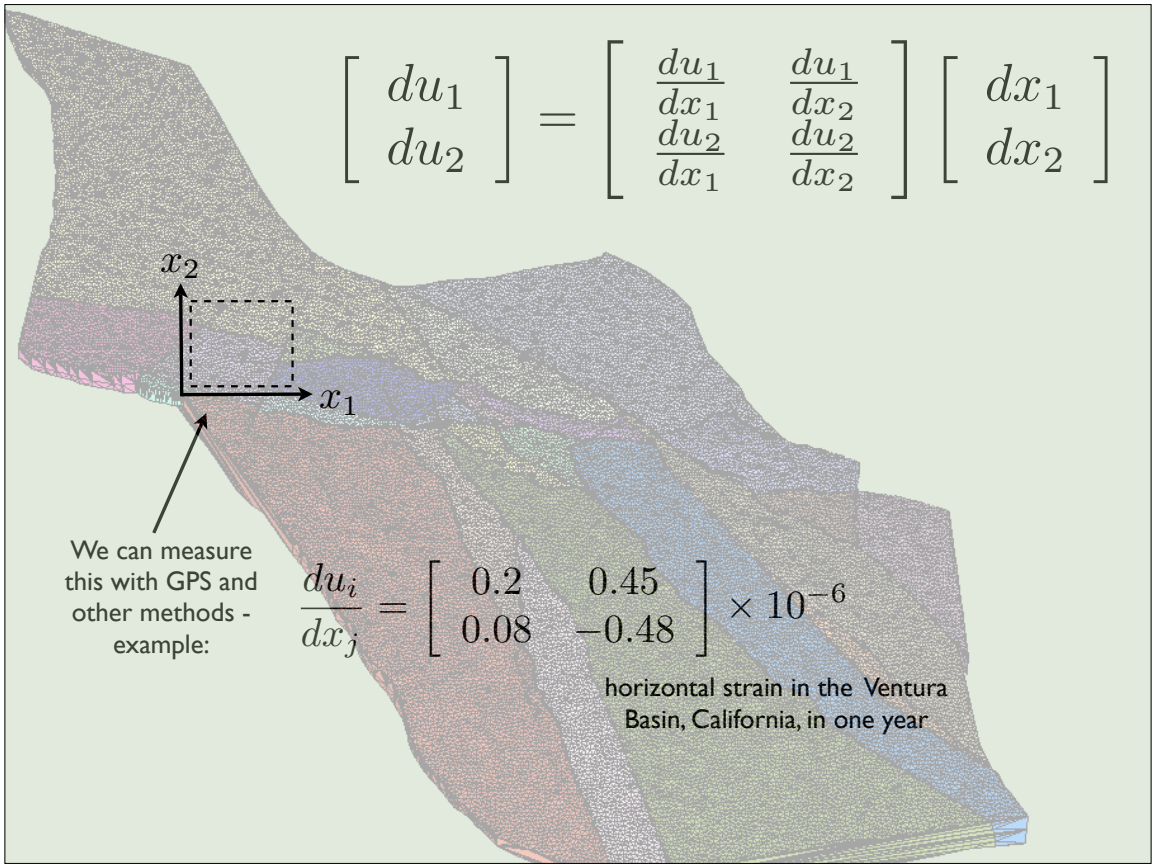


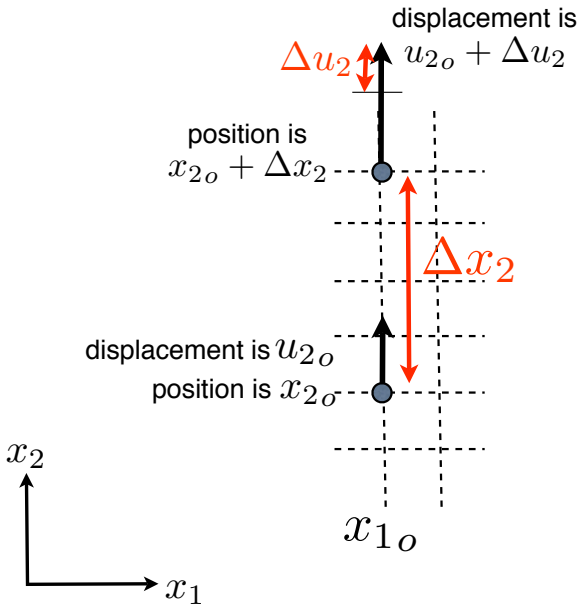
Congratulations! You have just come up with the "displacement gradient tensor".

$$\begin{bmatrix} \frac{\Delta u_1}{\Delta x_1} & \frac{\Delta u_1}{\Delta x_2} \\ \frac{\Delta u_2}{\Delta x_1} & \frac{\Delta u_2}{\Delta x_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} \end{bmatrix}$$

Today:

- normal strain
- shear strain
- rotation
- how to get the strain matrix from the matrix above

Normal strain: elongation or contraction



$$\epsilon_{22} = \frac{\Delta u_2}{\Delta x_2}$$

same as

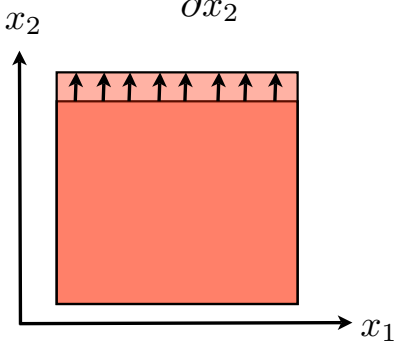
$$\epsilon_{22} = \frac{\partial u_2}{\partial x_2}$$

Normal strain: elongation or contraction

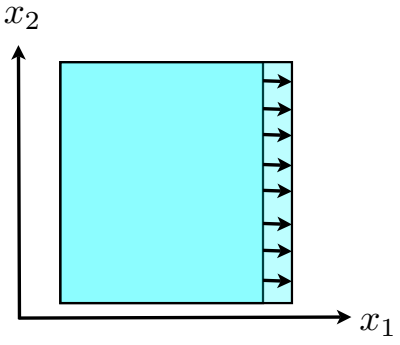
$$\epsilon_{11} = \frac{\partial u_1}{\partial x_1} \qquad \epsilon_{22} = \frac{\partial u_2}{\partial x_2}$$

- positive for lengthening
- negative for shortening

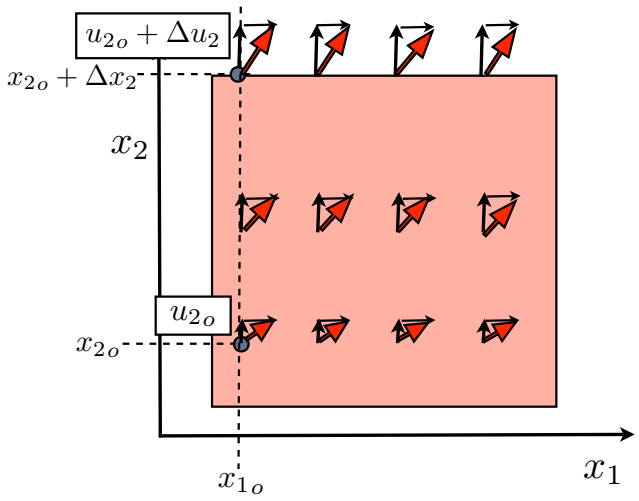
$$\epsilon_{22} = \frac{\partial u_2}{\partial x_2}$$



$$\epsilon_{11} = \frac{\partial u_1}{\partial x_1}$$



Normal strain: elongation or contraction
A 2D view.



$$\epsilon_{22} = \frac{\Delta u_2}{\Delta x_2}$$

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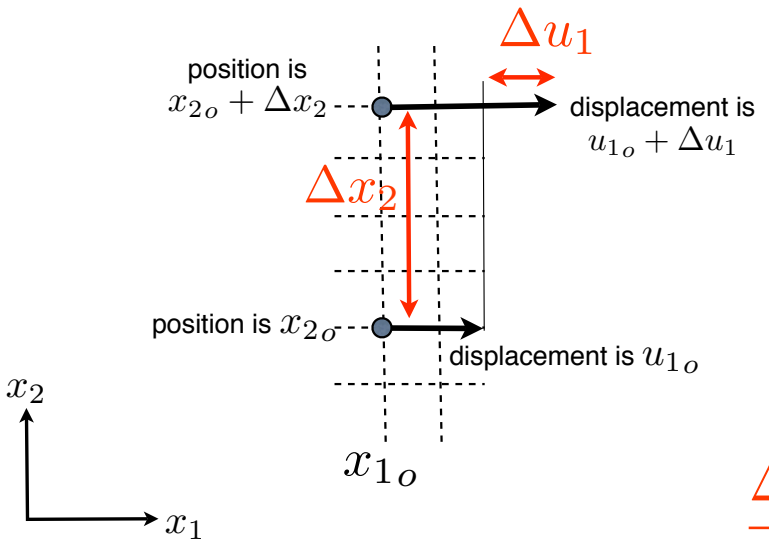
displacement gradients matrix strain matrix

$$\begin{bmatrix} \frac{\Delta u_1}{\Delta x_1} & \frac{\Delta u_1}{\Delta x_2} \\ \frac{\Delta u_2}{\Delta x_1} & \frac{\Delta u_2}{\Delta x_2} \end{bmatrix} \neq \begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix}$$

ϵ_{11} lives in row 1, column 1 of the displacement gradient matrix **and** the strain matrix

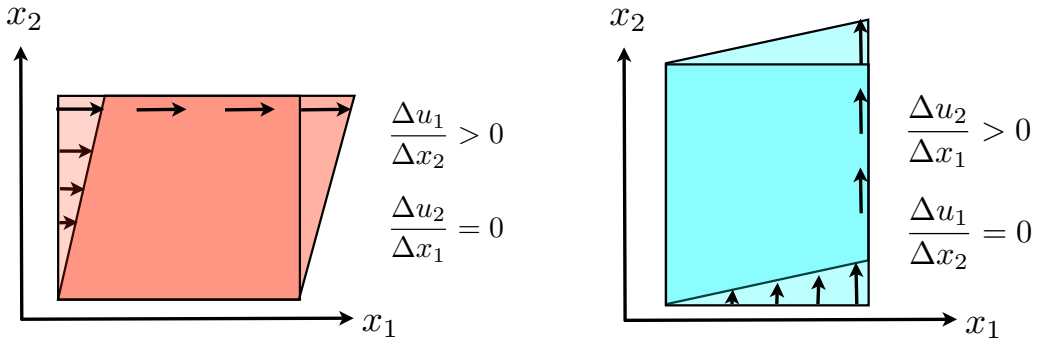
ϵ_{22} lives in row 2, column 2 of the displacement gradient matrix **and** the strain matrix

The other possibility: displacement component is perpendicular to Δx



$$\frac{\Delta u_1}{\Delta x_2} = ?$$

This is "simple shear" strain



Shear strain is defined as:

$$\epsilon_{12} = \epsilon_{21} = \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right)$$

displacement gradients matrix

strain matrix

$$\begin{bmatrix} \frac{\Delta u_1}{\Delta x_1} & \frac{\Delta u_1}{\Delta x_2} \\ \frac{\Delta u_2}{\Delta x_1} & \frac{\Delta u_2}{\Delta x_2} \end{bmatrix} \neq \begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix}$$

these are NOT shear strain

$$\epsilon_{12} = \epsilon_{21} = \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right)$$

ϵ_{12} is in row 1, column 2 of the strain matrix

ϵ_{21} is in row 2, column 1 of the strain matrix

ϵ_{21} and ϵ_{12} are the same exact number.

Why not just

$$\epsilon_{12} = \frac{\Delta u_1}{\Delta x_2} \qquad \epsilon_{21} = \frac{\Delta u_2}{\Delta x_1} \quad ?$$

Answer: Some rotation is hidden in those displacement derivatives.

Class activity: find out what rotation is and define it in terms of $\frac{\Delta u_1}{\Delta x_2}$ and $\frac{\Delta u_2}{\Delta x_1}$.

Group activity - This will be part 1 of your Homework 3 (hold onto it)

Trace axes and points onto the transparency.

Rotate the transparency (not by a large amount).

1) For the point on the x_1 axis: what is du_2 ? What is dx_1? (dx_2 and du_1 should be small enough to ignore.)

2) For the point on the x_2 axis: what is du_1? What is dx_2? (careful with signs)

Remember the definition of tangent of θ and that for a small angle, θ in radians = tan(θ)

3a) What is the angle of rotation (θ) of the x_1 axis? Of the x_2 axis?

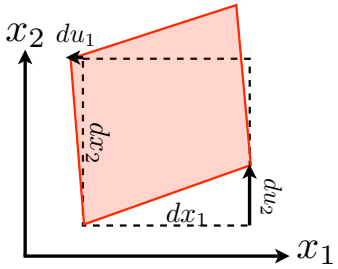
3b) We want to define counter-clockwise rotation as positive, then what are the answers to 3a?

3c) Write out the mean of the two angles in 3b (in terms of du's and dx's).

You have just found the mean rotation angle, "w".

4) In terms of du's and dx's, what is (shear strain + w)? What is (shear strain - w)?

Usually we have to deal with strain that looks like this:



The x_1 - parallel and x_2 - parallel sides have been rotated by different amounts. You can't make this happen with a rigid transparency because rotation and strain have both occurred.

5a) Compute: du_1/dx_2 and du_2/dx_1
5b) Compute the rotation w.
5c) Compute the shear strain ϵ_{12}.

6) Show me that for "simple shear" strain, ϵ_{12} = w (or -w).