How to make an earthquake:
Build up enough shear stress to exceed the frictional strength of a fault, *over a large enough spatial surface area of a frictionally unstable ("velocity weakening") fault*

(1) Building up shear stress (interseismic period):

We must define strain, elasticity, and stress (shear stress and normal stress). First: strain and how we measure it with GPS.

(From Monday's lecture)
we need to define strain quantitatively so we can get to stress

Congratulations! Now you know what the "displacement gradient matrix" is.
This is ALMOST the strain matrix.

$$\begin{array}{ccc} & \text{column 1} & \text{column 2} \\ \text{row 1} & \dfrac{\Delta u_1}{\Delta x_1} & \dfrac{\Delta u_1}{\Delta x_2} \\ \text{row 2} & \dfrac{\Delta u_2}{\Delta x_1} & \dfrac{\Delta u_2}{\Delta x_2} \end{array}$$

A matrix: a bunch of numbers arranged in rows and columns.

This is a matrix with 2 rows and 2 columns.

Do not fear the matrix - we have to use it to describe strain and stress in the Earth.

Congratulations! Now you know what the "displacement gradient matrix" is.
This is ALMOST the strain matrix.

$$
\begin{array}{ccc}
 & \text{column 1} & \text{column 2} \\
\text{row 1} & \dfrac{\Delta u_1}{\Delta x_1} & \dfrac{\Delta u_1}{\Delta x_2} \\
\text{row 2} & \dfrac{\Delta u_2}{\Delta x_1} & \dfrac{\Delta u_2}{\Delta x_2}
\end{array}
$$

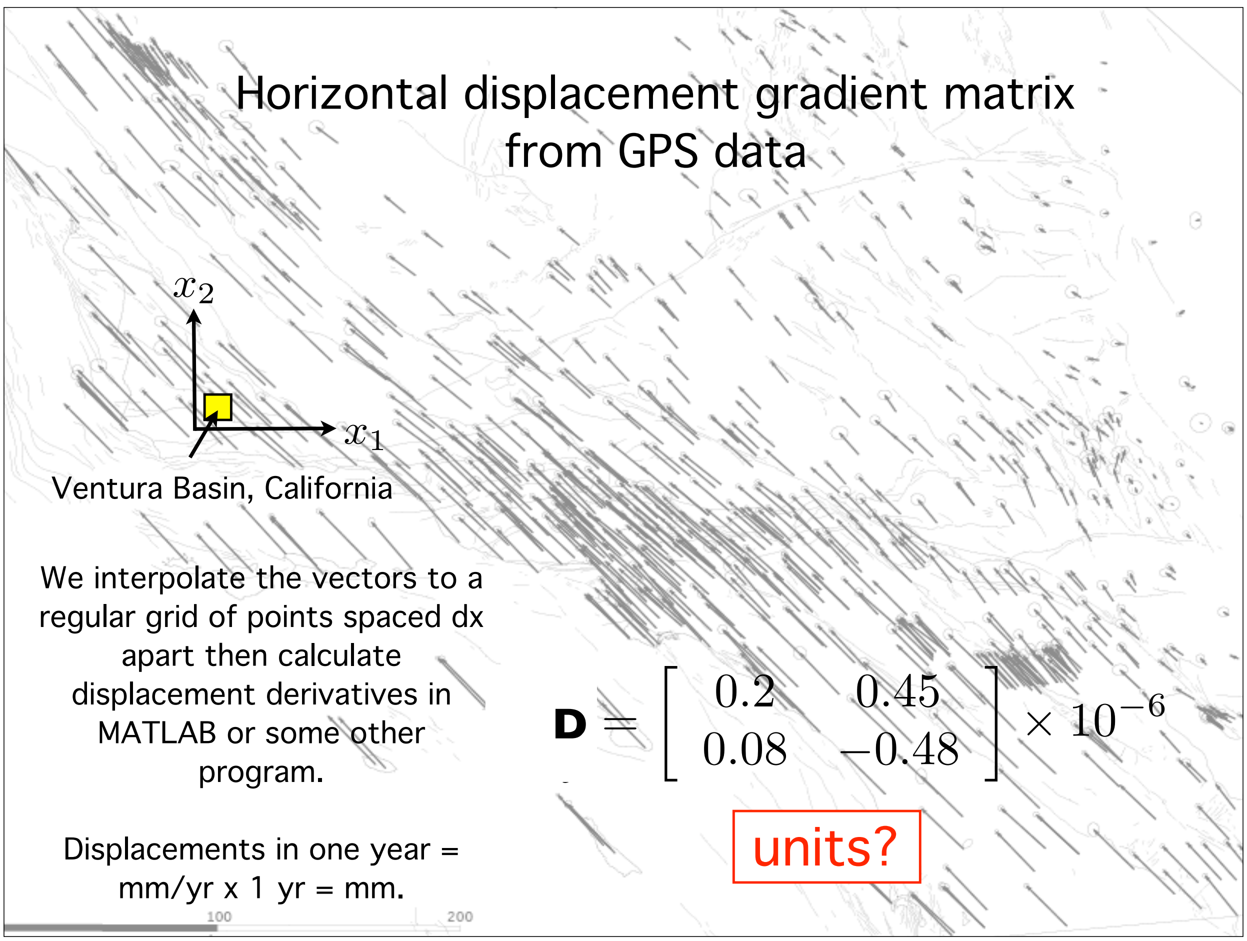
Suppose we named this matrix D.

Convention is to use boldface: **D**

Individual numbers in the matrix are indicated with subscripts showing the row and the column that the number is in:

D_{12} is the entry in ROW 1 COLUMN 2

Horizontal displacement gradient matrix from GPS data

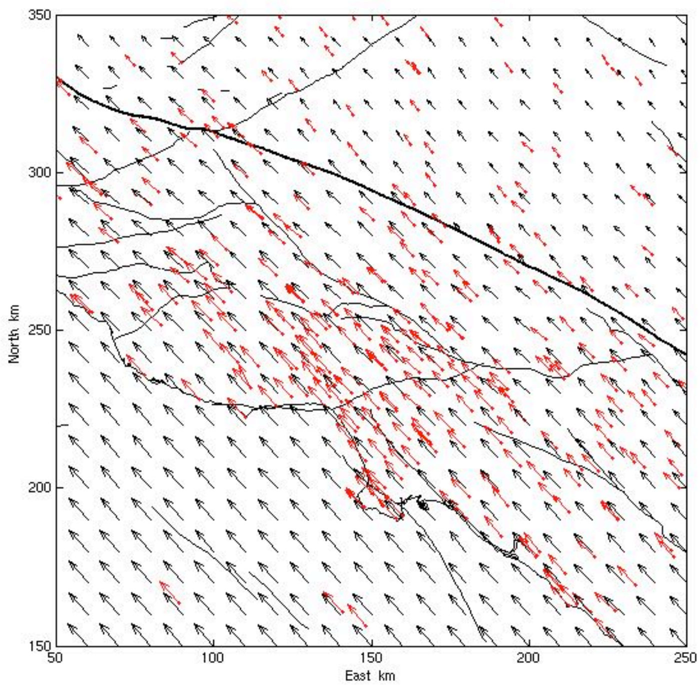


We interpolate the vectors to a regular grid of points spaced dx apart then calculate displacement derivatives in MATLAB or some other program.

Displacements in one year = mm/yr x 1 yr = mm.

$$\mathbf{D} = \begin{bmatrix} 0.2 & 0.45 \\ 0.08 & -0.48 \end{bmatrix} \times 10^{-6}$$

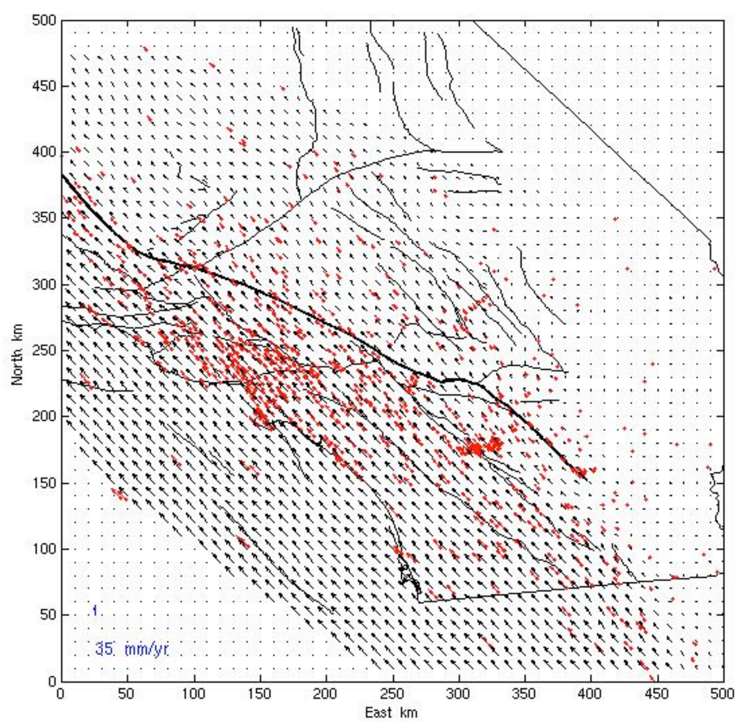
units?



red is GPS data

black is interpolated to a uniform grid

other lines are faults and the California coast

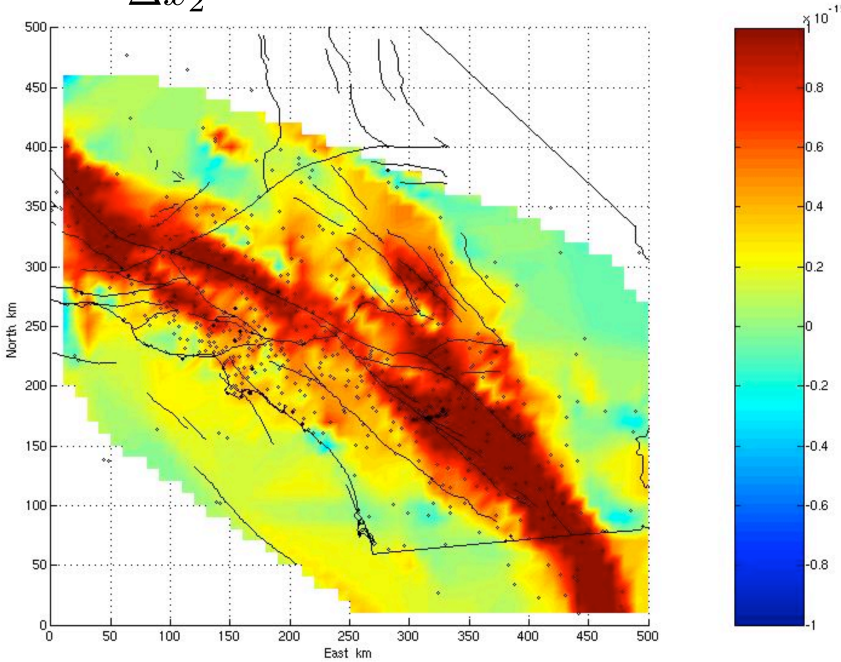


red is GPS data

black is interpolated to a uniform grid

other lines are faults and the California coast

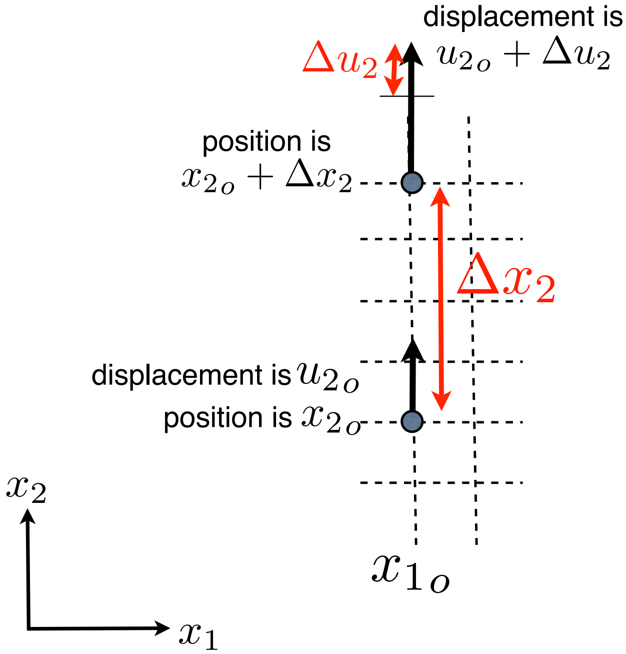
Here is $\dfrac{\Delta u_1}{\Delta x_2}$ from the interpolated vector field



We can also get $\dfrac{\Delta u_2}{\Delta x_1}$ $\dfrac{\Delta u_1}{\Delta x_1}$ and $\dfrac{\Delta u_2}{\Delta x_2}$

On to <u>strain</u>

"Normal" strain: elongation or contraction



$$\epsilon_{22} = \frac{\Delta u_2}{\Delta x_2}$$

for small Δx_2
this is written as

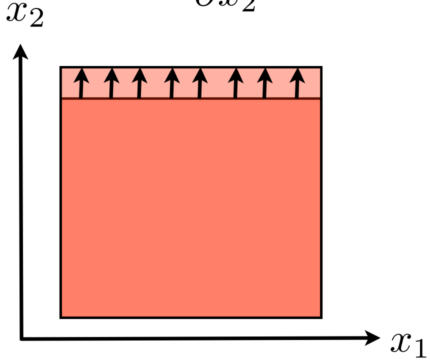
$$\epsilon_{22} = \frac{\partial u_2}{\partial x_2}$$

Normal strain: elongation or contraction

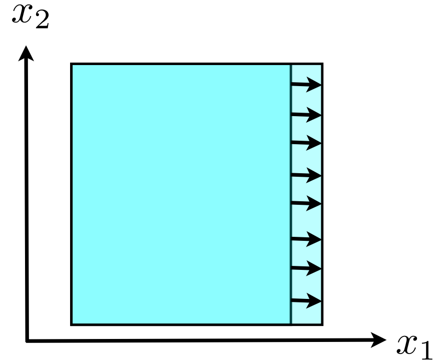
$$\epsilon_{11} = \frac{\partial u_1}{\partial x_1} \qquad \epsilon_{22} = \frac{\partial u_2}{\partial x_2}$$

- positive for lengthening
- negative for shortening

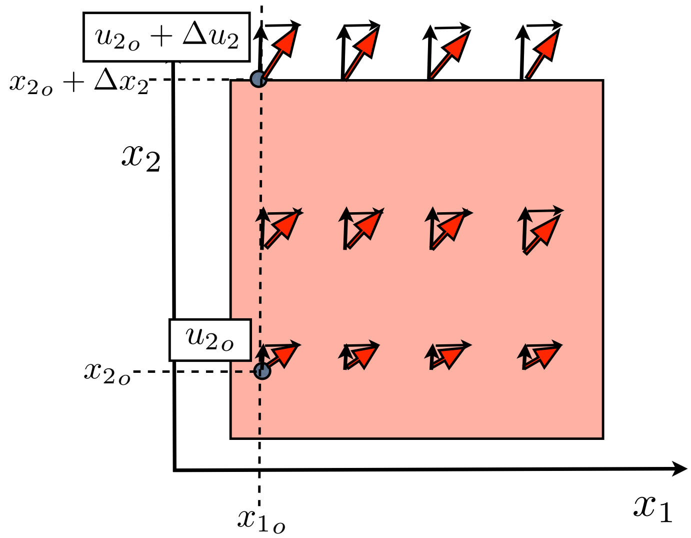
$$\epsilon_{22} = \frac{\partial u_2}{\partial x_2}$$



$$\epsilon_{11} = \frac{\partial u_1}{\partial x_1}$$



The displacement derivatives give the normal strain, even if translation is also going on



$$\epsilon_{22} = \frac{\Delta u_2}{\Delta x_2}$$

$\boxed{\text{displacement gradients}}$ matrix $\boxed{\text{strain}}$ matrix

ϵ_{11}

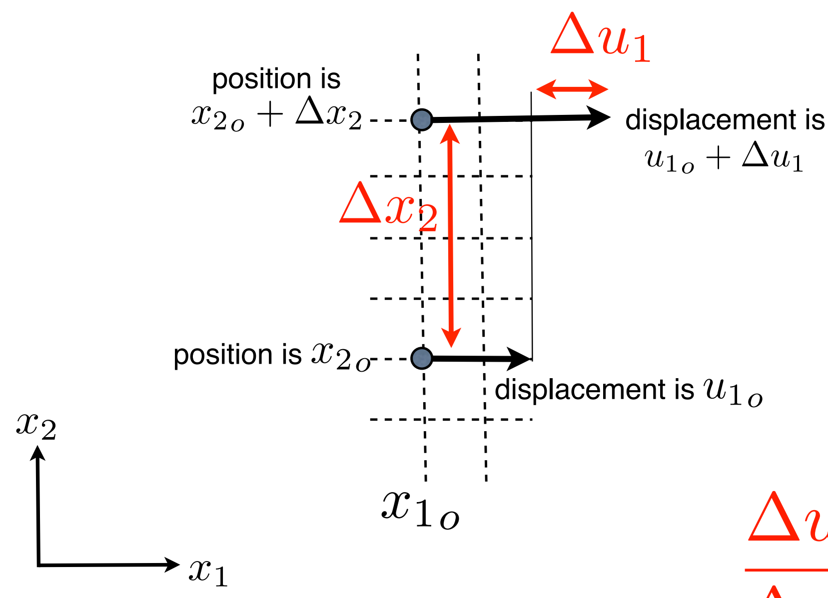
$$\begin{bmatrix} \frac{\Delta u_1}{\Delta x_1} & \frac{\Delta u_1}{\Delta x_2} \\ \frac{\Delta u_2}{\Delta x_1} & \frac{\Delta u_2}{\Delta x_2} \end{bmatrix} \neq \begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix}$$

ϵ_{22}

ϵ_{11} lives in row 1, column 1 of the displacement gradient matrix **and** the strain matrix

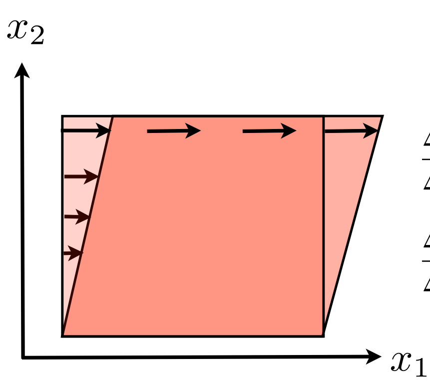
ϵ_{22} lives in row 2, column 2 of the displacement gradient matrix **and** the strain matrix

The other possibility: displacement component is perpendicular to Δx



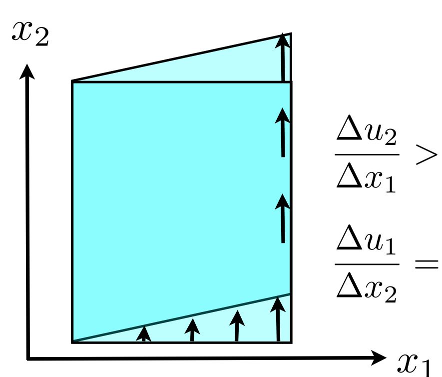
$$\frac{\Delta u_1}{\Delta x_2} = ?$$

This is called "simple shear". It is actually part shear strain and part rotation.



$$\frac{\Delta u_1}{\Delta x_2} > 0$$

$$\frac{\Delta u_2}{\Delta x_1} = 0$$



$$\frac{\Delta u_2}{\Delta x_1} > 0$$

$$\frac{\Delta u_1}{\Delta x_2} = 0$$

Shear strain is defined as:

$$\epsilon_{12} = \epsilon_{21} = \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right)$$

$\boxed{\text{displacement gradients}}$ matrix $\boxed{\text{strain}}$ matrix

$$\begin{bmatrix} \frac{\Delta u_1}{\Delta x_1} & \cancel{\frac{\Delta u_1}{\Delta x_2}} \\ \cancel{\frac{\Delta u_2}{\Delta x_1}} & \frac{\Delta u_2}{\Delta x_2} \end{bmatrix} \neq \begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix}$$

these are NOT shear strain

$$\epsilon_{12} = \epsilon_{21} = \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right)$$

ϵ_{12} is in row 1, column 2 of the strain matrix
ϵ_{21} is in row 2, column 1 of the strain matrix
ϵ_{21} and ϵ_{12} are the same exact number.

Why not just

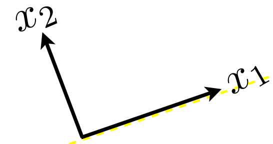
$$\epsilon_{12} = \frac{\Delta u_1}{\Delta x_2} \qquad \epsilon_{21} = \frac{\Delta u_2}{\Delta x_1} \quad ?$$

Some rotation is hidden in those displacement derivatives.

Friday activity: find out what rotation is and define it in terms of $\frac{\Delta u_1}{\Delta x_2}$ and $\frac{\Delta u_2}{\Delta x_1}$.

Why? <u>So we can get rid of it.</u> Strain is all that matters for generating elastic stress and earthquakes.

Shear strain in one year along the San Andreas Fault, from GPS



$$\frac{\Delta u_2}{\Delta x_1} \approx \left(\frac{-0.020\ m}{10000\ m}\right)$$

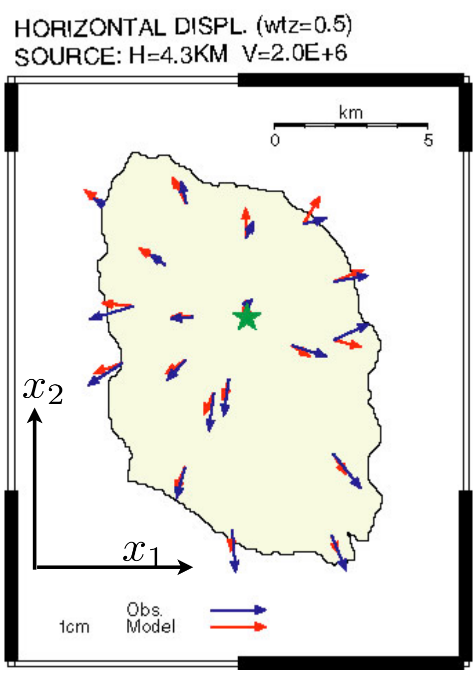
$$\frac{\Delta u_1}{\Delta x_1} = 0$$

and other derivatives = 0

$$\epsilon_{12} \approx ?$$

Fault slip rate here is about 20 mm/year but we see only part of this relative motion close to the fault (arctan function)

Mt Fuji 2002-2003



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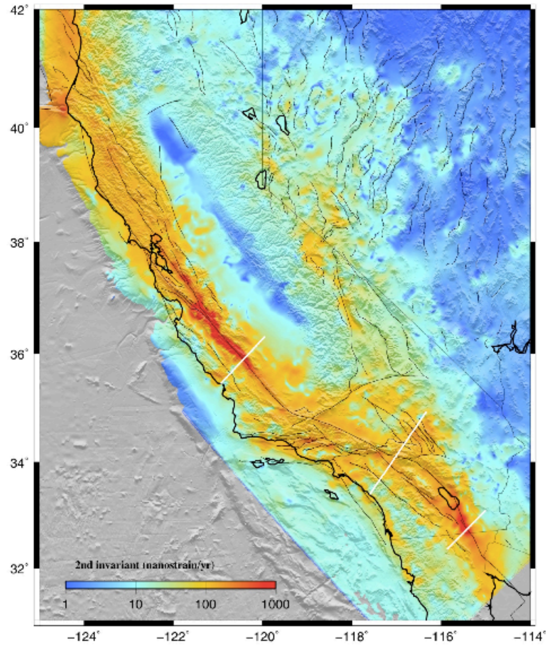
ϵ_{11}?

ϵ_{22}?

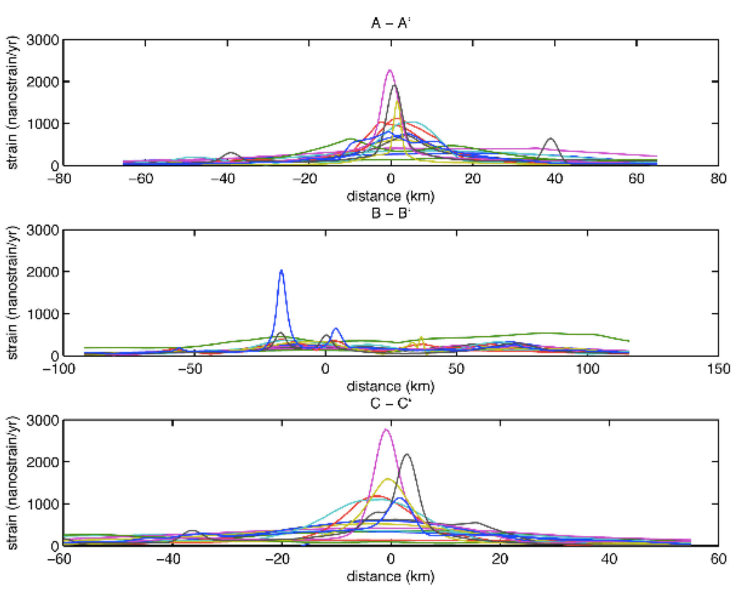
Real-life example of strain at a volcano

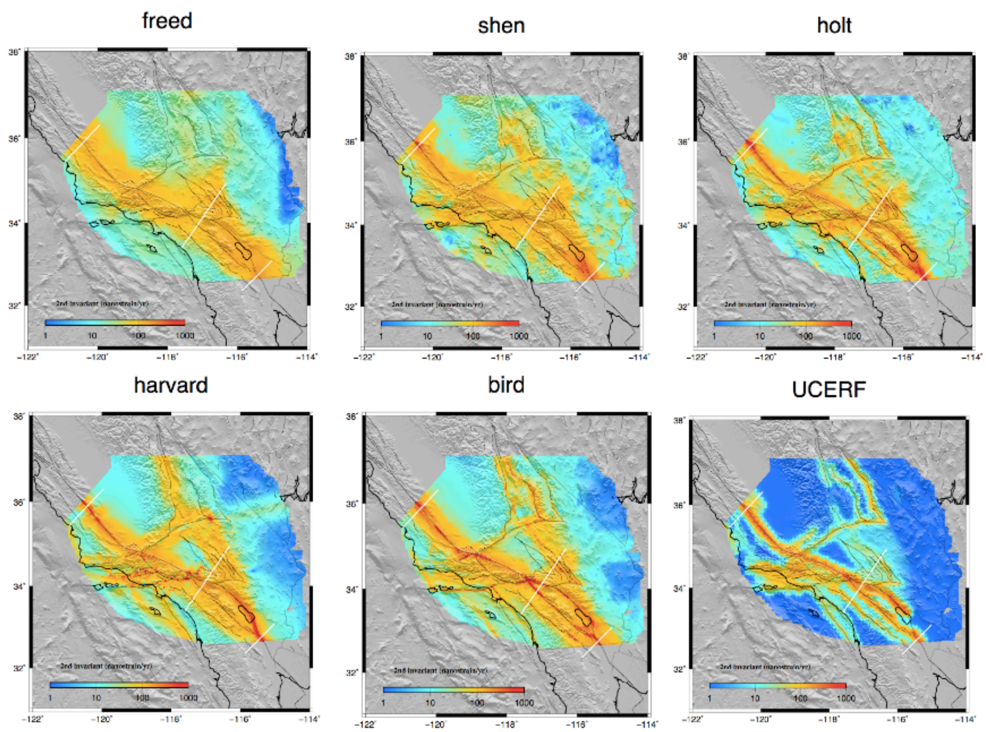
Mt St. Helens 2004 (USGS)



compute displacement derivatives from GPS displacement field and then remove rotation.
plot some scalar quantity in color to represent strain
here they show the "second invariant" of the strain matrix
which is close to the maximum possible shear strain

What is the true strain rate?





Group activity - for Friday March 2.

(your names)

(your answers)

Trace axes and square onto the transparency. Add points as shown.

Rotate the transparency (not by a large amount).

1) For the point on the x_1 axis: what is du_2 ? What is dx_1? (dx_2 and du_1 should be small enough to ignore.)

2) For the point on the x_2 axis: what is du_1? What is dx_2? (careful with signs)

Remember the definition of tangent of θ and that for a small angle, θ in radians = tan(θ)

3a) What is the angle of rotation (θ) of the x_1 axis? Of the x_2 axis? Be careful of signs for the du's and dx's.

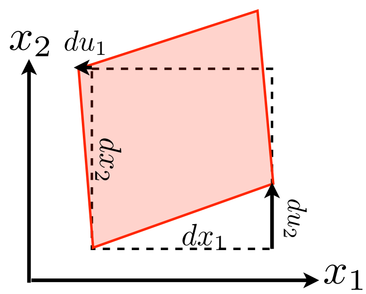
3b) If we want to <u>define counter-clockwise rotation as positive</u>, then what are the answers to 3a?

3c) Write out the mean of the two angles in 3b (in terms of du's and dx's).
You have just found the mean rotation angle, "w".

4) In terms of du's and dx's, what is (shear strain + w)? What is (shear strain - w)?

Usually we have to deal with strain that looks like this:

(your answers)

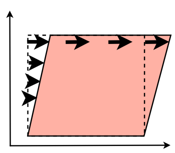


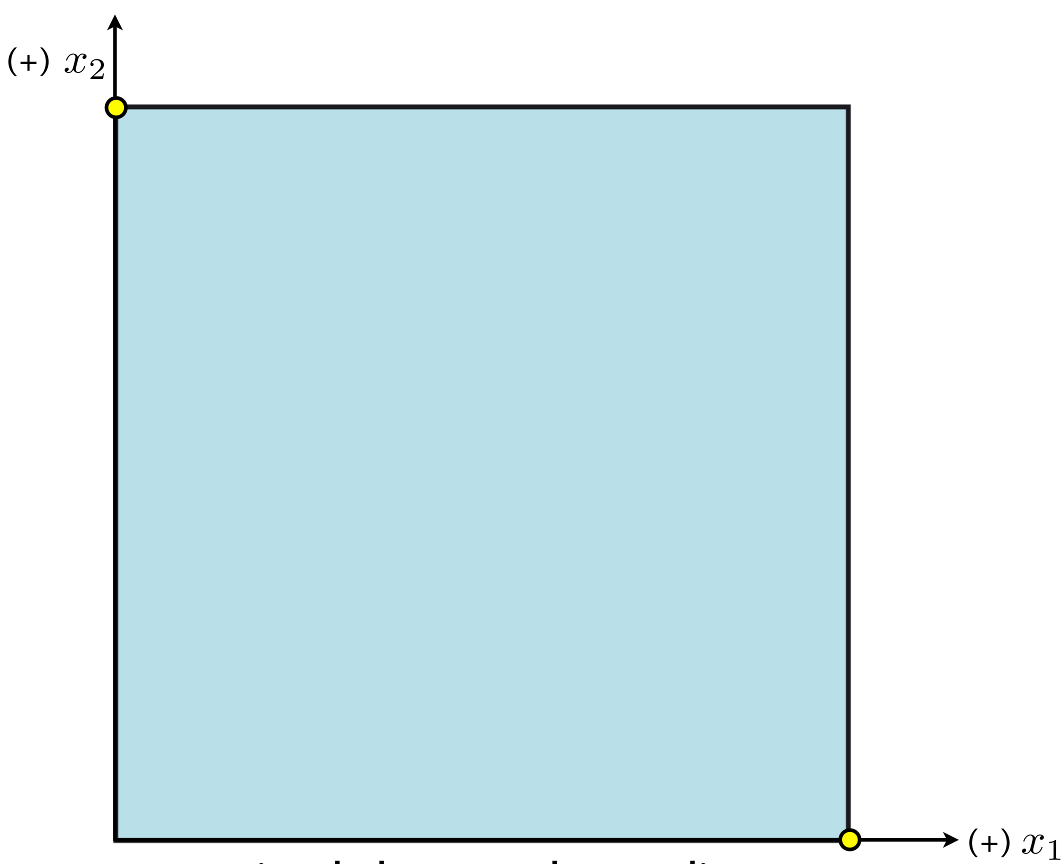
The x_1 - parallel and x_2 - parallel sides have been rotated by different amounts. You can't make this happen with a rigid transparency because rotation and strain have both occurred.

5a) Compute: du_1/dx_2 and du_2/dx_1
5b) Compute the rotation w.
5c) Compute the shear strain ϵ_{12}.

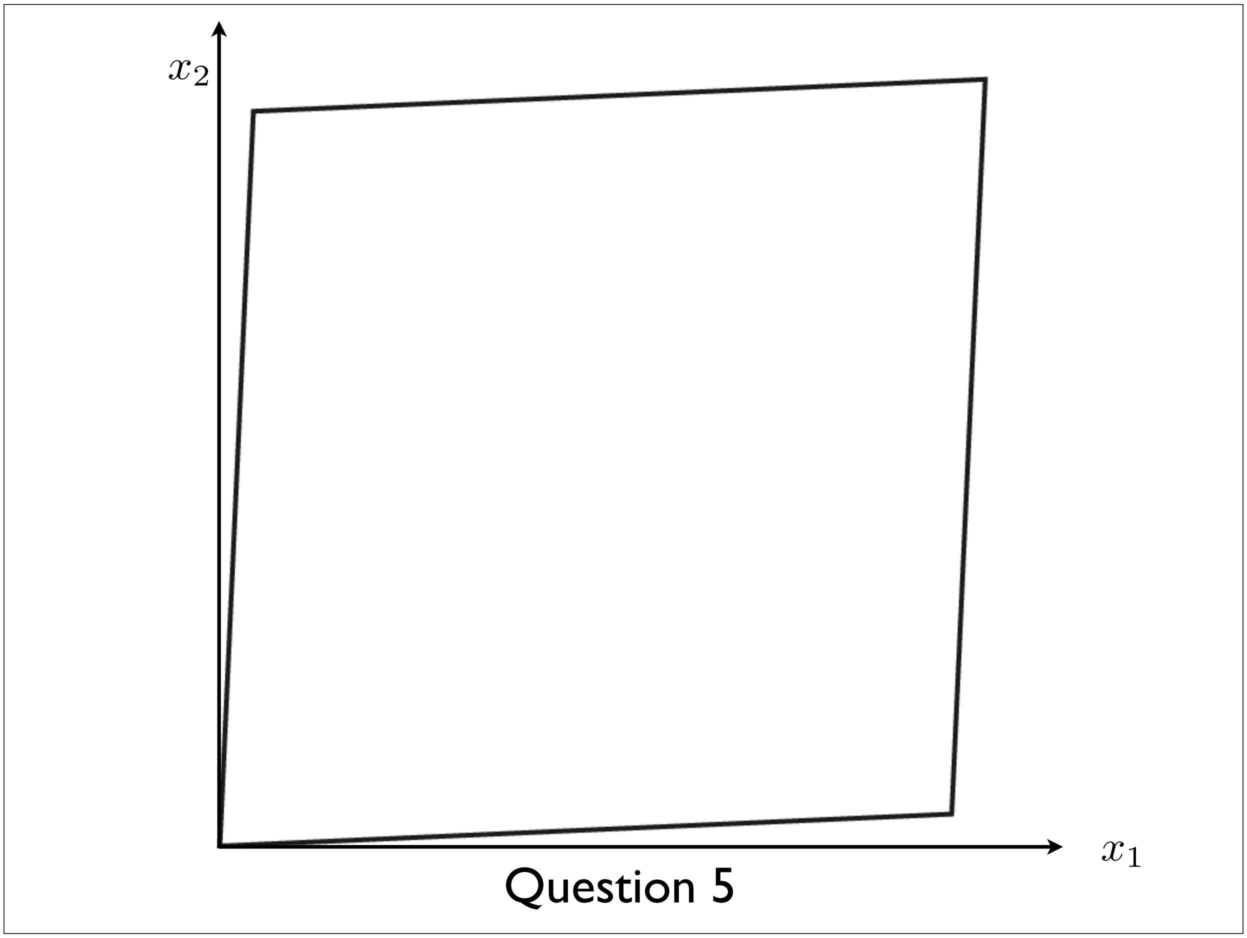
6) Do the same for the case marked "Question 6".

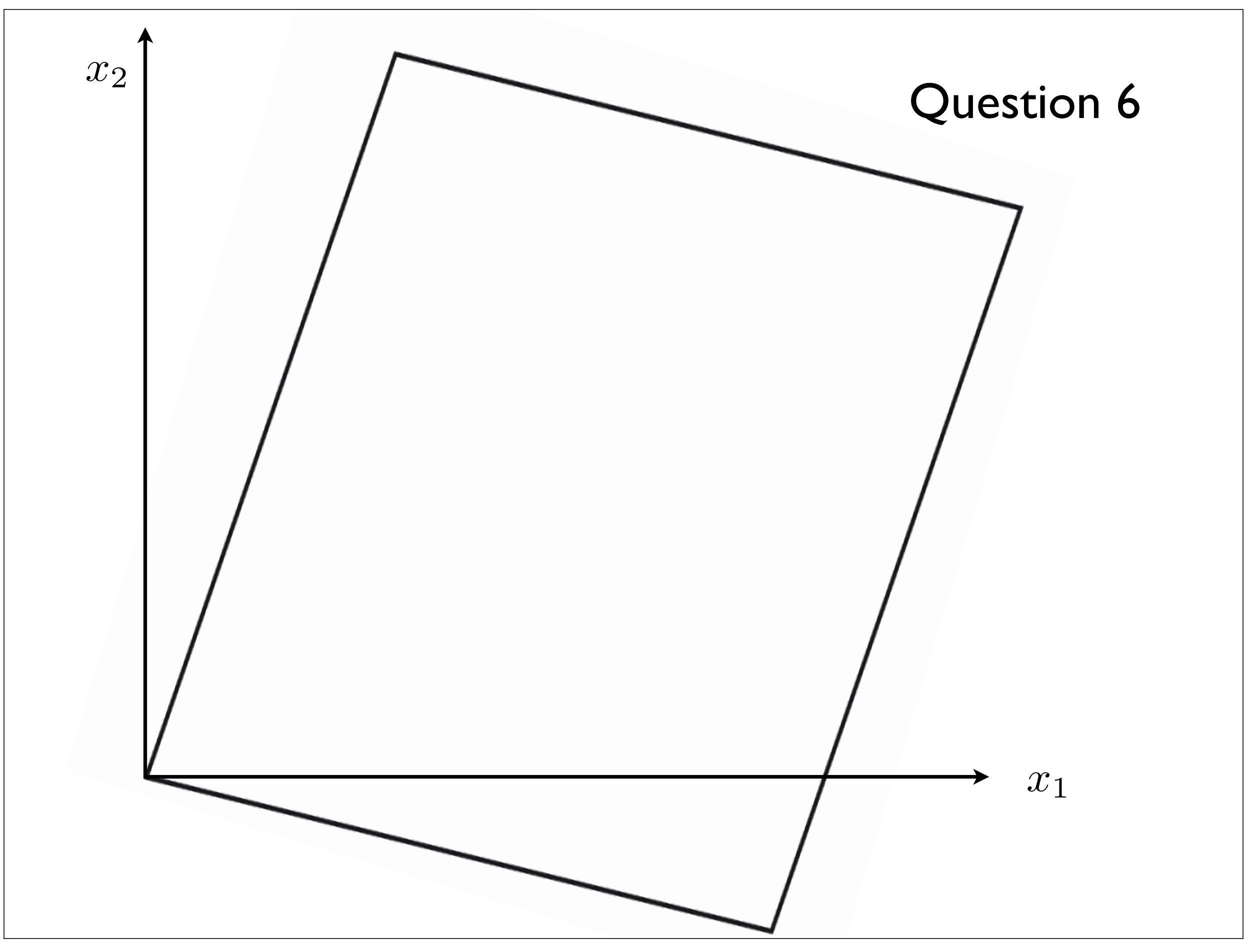
7) Show me that for "simple shear" strain, ϵ_{12} = w (or -w).





pre-strained shape and coordinate axes


x_2
x_1
Question 5


x_2
Question 6
x_1

3c) Write out the mean of the two angles in 3b (in terms of du's and dx's).

You have just found the mean rotation angle, "w".

$$\frac{1}{2}\left(\frac{\Delta u_2}{\Delta x_1} - \frac{\Delta u_1}{\Delta x_2}\right)$$

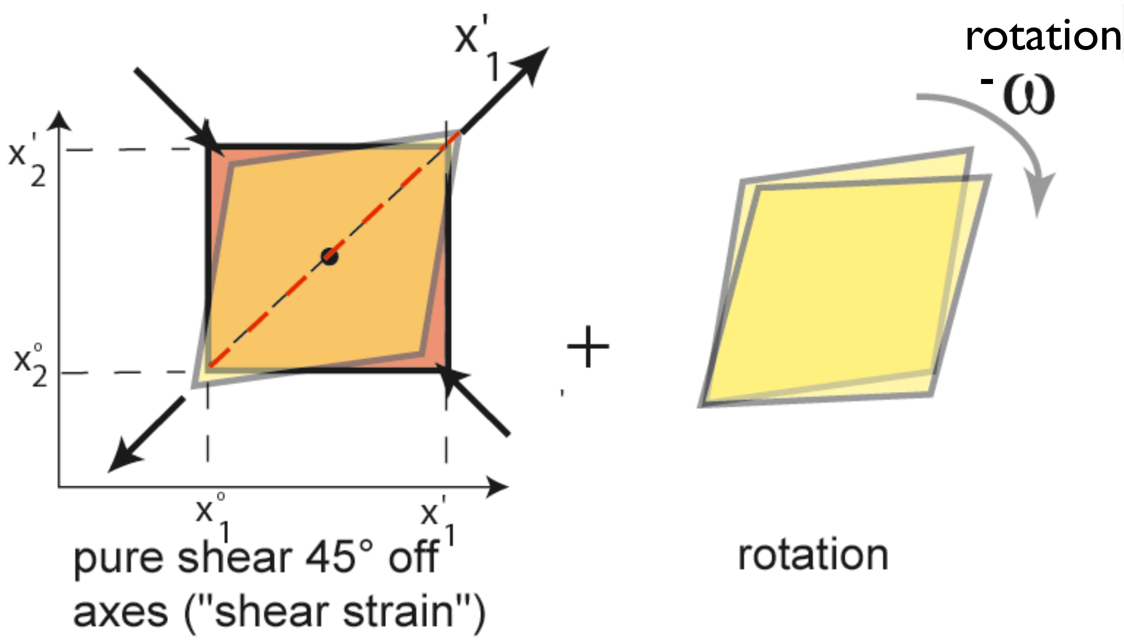
4) In terms of du's and dx's, what is (shear strain + w)?

$$\frac{1}{2}\left(\frac{\Delta u_2}{\Delta x_1} + \frac{\Delta u_1}{\Delta x_2}\right) + \frac{1}{2}\left(\frac{\Delta u_2}{\Delta x_1} - \frac{\Delta u_1}{\Delta x_2}\right) = \frac{\Delta u_2}{\Delta x_1}$$

What is (shear strain - w)?

$$\frac{\Delta u_1}{\Delta x_2}$$

"Simple shear" is actually shear strain plus (or minus) rotation



Group activity - Q 5

$$\begin{bmatrix} \frac{\Delta u_1}{\Delta x_1} & \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right) \\ \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right) & \frac{\Delta u_2}{\Delta x_2} \end{bmatrix}$$

(-w21)

$$\begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} - \frac{\Delta u_2}{\Delta x_1}\right) \\ \frac{-1}{2}\left(\frac{\Delta u_1}{\Delta x_2} - \frac{\Delta u_2}{\Delta x_1}\right) & 0 \end{bmatrix}$$

(w21)

strain matrix + *rotation matrix*

what we want!

= *displacement gradient matrix*

$$\begin{bmatrix} \frac{\Delta u_1}{\Delta x_1} & \frac{\Delta u_1}{\Delta x_2} \\ \frac{\Delta u_2}{\Delta x_1} & \frac{\Delta u_2}{\Delta x_2} \end{bmatrix}$$

what we can measure

<u>Easy</u> recipe to get strain and rotation matrices from D!

strain **E** = 1/2(**D** + **D**^{T})

rotation **W** = **D-E**

D^T is the "transpose" of **D**:

D

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

D^T

$$\begin{bmatrix} D_{11} & D_{21} \\ D_{21} & D_{22} \end{bmatrix}$$

these two have switched places!

strain matrix **E**

$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix}$$ is symmetric (same as its transpose:)

rotation matrix **W**

$$\begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix}$$ is antisymmetric

displacement gradient matrix **D**

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$ is neither - all 4 numbers are different