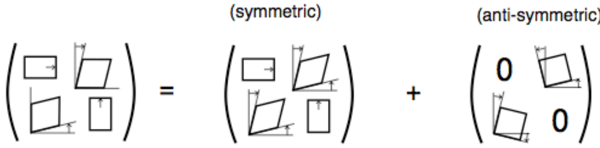
A couple more things about strain:

(1) strain rate $\dot{\epsilon}$

- strain per unit time
- denoted with a dot
- units?

$\dot{\epsilon}$ is obtained from derivatives of velocity (not displacement) with respect to position

$$\underbrace{\begin{pmatrix} \frac{\partial V_x}{\partial x} & \frac{\partial V_x}{\partial y} \\ \frac{\partial V_y}{\partial x} & \frac{\partial V_y}{\partial y} \end{pmatrix}}_{} = \underbrace{\begin{pmatrix} \frac{\partial V_x}{\partial x} & \frac{1}{2}\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right) & \frac{\partial V_y}{\partial y} \end{pmatrix}}_{[\dot{\epsilon}]\ \text{(symmetric)}} + \underbrace{\begin{pmatrix} 0 & \frac{1}{2}\left(\frac{\partial V_x}{\partial y} - \frac{\partial V_y}{\partial x}\right) \\ -\frac{1}{2}\left(\frac{\partial V_x}{\partial y} - \frac{\partial V_y}{\partial x}\right) & 0 \end{pmatrix}}_{[\dot{\omega}]\ \text{(anti-symmetric)}}$$



$$= \begin{pmatrix} \dot{\varepsilon}_{xx} & \dot{\varepsilon}_{xy} \\ \dot{\varepsilon}_{xy} & \dot{\varepsilon}_{yy} \end{pmatrix} + \begin{pmatrix} 0 & \dot{\omega} \\ -\dot{\omega} & 0 \end{pmatrix}$$

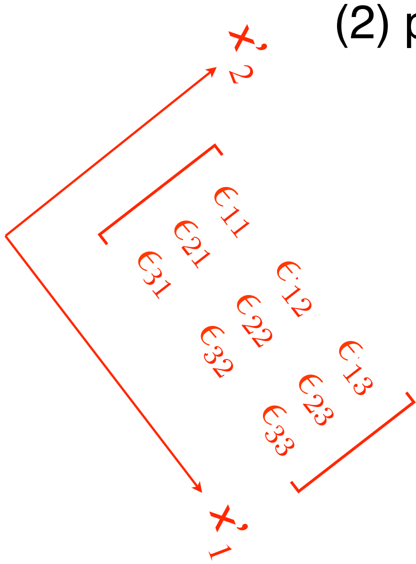
strain rate matrix | rotation rate matrix

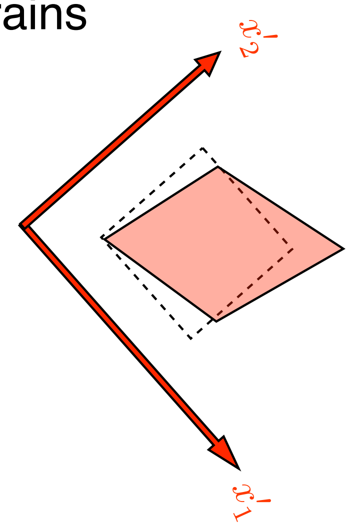
Where the dot over a term indicates a time derivitive and ω is amount of rotation.

velocity gradients ⇒ deformation rate $[\dot{\epsilon}]$ and rotation rate $[\dot{\omega}]$

A couple more things about strain

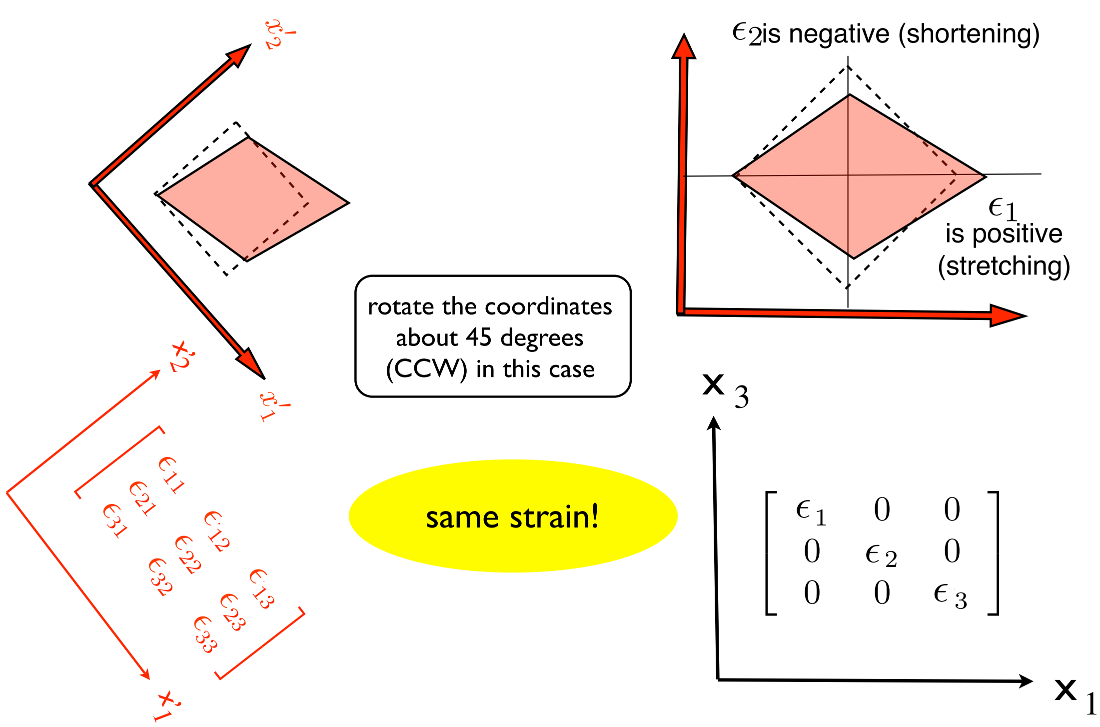
(2) principal strains





suppose you know the strain matrix, which was figured out in the (x'_1, x'_2, x'_3) coordinate system

By <u>rotating the coordinate system</u> you can express 2D strain with just two normal strains ("principal strains")



Why does this matter? It becomes clear later on, when we try to apply Hooke's Law in the Earth, without resorting to linear algebra.

F = k times "x" = (constant) x (spring displacement)

"x" = ϵ L

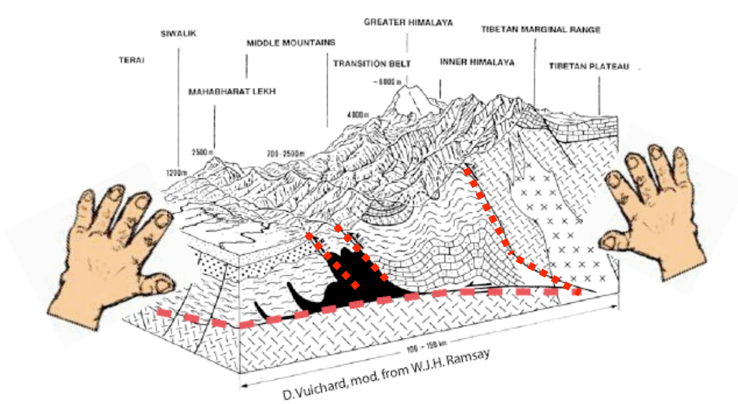
F = (kL)ϵ = (constant) x (spring strain ϵ)

(where L is the length of the spring)

Stress = σ = (constant) x (strain)

Stress is just force per unit area acting on a surface

It is just "some constant" times strain



This means that stress is a tensor (that is, a matrix), too.

What are the units of force / area?

Normal stress causes normal strain

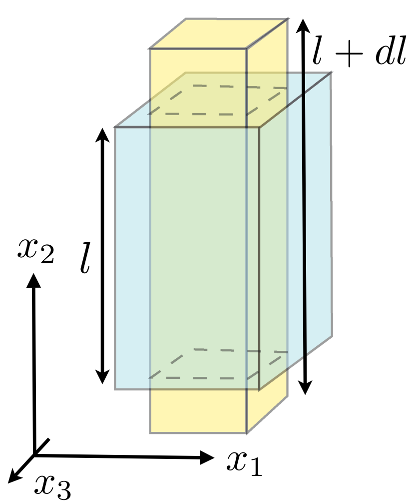
What are typical normal stresses in the Earth? How "big" is a Pascal?

Sign convention.

The mean of the three normal stresses is (-pressure).

Lithostatic pressure in the Earth...

Strain in the Earth due to normal stress:
Hooke's Law



$$\epsilon_{22} = \frac{dl}{l} = \frac{du_2}{dx_2}$$

$$\boxed{\begin{aligned}\sigma_{22} &= E\epsilon_{22} \\ \epsilon_{22} &= \frac{\sigma_{22}}{E}\end{aligned}}$$

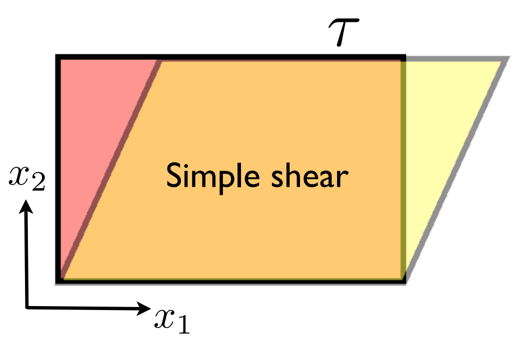
What is E?

E is the "Young's Modulus"

In rocks, E is very, very large.

typical value is 7.5 x 10^{10} Pascals

Shear stress (sometimes called τ)
causes shear strain



$$\tau_{21} = 2G\epsilon_{21}$$
$$\epsilon_{21} = \frac{\tau_{21}}{2G}$$

G is the "Shear Modulus"

In rocks, G is also very, very large.

typical value is 3 x 10^{10} Pascals