$$\begin{bmatrix} 0.2 & 0.45 \\ 0.08 & -0.48 \end{bmatrix} \times 10^{-6}$$

From GPS displacements over one year in the Ventura Basin region, California.
Is this strain or displacement gradient matrix (is it symmetric?)

This is D: $\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$

Easy recipe to get strain and rotation matrices!

$$E = 1/2(D + D^T)$$
$$W = D\text{-}E$$

What is E? What is W?

Graphically, this shows what these matrices do to an initially square piece of rock

$$\begin{pmatrix} \frac{\partial V_x}{\partial x} & \frac{\partial V_x}{\partial y} \\ \frac{\partial V_y}{\partial x} & \frac{\partial V_y}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\partial V_x}{\partial x} & \frac{1}{2}\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right) & \frac{\partial V_y}{\partial y} \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2}\left(\frac{\partial V_x}{\partial y} - \frac{\partial V_y}{\partial x}\right) \\ -\frac{1}{2}\left(\frac{\partial V_x}{\partial y} - \frac{\partial V_y}{\partial x}\right) & 0 \end{pmatrix}$$

(symmetric) (anti-symmetric)

D ϵ ω

(different variable names: he uses V_x instead of u_1 and V_y instead of u_2, also x instead of x_1 and y instead of x_2)

A few more things about strain:

(1) what are the **units** of strain?

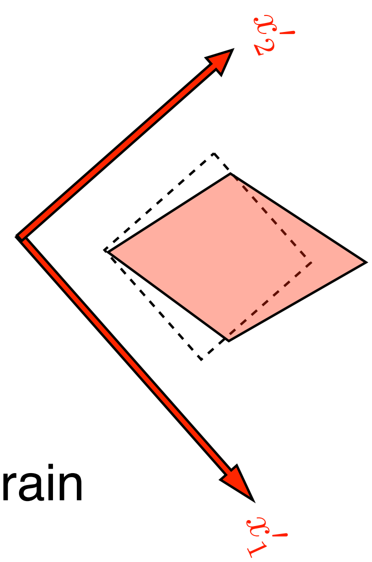
(2) strain <u>rate</u> $\dot{\epsilon}$

- strain per unit time
- denoted with a dot
- units?

A few more things about strain

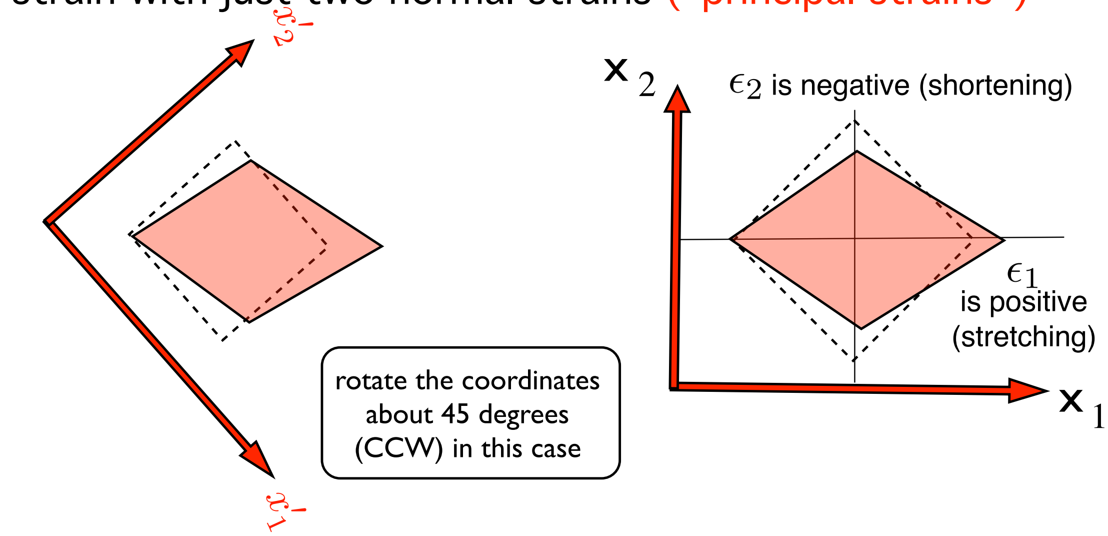
(3) Principal strains

$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix}'$$



Suppose you know the strain matrix in the (x'_1, x'_2) coordinate system

By <u>rotating the coordinate system</u> you can express 2D strain with just two normal strains ("principal strains")

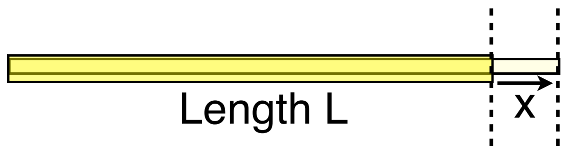


$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix}' \quad \text{same strain!} \quad \begin{bmatrix} \epsilon_{11} & 0 \\ 0 & \epsilon_{22} \end{bmatrix}$$

Moving on from strain to stress

$F = kx$

F = k times "x" = (constant) x (spring displacement)



$$\epsilon_{11} = x/L$$
$$x = \epsilon_{11} L$$

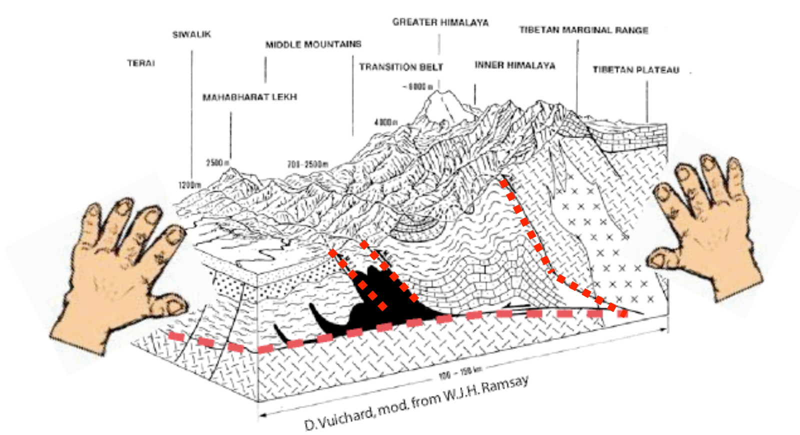
F = (kL)ϵ = (constant) x (spring strain ϵ)

For a force per unit area (stress) applied at the end of an elastic block:

Stress = σ = (constant) x (strain)

Stress is just force per unit area acting on a surface

For elastic rock, it is just "some constant" times strain



This means that stress is a matrix, too.

Units are force per area, so Newtons per square meter (Pascals).

$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix}$$

Just as we have a strain matrix, we also have a stress matrix

3 rows, 3 columns in 3D
2 rows, 2 columns in 2D

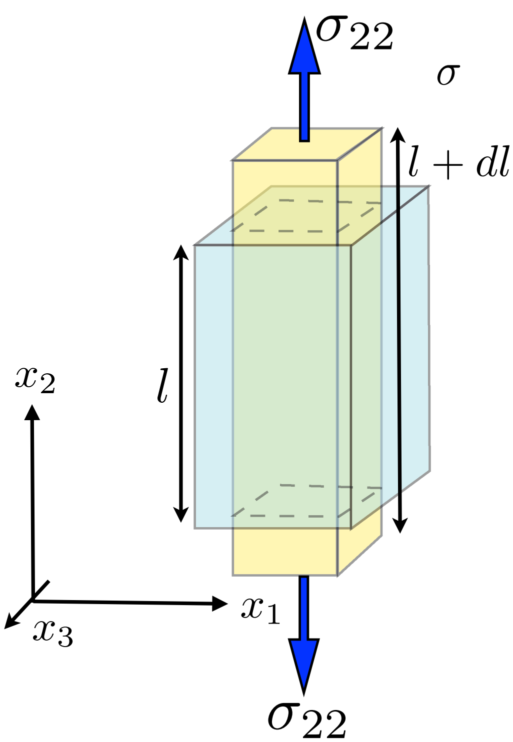
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Just like the strain matrix, the stress matrix is <u>symmetric</u>. stresses with identical subscripts are normal stresses; others are shear stresses

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

(2 rows, 2 columns in 2D)

<u>Normal</u> stress causes <u>normal</u> strain



$$\epsilon_{22} = \frac{dl}{l} = \frac{du_2}{dx_2}$$

$$\boxed{\begin{aligned} \sigma_{22} &= E\epsilon_{22} \\ \epsilon_{22} &= \frac{\sigma_{22}}{E} \end{aligned}}$$

$$\sigma_{22}$$

is normal stress: force per unit area acting in x_2 direction on the plane that is normal to the x_2 direction.

What is E?

E is the "Young's Modulus"

In rocks, E is very, very large.

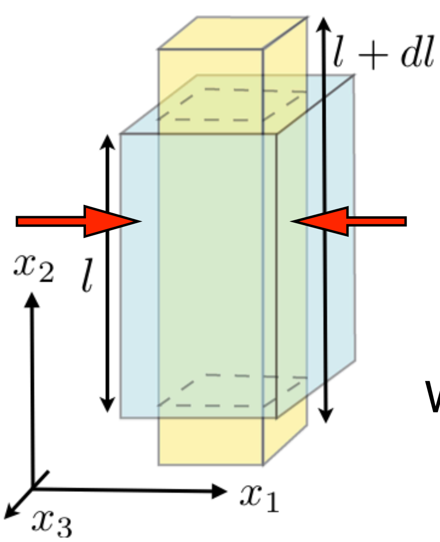
typical value is 7.5 x 10^{10} Pascals

$$\boxed{\begin{aligned}\sigma_{22} &= E\epsilon_{22} \\ \epsilon_{22} &= \frac{\sigma_{22}}{E}\end{aligned}}$$

How big must normal stress be to lengthen the block by 1%?

sign convention: positive normal stress is tension: causes lengthening.

But look at what else happened when this bar was elongated due to σ_{22}



blue = before

yellow = after

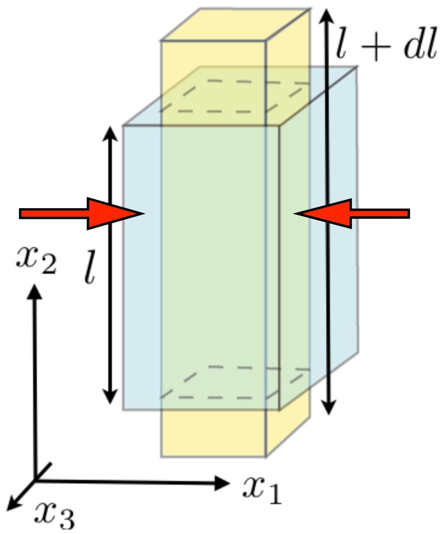
$$\boxed{\begin{aligned}\sigma_{22} &= E\epsilon_{22} \\ \epsilon_{22} &= \frac{\sigma_{22}}{E}\end{aligned}}$$

Are ϵ_{11} and ϵ_{33} 0?

What are the signs of ϵ_{11} and ϵ_{33} ?

(note that σ_{22} is the only non-zero stress in this case.)

A normal stress actually causes three normal strains



blue = before
yellow = after

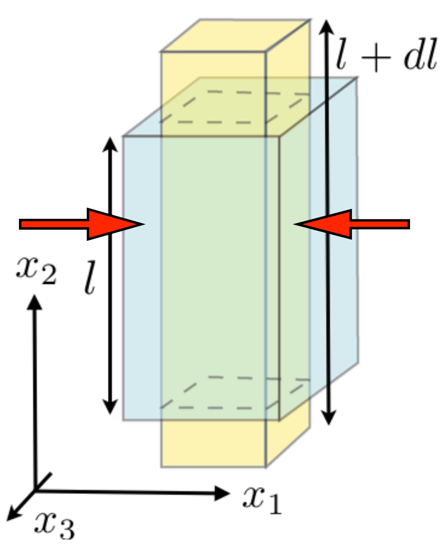
$$\boxed{\begin{aligned}\sigma_{22} &= E\epsilon_{22} \\ \epsilon_{22} &= \frac{\sigma_{22}}{E}\end{aligned}}$$

$$\epsilon_{33} = \epsilon_{11} = \frac{-\nu\sigma_{22}}{E}$$

ν is "Poisson's ratio" (unitless)

Typical ν is 0.25.

% volume change is the sum of all three normal strains. What is it if Poisson's Ratio is 0.5?



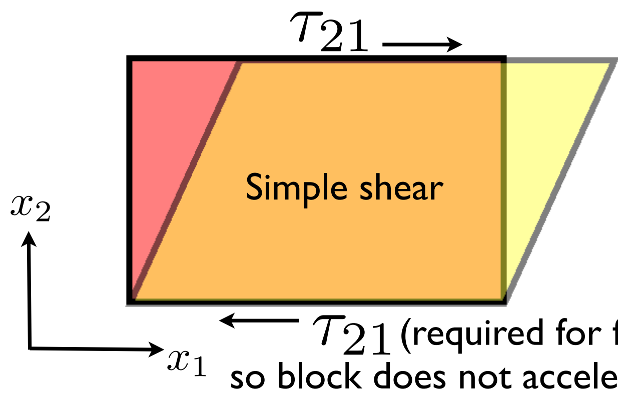
blue = before
yellow = after

$$\boxed{\begin{aligned}\sigma_{22} &= E\epsilon_{22} \\ \epsilon_{22} &= \frac{\sigma_{22}}{E}\end{aligned}}$$

$$\epsilon_{33} = \epsilon_{11} = \frac{-\nu\sigma_{22}}{E}$$

ν is "Poisson's ratio" (unitless)

Shear stress (sometimes called τ) causes simple shear (half shear strain and half rotation)

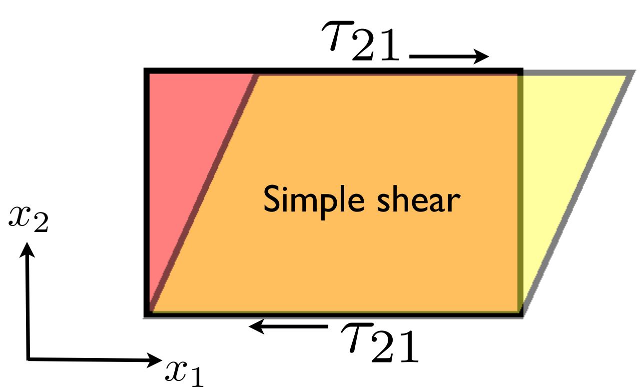


(required for force balance so block does not accelerate in the x_1 direction)

σ_{21} is force per unit area acting on the x_2 face in the x_1 direction

first index

second index



$$\boxed{\begin{aligned} \tau_{21} &= 2G\epsilon_{21} \\ \epsilon_{21} &= \frac{\tau_{21}}{2G} \end{aligned}}$$

G is the "Shear Modulus"

In rocks, G is also very, very large.

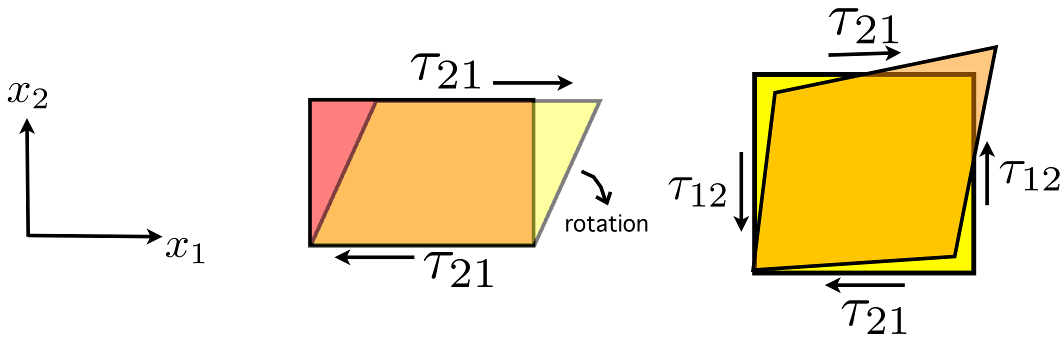
typical value is 3×10^{10} Pascals

ONE shear stress causes ONE shear strain (easy!)

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

Recall that this matrix is symmetric, so

$\sigma_{12} = \sigma_{21}$ $\qquad\qquad$ $\tau_{12} = \tau_{21}$

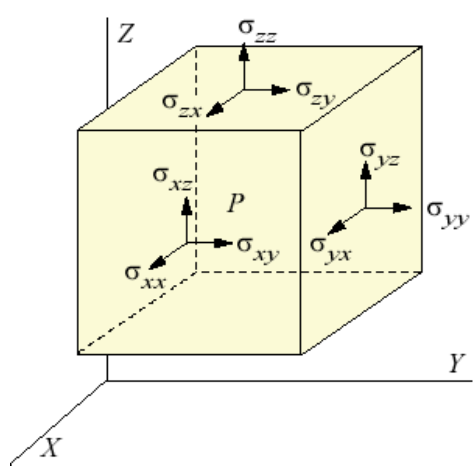


Why? Because without σ_{12} we have unbalanced clockwise rotation of the block. σ_{12} provides a counterbalancing clockwise rotation.

All torques and forces are balanced if the block is at rest.

Components of 2D stress matrix, graphically (on the board)

Components of 3D stress matrix



When you are standing on a flat surface, what is the normal stress you exert on the ground?

What is the shear stress?

How could you exert a non-zero shear stress on the ground?

What are typical normal stresses in the Earth? How "big" is a Pascal? (Sign convention.)

The mean of the three normal stresses is (-pressure).

Lithostatic pressure in the Earth...

Webpage with basic explanations of stress, some jargon defined:
http://www.uwgb.edu/dutchs/structge/stress.htm
basic elasticity concepts:
http://www.uwgb.edu/dutchs/structge/strsparm.htm

Lithostatic pressure in the Earth...

$$\boxed{P = \rho g h}$$

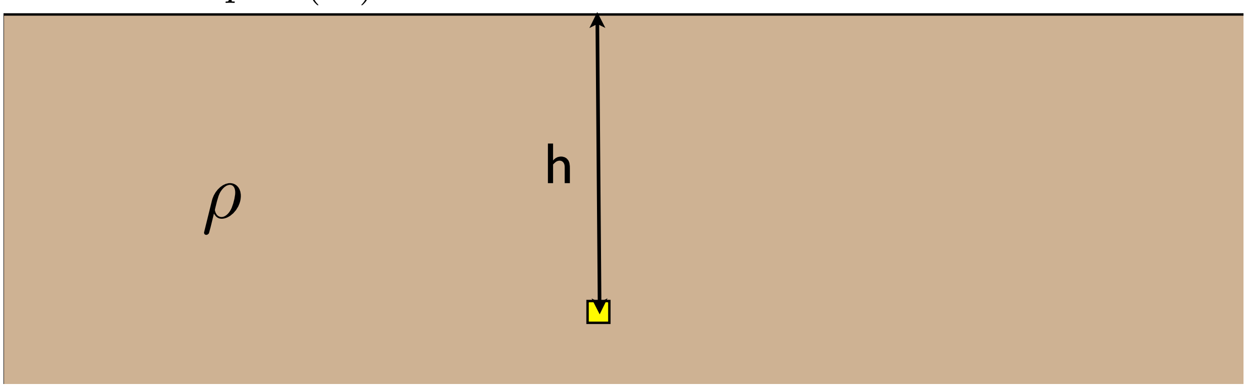
$$\rho = 2500\ kg/m^3$$
$$g = 10\ m/s^2$$
$$h = \ depth\ (m)$$

Moho (crust-mantle boundary) is 30 km down

Cascadia SZ (80 km below us)

Deep ocean floor = 4 km

(EQ works for water or ice sheets too with correct density)



In a fluid normal stresses are -P ("hydrostatic pressure") shear stresses are 0

$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} = \begin{bmatrix} \bar{\sigma} & 0 \\ 0 & \bar{\sigma} \end{bmatrix} = \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix}$$

Below the Earth's upper crust normal stresses are *close* to -P and shear stresses are small

$$\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \approx \begin{bmatrix} \bar{\sigma} & 0 \\ 0 & \bar{\sigma} \end{bmatrix} = \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix}$$