation to the physical solution might be stable, sometimes the parasitic modes can be unstable.



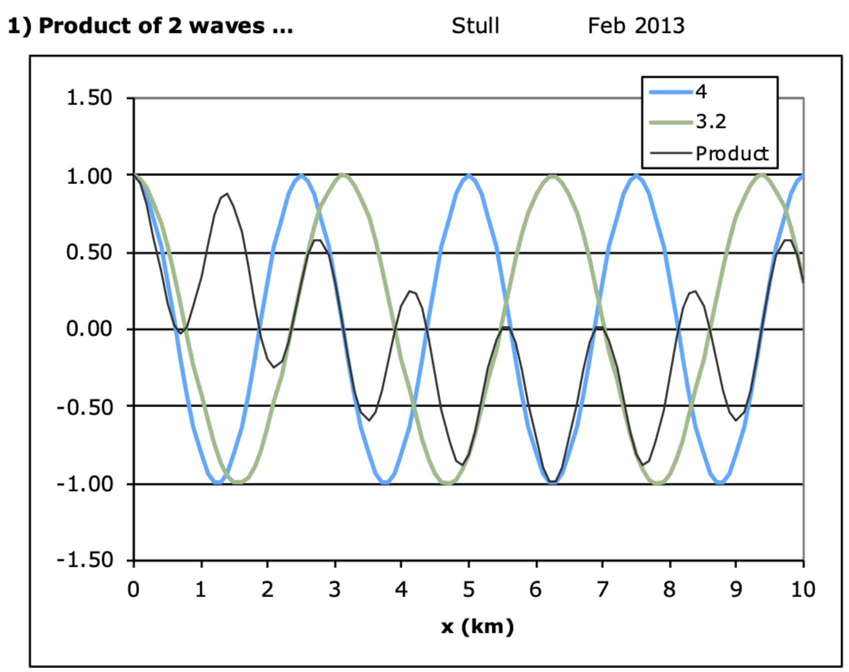
Key Points from Error Analysis: Phase and Group Speed Errors

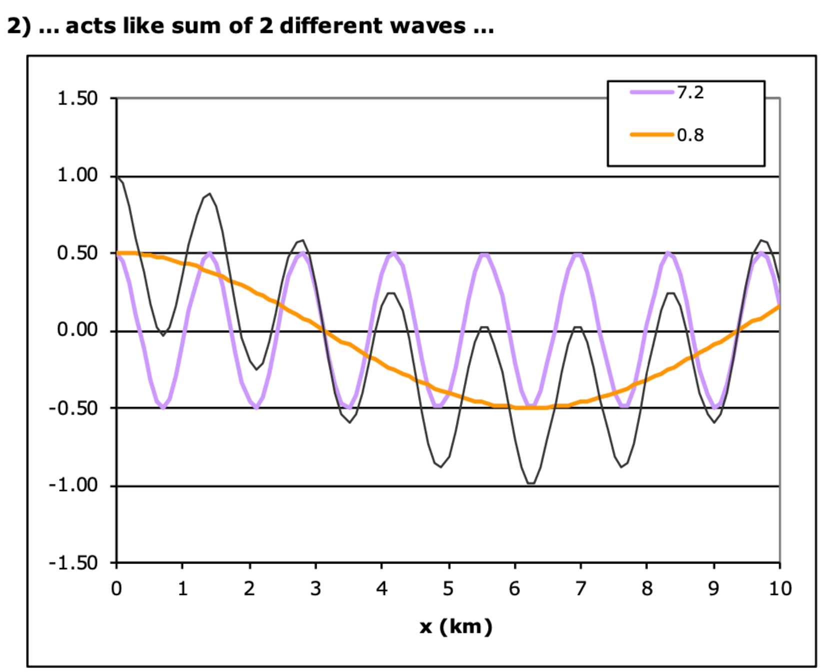
- often affects smaller wavelengths more strongly
- Thus, it is very desirable to filter out (or smooth) short wavelengths.

- **Significance**: WRF-ARW for odd-order (3, 5, etc.) spatial differencing has good implicit damping of the smaller wavelengths.

Key Points from Error Analysis:
Nonlinear Issues. Very complex. Overview:

- Eqs. of motion contain MANY nonlinear terms that contain the product of 2 or more dependent variables: e.g., advection U ∂T/∂x
- If U and T are thought of as consisting of sums of sines and cosines (i.e., DFT), then the nonlinear terms consist of sums of many terms, each of which are products of different wavelengths.
- But products of different wavelengths appear to the NWP model as sums of other wavelengths, some of which are very short, and others that are so short that they are folded/aliased into longer wavelengths.
- Thus, there is an unphysical build-up of energy in the smaller wavelengths.
- **Significance**: Need to filter out the smaller wavelengths.





Many NWP Topics NOT Discussed

- Smoothing & Filtering
- Spectral Methods
- Pseudo-spectral Methods
- Time-splitting for Acoustic Modes
- Lateral Boundary Conditions
- Finite Element Methods
- Artificial Intelligence (alternative to NWP)

But you’ve been exposed to a wide range of NWP textbooks, for further studies.

The End

Overall motivation: to not use NWP as a "black box"

No final exam.

Meanwhile, I will mark your homeworks as your submit them. Please submit by 27 April.

Also, if you get some of the HW wrong, I give you feedback and allow you to resubmit until you get it right.

Any Questions?

Instructor Eval.

ATSC_V 507-201 - Numerical Weather Prediction

Students

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