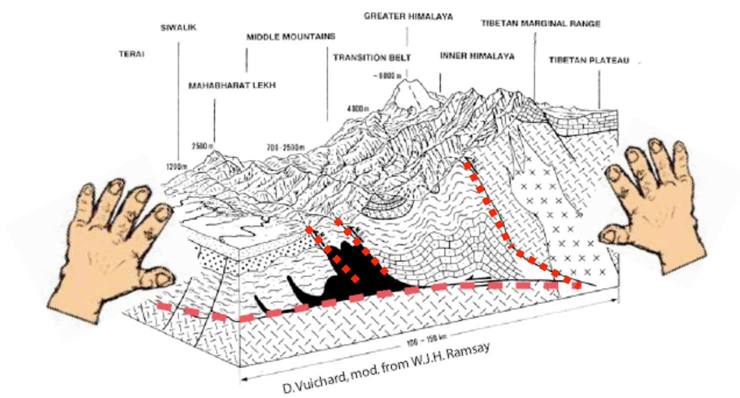
Stress is just force per unit area acting on a surface

It is just "some constant" times strain



This means that stress is a tensor (that is, a matrix), too.

What are the units of force / area?

$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix} \text{ x elastic constants } = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

If we choose the right coordinate system we can express strain in terms of normal strains only ("principal strains")

$$\begin{bmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{bmatrix}$$

ϵ_1 is the most negative normal strain

ϵ_2 is the most positive normal strain

The same is true of stresses.

$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

σ_1 is the most negative normal stress

σ_2 is the most positive normal stress

Normal stress causes normal strain

What are typical normal stresses in the Earth? How “big” is a Pascal?

Sign convention.

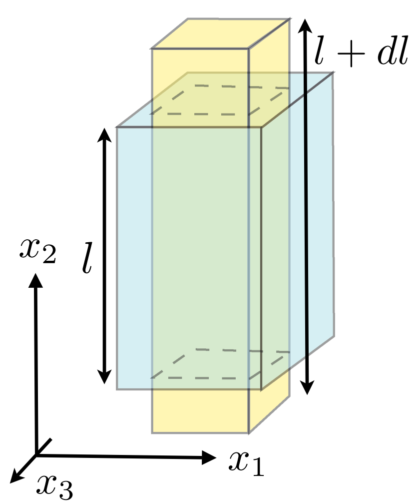
$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

σ_1 is the most negative normal stress

σ_2 is the most positive normal stress

differential stress: maximum principal stress minus minimum principal stress
maximum possible shear stress is 1/2 differential stress

Strain in the Earth due to normal stress: Hooke’s Law

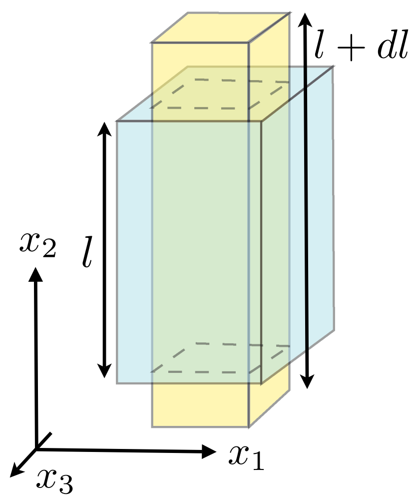


$$\epsilon_{22} = \frac{dl}{l} = \frac{du_2}{dx_2}$$

$$\boxed{\begin{aligned} \sigma_{22} &= E\epsilon_{22} \\ \epsilon_{22} &= \frac{\sigma_{22}}{E} \end{aligned}}$$

Note the contraction in the x_1-x_3 plane: this depends on another parameter - the Poisson’s ratio

Only ϵ_{22} is not 0
("uniaxial stress")



$$\sigma_{22} = E\epsilon_{22}$$

$$\epsilon_{22} = \frac{\sigma_{22}}{E}$$

$$\epsilon_{33} = \epsilon_{11} = \frac{-\nu\sigma_{22}}{E}$$

E is the "Young's Modulus"

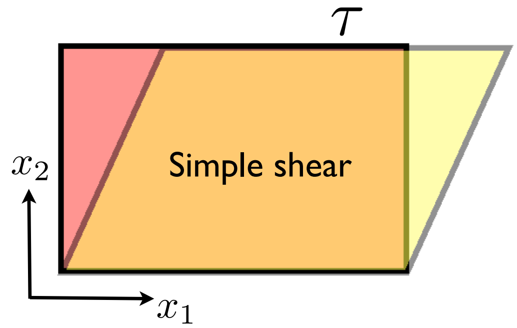
In rocks, E is very, very large.

typical value is 7.5 x 10^{10} Pascals

$$\sigma_{22} = E\epsilon_{22}$$
$$\epsilon_{22} = \frac{\sigma_{22}}{E}$$

Typical ν is 0.25
0.5 means the material is
incompressible (like water)

Shear stress (sometimes called τ) causes shear strain



$$\tau_{21} = 2G\epsilon_{21}$$
$$\epsilon_{21} = \frac{\tau_{21}}{2G}$$

G is the "Shear Modulus"

In rocks, G is also very, very large.

typical value is 3×10^{10} Pascals

Lithostatic pressure in the Earth...

$$P = \rho g h$$

$\rho = 2500\ kg/m^3$

$g = 10\ m/s^2$

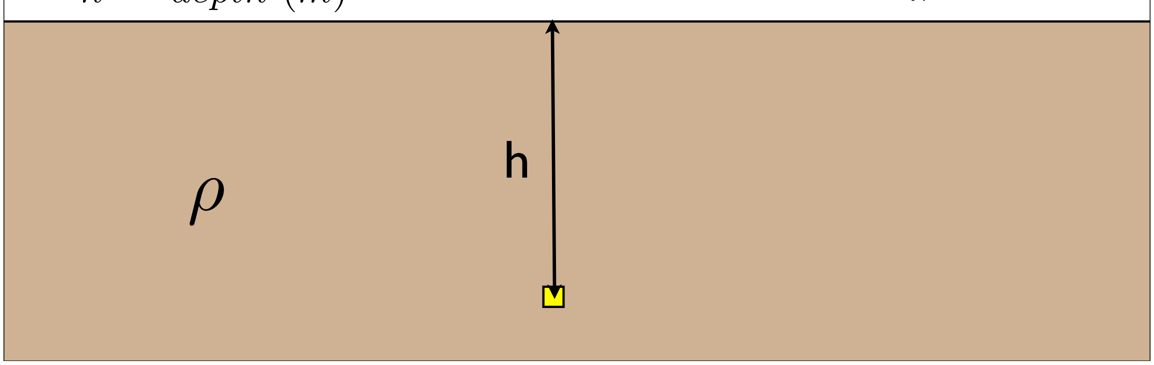
$h =\ depth\ (m)$

Moho is 30 km down

Cascadia SZ (80 km below us)

Deep ocean floor = 4 km

(EQ works for water or ice sheets too with correct density)

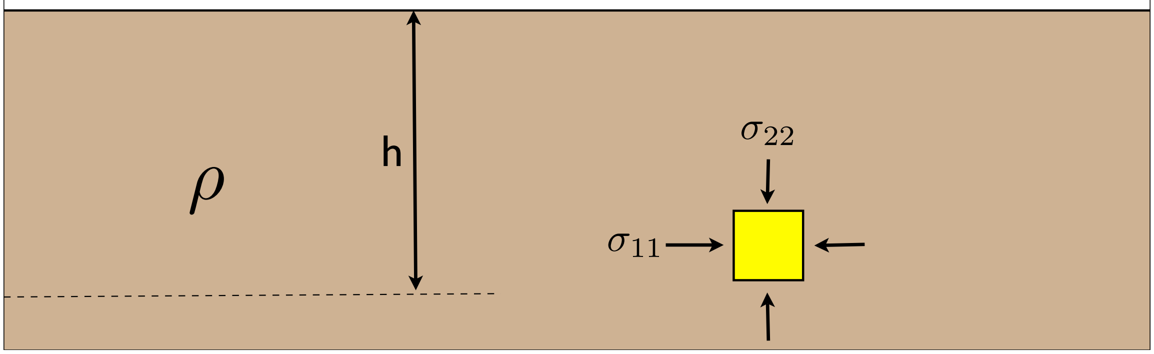


In a fluid normal stresses are -P ("hydrostatic pressure") shear stresses are 0

$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} = \begin{bmatrix} \bar{\sigma} & 0 \\ 0 & \bar{\sigma} \end{bmatrix} = \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix}$$

Below the Earth's upper crust normal stresses are *close* to -P and shear stresses are small

$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \approx \begin{bmatrix} \bar{\sigma} & 0 \\ 0 & \bar{\sigma} \end{bmatrix} = \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix}$$



In the upper crust, the mean of the normal stresses ($\bar{\sigma}$) equals -P.

The shear stresses and the (normal stresses minus their average) make up the deviatoric stress matrix

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}_{DEV} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} - \begin{bmatrix} \bar{\sigma} & 0 \\ 0 & \bar{\sigma} \end{bmatrix}$$

At a typical hypocentre = 7 km

$\bar{\sigma}$ = ?

Deviatoric stresses are smaller.
How much? It is a subject of debate!

Rocks in the upper crust are brittle: this limits the maximum deviatoric stresses

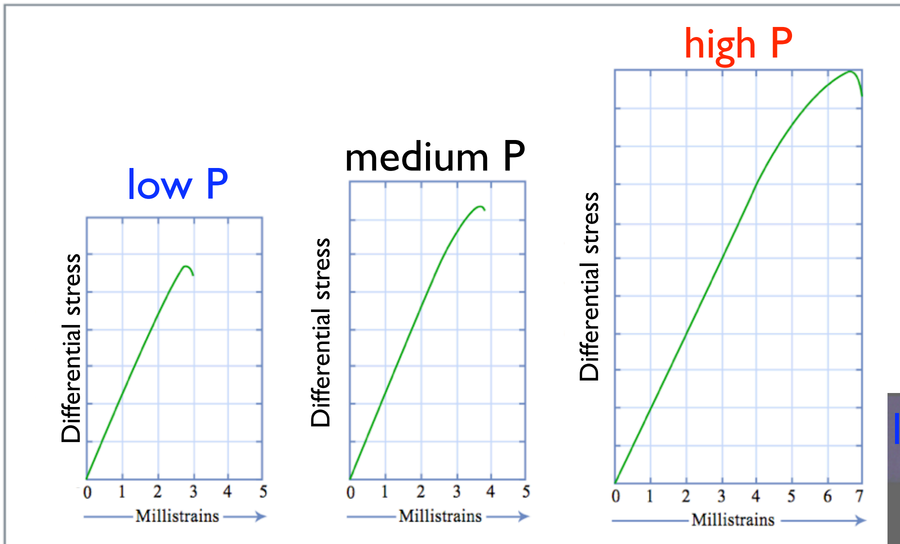
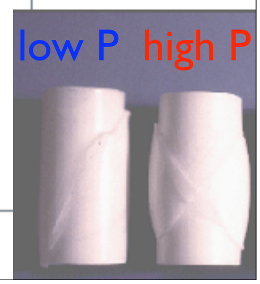


Figure 5.8. Stress-strain curves for Rand quartzite at various confining pressures.

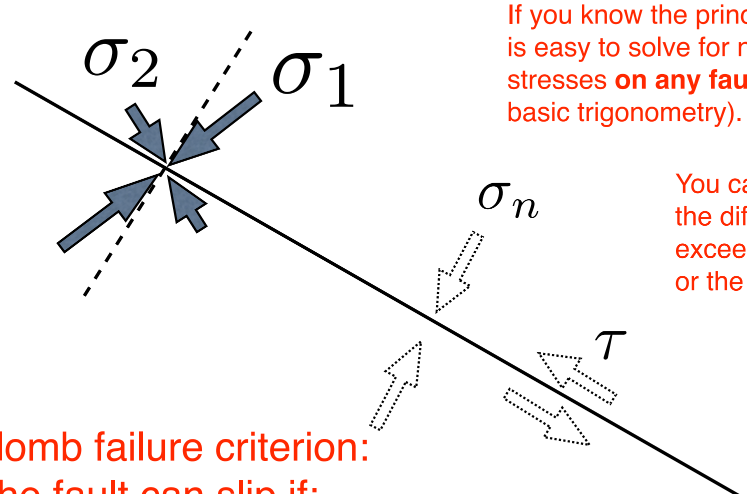
rocks are stronger in compression than in tension!

However, even at high pressures their strength has a limit:

100's of MPa.



$$\sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$



If you know the principal stresses it is easy to solve for normal and shear stresses **on any fault plane** (using basic trigonometry).

You can very easily estimate the differential stress. It can't exceed a few hundred MPa or the rock will break.

Coulomb failure criterion:
The fault can slip if:

$$\tau = \mu \sigma_n$$

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The fault can slip if:

$$\tau = \mu \sigma_n$$

Right before an earthquake,
At 7 km depth what is τ?

$\sigma_n = \rho g h$
$\mu = 0.7$
$\rho = 2500\ kg/m^3$

Now, estimate τ from shear strain accumulated between earthquakes and shear modulus G:

$t = 200\ yr$
$\dot{\epsilon}_{12} = 5 \times 10^{-7}\ /yr$
$\epsilon_{12} = \dot{\epsilon}_{12} \times t$

$G = 30\ GPa = 3 \times 10^{10}\ Pa$
$\tau_{12} = 2G\epsilon_{12}$

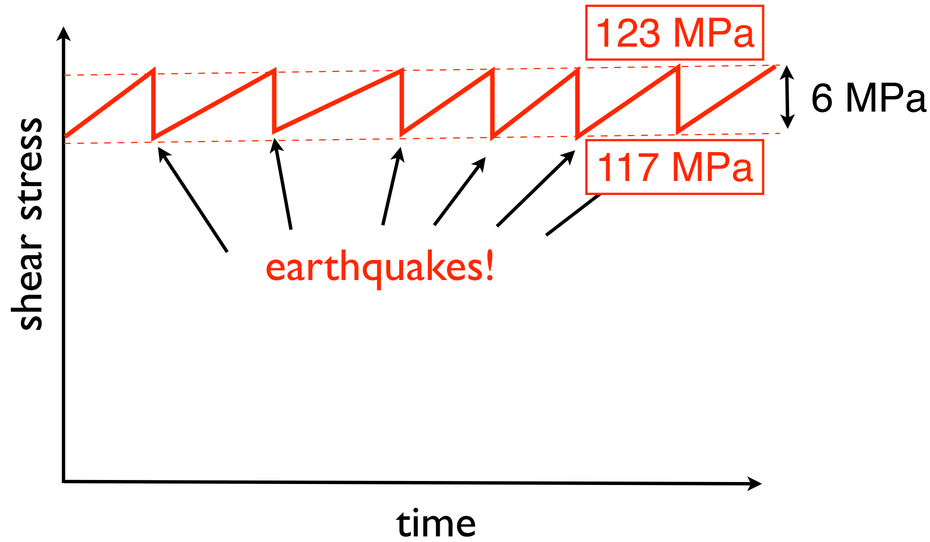
$\tau = 123\ MPa$ from $\tau = \mu \sigma_n$

$\tau = 6\ MPa$ from $\tau = 2G\epsilon_{12}$

Which answer is right?

How could each answer be mistaken?

For many faults, earthquake shear stress drop is much smaller than “background” shear stress



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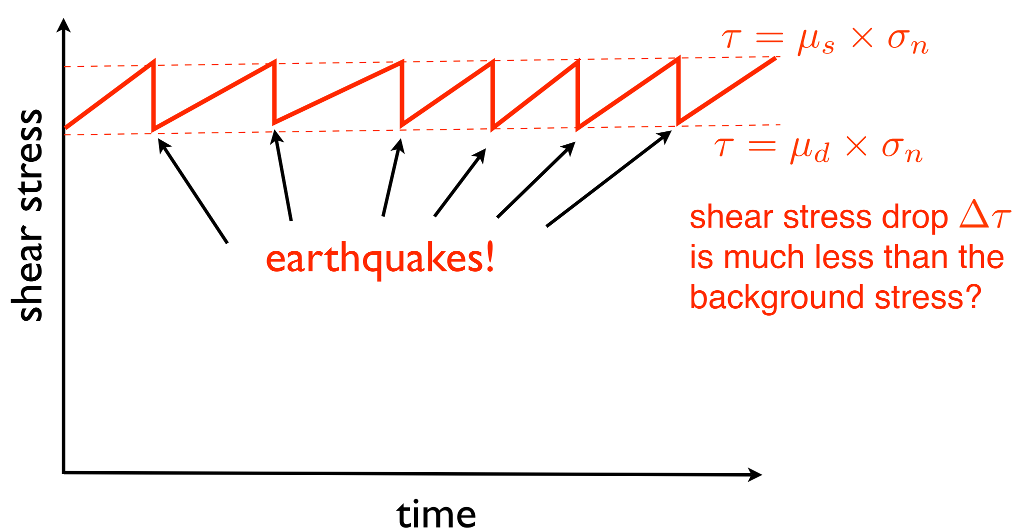

Fault friction is actually more complicated than this because it depends on slip speed and the presence of water in the fault zone!