Important Concepts in GPR

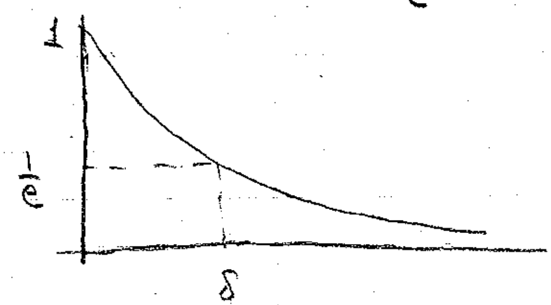
How the EM wave travels in the earth depends upon the ratio

$$\frac{\sigma}{\omega\epsilon}$$

If $\boxed{\frac{\sigma}{\omega\epsilon} << 1}$ Low loss and the energy propagates as a wave

with velocity $\boxed{v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{\sqrt{\epsilon_r}}}$ c = speed of light, $\mu_r = 1$

and decays as $e^{-\alpha z}$



$$\alpha = \frac{\mu\sigma v}{2} = \frac{\mu\sigma c}{2\sqrt{\epsilon_r}}$$

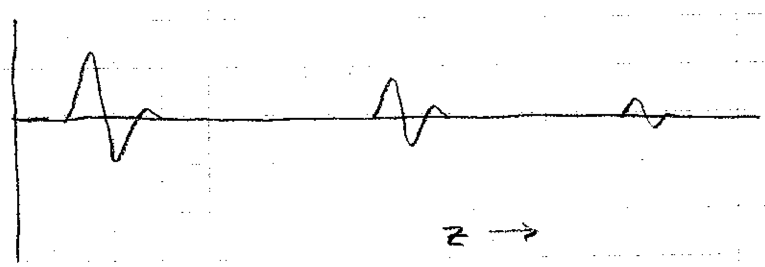
$$\delta = \frac{2\sqrt{\epsilon_r}}{\mu\sigma c} = \frac{5.31\sqrt{\epsilon_r}}{\sigma}$$

σ mS/m

$$\boxed{\delta = \frac{5.31\sqrt{\epsilon_r}}{\sigma} \text{ m}}$$

(σ measured in milli Siemens/m)

So



Pulse travels with the same shape but decreases in amplitude

If

$$\boxed{\frac{\sigma}{\omega\epsilon} >> 1}$$

then the waves diffuse into the earth. This is the regime of electromagnetics for most other geophysical surveys. (airborne and ground survey systems for minerals or deep penetration)

For these signals

velocity $\quad v = \sqrt{\frac{2\omega}{\mu\sigma}}$

decay constant $\quad e^{-\alpha z} \qquad \alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$

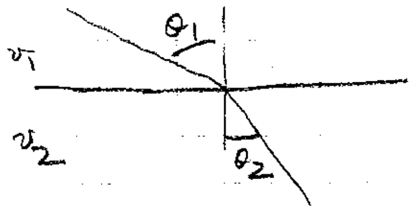
skin depth $\quad \delta = \sqrt{\frac{2}{\omega\mu\sigma}} \simeq 500\sqrt{\frac{1}{\sigma f}} \qquad \omega = 2\pi f$

In the diffusive range the waves are mostly attenuated by the time they travel one wavelength into the medium.

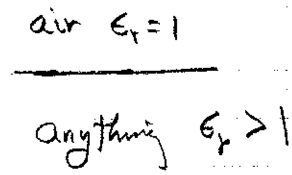
wavelength $\quad \lambda = 2\pi\delta$

after one wavelength, amplitude is $\quad e^{-2\pi}$

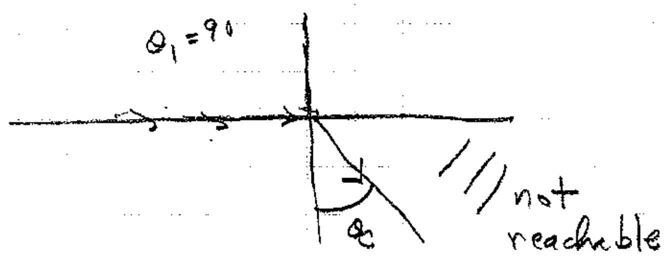
Snell's Law:



$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} \qquad v = \frac{c}{\sqrt{\epsilon_r}}$$



So energy entering the earth is limited in the angle at which the earth is illuminated

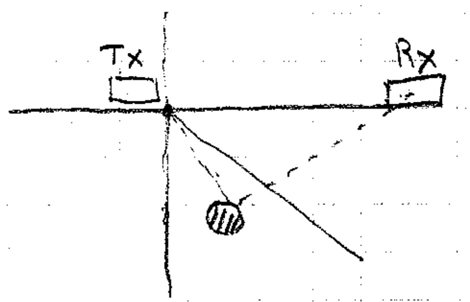


$\theta_1 = 90 \quad \theta$

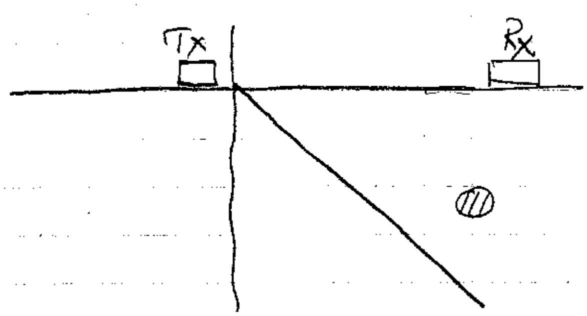
$$\frac{1}{v_1} = \frac{\sin\theta_c}{v_2}$$

$$\boxed{\sin\theta_c = \frac{v_2}{v_1}}$$

Suppose two cases



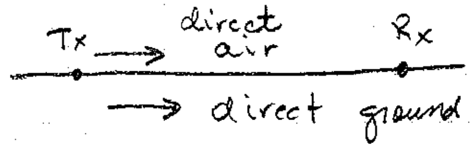
Object is inside the cone of illumination, the there will be some scattered energy.



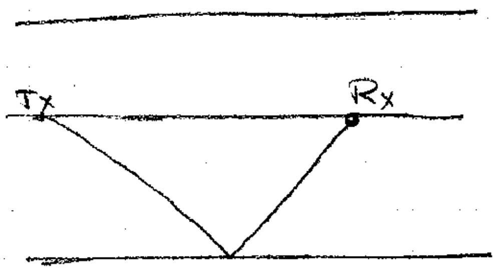
Object is outside the cone of illumination there will be no scattered energy.

Ray Paths.

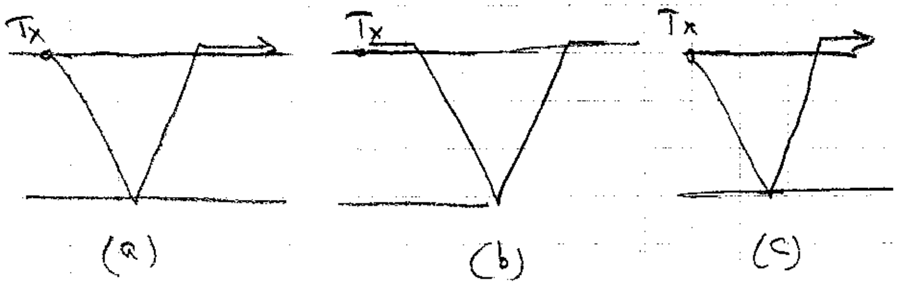
The ray paths



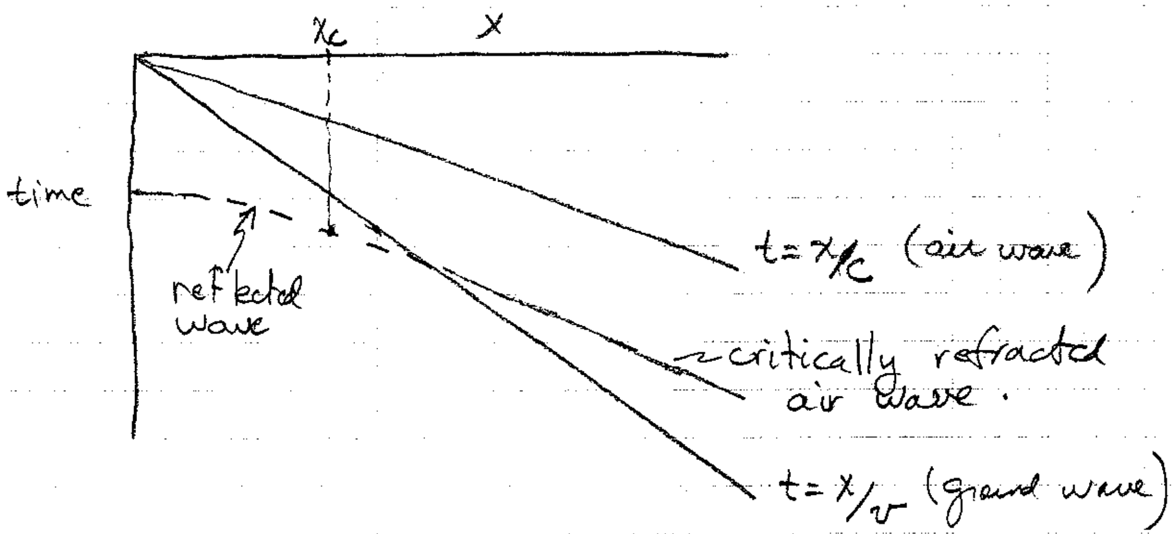
Reflection



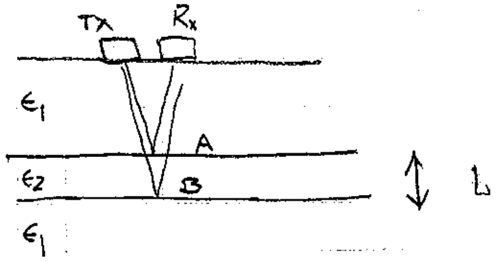
Refraction



The arrivals as a function of separation distance are



Resolution



Suppose we had a thin layer and we wanted to detect it

Reflection coefficient at the top (A)

$$r = \frac{\sqrt{\epsilon_2} - \sqrt{\epsilon_1}}{\sqrt{\epsilon_2} + \sqrt{\epsilon_1}}$$

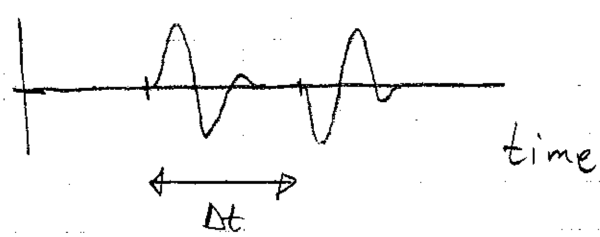
(going from medium (1) to medium (2))

Reflection coefficient at the bottom (B) is

$$\frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = -r$$

(going from medium (2) to medium (1))

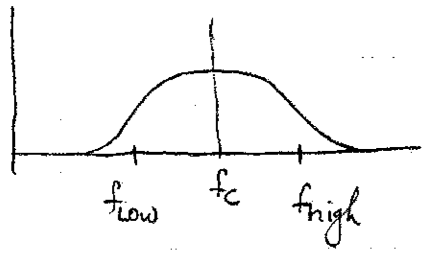
So the radar signal looks like

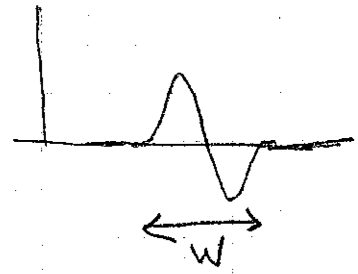


Separation in time is equal to the 2-way travel time through the layer.

$$\Delta t = \frac{2L}{v_2}$$

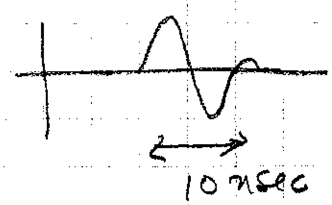
Remember the GPR wavelet



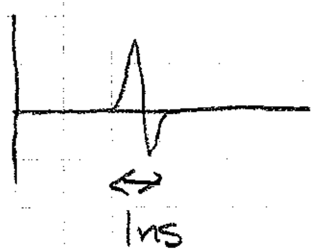


Width in time $\boxed{W \simeq \frac{1}{f_c}}$ seconds

$f_c = 100$ MHz



$f_c = 1000$ MHz $= 1$ GHz

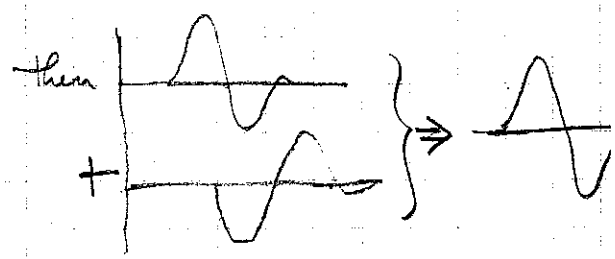


To detect the thin layer

$$\Delta t = \frac{2L}{v_2} > \frac{W}{2}$$

or

$$\boxed{L > \frac{W v_2}{4}}$$

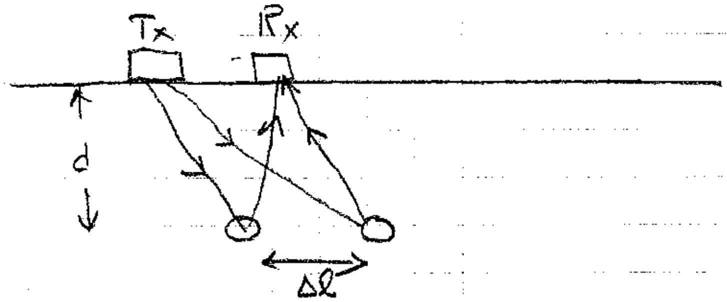


Suppose $\epsilon_2 = 25$

for	f_c	W	L	
	100 MHz	10 nsec	.15 m	= 15 cm
	1000 MHz	1 nsec	.015 m	= 1.5 cm

So, vertical resolution depends upon the frequency of the GPR unit. For a 1GHz system you can find layers that are a few cm in thickness.

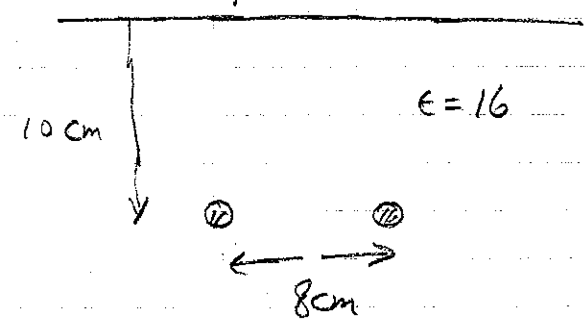
Horizontal Resolution



Again, Δl must be large enough so there is a significant difference between the arrival times from the two objects. We find that

$$\boxed{\Delta l > \sqrt{\frac{d\, v\, W}{2}}} \qquad v = \frac{c}{\sqrt{\epsilon}} \qquad W = \frac{1}{f_c}$$

Example: Two utility cables buried 10 cm below the surface. Can we see them?

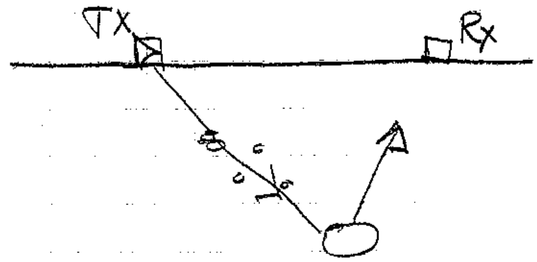


f_c	W	Δl
100 MHz	10 nsec	.19 m = 19 cm
1000 MHz	1 nsec	.06 m = 6 cm

So you could resolve the two cables with a 1000 MHz system but not with a 100 MHz system.

Attenuation: How far can signals travel?

See cartoon plot: Important elements



(1) Spherical spreading of the wave

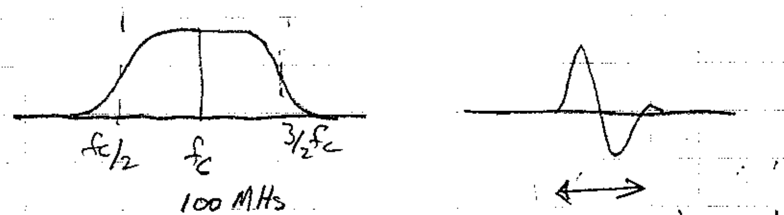
(2) Scattering by inhomogeneities in the background

(3) Ohmic losses $\delta = \frac{5.31\sqrt{\epsilon_r}}{\sigma}$

(4) Scattering from the object

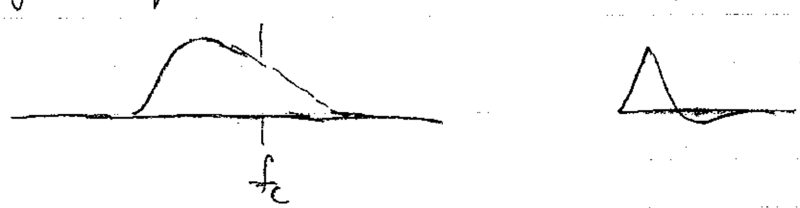
Net result: higher frequencies are attenuated faster than lower frequencies

Remember



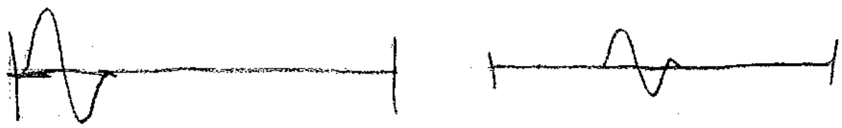
$W \sim \frac{1}{f_c} = 10$ nsec

but if high frequencies are attenuated

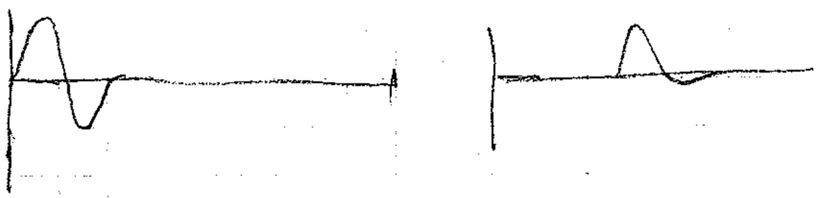


The larger the conductivity the greater the attenuation effects

For low loss media: the wavelet keeps its shape but looses amplitude as it travels



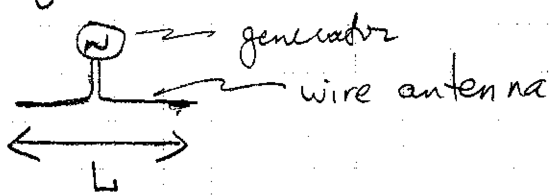
For high loss media, the wavelet also changes shape



So conductivity of the background is very important

<u>Other important points about GPR.</u>

(1) <u>Antenna Size</u>. The transmitter for a GPR is an electric dipole



$$L \sim \frac{\lambda}{2} = \frac{c}{2f_c} \qquad \lambda = cT = \frac{c}{f} \qquad T = \text{period},\ f = \text{freq.}$$

So higher frequency radars can have smaller antennae

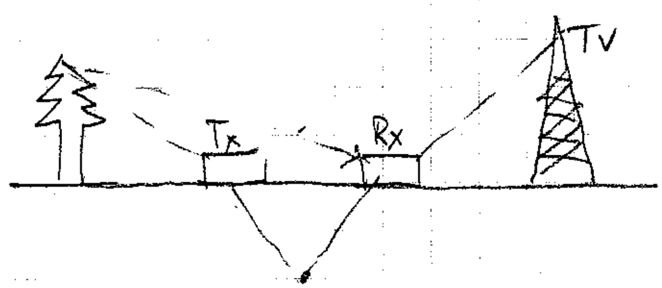
f_c	L
100 MHz	1.5 m
1000 MHz	.15 m = 15 cm

(2) Antennas have a directionality with respect to how they put energy into the earth.

Also the source function for the antenna depends upon the ground permittivity.

This is worthwhile remembering but is too complicated to explore in detail here.

(3) Antennas may be shielded or unshielded.



If antenna is not shielded then you could see reflections from trees, fences ..., and get direct signals from TV's or cell phones.

Small units can be shielded. More difficult for larger units.