

# Earthquake Magnitude

## HISTORY OF MAGNITUDES

> first devised by Charles Richter in 1935 for S California: ``Richter Scale'' or ``local'' magnitude

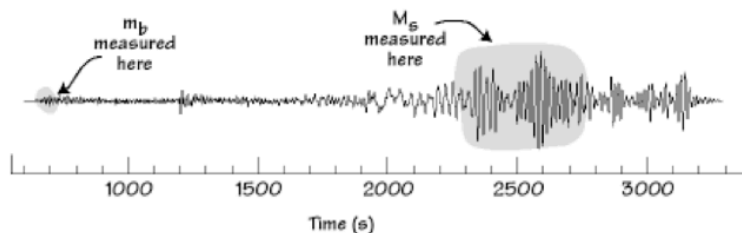
$$M_L = \log A + 2.76 \log \Delta - 2.48$$

> valid for Wood-Anderson seismograph (resonant frequency = 0.8 Hz) and uses S-wave amplitude

> sometimes still reported since good indicator of structural damage

> later modifications include global body wave and surface wave scales:

$$m_b = \log(A/T) + Q(h, \Delta) \quad M_s = \log(A/T) + 1.6 \log \Delta + 3.3$$



## Earthquake **magnitude** scales: Logarithmic measure of earthquake size

- *amplitude of biggest wave*: Magnitude 6 quake  $10 \times$  Magnitude 5
- *energy*: Magnitude 6 quake is about  $32 \times$  Magnitude 5

Richter Magnitude is calculated from the maximum amplitude of waves recorded on a seismogram, and distance to the earthquake.

$$10^{M_L} = 10^{(\log A + 2.76 \log \Delta - 2.48)}$$

$$10^{M_L} = 10^{(\log A)} \boxed{10^{(2.76 \log \Delta - 2.48)}}$$

this is a constant,  
we can call it "k".

$$10^{M_L} = A k$$

How does the maximum shaking amplitude A of a M6 quake compare to a M4 quake (same hypocenter, same seismograph location)?

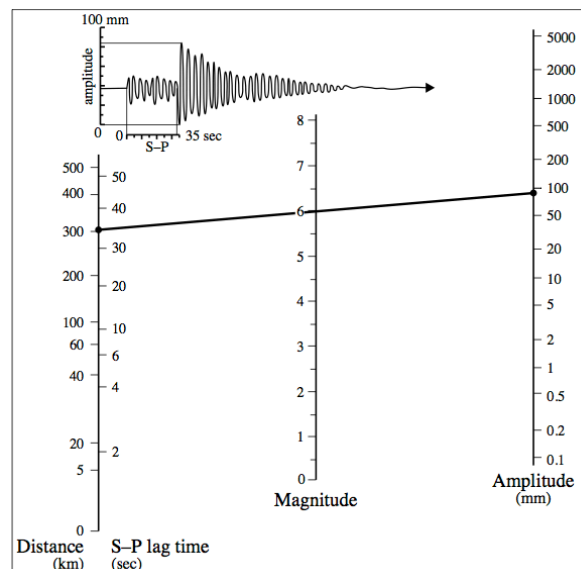
By the same token, you can easily compare the energy release (“seismic moment”) of different magnitude earthquakes:

$$32^{M_L} = E k$$

How does the energy (moment) of a M6 quake compare to that of a M4 quake?

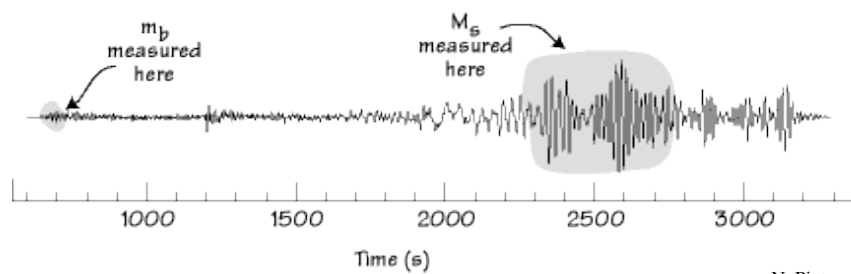
A quick method (pre electronic calculators), involved using a graphical construct (called a Nomogram) that takes care of the mathematics by constructing the axes in a particular fashion.

The **Nomogram** allows one to compute the magnitude by plotting the distance between the quake and observatory on the left axis, and the seismometer deflection in millimetres on the right axis. (The amplitude of the deflections are what would have been recorded by a Wood-Anderson seismometer – the actual ground motions have been multiplied by 2000 which is the amplification of the Wood - Anderson seismometer at these frequencies). The points on the left and right axes are connected by a straight line, and the intersection on the middle axis is the earthquake magnitude.



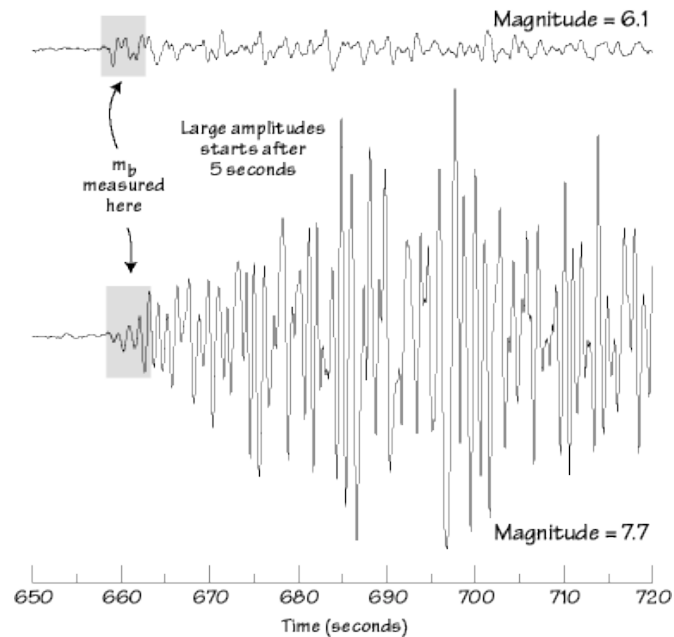
Magnitude	Symbol	Wave	Period
Local (Richter)	$M_L$	S or Surface Wave*	0.8 s
Body-Wave	$m_b$	P	1 s
Surface-Wave	$M_s$	Rayleigh	20 s
Moment	$M_w$	Rupture Area, Slip	> 100 s

\*whichever's biggest at a period of 0.8s (typically the S wave), and ALSO always using a Wood-Anderson seismograph (or converting the amplitude so the seismogram looks just like one from a WA seismograph)



N. Pinter

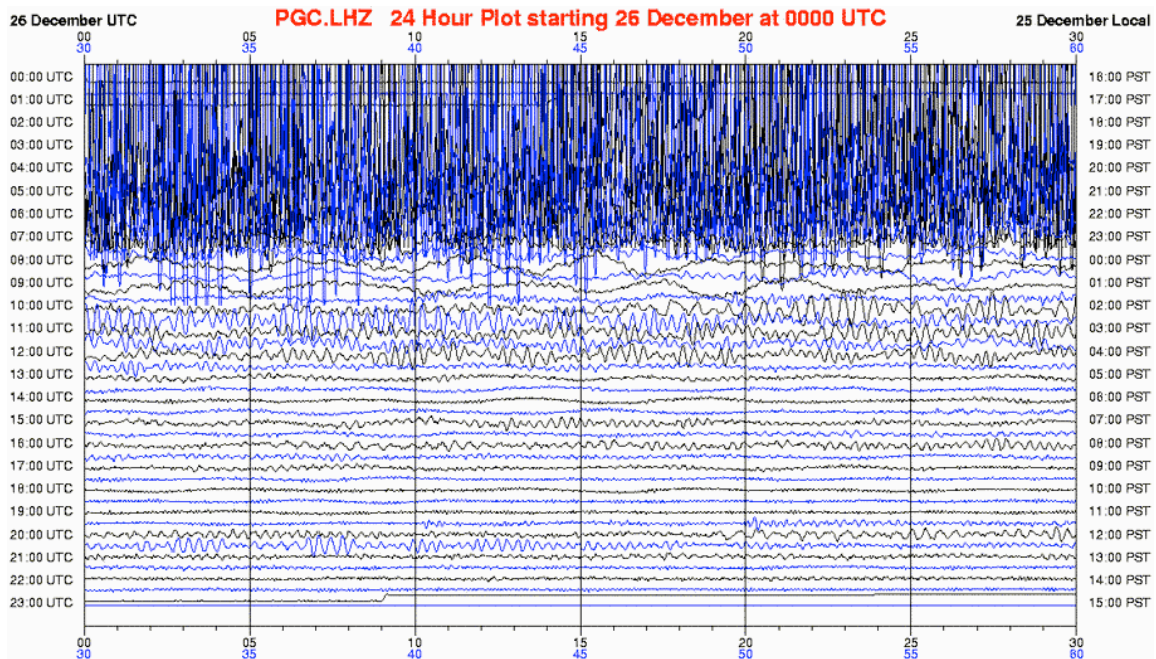
## $M_b$ and $M_L$ are inadequate for large earthquakes



N. Pinter

# M 9.2 2004 Sumatra Earthquake

measured in Victoria BC



## MAGNITUDE DISCREPANCIES

Earthquake	Body wave magnitude $m_b$	Surface wave magnitude $M_s$	Fault area ( $\text{km}^2$ ) length $\times$ width	Average dislocation (m)	Moment (dyn-cm) $M_0$	Moment magnitude $M_w$
Truckee, 1966	5.4	5.9	$10 \times 10$	0.3	$8.3 \times 10^{24}$	5.8
San Fernando, 1971	6.2	6.6	$20 \times 14$	1.4	$1.2 \times 10^{26}$	6.7
Loma Prieta, 1989	6.2	7.1	$40 \times 15$	1.7	$3.0 \times 10^{26}$	6.9
San Francisco, 1906		8.2	$320 \times 15$	4	$6.0 \times 10^{27}$	7.8
Alaska, 1964	6.2	8.4	$500 \times 300$	7	$5.2 \times 10^{29}$	9.1
Chile, 1960		8.3	$800 \times 200$	21	$2.4 \times 10^{30}$	9.5

> note discrepant body-wave and surface wave magnitudes (due to empirical nature, no account for radiation pattern, local ground effects, etc.

> body-wave magnitudes saturate at  $\sim 6.2$ , surface wave magnitudes at  $\sim 8.4$

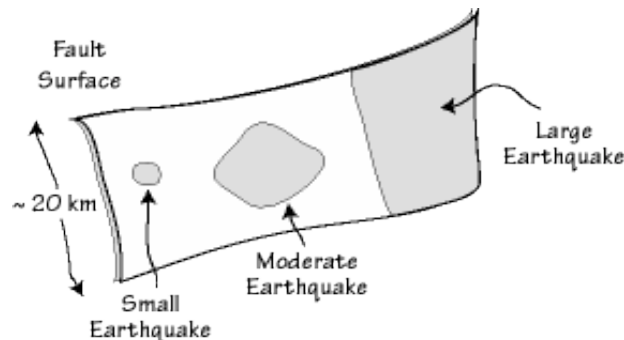
# Earthquake magnitude scales

Magnitude	Symbol	Wave	Period
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Moment	$M_w$	Rupture Area, Slip	> 100 s

$M_w$  is calculated from the **earthquake energy release**, which can be done with **many** different kinds of data, such as very long-period surface wave recordings from broadband seismometers and even GPS measurements of permanent ground displacement

**$M_w$  is best for large earthquakes**

## Energy released by an earthquake (seismic moment)



C. Ammon

$$M_o = A \mu s$$

Seismic moment (Newton m)

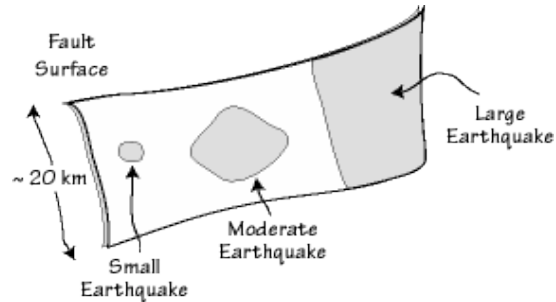
$A$  is the area of the fault that slipped in the earthquake ( $m^2$ )

$\mu$  is that SAME constant ("shear modulus" or "rigidity") that appeared in the seismic wave speed equations on Monday. Pascals, i.e., Newtons /  $m^2$

$s$  is the slip (m)

Moment magnitude  $M_w$  comes from seismic moment  $M_o$

$$M_o = A \mu s$$



C. Ammon

= unwieldy large number, for example, “ $10^{21}$  Newton meters”

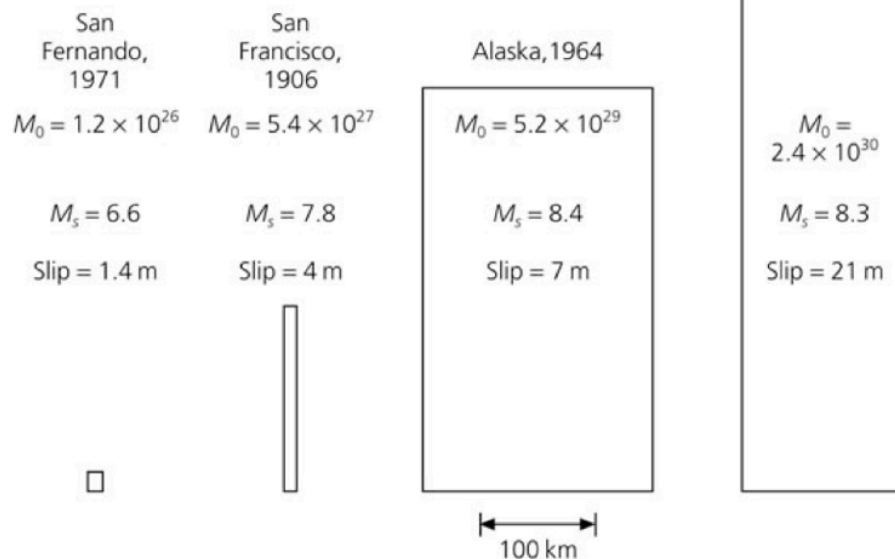
To get a number that looks like a Richter magnitude, we use this equation:

$$M_w = \frac{2}{3} (7 + \log M_o) - 10.73$$

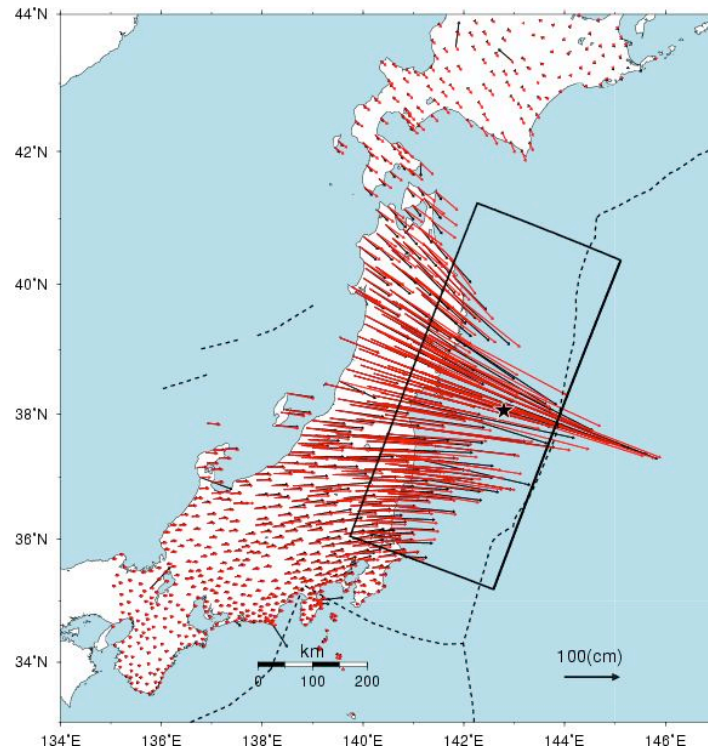
$$M_w = \frac{2}{3} \log M_o - 6.07$$

## RELATIVE AREAS, SLIP MOMENTS OF SOME EARTHQUAKES

$M_o$  units here are “dyne cm”.  
 1 dyne cm =  $10^{-7}$  Newton m.  
 I PREFER Newton m.

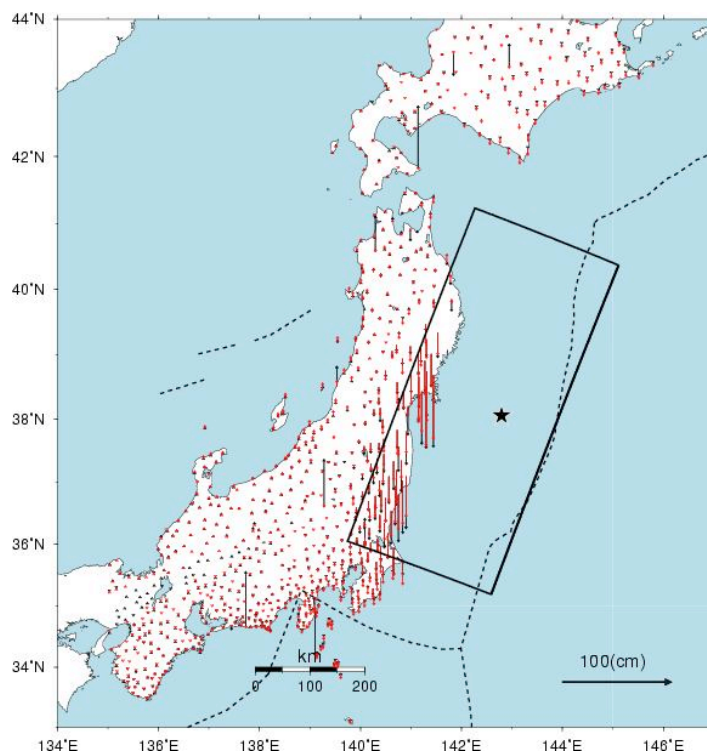


## Moment magnitude $M_w$ of the 2011 Tohoku earthquake from GPS



Horizontal  
displacements

[http://www.tectonics.caltech.edu/slip\\_history/2011\\_taiheiyo-oki/](http://www.tectonics.caltech.edu/slip_history/2011_taiheiyo-oki/)

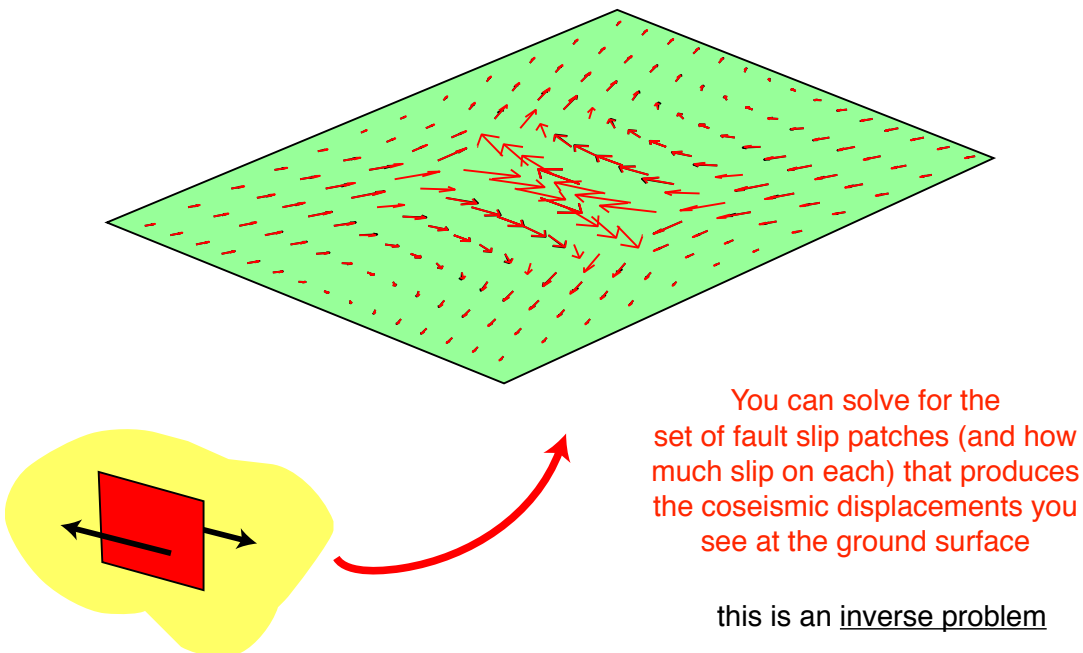


Vertical  
displacements

[http://www.tectonics.caltech.edu/slip\\_history/2011\\_taiheiyo-oki/](http://www.tectonics.caltech.edu/slip_history/2011_taiheiyo-oki/)

- we know how to calculate surface displacements resulting from slip on a fault.
- here, we are given surface displacements and we want to know slip on the fault (and sometimes, where the fault is and its orientation)
- this is another example of an “inverse problem”, like the earthquake location problem. (we have lots of these in geophysics)

Let's say you know how to calculate how much the ground deforms if there is 1 m of slip on any 1 km patch of a fault (you do: Okada, 1985)



## Slip inversions from surface displacement data

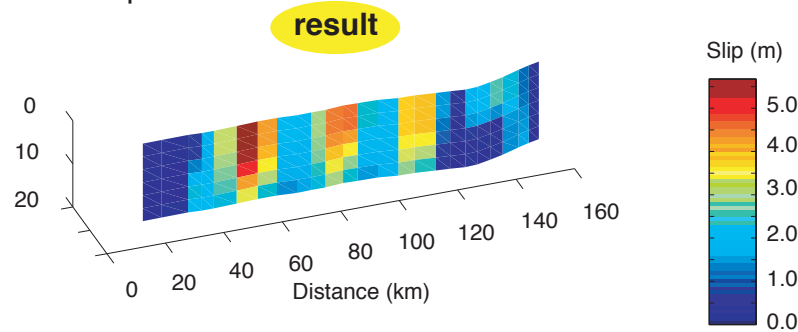
- minimize L2 norm of [*misfit between modeled and measured surface displacements weighted by meas. errors*]  
(“weighted residual sum of squares”)
- smooth slip distribution (also minimize  $\Sigma$  curvature of slip distribution)
- no backward slip

$$d_i = \sum_j \Gamma_{ij} s_j \quad \begin{matrix} j \text{ patches} \\ i \text{ displ. dof} \end{matrix}$$

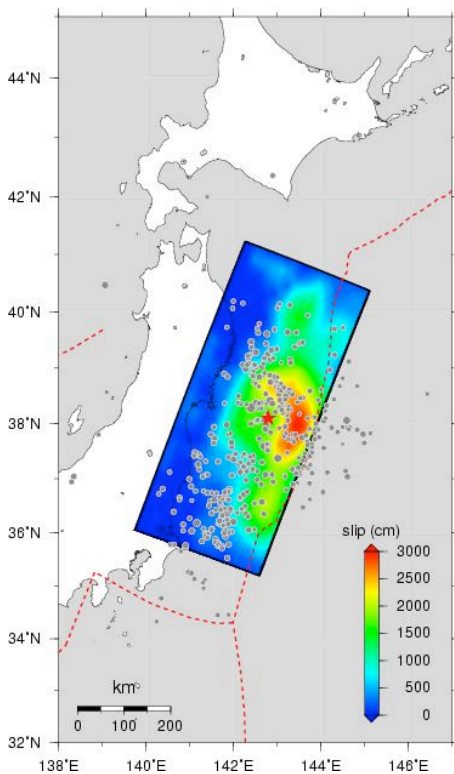
$$s = (\Gamma^T \Gamma + \beta^2 H^T H)^{-1} (\Gamma^T d)$$

$\Gamma$  = Green's function

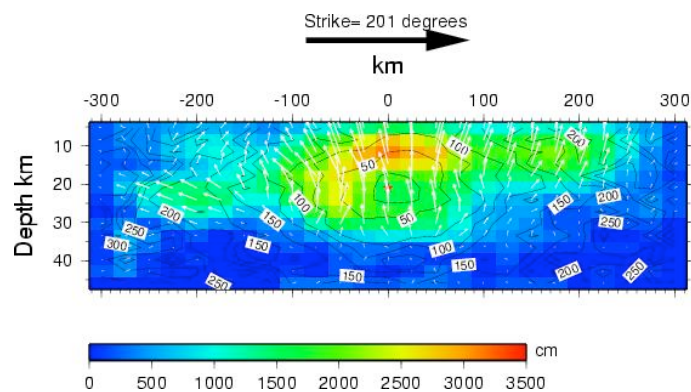
$H$  = curvature operator



bvls method, Stark and Parker, 1995



Caltech's estimate of slip and moment for the March 2011 Tohoku Earthquake

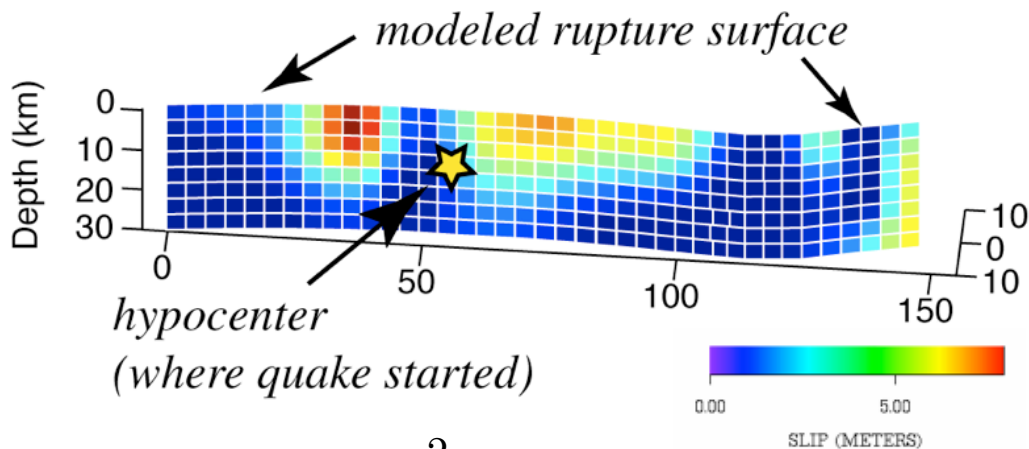


$$M_o = A \mu s$$

$$M_w = \frac{2}{3} \log M_o - 6.07$$

$$M_w = 9.0$$

## My estimate of slip and moment for the 1999 Izmit, Turkey Earthquake



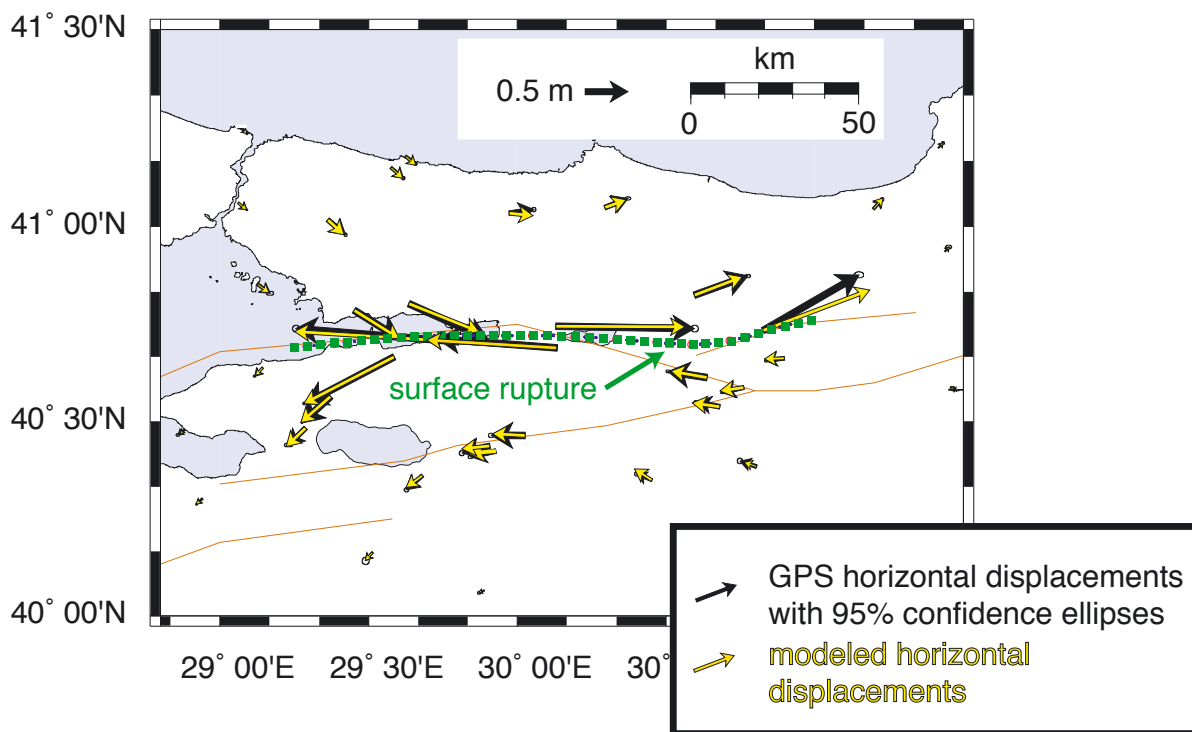
$$M_o = A \mu s \quad M_w = \frac{2}{3} \log M_o - 6.07$$

Sum moment for all 312 4-km “patches”

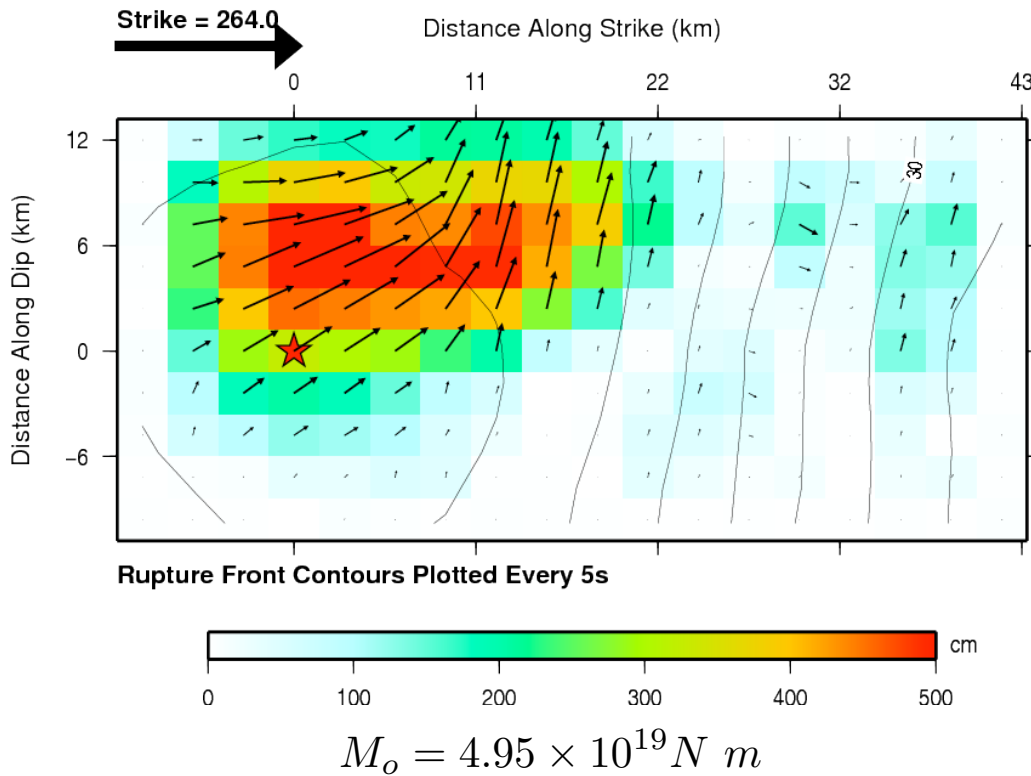
The seismic moment from this is  $2.3 \times 10^{20}$  Newton m,  
which is a moment magnitude of 7.4

## Moment magnitude from GPS data:

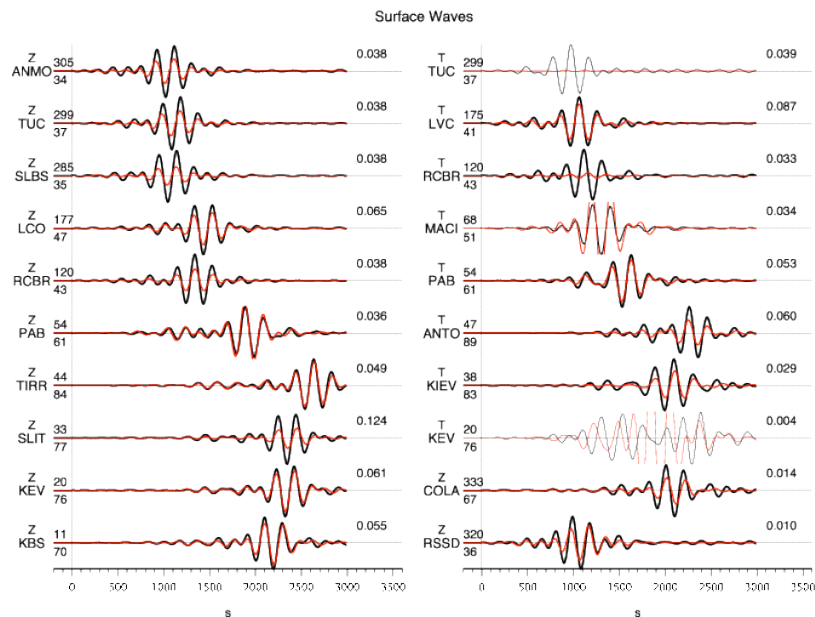
$M_w = 7.5$  Izmit, Turkey earthquake



## Slip for the 2009 Haiti earthquake



## Slip and moment for the 2009 Haiti earthquake are from an inversion of surface wave data



Slip versus time from an inversion of seismic records (strong motion data, frequencies between 0.01 and 0.2 Hz)

[http://www.youtube.com/watch?v=A\\_dVf9Lr9qE](http://www.youtube.com/watch?v=A_dVf9Lr9qE)

S.-J. Lee et al., 2011.