EOSC252

Error Analysis

1 Significant figures

In scientific analysis, knowing the answer is only half work: A number is meaningless unless you also know how accurate and precise it is. The term "significant figures (SF)" refers to the meaningful part of a measured or calculated quantity. It is very important that we all use the same language and rules to discuss these quantities, to avoid confusion. There are conventions which should be followed, to make sure you express measurements or results correctly.

1. **Any digit that is not zero is significant**: 549 has 3 SF, 1.899 has 4 SF.
2. **Zeros between non zero digits are significant**: 4023 has 4 SF.
3. **Zeros to the left of the first non zero digit are *non significant***: 0.00023 has 2 SF
4. **For numbers with decimal points, zeros to the right of a non zero digit are significant**: 2.00 has 3 SF, 0.040 has 2 SF.
5. **For numbers without decimal points, trailing zeros may or may not be significant**: **400** indicates only one SF. To indicate that the trailing zeros are significant, a decimal point must be added. **400.** has 3 SF.
6. **Exact numbers have an infinite number of SF**. For example, if there is two oranges on the table then the number of oranges is 2.00000... Defined numbers or physical constants, such as π, also have an infinite number of SF.

1.1 Expressing uncertainty

In general, the last SF in any result should be of the same order of magnitude (i.e. in the same decimal position) as the uncertainty. **Also, the uncertainty should be rounded to one or two SF**.

For example,

9.82 ± 0.02
10.0 ± 1.5
4 ± 1

1.2 Addition and subtraction with significant figures

When combining measurements with different degrees of accuracy and precision, the accuracy of the final answer can be no greater than the least accurate measurement. This principle can be translated into a simple rule for addition and subtraction: When measurements are added or subtracted, the answer can contain no more decimal places than the least accurate measurement.

$$M = 150.0\,g + 0.50\,g = 150.5\,g$$

1.3 Multiplication and division with significant figures

The same principle governs the use of significant figures in multiplication and division; the final result can be no more accurate than the least accurate measurement. In this case, however, we count the significant figures in each measurement, not the number of decimal places: When measurements are multiplied or divided, the answer can contain no more significant figures than the least accurate measurement.

$$\delta = \frac{M}{V} = \frac{150.5\,g}{24.2\,cm^3} = 6.22\,g/cm^3$$

2 Measurement errors

Measurement errors are errors intrinsic to the use of measuring devices. In general, the measurement errors are half the smallest division of the instruments when using analog instruments, and equal to the smallest division for digital instruments.

3 Propagation of measurement errors

The propagation of measurment errors through calculations can be acheived using a technique base on derivatives. This technique can be applied to any error calculation. Assuming $X = \mathrm{F}(A, B)$ and A and B are two uncorrelated measurments, the error of X is expressed as the following:

$$\Delta X = \sqrt{\left(\frac{\partial \mathrm{F}}{\partial A}\Delta A\right)^2 + \left(\frac{\partial \mathrm{F}}{\partial B}\Delta B\right)^2}$$

If you are not familiar with calculus, here are some simple results for typical operations.

3.1 Additions and subtractions

The resulting error is obtained by adding the absolute errors.

$$(X \pm x) = (A \pm a) + (B \pm b) - (C \pm c)$$
$$x = \sqrt{a^2 + b^2 + c^2}$$

3.2 Multiplication and division

The resulting error is obtained by adding the relative errors.

$$(X \pm x) = \frac{(A \pm a)(B \pm b)}{(C \pm c)}$$

$$x = X\sqrt{\left(\frac{a}{A}^{2} + \frac{b}{B}^{2} + \frac{c}{C}^{2}\right)}$$

- Multiplication or division by a constant

$$X \pm x = B\,(A \pm a)$$
$$x = X\left(\frac{a}{A}\right) = Ba$$

3.3 Exponentiation

$$X \pm x = (A \pm a)^b$$
$$x = X\left(\frac{a}{A}\right)b$$