Today (Feb 4):

- complete: strain and rotation from displacement gradient matrix
- examples
- GPS

Group activity - This will be part 1 of your Homework 3 (hold onto it)

Trace axes and points onto the transparency.

Rotate the transparency (not by a large amount).

1) For the point on the x_1 axis: what is du_2 ? What is dx_1? (dx_2 and du_1 should be small enough to ignore.)

2) For the point on the x_2 axis: what is du_1? What is dx_2? (careful with signs)

Remember the definition of tangent of θ and that for a small angle, θ in radians = tan(θ)

3a) What is the angle of rotation (θ) of the x_1 axis? Of the x_2 axis?

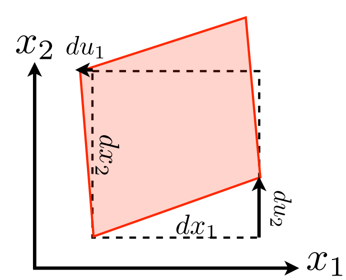
3b) We want to define counter-clockwise rotation as positive, then what are the answers to 3a?

3c) Write out the mean of the two angles in 3b (in terms of du's and dx's).

You have just found the mean rotation angle, "w".

4) In terms of du's and dx's, what is (shear strain + w)? What is (shear strain - w)?

Usually we have to deal with strain that looks like this:



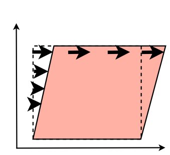
The x_1 - parallel and x_2 - parallel sides have been rotated by different amounts. You can't make this happen with a rigid transparency because rotation and strain have both occurred.

5a) Compute: du_1/dx_2 and du_2/dx_1

5b) Compute the rotation w.

5c) Compute the shear strain ϵ_{12}.

6) Show me that for "simple shear" strain, ϵ_{12} = w (or -w).



3c) Write out the mean of the two angles in 3b (in terms of du's and dx's).

You have just found the mean rotation angle, "w".

$$\frac{1}{2}\left(\frac{\Delta u_2}{\Delta x_1} - \frac{\Delta u_1}{\Delta x_2}\right)$$

4) In terms of du's and dx's, what is (shear strain + w)?

$$\frac{1}{2}\left(\frac{\Delta u_2}{\Delta x_1} + \frac{\Delta u_1}{\Delta x_2}\right) + \frac{1}{2}\left(\frac{\Delta u_2}{\Delta x_1} - \frac{\Delta u_1}{\Delta x_2}\right) = \frac{\Delta u_2}{\Delta x_1}$$

What is (shear strain - w)?

$$\frac{\Delta u_1}{\Delta x_2}$$

$$\begin{bmatrix} \frac{\Delta u_1}{\Delta x_1} & \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right) \\ \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right) & \frac{\Delta u_2}{\Delta x_2} \end{bmatrix} \quad \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} - \frac{\Delta u_2}{\Delta x_1}\right) \\ \frac{-1}{2}\left(\frac{\Delta u_1}{\Delta x_2} - \frac{\Delta u_2}{\Delta x_1}\right) & 0 \end{bmatrix}$$

(-w)

(w)

strain matrix + rotation matrix

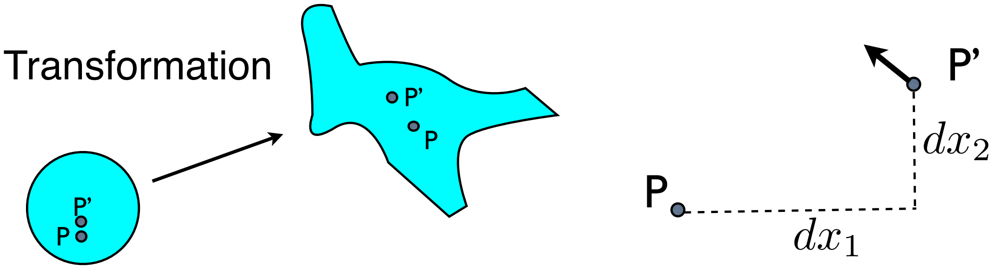
what we want!

= displacement gradient matrix

$$\begin{bmatrix} \frac{\Delta u_1}{\Delta x_1} & \frac{\Delta u_1}{\Delta x_2} \\ \frac{\Delta u_2}{\Delta x_1} & \frac{\Delta u_2}{\Delta x_2} \end{bmatrix}$$

what we can measure

Transformation



$$\begin{bmatrix} du_1 \\ du_2 \end{bmatrix} = \begin{bmatrix} \text{Displacement gradient matrix} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix}$$

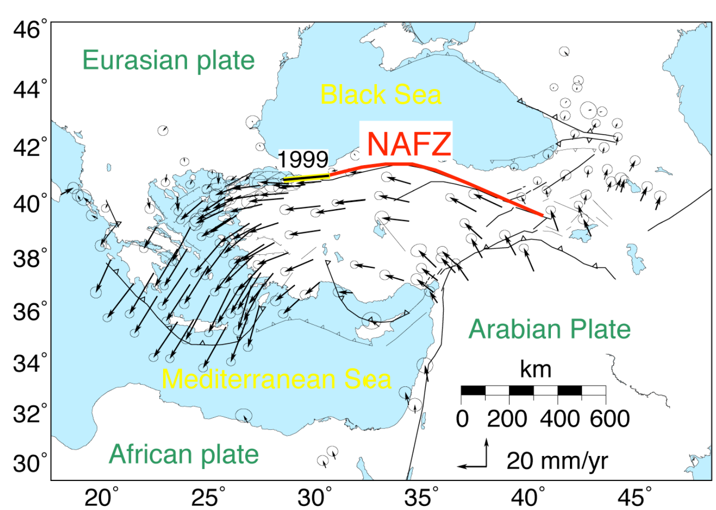
- gives displacement of P' relative to P WHICH WE CAN MEASURE

$$\begin{bmatrix} du_1 \\ du_2 \end{bmatrix}_S = \begin{bmatrix} \text{strain matrix} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix}$$

- gives displacement of P' relative to P due to strain only (not rotation)

Real-life example of **rotation**: Turkey

GPS horizontal velocities relative to Eurasia 1988-1997



from McClusky et al., JGR, 2000

Earthquake cycle model - strike-slip fault

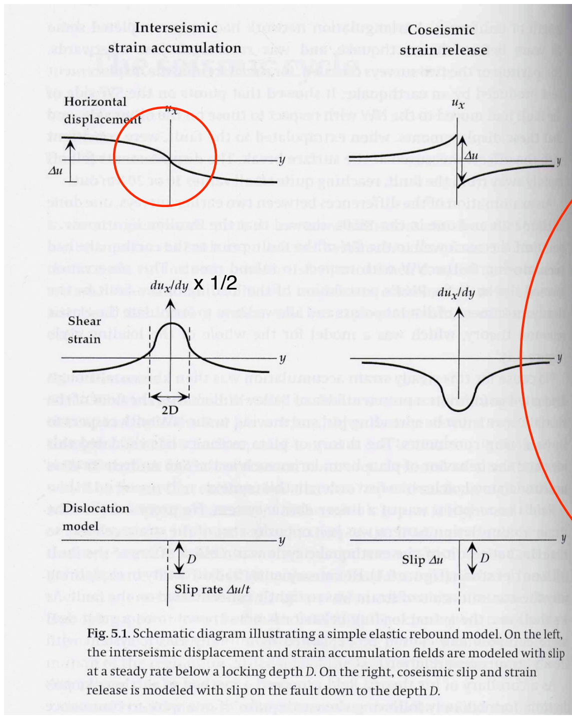
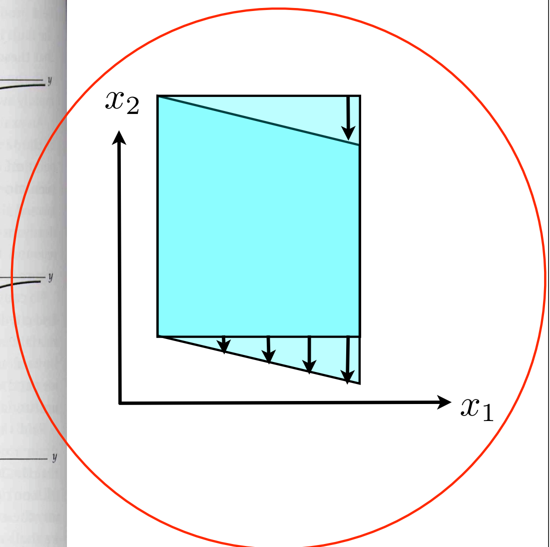


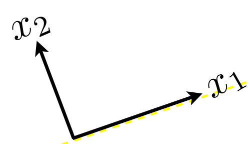
Fig. 5.1. Schematic diagram illustrating a simple elastic rebound model. On the left, the interseismic displacement and strain accumulation fields are modeled with slip at a steady rate below a locking depth D. On the right, coseismic slip and strain release is modeled with slip on the fault down to the depth D.



From Scholz, Mechanics of Earthquakes and Faulting, 2002

Real-life example of simple shear strain around a fault

Shear strain in one year along the San Andreas Fault



$$\frac{\Delta u_2}{\Delta x_1} \approx \left(\frac{-0.020\ m}{10000\ m}\right)$$

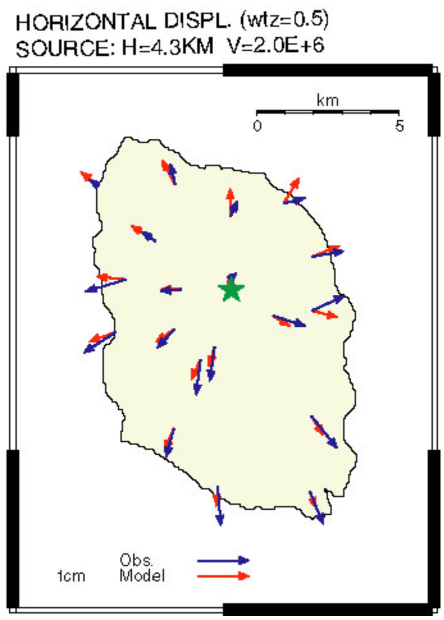
$$\frac{\Delta u_1}{\Delta x_1} = 0$$

and other derivatives = 0

$$\epsilon_{12} \approx ?$$

Relative plate motion is 34 mm/year but we see only part of this relative motion close to the fault (arctan function)

Mt Fuji 2002-2003



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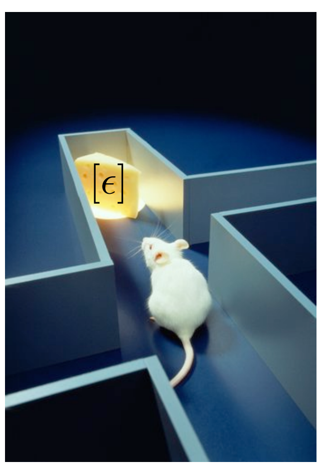
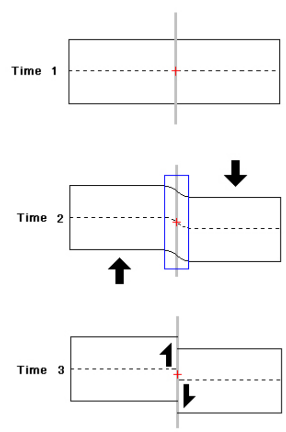
Real-life example of
normal strain
around a volcano

Mt St. Helens 2004 (USGS)

For figuring out elastic force acting the fault surface, pushing it toward failure, all that matters is strain





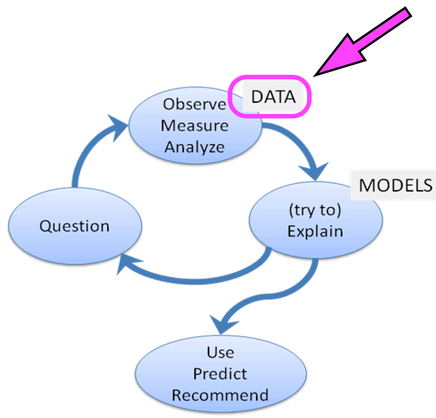
Translation and rotation do not distort the volume (*pull the spring*) and they do not matter at all to us.

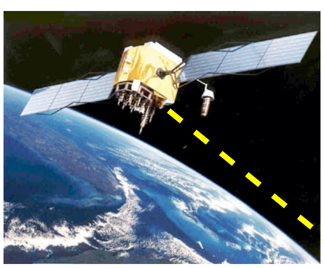
GPS data give us the displacements and the displacement gradient matrix.
We know how to get strain from this matrix now!

Satellite geodesy - widespread since the early 1990's:

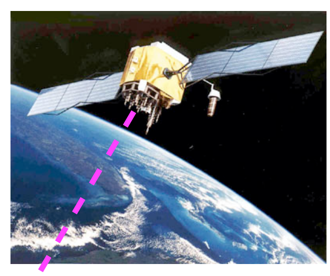
GPS

InSAR

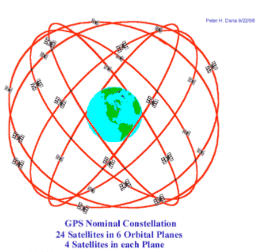


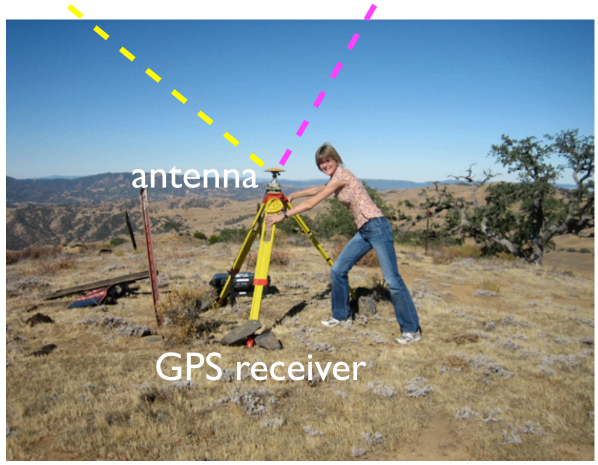


Satellite geodesy for crustal deformation monitoring- since the 1990's



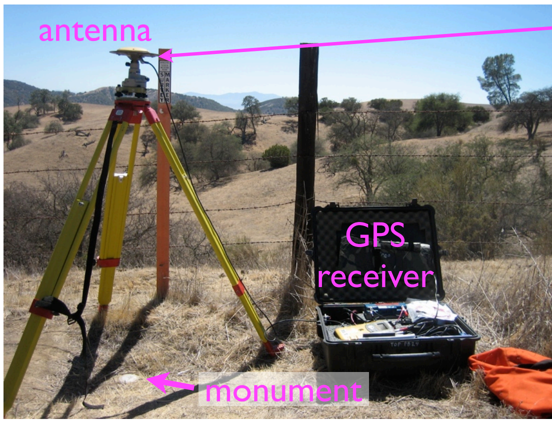
GPS satellites are in polar orbits

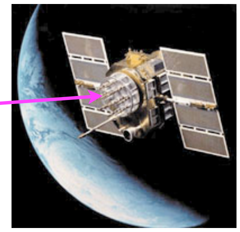




32 of these as of March 2008

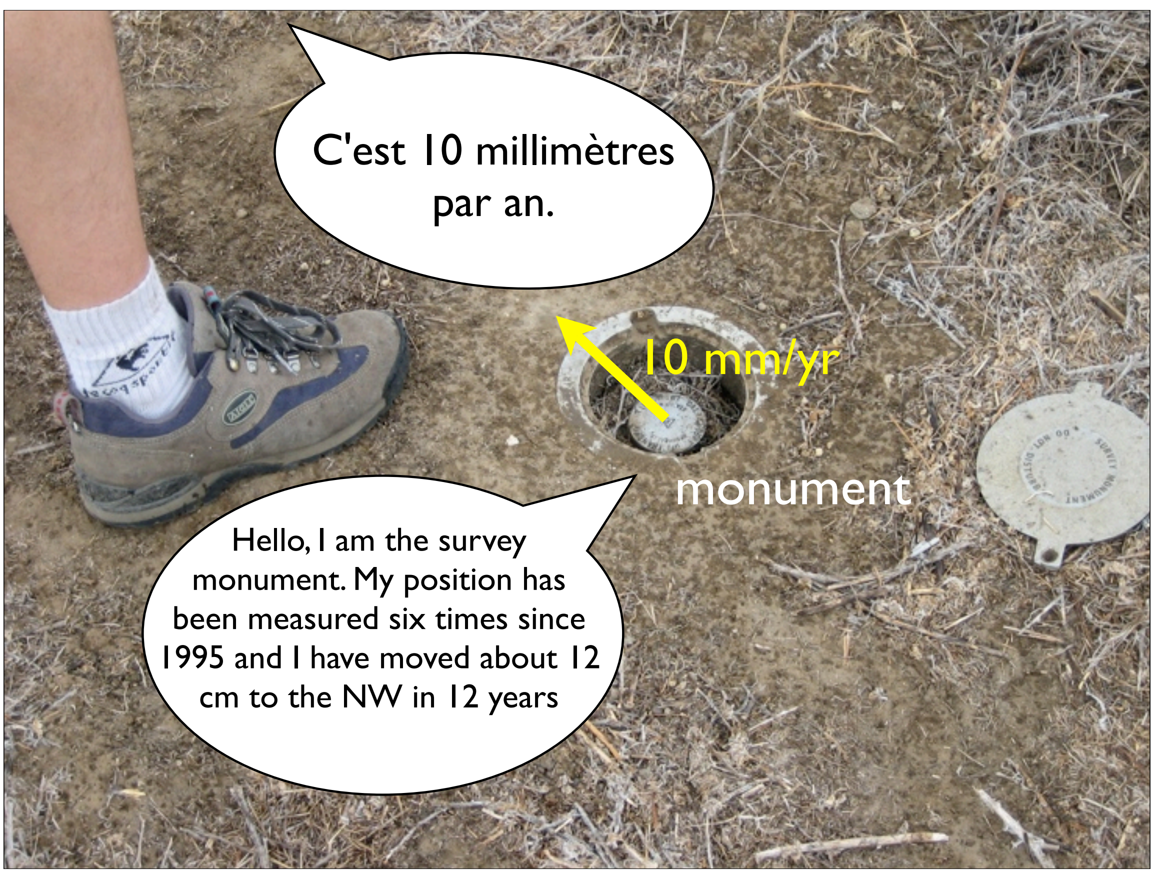
(edited since lecture, not examinable - EHH)



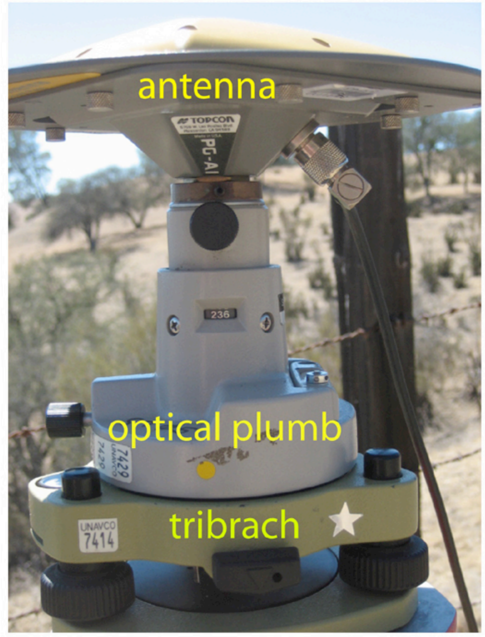


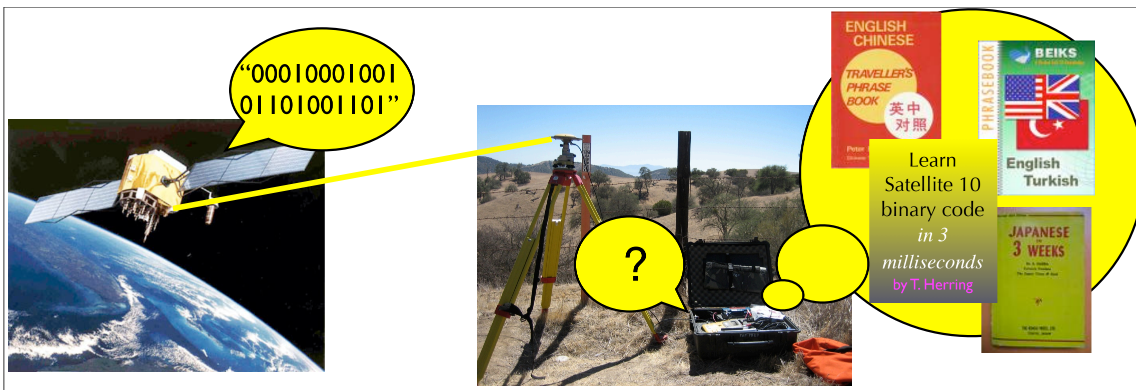
Each satellite broadcasts its own tunes at about 1500 Hz and 1200 Hz

repeated position measurements are made over time give velocity to within less than 1 mm/year!



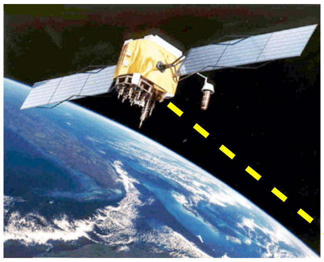
GPS receiver - not the car kind
• ~ $5,000 (with antenna etc.)
• "dual frequency" (both GPS frequencies are recorded)
• lots of memory and sophisticated software to process the data





When GPS receiver gets a signal

- It compares that signal with all the known codes (there are currently 37).
- The receiver determines which satellite it is.
- It knows when the signal was sent (comes with a time stamp) and when it was received, multiplies the difference by the speed of light to find the distance (pseudorange).
- Once it has done that for 3 satellites, it can determine the location.



Satellite sends signal at t_1, signal is received at t_2

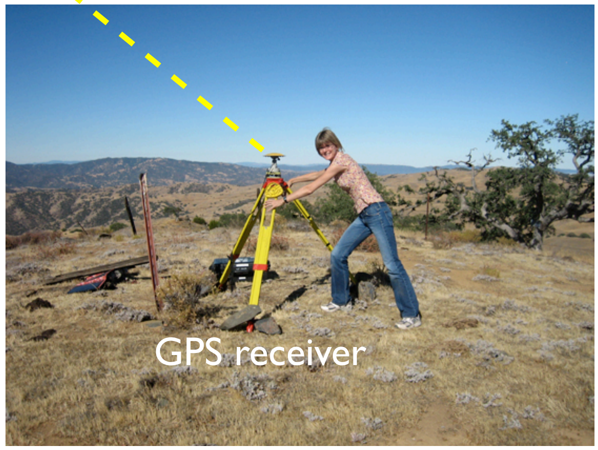
Time Difference (in seconds) * 2.99792458 x 10^8 meters/second = Distance (in meters)

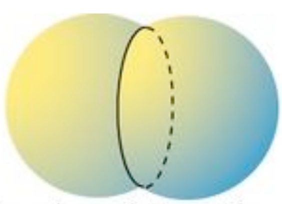
GPS signal tells you how far it is from the satellite (“pseudorange”).

What is a typical time difference?

GPS satellites are ~2×10^7 meters above the Earth.

about 70 milliseconds

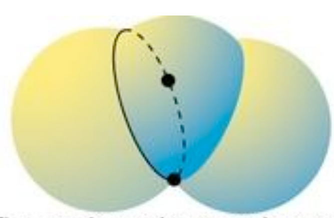




Two spheres intersect in a circle

Intersecting spheres of radius r

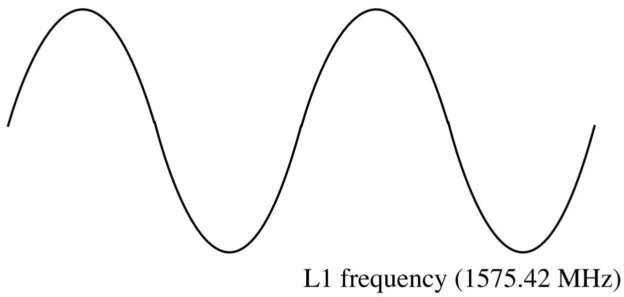
3 spheres intersect at two points - but only 1 point is on the Earth



Three spheres intersect in two points

Extra tricks are required to get position accuracy of 1 mm

Actually each satellite is broadcasting 3 tunes at once ... one really low-pitched tune with the time stamp (50 Hz) the C/A code (song) and the P(Y) code song (1 and 10 MHz)



Two microwave carrier signals.
The L1 frequency (1575.42 MHz) carries the navigation message and the SPS code signals. The L2 frequency (1227.60 MHz) is used to measure the ionospheric delay by PPS equipped receivers.

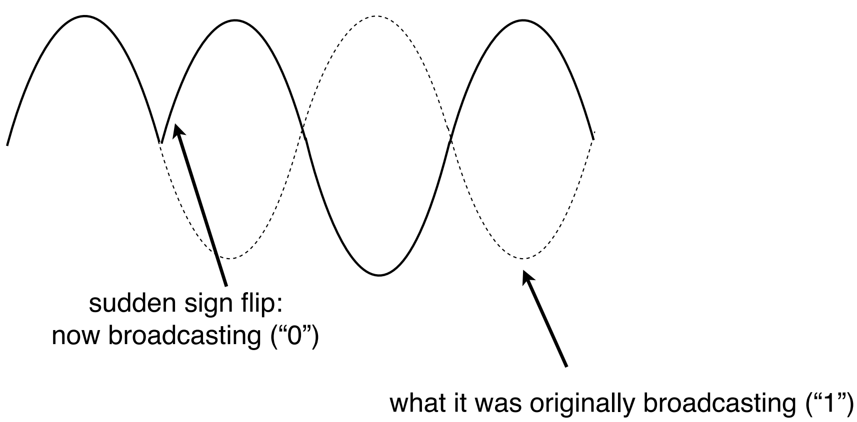
The C/A Code (Coarse Acquisition) modulates the L1 carrier phase. The C/A code is a repeating 1 MHz Pseudo Random Noise (PRN) Code. (This is the one used by your car's GPS)

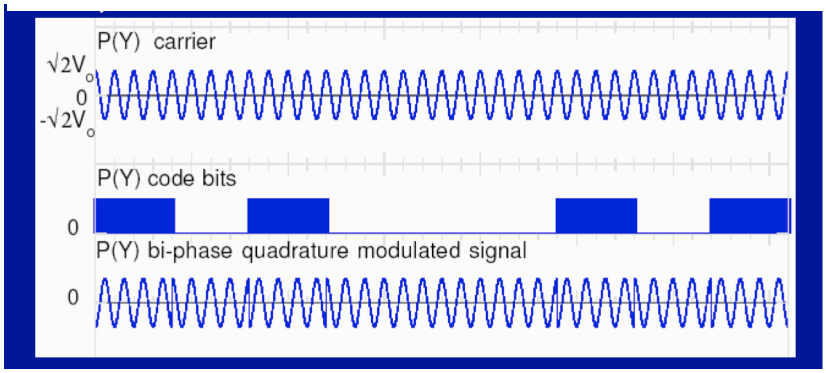
The P-Code (Precise) modulates both the L1 and L2 carrier phases. The P (Y)-Code is 10 MHz the basis for the PPS. Shorter time intervals of 0's and 1's allow a more precise pseudorange.

The only lyrics are "0" or "1" (binary code)

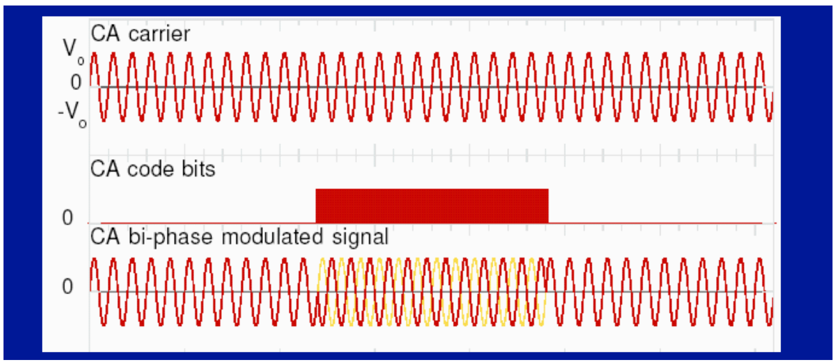
0 and 1 are indicated by a sudden flip in the polarity of the signal...

0 or 1: indicated by a sudden flip in the polarity of the signal





about 0.3 microseconds



about 3 microseconds

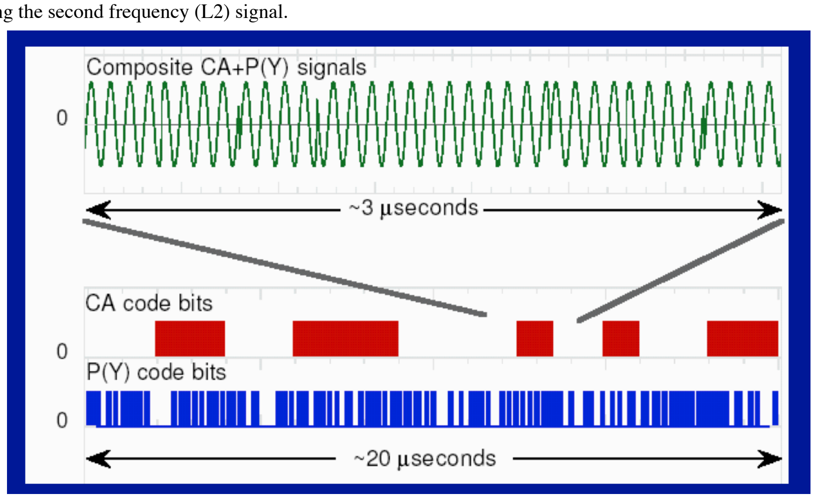
"about 70 milliseconds" = 2 x 10^7 meters. Positioning can be to within 0.001 m. How??

The P (Y)-code is 10 MHz the basis for precise positioning. Shorter time intervals of 0's and 1's allow a more precise pseudorange. We can also measure the phase of the signal (not the code) and get the "phase pseudorange".

The C/A code (1 MHz) is used for coarse positioning.

In tectonic geodesy we measure position every minute or even every second for a long time, then compute a mean. This makes the measurement even more precise.

We also correct the data for weather. Water in the atmosphere slows the microwave signal down but this is corrected for using the second frequency (L2) signal.



For example, here are the first 1000 numbers of the code for satellite 1

```
00001000101001110000111001001000100001000101011000111101110010101101100111101011
00101100101001100111111011001111001001100110100011100010010001011000101101110000
001101100100010001011010001010010000000011111000110001011111011111100110111001011
01111000111111010100101000010101001110000110100111011000111101111100001111111111
01001001001001100111010101111100001000101101001111110000100110111100111000110110
10110110101000010110100101000101001000111001110001010010111010111010101000001011
0111001101100110100000000000001110111011000110110101010110110001110001100110011111
01011111001110101010000011111100100101000000111010001111011010010110110000010010
01001100001101100001111011101110001101110110100111001000110101010000110110100101
110010111111111011000111000000110111000110000000100000000100000110101000101011110
11000111011010001100101011111001111010000000110111100110011101011110000011110110
010001001010111000000000100001010101001111101100111011011111100101111000100110101
```

How do you compare codes?

100111101110100010011011111111101

000010001010011100001110010010001

Every time the numbers agree, add 1.

Every time the numbers disagree, subtract 1.

This example: same satellite codes, but shifted

Not so good - score of -3.

0110001010101100100010010000011000001111100000

1100010101011001000100100000110000011111000001

But if you recognize they are shifted by 1:

0110001010101100100010010000011000001111100000

 1100010101011001000100100000110000011111000001

Agreement is perfect.

Each red dot tells you the position of a GPS receiver on a single day.

Churchill is moving 1.9 cm/yr west, 0.6 cm/yr south, and 1.1 cm/yr up.