

Instability happens if friction decrease during sliding is steeper than elastic force decrease...

Instability if

$$k < \left| \frac{\sigma_n(b-a)}{D_c} \right|$$

(think in terms of absolute values - i.e. both quantities with the same sign)

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Friday, what happened in the “stiff spring” experiment?

Instability happens if friction decrease during sliding is steeper than elastic force decrease...

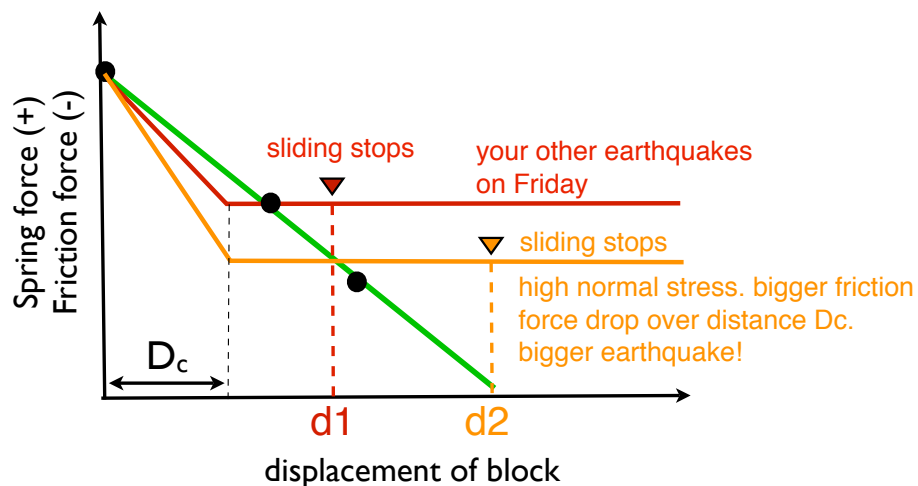
Instability if

$$k < \frac{\sigma_n(b-a)}{D_c}$$

- stiff spring?
- high normal stress?
- weak spring?
- low normal stress?

What about those big earthquakes you made by adding weight to the block?

- high normal stress (added a weight)
- no change to the spring stiffness (same old spring)



## What does this mean in the Earth?

$$k < \frac{\sigma_n(b-a)}{D_c}$$

$D_c$  and (a-b) : from lab experiments

$\sigma_n$  : from (approx.)  $\rho gh$

k: spring: k related block slip to elastic force decrease of the spring.

Earth: k relates fault slip to elastic shear force decrease of the rock

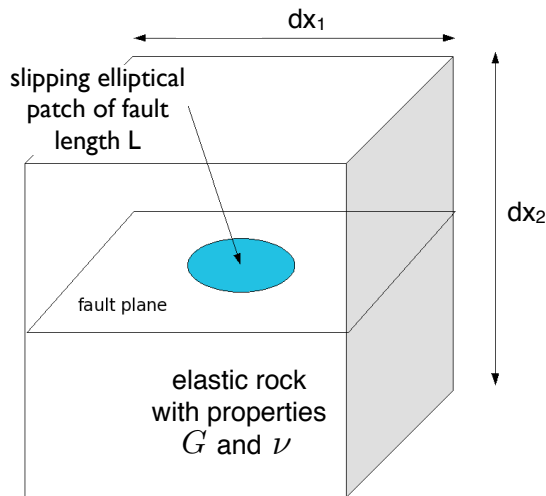
In the Earth, we use an equation for the stiffness k of a small, elliptical crack which comes from elasticity theory (and is proven by experiments)

### k relates slip (offset) to the shear stress change

Equation for stiffness k for a small, elliptical crack is:

$$k = \frac{G}{(1-\nu)L}$$

$G$  = shear modulus  
 $\nu$  = Poisson's ratio  
 $L$  = length of slipping area



**Earthquake machine:**  
k is the spring stiffness

**Earth:** k is the elastic force due to offset along crack (length L)

$$k < \frac{\sigma_n(b-a)}{D_c}$$

$$\frac{G}{(1-\nu)L} < \frac{\sigma_n(b-a)}{D_c}$$

$k$  = crack stiffness  
 $G$  = shear modulus  
 $\nu$  = Poisson's ratio  
 $L$  = length of slipping area  
 $(b-a)$  = friction weakening parameter  
 $\sigma_n$  = normal stress  
 $D_c$  = friction weakening distance

$$L > \frac{D_c G}{(1-\nu)(b-a)\sigma_n}$$

This tells us that the slipping patch of fault must be bigger than a critical size to go unstable, even for a velocity weakening fault

The slipping patch of fault must be bigger than a critical size to go unstable

Determine the **minimum earthquake size** (magnitude and moment) assuming:  $G=30$  GPa, normal stress = 150 MPa,  $b-a = 0.01$ ,  $\nu=0.25$ , and  $D_c = 10^{-4}$  m.

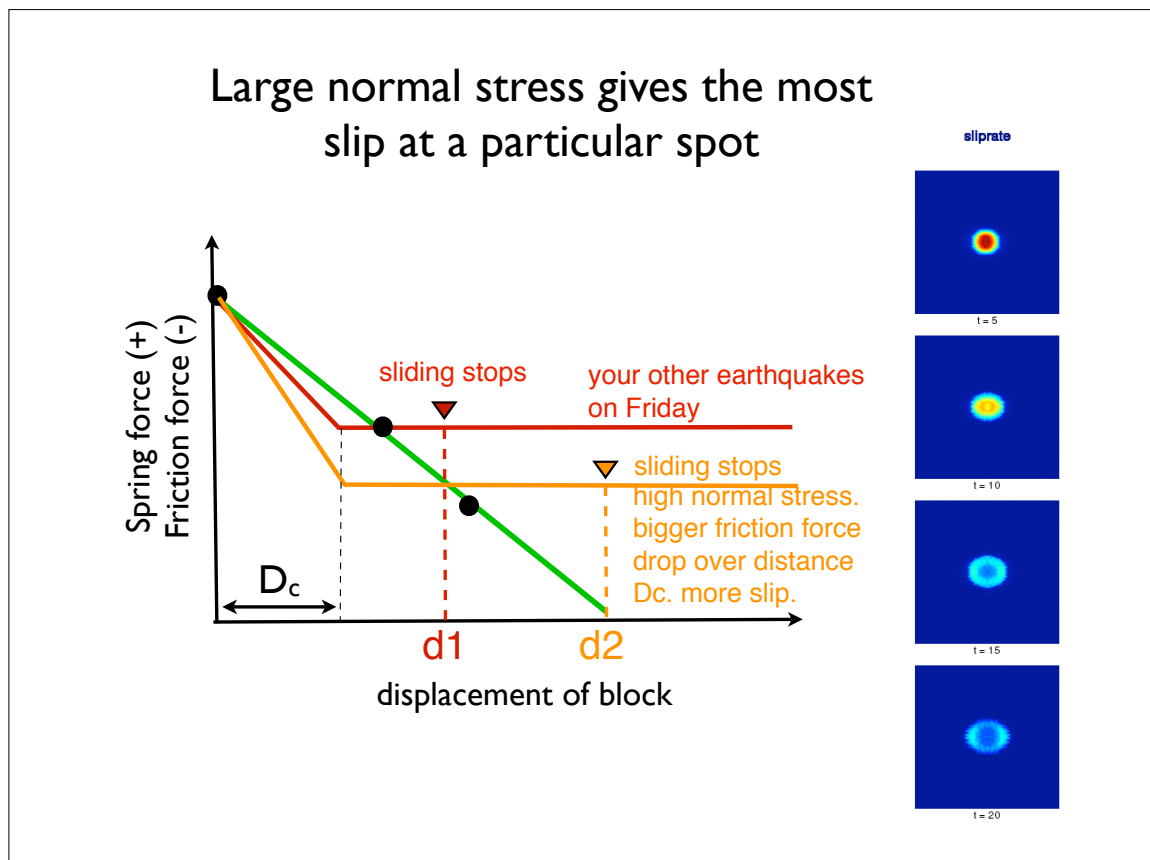
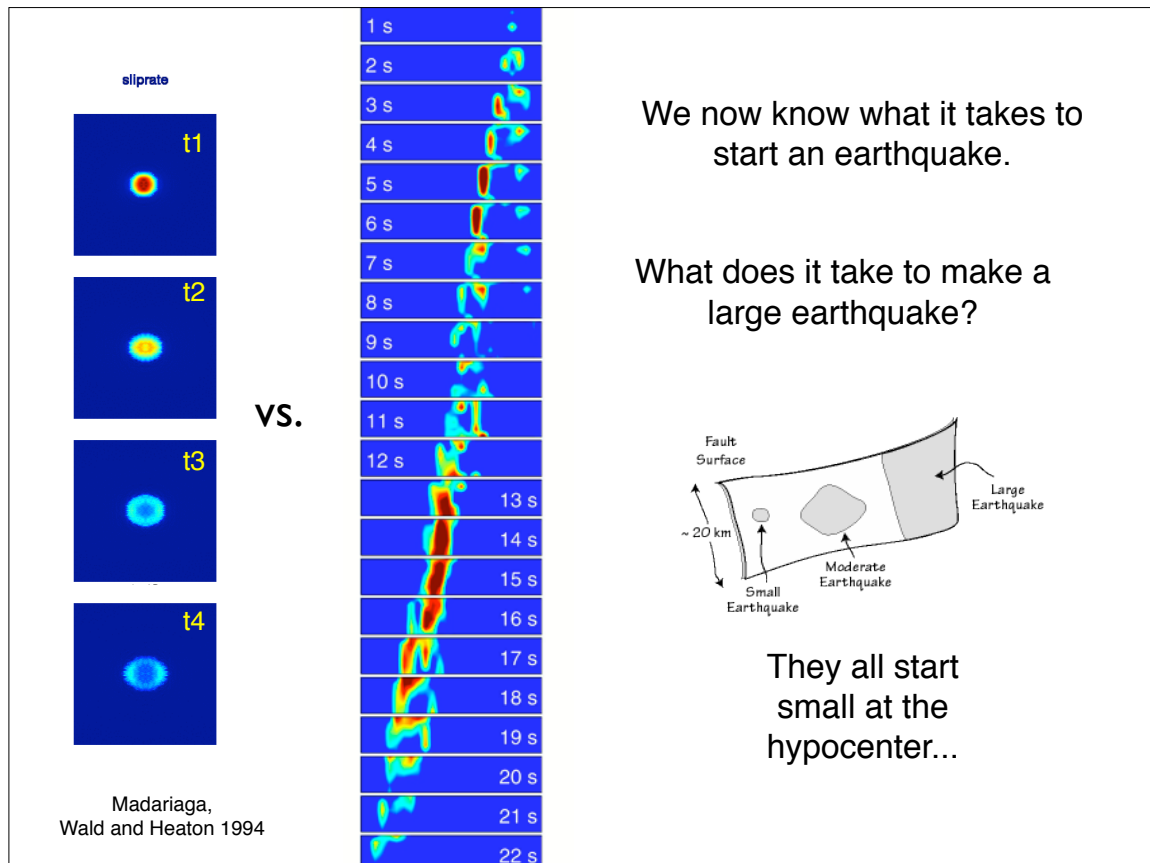
$$L > \frac{D_c G}{(1-\nu)(b-a)\sigma_n}$$

$$\log M_o = 1.5(M_w + 6.0333)$$

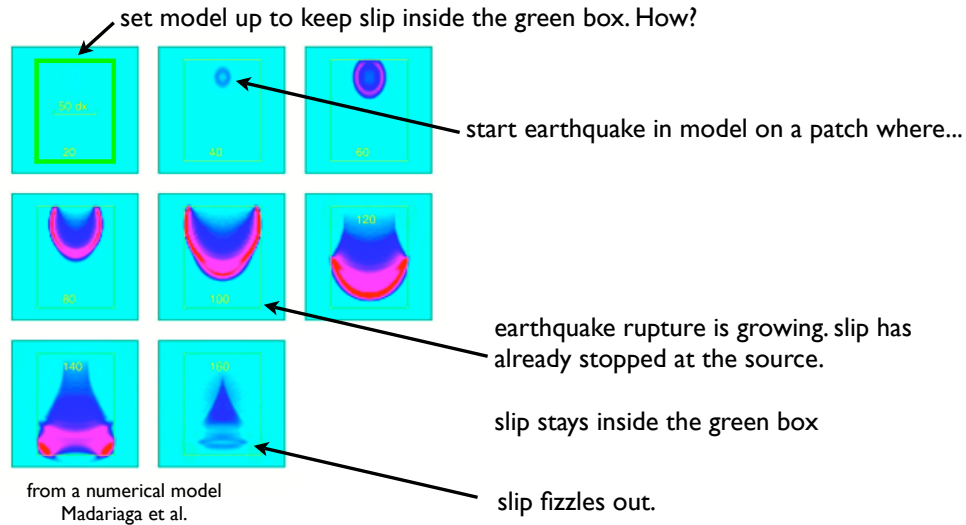
$$M_w = \log M_o / 1.5 - 6.0333$$

$$M_o = A_s G$$

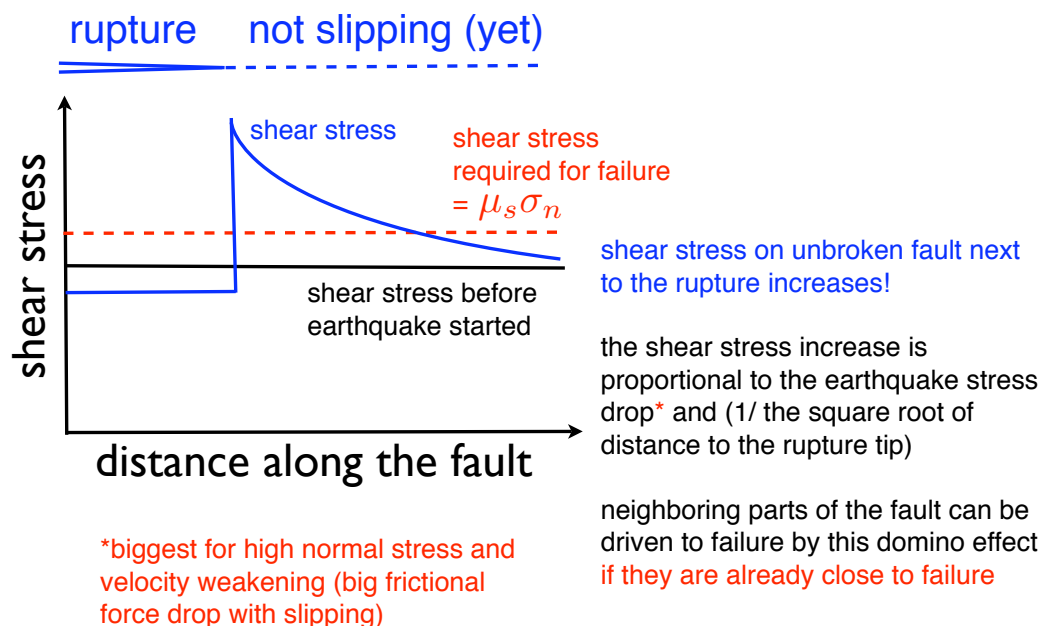
Get  $L$  and then get moment and moment magnitude.  
 Assume offset (slip) is 0.01 times  $L$ .



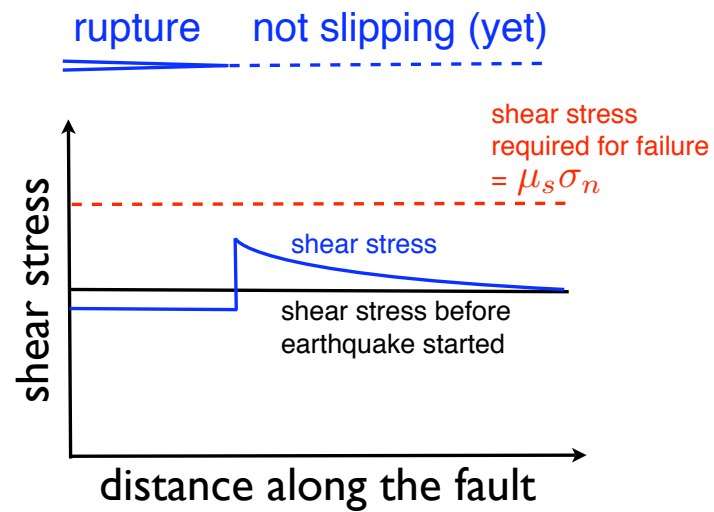
In a large earthquake, the slipping patch grows:  
the rupture propagates into previously unbroken  
parts of the fault



## Good conditions for rupture propagation

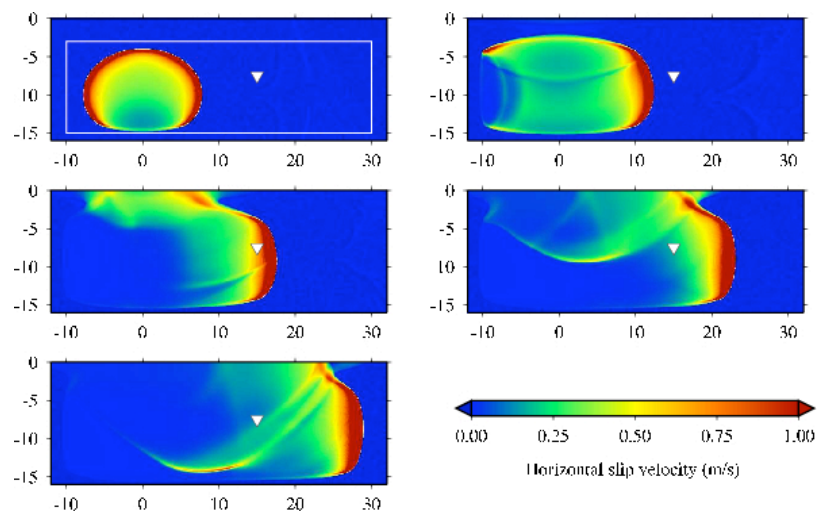


## Bad conditions for rupture propagation

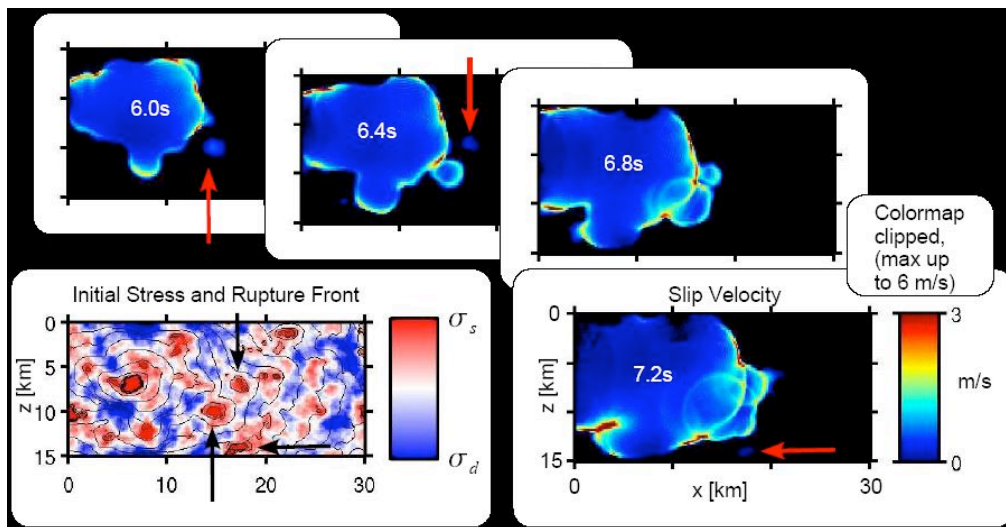


Neighboring parts of the fault can be driven to failure by this domino effect  
if they are already close to failure

Two ways for the fault to be close to failure:



## Stress is heterogeneous on real faults



Rupture propagation model from J. Ampuero et al.

- Long, continuous fault (no need to jump from segment to segment)
- Shear stress near the Coulomb threshold along this fault
- Large normal stress and velocity weakening friction --> big frictional force drop and large “kick” to adjacent parts of the fault

