MAG3D

A Program Library for Forward Modelling and Inversion of Magnetic Data over 3D Structures

VERSION 4.0

Developed under the consortium research project

JOINT/COOPERATIVE INVERSION OF GEOPHYSICAL AND GEOLOGICAL DATA

UBC–Geophysical Inversion Facility
Department of Geophysics and Astronomy
University of British Columbia
Vancouver, British Columbia

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Description

MAG3D is a program library (version 4.0 as of August 2005) for carrying out forward modelling and inversion of surface, airborne, and/or borehole magnetic data in the presence of a three dimensional Earth. The program library carries out the following functions:

- Forward modelling of the magnetic field anomaly response to a 3D volume of susceptibility contrast. Data are assumed to be the anomalous magnetic response to buried susceptible material, not including Earth's ambient field.
- The model is specified using a mesh of rectangular cells, each with a constant value of susceptibility, and topography is included. The magnetic response can be calculated anywhere within the model volume, including above the topography, simulating ground or airborne surveys, and inside the ground simulating borehole surveys.
- Assumptions:
  - This code assumes susceptibilities are "small". This means results will be wrong when susceptibilties are high enough to cause self-demagnetization.
  - There is no method for incorporating remanent magnetization in this code.
- Inversion of surface, airborne, and/or borehole magnetic data to generate 3D models of susceptibility contrast.
  - The inversion is solved as an optimization problem with the simultaneous goals of *(i)* minimizing an objective function on the model and *(ii)* generating synthetic data that match observations to within a degree of misfit consistent with the statistics of those data.
  - To counteract the inherent lack of information about the distance between source and measurement, the formulation incorporates a depth or distance weighting term.
  - By minimizing the model objective function, distributions of subsurface susceptibility contrast are found that are both close to a reference model and smooth in three dimensions. The degree to which either of these two goals dominates is controlled by the user by incorporating *a priori* geophysical or geological information into the inversion. Explicit prior information may also take the form of upper and lower bounds on the susceptibility contrast in any cell (as of version 4.0).
  - The regularization parameter (controlling relative importance of objective function and misfit terms) is determined in either of three ways, depending upon how much is known about errors in the measured data.
- The large size of useful 3D inversion problems is mitigated by the use of wavelet compression. Parameters controlling the implementation of this compression are available for advanced users.

The research was funded principally by the mineral industry consortium "Joint and Cooperative Inversion of Geophysical and Geological Data" (1991 - 1997) which was sponsored by NSERC (Canada's **N**ational **S**cience and **E**ngineering **R**esearch **C**ouncil) and the following 11 companies: BHP Minerals, CRA Exploration, Cominco Exploration, Falconbridge, Hudson Bay Exploration and Development, INCO Exploration & Technical Services, Kennecott Exploration Company, Newmont Gold Company, Noranda Exploration, Placer Dome, and WMC.

Since then, improvements have been implemented as time and resources permit, especially in the context of the "ICIS consortium" project, 2005-2006 which supported development of version 4.0.

The theoretical framework for MAG3D is provided in the following papers (see the UBC-GIF website publications page for details):

- Li, Y. and Oldenburg, D. W., 1996, 3-D inversion of magnetic data: Geophysics, **61**, no. 02, 394-408.
- Li, Yaoguo and Oldenburg, Douglas W., 1998, Separation of regional and residual magnetic field data: Geophysics, **63**, no. 02, 431-439.
- Li, Y. and Oldenburg, D. W., 2000, Joint inversion of surface and three-component borehole magnetic data, Geophysics, Geophysics, **65** , #2, pp540-552.

Two short papers including examples of applying MAG3D in mineral exploration contexts are:

- *Cost effectiveness of geophysical inversions in mineral exploration: Applications at San Nicolas*, Nigel Phillips, Doug Oldenburg, and Jiuping Chen, Yaoguo Li, Partha Routh, 2001, The Leading Edge, Volume 20, Issue 12 p. 1351
- *Applications of Geophysical Inversions in Mineral Exploration Problems,* Oldenburg D.W., Li Y., Farquharson C.G., Kowalczyk P., Aravanis T., King A., Zhang P., and Watts A. (1998), The Leading Edge, 17, 461 - 465.

Software package contents

The package that can be licensed includes the following components:

- **Executable programs** for performing 3D forward modelling and inversion of magnetic surveys. The MAG3D library (WindowsXX or Linux platforms) consists of three major programs and one utility:
  - MAGFOR3D: performs forward modelling.
  - MAGSEN3D: calculates sensitivity and the depth weighting function.
  - MAGINV3D: performs 3D magnetic inversion.
  - MAGPRE3D: multiplies the sensitivityfile by the model to get the predicted data.
- **A graphical user interface** is supplied for the WindowsXX platforms ***only*** . Facilities include
  - MAG3D-GUI.EXE: a primary interface for setting up the inversion and monitoring the progress of calculations;
  - GM-DATA-VIEWER: a utility for viewing raw surface or airborne data (but not borehole data), error distributions, and for comparing observed to predicted data directly or as difference maps;
  - MESHTOOLS3d: a utility for displaying resulting 3D models as volume renderings. Susceptibility volumes can be sliced in any direction, or isosurface renderings can be generated.
- **Documentation** is elsewhere via the menu to the left.
- **Example data sets and excercises** are provided on the IAG CD-ROM.

Licensing

A **constrained educational** version of the program is avaliable with the IAG CD-ROM. The educational version is fully functional so that users can learn how to carry out effective and efficient 3Dinversions of magnetic data. **However, RESEARCH OR COMMERCIAL USE IS NOT POSSIBLE because the educational version will NOT work with more than 200 data points or 12,000 cells in the 3D mesh.**

Licensing for an **unconstrained academic** version is available - see the licensing policy document (on the UBC-GIF website).
NOTE: all academic licenses will be **time-limited to one year**. You can re-apply after that time. This ensures that everyone is using the most recent versions of codes.

Licensing for **commercial use** is managed by distributors, not by the UBC-GIF research group. Details are in the licensing policy document.

For learning and documentation:

For links to documentation, related utilities, and examples, see the menu to the left.

Installing MAG3D version 4.0 - educational version
For users with a copy of IAG only!

1. **Copy all files in this folder onto your computer**.
   Place them all together in a new folder such as c:\ubcgif\mag3d\
2. No further installation is necessary.
3. Follow instructions in one of this CD-ROM's exercises, or refer to the program documentation.
4. Recall that this is an educational version. **Codes will not work with more than 200 data points or 12,000 cells in the 3D mesh.**

MAG3D manual:
Introduction

MAG3D is a program library for forward modelling and inversion of magnetic data over 3d structures. It was developed under the consortium research project JOINT/COOPERATIVE INVERSION OF GEOPHYSICAL AND GEOLOGICAL DATA, by the UBC–Geophysical Inversion Facility

MAG3D Ver 4.0 (June 2005) - changes to the code and the manual

As might be expected, more recent UBC-GIF codes have features that have been found important for solving practical problems but these features were not included in earlier program libraries. The upgrades described below address this issue. The revised codes are more uniform in capabilities and are more computational efficient.

Improvements since version 3.1:

- The user can input lower and upper bounds. This feature can be important in helping control the range of acceptable models.
- A new preconditioner for solving the Gauss-Newton system of equations results in significantly improved performance.
- All values, except for the stored sensitivity matrix, are now in double precision. This results in more accurate calculations.
- A file **sensitivity.txt** is output after running magsen3d.exe. It contains the average sensitivity for each cell. This file can be used for depth of investigation analysis or for use in designing special model objective function weighting.
- A file **maginv3d_nopos.sus** is output during the first part of the inversion. It contains the susceptibilities without the bounds constraints imposed.
- A file **maginv3d_XX.sus** is output after each beta iteration.
- In **maginv3d.exe**, the user can enter either alpha values or length scales.
- The reference model is included in the calculation of the model norm.
- The reference model is now scaled by the depth weighting before starting the no-positivity iterations.
- Sensitivity calculations carried out by **magsen3d.exe** are now more efficient.

Changes to run time files for MAG3D Version 4.0

- To implement the bound constraints the input file for maginv3d has an extra line where the user can choose one of three options:
  - a filename: two-column file with lower and upper bounds for each cell
  - 2 numbers: constant values for lower and upper bounds
  - NULL: default bounds of 0 and 1.
- There are several additional output files created; see description above.

Changes to this manual for MAG3D Version 4.0:

- This introductory note.
- Additions to section 1.4 "Inversion Methodology" are included in the Appendix.
- Changes to the format for input file "maginv3d.inp" are described in the Appendix.
- The optional "bounds.sus" file that can be used to specify lower and upper bounds for susceptibility values in each model cell is described after page 22.

Notes on computation speed

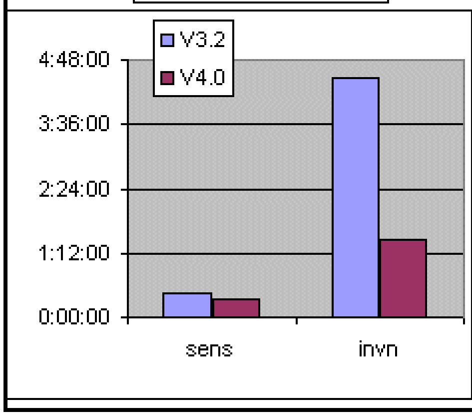
- Time to run computations using MAG3D Ver3.2 and Ver4.0 are compared in the figure below for a moderately complicated problem. This was a real example involving 1564 data points and 417,582 cells in the model, and moderate topography.
- The Version 4.0 code is significantly faster in all cases.
- Speed also depends strongly on the computer. Times when using two computers are shown. Both were Pentium IV processors with 1 Gbyte of RAM. The significant difference however concerns the size of the CPU's cache memory. Most good computers sold since roughly 2005 have 1.0Mb of cache memory and this results in a significant improvement in time to complete computation-intensive jobs.

- The complexity of the problem also affects computation times. More complex problems include topography, complicated distributions of susceptibility, weighting functions, reference models, bounds, etc.
- These results are presented for illustration only. The time to compute any given problem is strongly dependent upon the number of data points, the size of the mesh, and how you set up all parameters for the inversion, including data, constraints, regularization, compression, etc.

MAG3D computation times on two computers, for both versions 3.2 and 4.0.
# of data: 1564 **# of cells:** 417,582

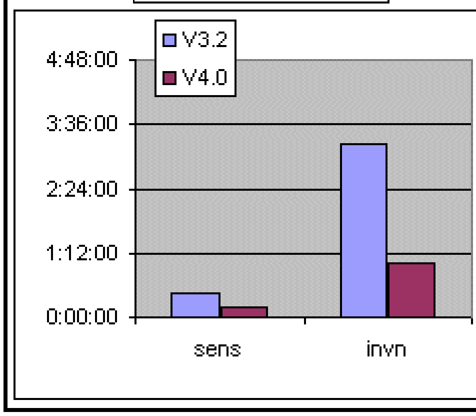
CPU1: clock = 3.0 Ghz, cache=0.5Mb

	V3.2	V4.0
sens	0:26:38	0:20:03
invn	4:28:41	1:27:14



CPU2: clock = 3.2 Ghz, cache=1.0Mb

	V3.2	V4.0
sens	0:27:14	0:11:54
invn	3:14:10	1:00:22



The remaining sections of the manual are on separate pages:

- Background theory,
- Elements of the library,
- Executing the programs,
- Examples,
- Graphical user interface.

This page: | Introduction | Forward modelling | borehole | Inversion | Depth weighting | Wavelet compression | Regularization | Example |

1.1 Introduction

This manual presents theoretical background, numerical examples, and explanation for implementing the program library MAG3D. This suite of algorithms, developed at the UBC- Geophysical Inversion Facility, is needed to invert magnetic responses over a three-dimensional susceptibility distribution. The manual is designed so that a geophysicist who is familiar with the magnetic experiment, but who is not necessarily versed in the details of inverse theory, can use the codes and invert his or her data.

A magnetic experiment involves measuring the anomalous magnetic field produced by magnetically susceptible materials beneath the surface, which have been magnetized by the earth's main magnetic field . The material with susceptibility $\kappa(x, y, z)$ is magnetized when the
earth's main field with flux intensity $\vec{B}_0$ impinges upon the subsurface formation. The magnetized material gives rise to a magnetic field, $\vec{B}_a$, which is superimposed on the inducing field to produce a total, or resultant, field. By measuring the resultant field and removing the inducing field from the measurements through numerical processing, one obtains the distribution of the anomalous field due to the susceptible material. Very often, the susceptible materials underground possess a certain amount of natural remanent magnetization. In this program library, however, we make the assumption that no remanent magnetization is present and restrict our attention to induced magnetization.

The data from a typical magnetic survey is a set of magnetic field measurements acquired over a 2D grid above the surface or along a number of boreholes within the volume of interest. These data are first processed to yield an estimate of the anomalous field due to the susceptible material in the area. The goal of the magnetic inversion is to obtain, from the extracted anomaly data, quantitative information about the distribution of the magnetic susceptibility in the ground. Thus it is assumed that the input data to the inversion program is the extracted residual anomaly and the programs in the library are developed accordingly.

1.2 Forward Modelling

General Formulation

For a given inducing field $\vec{B}_0$, the magnetization $\vec{J}$ depends upon the susceptibility through a differential equation. However, to the first order approximation when the actual susceptibility is very small, as is most often the case with material encountered in mineral explorations, the magnetization $\vec{J}$ is proportional to the susceptibility and is given by the product of susceptibility with inducing magnetic field $\vec{H}_0$,

$$\vec{J} = \kappa \vec{H}_0 \qquad (1)$$

where $\vec{H}_0 = \vec{B}_0 / \mu_0$ and μ_0 is the free space magnetic permeability. This essentially ignores the self-demagnetization effect by which the secondary field reduces the total inducing field within the susceptible region and results in a weaker magnetization than that given by eq. (1).

The anomalous field produced by the distribution of magnetization $\vec{J}$ is given by the following integral equation with a dyadic Green's function,

$$\vec{B}_a(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \nabla\nabla \frac{1}{|\vec{r} - \vec{r}'|} \cdot \vec{J} dv. \qquad (2)$$

where $\vec{r}$ is the position of the observation point. ***V*** represents the volume of magnetization. The above equation is valid for observation locations above the earth's surface. It is also valid in the boreholes provided we assume that the magnetic permeability is μ_0.

When the susceptibility is constant within a volume of source region, the above equation can be written in matrix form as

$$\vec{B} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ \end{pmatrix} \vec{J}$$

$$\vec{B}_a = \mu_0 \begin{pmatrix} T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix} \kappa \vec{H}_0 \qquad (3)$$

$$\equiv \mu_0 \kappa \mathbf{T} \vec{H}_0,$$

where T_{ij} is given by

$$T_{ij} = \frac{1}{4\pi} \int_V \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} \frac{1}{|\vec{r} - \vec{r}\,'|} dv, \qquad i = 1,3;\ j = 1,3, \qquad (4)$$

where x_1, x_2, and x_3 represent *x, y,* and *z*, respectively. The expressions of T_{ij} for a cuboidal source volume can be found in Bhattacharyya (1964) and Sharma (1966). (Here we assume that the effect of borehole cavity can be neglected.) Since **T** is symmetric and its trace is equal to -1 when the observation is inside the cell and is 0 when the observation is outside the cell, only five independent elements need to be calculated.

Once **T** is formed, the magnetic anomaly $\vec{B}_a$ and its projection onto any direction of measurement is easily obtained by the inner product with the directional vector. The projection of the field $\vec{B}_a$ onto different directions yields different anomalies commonly obtained in the magnetic survey. For instance, the vertical anomaly is simply B_{az}, the vertical component of $\vec{B}_a$, whereas the total field anomaly is, to first order, the projection of $\vec{B}_a$ onto the direction of the inducing fiels $\vec{B}_0$.

Borehole Data

In a borehole experiment, the three components are measured in the directions of local coordinate axes (x_1, y_1, z_1) define according to the borehole orientation. Assuming that the borehole dip θ is measured downward from the horizontal surface and azimuth φ is measured eastward from the north, a commonly used convention has the z_1–axis pointing downward along borehole, x_1–axis pointing perpendicular to the borehole in the direction of the azimuth. The y_1–axis completes the right-handed coordinate system and is 90^o clockwise from the azimuth and perpendicular to the borehole. Based upon the above definition the rotation matrix that transforms three components of a vector in the global coordinate system to the components in the local coordinates is given by

$$\mathbf{R} = \begin{pmatrix} \cos\varphi\sin\theta & \sin\varphi\sin\theta & -\cos\theta \\ -\sin\varphi & \cos\theta & 0 \\ \cos\varphi\cos\theta & \sin\varphi\cos\theta & \sin\theta \end{pmatrix}. \qquad (5)$$

If a vector is define in local coordinates as $(l_1, l_2, l_3)^T$, and in global coordinates as $(g_1, g_2, g_3)^T$, then the following two relations hold;

$$(l_1, l_2, l_3)^T = \mathbf{R}(g_1, g_2, g_3)^T, \qquad (6)$$
$$(g_1, g_2, g_3)^T = \mathbf{R}^T(l_1, l_2, l_3)^T.$$

The rotation matrix **R** therefore allows measured components in local coordinates to be rotated into global coordinate, or the components of the regional field to be rotated into local coordinates for use in regional removal.

Numerical Implementation

We divide the region of interest into a set of 3D cuboidal cells by using a 3D orthogonal mesh and assume a constant susceptibility within each cell. By eq.(1), we have an uniform magnetization within each cell and its field anomaly can be calculated using eqs.(3) and (6). The actual anomaly that would be measured at an observation point is the sum of field produced by all cells having a non-zero susceptibility value. The calculation involves the evaluation of eq. (3) in a 3D rectangular domain define by each cell. The program that performs this calculation is MAGFOR3D. As input parameters, the coordinates of the observation points and the inclination and declination of the anomaly direction must be specifie for each datum. For generality, each component in a multi-component data set is specified as a separate datum with its own location and direction of projection.

As an illustration of the forward modelling program, we calculate the total field anomaly above the surface and three-component anomaly in boreholes produced by a synthetic model. The model consists of two vertical prisms buried at different depths. Figure 1 displays one cross-section and one plan-section of the model. The inducing field has inclination $I=65^o$ and declination $D=25^o$. The surface anomaly is calculated at an interval of 25m over seven east-west lines spaced 100 m apart. The borehole data are calculated in three vertical holes at an interval of 25m. Figure 2 shows the contour map of the surface data and the depth profiles of the borehole data.

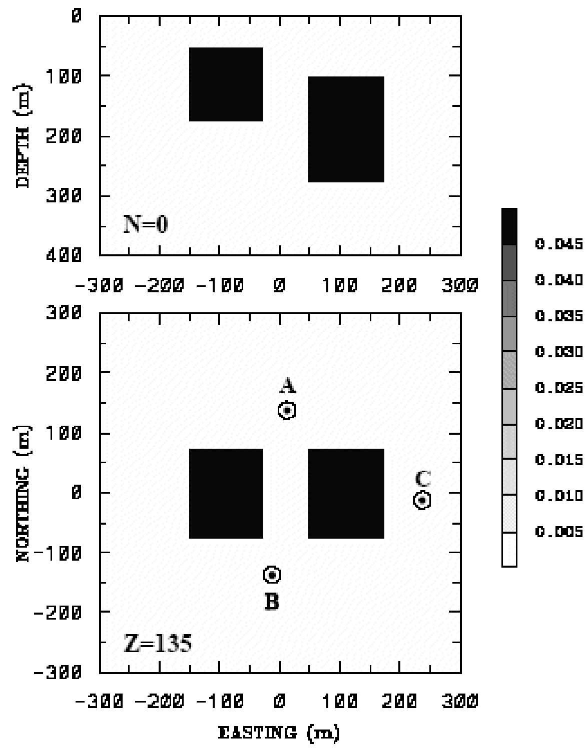


Figure 1.
Two sections of a susceptibility model consisting of two prisms buried in a nonsusceptible background. The collar positions of three vertical boreholes are indicated on the plan-section.

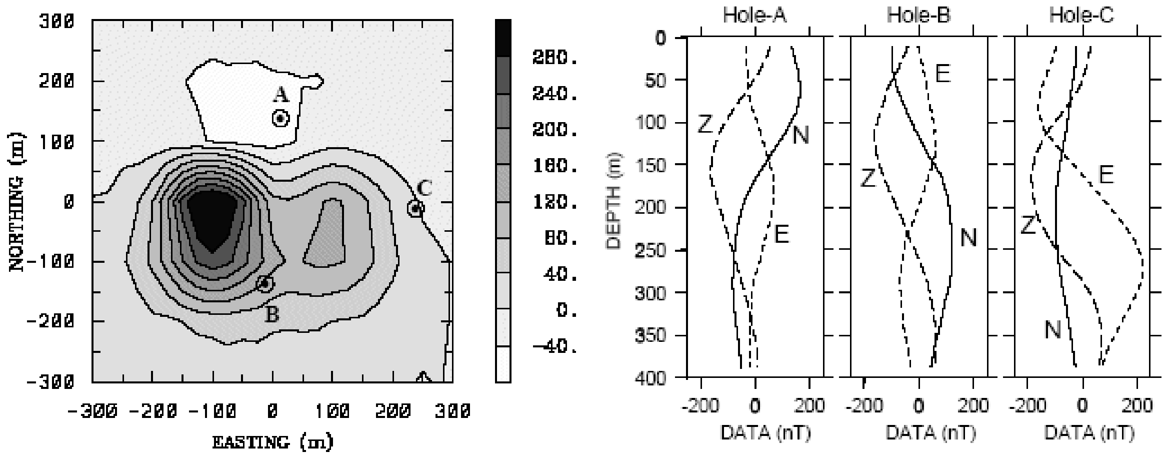


Figure 2. The panel on the left is the total field anomaly on the surface produced by the model in Figure 1 under an inducing field with declination and inclination noted above. The panels on the right are the three-component anomalies in three vertical boreholes. Different components are identifie by the labels beside the curves.

1.3 Inversion Methodology for Ver 3.1.

(Notes regarding modifications for Ver 4.0 are provided below). Let the set of extracted anomaly data be $\vec{d} = (d_1, \cdots, d_N)^T$ and the susceptibility of cells in the model be $\vec{\kappa} = (\kappa_1, \cdots, \kappa_M)^T$. The two are related by the sensitivity matrix

$$\vec{d} = \mathbf{G}\vec{\kappa} \qquad (7)$$

The matrix has elements g_{ij} which quantify the contribution to the i^{th} datum due to a unit susceptibility in the j^{th} cell (see Section 1.2). The program MAGSEN3D performs the calculation of the sensitivity matrix , which is to be used by the subsequent inversion. The sensitivity matrix gives the forward mapping from a model to data during the entire inverse process. We will discuss its efficient representation via the wavelet transform in a separate section.

The inverse problem is formulated as an optimization problem where an objective function of the model is minimized subject to the constraints in eq.7. For magnetic inversion the first question that arises concerns definition of the "model". We choose κ as the model since the anomalous field is directly proportional to the susceptibility. This is the choice for the inversion program MAGINV3D. For generality, we introduce the generic symbol *m* for the model element. Having define a "model" we next construct an objective function which, when minimized, produces a model that is geophysically interpretable. The details of the objective function are problem dependent but generally we need the flexibility to be close to a reference model m_0 and also require that the model be relatively smooth in three spatial directions. Here we adopt a right handed Cartesian coordinate system with positive north and positive down. Let the model objective function be

$$\phi_m(m) = \alpha_s \int_V w_s \{w(\vec{r})[m(\vec{r}) - m_0]\}^2 dv + \alpha_x \int_V w_x \left\{ \frac{\partial w(\vec{r})[m(\vec{r}) - m_0]}{\partial x} \right\}^2 dv + \alpha_y \int_V w_y \left\{ \frac{\partial w(\vec{r})[m(\vec{r}) - m_0]}{\partial y} \right\}^2 dv + \alpha_z \int_V w_z \left\{ \frac{\partial w(\vec{r})[m(\vec{r}) - m_0]}{\partial z} \right\}^2 dv. \quad (8)$$

where the functions w_s , w_x, w_y and w_z are spatially dependent while α_s, α_x, α_y, and α_z are coefficient which affect the relative importance of different components in the objective function. Here the function $w(\vec{r})$ is a generalized depth weighting function. The purpose of this function is to counteract the geometrical decay of the sensitivity with the distance from the observation location so that the recovered susceptibility is not concentrated near the observation locations. The details of the depth weighting function will be discussed in the next section.

The objective function in eq. (8) has the flexibility to construct many different models. The reference model m_0 may be a general background model that is estimated from previous investigations or it could be the zero model. The reference model would generally be included in the first component of the objective function but it can be removed if desired from the remaining terms; often we are more confident in specifying the value of the model at a particular point than in supplying an estimate of the gradient. The relative closeness of the final model to the reference model at any location is controlled by the function w_s. For example, if the interpreter has high confidenc in the reference model at a particular region, he can specify w_s to have increased amplitude there compared to other regions of the model. The weighting functions w_x, w_y, and w_z can be designed to enhance or attenuate structures in various regions in the model domain. If geology suggests a rapid transition zone in the model, then a decreased weighting for flatness can be put there and the constructed model will exhibit higher gradients provided that this feature does not contradict the data.

To perform a numerical solution, we discretize the model objective function in eq. (8) using a finite difference approximation on the mesh defining the susceptibility model. This yields

$$\begin{aligned} \phi_m(m) &= (m - m_0)^T \left(\mathbf{W}_s^T \mathbf{W}_s + \mathbf{W}_x^T \mathbf{W}_x + \mathbf{W}_y^T \mathbf{W}_y + \mathbf{W}_z^T \mathbf{W}_z \right) (m - m_0) \\ &\equiv (m - m_0)^T \left(\mathbf{W}_m^T \mathbf{W}_m \right) (m - m_0) \\ &= ||\mathbf{W}_m (m - m_0)||^2 \end{aligned} \quad (9)$$

where m and m_0 are M-length vectors. The individual matrices W_s , W_x, W_y and W_z are straight- forwardly calculated once the model mesh and the weighting functions $w(\vec{r})$ and w_s , w_x, w_y, w_z are defined The cumulative matrix $\mathbf{W}_m^T \mathbf{W}_m$ is then formed.

The next step in setting up the inversion is to define a misfit measure. Here we use the 2–norm measure

$$\phi_d = \left|\left| \mathbf{W}_d \left(\mathbf{G}\vec{\kappa} - d^{obs} \right) \right|\right|^2 \quad (10)$$

For the work here we assume that the contaminating noise on the data is independent and Gaussian with zero mean. Specifying W_d to be a diagonal matrix whose i^{th} element is $1/\sigma_i$, where σ_i is the standard deviation of the i^{th} datum, makes ϕ_d a chi-squared variable distributed with ***N*** degrees of freedom. Accordingly $E[\chi^2] = N$ provides a target misfit for the inversion.

The inverse problem is solved by finding a model *m* which minimizes ϕ_m and misfits the data by a pre-determined amount. Since the susceptibility is positive by definition we also need to impose the constraint that all model elements be positive. Thus the solution is obtained by the following minimization problem,

$$\begin{aligned} &\text{minimize :} \quad \phi = \phi_d + \mu \phi_m, \\ &\text{subject to :} \quad \vec{m} > \vec{0}, \end{aligned} \quad (11)$$

where μ is a tradeoff parameter that controls the relative importance of the model norm and data misfit. When the standard deviations of data errors are known, the acceptable misfit is given by the expected value ϕ_d^* and we will search for the value of μ that produces the expected misfit. Otherwise, an estimated value of μ will be prescribed. The details of various aspects of choosing a tradeoff parameter will be discussed in a following section.

We use a primal logarithmic barrier method with the conjugate gradient technique as the central solver. In the logarithmic barrier method, the positivity constraint is implemented as a logarithmic barrier term. The new objective function is given by

$$\phi(\lambda) = \phi_d + \mu\phi_m - 2\lambda \sum_{j=1}^{M} \ln(m_j), \quad (12)$$

where λ is the barrier parameter, and the tradeoff parameter is fixed during the minimization. As the name suggests, the logarithmic barrier term forms a barrier along the boundary of the domain of feasible solutions, and prevents the minimization from crossing over to the infeasible region. The method solves a sequence of nonlinear minimizations with decreasing λ and, as λ approaches zero, the sequence of solutions approach the solution of eq. (11).

This methodology provides a basic framework for solving 3D magnetic inversion with arbitrary observation locations. The basic components are the forward modelling, a model objective function that incorporates a "depth" weighting, a data misfit function, a tradeoff parameter that ultimately determines how well the data will be fit, and the logarithmic barrier method to obtain the solution with positivity. Without getting into the algorithmic details, we discuss three of these basic components in the next sections, namely, the depth weighting, efficient forward mapping, and the choice of the tradeoff parameter. An understanding of these components is necessary for the user to have a global view of the algorithm and to use the program library correctly.

Notes regarding MAG3D Ver.4.0.

In MAG3D versions 3.1 and earlier the condition of positivity in the model was imposed by implementing a modification to the model objective function. In Version 4.0 a more general condition can be imposed such that lower and upper bounds on values of susceptibility in the model can be defined. This is useful because often there are well-defined bounds on the susceptibility contrast based on direct sampling or other geological information.
The procedure for implementing upper and lower bounds on model values is the same as that used in GRAV3D Version 2.0 and later. In MAG3D the positivity constraint is no longer necessary if upper and lower bounds are defined. However, if the user chooses to not define upper and lower bounds, the program employs default bounds for susceptibilities in every cell of 1.0 and 0.0 respectively (S.I. units). While it is true that some rocks have susceptible greater than 1.0, MAG3D Version 4.0 still assumes small susceptibilities, as this code has done since the original version. When there are very high susceptibilities, the relation between incident and induced magnetization is no longer linear, and the problem becomes more complicated. Inverting data in the presence of very high susceptibilities is still a topic of research, and MAG3D Version 4.0 does not allow for high susceptibilities in the solution.

As a result of implementing upper and lower bounds, equations (11) and (12) above are different for MAG3D Version 4.0. Instead of imposing positivity, the solution is obtained by the following minimization problem, which replaces equation (11):

$$\begin{aligned} &\text{minimize}: \quad \phi = \phi_d + \mu\phi_m, \\ &\text{subject to}: \quad \kappa_{min} \le \kappa \le \kappa_{max} \end{aligned} \quad (11b)$$

where κ_{min} and κ_{max} are vectors containing the lower and upper bounds on the model values, and κ is the vector containing model values.
As before, we use the primal logarithmic barrier method with the conjugate gradient technique as the central solver. In the logarithmic barrier method, the bounds constraints are implemented as a logarithmic barrier term. The new objective function is given by

$$\phi(\lambda) = \phi_d + \mu\phi_m - 2\lambda \sum_{j=1}^{M} \left[\ln(\kappa_j - \kappa_j^{min}) + \ln(\kappa_j^{max} - \kappa_j)\right] \quad (12b)$$

where λ is the barrier parameter, and the regularization parameter μ is fixed during the minimization. As the name suggests, the logarithmic barrier term forms a barrier along the boundary of the feasible domain and prevents the minimization from crossing over to the infeasible region. The method solves a sequence of nonlinear minimizations with decreasing λ and, as λ approaches zero, the sequence of solutions approach the solution of eq (11b).

1.4 Depth Weighting

"Depth" weighting for surface or airborne data

It is a well known fact that static magnetic data have no inherent depth resolution. A numerical consequence of this is that when an inversion is performedwhich minimizes $\int m(\vec{r})^2 dv$ subject to fitting the data, the constructed susceptibility is concentrated close to the observation locations. This is a direct manifestation of the kernel's decay with the distance between the cell and observation locations. Because of the rapidly diminishing amplitude, the kernels of magnetic data are not sufficient to generate a function that possess significant structure at locations that are far away from observations. In order to overcome this, the inversion needs to introduce a weighting to counteract this natural decay. Intuitively, such a weighting will approximately cancel the decay and give cells at different locations equal probability to enter into the solution with a non-zero susceptibility.

For surface data, the sensitivity decays predominantly in the depth direction. Numerical experiments indicate that the function of the form $(z + z_0)^{-3}$ closely approximates the kernel's decay directly under the observation point provided that a reasonable value is chosen for z_0. The value of –3 in the exponent is consistent with the fact that, to first order, a cubic cell acts like an dipole source whose magnetic field decays as inverse distance cubed. The value of z_0 can be obtained by matching the function $1/(z + z_0)^{-3}$ with the field produced at a observation point by a column of cells . Thus we use a depth weighting function of the form

$$w(\vec{r}_j) = \left[\frac{1}{\Delta z_j} \int_{\Delta z_j} \frac{dz}{(z + z_0)^{\beta}} \right]^{1/2}, \qquad j = 1, \cdots, M \qquad (13)$$

for the inversion of surface data, where β=3.0 and $\vec{r}_j$ is used to identify the jth cell and Δz_j is its thickness. This weighting function is first normalized so that the maximum value is unity. Numerical tests indicate that when this weighting is used, the susceptibility model constructed by minimizing a model objective function in eq.(8), subject to fitting the data, places the recovered anomaly at approximately the correct depth.

Note that if the data set involves highly variable observation heights the normal depth weighting function might not be most suitable. Distance weighting used for borehole data may be more appropriate - as explained next.

"Depth" weighting for borehole data

For data sets that contain borehole measurements, the sensitivities do not have a predominant decay direction, therefore a weighting function that varies in three dimensions is needed. We generalize the depth weighting used in surface data inversion to form such a 3D "depth" weighting function called distance weighting:

$$w(\vec{r}_j) = \frac{1}{\sqrt{\Delta V_j}} \left\{ \sum_{i=1}^{N} \left[\int_{\Delta V_j} \frac{dv}{(R_{ij} + R_0)^{\beta}} \right]^2 \right\}^{1/4}, \qquad j = 1, \cdots, M \qquad (14)$$

where β=3.0 and ΔV_j is the volume of jth cell, R_{ij} is the distance between a point in and the ith observation, and R_0 is a small constant used to ensure that the integral is well define (chosen to be a quarter of the smallest cell dimension). Similarly, this weighting function is normalized to have a maximum value of unity. For inversion of borehole data, it is necessary to use this more general weighting.

Note: This weighting function is also advantageous if surface data with highly variable observation heights are inverted.

The weighting function is directly incorporated in the sensitivity file generated by program MAGSEN3D. This program allows user to specify whether to use the depth weighting or the distance weighting for surface data. When borehole data are present, however, distance weighting must be used.

1.5 Wavelet Compression of Sensitivity Matrix

The two major obstacles to the solution of large scale magnetic inversion problem are the large amount of memory required for storing the sensitivity matrix and the CPU time required for the application of the sensitivity matrix to model vectors. The MAG3D program library overcomes these difficultie by forming a sparse representation of the sensitivity matrix using a wavelet transform based on compactly supported, orthonormal wavelets. For more details, the users are referred to Li and Oldenburg (1997). In the following, we give a brief description of the method necessary for the use of the MAG3D library.

Each row of the sensitivity matrix in a 3D magnetic inversion can be treated as a 3D image and a 3D wavelet transform can be applied to it. By the properties of the wavelet transform, most transform coefficient are nearly or identically zero. When the coefficient with small magnitude are discarded (the process of thresholding), the remaining coefficient still contain much of the necessary information to reconstruct the sensitivity accurately. These retained coefficients form a sparse representation of the sensitivity in the wavelet domain. The need to store only these large coefficients means that the memory requirement is reduced. Further, the multiplication of the sensitivity with a vector can be carried out by a sparse multiplication in the wavelet domain. This greatly reduces the CPU time. Since the matrix-vector multiplication constitutes the core computation of the inversion, the CPU time for the inverse solution is reduced accordingly. The use of this approach increases the size of solvable problems by nearly two orders of magnitude.

Let **G** be the sensitivity matrix, and $\mathcal{W}$ be the symbolic matrix-representation of the 3D wavelet transform. Then applying the transform to each row of **G** and forming a new matrix consisting of rows of transformed sensitivity is equivalent to the following operation,

$$\tilde{\mathbf{G}} = \mathbf{G}\mathcal{W}^T. \qquad (15)$$

where $\tilde{\mathbf{G}}$ is called the transformed matrix. The thresholding is applied to individual rows of $\tilde{\mathbf{G}}$ by the following rule to form the sparse representation $\tilde{\mathbf{G}}^s$,

$$\tilde{g}_{ij}^s = \begin{cases} \tilde{g}_{ij}, & |\tilde{g}_{ij}| \geq \delta_i \\ 0, & |\tilde{g}_{ij}| < \delta_i \end{cases}, \quad i = 1, \cdots N \qquad (16)$$

where δ_i is the threshold level, and $\tilde{g}_{ij}$ and $\tilde{g}_{ij}{}^s$ are the elements of $\tilde{\mathbf{G}}$ and $\tilde{\mathbf{G}}^s$, respectively.

The threshold level δ_i are determined according to the allowable error of the reconstructed sensitivity, which is measured by the ratio of norm of the error in each row to the norm of that row, $r_i(\delta_i)$. It can be evaluated directly in the wavelet domain by the following expression:

$$r_i(\delta_i) = \sqrt{\frac{\sum_{|\tilde{g_{ij}}|<\delta_i} \tilde{g_{ij}}^2}{\sum_j \tilde{g_{ij}}^2}}, \qquad i = 1, \cdots, N. \qquad (17)$$

Here the numerator is the norm of the discarded coefficients. For each row we choose δ_i such that $r_i(\delta_i) = r^*$, where r^* is the prescribed reconstruction accuracy. However, this is a costly process. Instead, we choose a representative row, i_0, and calculate the threshold level δ_{i0} . This threshold is then used to define a relative threshold $\epsilon = \delta_{i_0} / \max_j |\tilde{g}_{i_0 j}|$. The absolute threshold level for each row is obtained by

$$\delta_i = \epsilon \max_j (|\tilde{g_{ij}}|), \qquad i = 1, \cdots, N. \qquad (18)$$

The program that implements this compression procedure is MAGSEN3D. The user is asked to specify the relative error r^* and the program will determine the relative threshold level ϵ . Usually a value of a few percent is appropriate for r^*. When both surface and borehole data are present, two different relative threshold levels are calculated by choosing a representative row for surface data and another for borehole data. For experienced users, the program also allows the direct input of the relative threshold level ϵ .

1.6 Choice of Tradeoff Parameter

The choice of the tradeoff parameter ultimately depends upon the magnitude of the error associated with the data. The inversion of noisier data requires heavier regularization, thus a greater value of μ is required. In this section, we discuss the various implementations for the choice of μ in the MAG3D library.

If the standard deviation associated with each datum is known, then the data misfit define by eq.(10) has a known expected value $\phi_d{}^*$, which is equal to the number of data when the errors are assumed to be independent Gaussian noise with zero mean. The value of μ should be such that the expected misfit is achieved. This entails a line search based on the misfit curve as a function of μ. Because of the positivity constraint, our problem is nonlinear. Thus for each μ a nonlinear solution using a logarithmic barrier method must be obtained. This is computationally demanding and we therefore have developed the following strategy to reduce the cost.

It is observed that, when plotted on a log-log scale, the misfit curves for 3D magnetic inversion with and without positivity often parallel each other in the vicinity of the expected misfit The curve with positivity must lie above the curve without positivity. Therefore, we can first perform a line search without positivity to find a μ_0 that gives rise to ϕ_d *. This search also generates the slope, s_0, of misfit curve at μ_0. This process is very efficient and the required CPU time is much smaller compared to the time required for the solution with positivity. We next assume that s_0 can be used to approximate the slope of the misfit curve when the positivity is imposed. A rigorous line search incorporating positivity starts with an initial guess of μ=0.5μ_0. This usually yields a misfit that is very close to the target value. If the misfit is not sufficientl close to ϕ_d *, however, a new guess for μ is obtained which makes use of the approximate slope s_0. The inversion with updated μ can be solved efficientl if the logarithmic barrier algorithm is started from an initial model close to the fina solution. That model is obtained by perturbing the solution corresponding to the previous μ away from the zero bound. The line search using this strategy is often successful in reaching the target ϕ_d * after testing two to four values of μ. This strategy is implemented in MAGINV3D as the first method for choosing the tradeoff parameter.

In practical applications the estimate of data error is often not available. Then the degree of regularization, hence the value of μ needs to be determined based on other criteria. A commonly used method in linear inverse problems is the generalized cross-validation (GCV) technique. The use of GCV in inverse problems with inequality constraints such as positivity is far more involved and numerically expensive to implement. However, applying GCV on the 3D magnetic inversion without positivity still produces a reasonable estimate of the data error. That error can serve as a starting point for further adjustment by the user based on his or her judgement. Since no other information is assumed, we have chosen to use the value of μ obtained in this manner directly in the final inversion, which has the positivity imposed. In this case, only one logarithmic barrier solution is needed. Numerical tests have indicated that this simplistic use of GCV is in fact surprisingly effective unless the data have a large negative bias or are distributed sparsely. MAGINV3D has implemented this approach as the third method for choosing the tradeoff parameter.

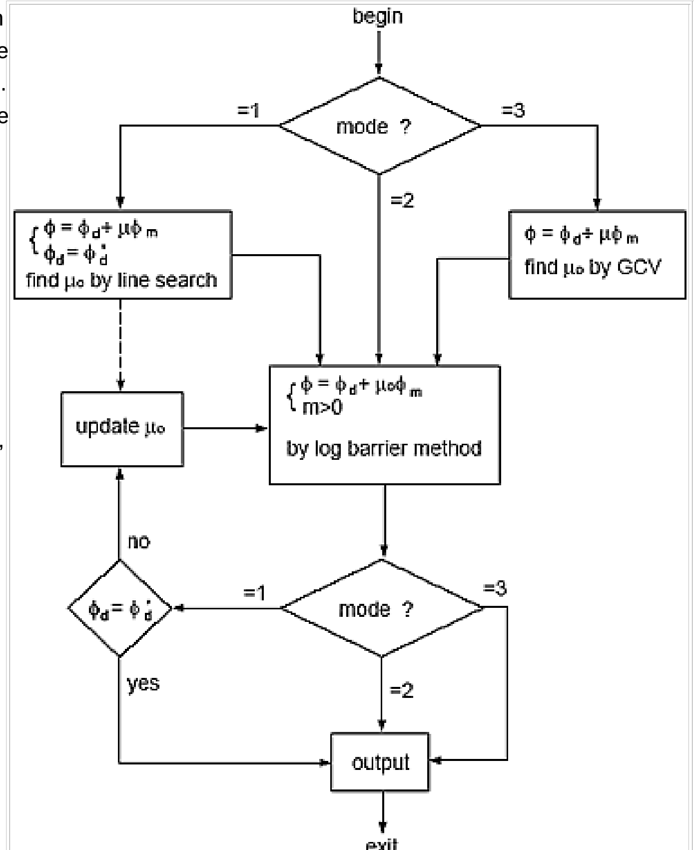


Figure 3. Flow chart illustrating the solution of the 3D magnetic inversion by MAGINV3D using different strategies for choosing the tradeoff parameter.

Figure 3 illustrates the structure of the program MAGINV3D. It has three options for determining the tradeoff parameters. The controlling parameter is *mode*. When *mode=1,* the line search based on known target value of data misfit is used. Two stages, as discussed above, are used and several solutions for different values of μ must be tested to obtain one that produces the target misfit. When *mode=2*, the user specifie a tradeoff parameter and a single solution is produced. When *mode=3*, the program first performs GCV analysis on the inversion without positivity and then uses the resultant value μ of in the final inversion.

1.7 Example of Inversion

We now invert the total field anomaly data in Figure 2 using program MAGINV3D. The model region is discretized into 24 by 24 by 16 cells. The cells are all cubes of 25 m on a side. A total of 344 data (175 surface data and 144 three-component borehole data) are inverted to recover the susceptibility in 9,216 cells.

For a model objective function, we choose α_s=0.0001, α_x=α_y=α_z=1.0, and a distance weighting with R_0=6.2 m. A zero reference model is used. Figure 4 shows one cross-section and one plan-section of the recovered model. The two target prisms are well define in both horizontal and vertical locations and their amplitudes are comparable to those of the true model.

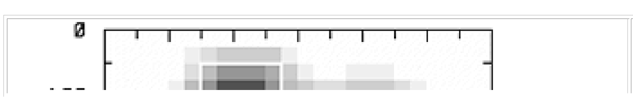
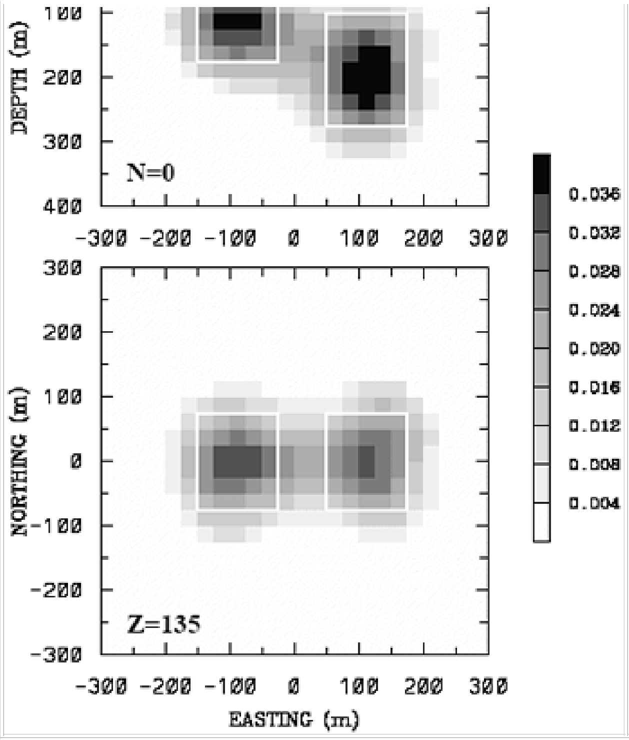


Figure 4. The susceptibility model recovered from the joint inversion of



total-field surface data and three-component borehole data. The distance weighting function is used in this inversion. This model clearly define both prisms and whose true positions are indicated by the solid whire lines.

1.8 References

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Li, Y. and Oldenburg, D. W., 1996, **3–D inversion of magnetic data, Geophysics**, 61, 394–408.

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Sharma, P. V., 1966, **Rapid computation of magnetic anomalies and demagnetization effects caused by bodies of arbitrary shape**: Pure Appl. Geophys., 64, 89–109.

MAG3D manual:
Elements of the program

Contents of this page:

2.1 Introduction

The MAG3D library consists of three major programs and one utility:

1. MAGFOR3D: performs forward modelling.
2. MAGSEN3D: calculates sensitivity and the depth weighting function.
3. MAGINV3D: performs 3D magnetic inversion.
4. MAGPRE3D: multiplies the sensitivityfile by the model to get the predicted data. This rarely used utility multiplies a model by the sensitivity matrix in maginv3d.mtx to produce the predicted data. This program is included so that users who are not familiar with the wavelet transform and the structure of maginv3d.mtx can utilize the available sensitivity matrix to carry out model studies.

Each of the above programs requires input files, as well as the specification of parameters, in order to run. However, some file are used by a number of programs. Before detailing the procedures for running each of the above programs, we first present information about these general files.

2.2 General Files for MAG3D Programs

There are seven general files which are used in MAG3D version4.0. All are in ASCII text format. **Input** files can have any name you like. Only program **output** files have restricted file names. Also the filename extensions are not important. Many prefer to use the *.txt filename convention so that files are more easily read and edited in the Windows environment. The files are:

1. mesh: 3D mesh defining the discretization of the 3D model region
2. topo.dat: specifies the surface topography
3. obs.loc: specifies the inducing field parameters, anomaly type and observation locations
4. obs.mag: specifies the inducing field parameters, anomaly type, observation locations, and the observed magnetic anomalies with estimated standard deviation
5. model.sus: susceptibility model file
6. w.dat: contains the 3D weighting functions
7. bounds.sus: optional file containing values for upper and lower bounds (new for version4.0)

FILE: mesh

This file contains the 3D mesh which define the model region. mesh has the following structure:

```
NE NN NV
E0 N0 V0
ΔE1 ΔE2 ... ΔENE
ΔN1 ΔN2 ... ΔNNN
ΔV1 ΔV2 ... ΔVNV
```

Parameter definitions:

NE Number of cells in the East direction.

NN Number of cells in the vertical direction.

NV Number of cells in the North direction.

`E0, N0, V0` Coordinates, in meters, of the southwest top corner, specifie in (Easting, Northing, Elevation). The elevation can be relative, but it needs to be consistent with the elevation used to specify the observation location in obs.loc or obs.mag and in topo.dat (see the relevantfile for description).

ΔEn Cell widths in the easting direction (from W to E).

ΔNn Cell widths in the northing direction (from S to N).

ΔVn Cell depths (top to bottom).

The mesh can be designed in accordance with the area of interest and the spacing of the data available in the area. In general, the mesh consists of a core region which is directly beneath the area of available data, and a padding zone surrounding this core mesh. Within the core mesh, the size of the cells should be comparable with the spacing of the data. There is no restriction on the relative position of data location and nodal points in horizontal direction. The cell width in this area is usually uniform.

The maximum depth of the mesh used for inversion should be large enough so that no magnetic material below that depth would produce a noticeable anomaly with the length scale covered by the data area. A rule of thumb is that the maximum depth should be at least half of the longest side of the data area. Based upon the user's knowledge of the survey area, one may adjust the maximum depth as necessary. The cell thickness in vertical direction usually increases slightly with depth. In the shallow region, the ratio of thickness to width of about 0.5 is good, especially when surface topography is present. At depth, a cell thickness close to the cell width is advisable. Once this core mesh is designed, it can be extended laterally by padding with a few cells, possibly of variable width. This padding is necessary when the extracted anomalies are close to the boundary of the core mesh or if there are influences from anomalies outside the area which cannot be easily removed. Problems with more than 20,000 to 30,000 model cells, and/or more than a few thousand data points would be considered large, and can be expected to require a considerable amount of computing memory and time.

The vertical position of the mesh is specified in elevation. This is to accommodate the inversion of a data set acquired over a topographic surface. When there is strong topographic relief and one wishes to incorporate it into the inversion, special care should be taken to design the mesh. A conceptually simple approach is first to design a rectangular mesh whose top (specifie by V_0) is just below the highest elevation point, and then to strip off cells that are above the topographic surface. This is the approach taken in MAG3D. The number of cells to be stripped off in each column is determined by the user-supplied topography file topo.dat. Only the remaining cells will be used in the forward modelling or included in the inversion as model parameters.

Example of the mesh file: The following is a 10 x 10 x 5 mesh where each cell is 50m by 50m by 50m.

```
10 10 5
0 0 0
50.0 50.0 50.0 50.0 50.0 50.0 50.0 50.0 50.0 50.0
50.0 50.0 50.0 50.0 50.0 50.0 50.0 50.0 50.0 50.0
50.0 50.0 50.0 50.0 50.0
```

FILE: topo.dat

This optional file is used to define the surface topography of the 3D model by the elevation at different locations. topo.dat has the following structure:

```
! comment
!
npt
E1 N1 elev1
E2 N2 elev2
Enpt Nnpt elevnpt
```

`Comments`: top lines beginning with ! are comments.

`npt` = number of points specifying

`Ei, Ni, elevi` = Easting, Northing and elevation of the i^{th} point. The elevation in this file and Vo in the meshfile must be specified relative to a common reference.

The lines in this file can be in any order as long as the total number is equal to npt. The topographic data need not be supplied on a regular grid. MAG3D assumes a set of scattered points for generality and uses triangulation-based interpolation to determine the surface elevation above each column of cells. To ensure the accurate discretization of the topography, it is important that the topographic data be supplied over the entire area above the model and that the supplied elevation data points are not too sparse.

If topo.dat is not supplied, the surface will be treated as being flat.

Example of topo.dat file:

```
!! topographic data
4
   0.0    0.0  50.0
   0.0 1000.0  50.0
1000.0    0.0 -50.0
1000.0 1000.0 -50.0
```

NOTE: Although the cells above the topographic surface are removed from the model, they must still be included in the model file , model.sus, as if they are a part of the model. For input model files these cells can be assigned any value. The recovered model produced by inversion program MAGINV3D also includes the cells that are excluded from the model, but these cells will have a value of -1.0 as identifier

FILE: obs.loc

This file is used to specify the inducing fieldparameters, anomaly type and observation locations. The following is the file structure of obs.loc:

```
! comments ...
!
Incl  Decl  geomag
Aincl Adecl idir
ndat
E1    N1    Elev1    [aincl1    adecl1]
E2    N2    Elev2    [aincl2    adecl2]
Endat Nndat Elevndat [aínclndat adeclndat]
```

Parameters are:

`! comments`	top lines beginning with ! are comments.
`Incl, Decl`	inclination and declination of the inducing magnetic field. The declination is specified +east with respect to the northing used in the mesh, obs.loc, and obs.mag files.
`geomag`	strength of the inducing fieldin nT.
`Aincl, Adecl`	inclination and declination of the anomaly projection.
`idir`	= 0 : multi-component data set. Observations have different inclinations and declinations, aincln and adecln, specifying the projection direction of the anomaly. In this case, Aincl and Adecl should be equal to Incl and Decl, respectively. = 1 : single-component data set. All observations have the same inclination and declination of the anomaly projection: Aincl, Adecl. If idir is missing, it is assumed to be equal to 1.
`ndat`	number of observations. When single component data are specified the number of observations is equal to the number of data locations. When multi-component data are specified the number of observations will exceed the number of data locations. For example, if three-component data are specifie at N locations, the number of observations is 3N.
`En, Nn, Elevn`	easting, northing and elevation of the observation, measured in meters. Elevation should be above the topography for surface data, and below the topography for borehole data. The observation locations can be listed in any order.
`aincln, adecln`	inclination and declination of the anomaly projection for observation n. This is used only when idir = 0. The brackets "[...]" indicate that these two field are optional depending on the value of idir.

The total fieldanomaly is calculated when Aincl equals Incl and Adecl equals Decl. The vertical field anomaly is calculated when Aincl=90^o and Adecl=0^o. The user can specify other (Aincl, Adecl) pairs to calculate the anomaly component in those directions. Easting, northing and elevation information should be in the same coordinate system as define in the mesh.

Example of obs.loc file: We provide two example file below. The first file is for calculating total field anomaly at 441 points. The inducing field has an inclination of 45^o and a declination of 45^o. The second file is for calculating multi-component anomalies in boreholes and each datum is specified by its own inclination and declination of anomaly projection.

File-1: single-component data.

```
! surface data
!
45.0 45.0 50000.0  !! incl, decl, geomag
45.0 45.0          !! aincl, adecl (direction of anomaly), idir
441                !! # of data
   0.00   0.00 1.0
   0.00  50.00 1.0
   0.00 100.00 1.0
:
1000.00  900.00 1.0
1000.00  950.00 1.0
1000.00 1000.00 1.0
```

File-2: multi-component data.

```
! borehole data
!
65.00 25.00 50000.00 !! incl, decl, geomag
65.00 25.00 0        !! aincl, adecl: anomaly, idir
144                  !! # of data
  -12.50 -137.50 -12.50  0.0 0.0
  -12.50 -137.50 -37.50  0.0 0.0
  -12.50 -137.50 -62.50  0.0 0.0
:
  237.50 -12.50 -337.50 90.0 0.0
  237.50 -12.50 -362.50 90.0 0.0
  237.50 -12.50 -387.50 90.0 0.0
```

FILE: obs.mag

This file is used to specify the inducing fieldparameters, anomaly type, observation locations, and the observed magnetic anomalies with estimated standard deviation. The values of parameters specifying the inducing field anomaly type and observation locations are identical to those in obs.loc. The output of the forward modelling program MAGFOR3D has the same structure except that the column of standard deviations for the error is omitted. The following is the file structure of obs.mag:

```
! comments ...
!
Incl  Decl  geomag
Aincl Adecl idir
ndat
E1    N1    Elev1    [aincl1    adecl1]    Mag1    Err1
E2    N2    Elev2    [aincl2    adecl2]    Mag2    Err2
:
Endat Nndat Elevndat [aincl ndat adeclndat] Magndat Errndat
```

Parameters are:

`! comments` top lines beginning with ! are comments.

`Incl, Decl` inclination and declination of the inducing magnetic field. The declination is specified +east with respect to the northing used in the mesh, obs.loc, and obs.mag files

`geomag` strength of the inducing fieldin nT.

`Aincl, Adecl` inclination and declination of the anomaly projection.

`idir` = 0 : multi-component data set. Observations have different inclinations and declinations, aincln and adecln, specifying the projection direction of the anomaly. In this case, Aincl and Adecl should be equal to Incl and Decl, respectively.
= 1 : single-component data set. All observations have the same inclination and declination of the anomaly projection: Aincl, Adecl. If idir is missing, it is assumed to be equal to 1.

`ndat` number of observations. When single component data are specified the number of observations is equal to the number of data locations. When multi-component data are specified the number of observations will exceed the number of data locations. For example, if three-component data are specifie at N locations, the number of observations is 3N.

`En, Nn, Elevn` easting, northing and elevation of the observation, measured in meters. Elevation should be above the topography for surface data, and below the topography for borehole data. The observation locations can be listed in any order.

aincln, adecln	inclination and declination of the anomaly projection for observation n. This is used only when idir = 0. The brackets "[...]" indicate that these two field are optional depending on the value of idir.
Magn	magnetic anomaly data, measured in nT.
Errn	standard deviation of Magn. This represents the absolute error. It CANNOT be zero or negative.

NOTE: It should be noted that the data Magn are **extracted anomalies** which are derived by removing the regional from the fieldmeasurements. Furthermore, the inversion program assumes that the anomalies are produced by a positive susceptibility (contrast) distribution. It is crucial that the data be prepared as such.

Examples of obs.mag file: The following two example data files The firs example file specifie a set of total fieldanomaly data, and the second example file provides a set of multi-component borehole data.

File-1: single-component data.

```
! surface data
!
45.0 45.0 50000.0  !! incl, decl, geomag
45.0 45.0          !! aincl, adecl (direction of anomaly), idir
441                !! # of data
   0.00   0.00 1.0   0.174732E+02 0.598678E+01
   0.00  50.00 1.0   0.265550E+02 0.613890E+01
   0.00 100.00 1.0   0.311366E+02 0.629117E+01
:
1000.00  900.00 1.0  -0.109595E+01 0.530682E+01
1000.00  950.00 1.0  -0.902209E+01 0.523738E+01
1000.00 1000.00 1.0  -0.397501E+01 0.518496E+01
```

File-2: multi-component data.

```
! borehole data
!
65.00 25.00 50000.00 !! incl, decl, geomag
65.00 25.00 0        !! aincl, adecl: anomaly, idir
144                  !! # of data
  -12.50 -137.50 -12.50  0.0 0.0 0.134759E+03 0.200000E+01
  -12.50 -137.50 -37.50  0.0 0.0 0.162606E+03 0.200000E+01
  -12.50 -137.50 -62.50  0.0 0.0 0.165957E+03 0.200000E+01
:
  237.50 -12.50 -337.50 90.0 0.0 0.662445E+02 0.200000E+01
  237.50 -12.50 -362.50 90.0 0.0 0.693134E+02 0.200000E+01
  237.50 -12.50 -387.50 90.0 0.0 0.608605E+02 0.200000E+01
```

FILE: model.sus

This file contains the cell values of the susceptibility model. The susceptibility must have values in SI units. The following is the file structure of model.sus:

$sus_{1,1,1}$
$sus_{1,1,2}$
:
$sus_{1,1,NV}$
$sus_{1,2,1}$
:
$sus_{i,j,k}$
:
$sus_{NN,NE,NV}$

Each $sus_{i,j,k}$ is the susceptibility at location [i j k].

[i j k]=[1 1 1] is define as the cell at the top-south-west corner of the model. The total number of lines in this file should equal NN x NE x NV, where NN is the number of cells in the North direction, NE is the number of cells in the East direction, and NV is the number of cells in the vertical direction.

The lines must be ordered so that k changes the quickest (from 1 to NV), followed by j (from 1 to NE), then followed by i (from 1 to NN). If the surface topography (topo.dat) file is supplied, the values above the surface will be ignored. These values should be assigned –1.0 to avoid confusion with the other model elements.

FILE: w.dat

This file contains the values for a user supplied weighting function. The following is the file structure for w.dat:

```
W.S_{1,1,1} ... W.S_{NN,NE,NV}
W.E_{1,1,1} ... W.E_{NN,NE-1,NV}
W.N_{1,1,1} ... W.N_{NN-1,NE,NV}
W.Z_{1,1,1} ... W.Z_{NN,NE,NV-1}
```

Parameters are:

$W.S_{i,j,k}$ = cell weights for the smallest model.
$W.E_{i,j,k}$ = cell weights for the interface perpendicular to the easting direction.
$W.N_{i,j,k}$ = cell weights for the interface perpendicular to the northing direction.
$W.Z_{i,j,k}$ = cell weights for the interface perpendicular to the vertical direction.

Within each part, the values are ordered in the same way as in model.sus, however, they can be all on one line, or broken up over several lines. Since the weights for a derivative term are applied to the boundary between cells, the weights have one fewer value in that direction. For instance, the weights for the derivative in easting direction has (NE-1)*NN*NV values, whereas the number of cells is NE*NN*NV.

If the surface topography (topo.dat) file is supplied, the cell weights above the surface will be ignored. These weights should be assigned a value of -1.0 to avoid confusion. If null is entered instead of the file w.dat, then all of the cell weights will be set equal 1.0.

FILE: bounds.sus

This file contains the cell values of the lower and upper bounds on the sought model. It is only required optionally by maginv3d. The bounds have the same dimension as the susceptibility contrast. The following is the file structure of bounds.sus:

```
lk_{1,1,1}    uk_{1,1,1}
lk_{1,1,2}    uk_{1,1,2}
:
lk_{1,1,NV}   uk_{1,1,NV}
lk_{1,2,1}    uk_{1,2,1}
:
lk_{i,j,k}    uk_{i,j,k}
:
lk_{NN,NE,NV} uk_{NN,NE,NV}
```

Parameters are:

$lk_{i,j,k}$ is the lower bound on cell [i j k].
$uk_{i,j,k}$ is the upper bound on cell [i j k].

The ordering of the cells is the same as that for model cells: [i j k]=[1 1 1] is defined as the cell at the top-south-west corner of the model. The total number of lines in this file should equal NN NE NV, where NN is the number of cells in the North direction, NE is the number of cells in the East direction, and NV is the number of cells in the vertical direction. The lines must be ordered so that k changes the quickest (from 1 to NV), followed by j (from 1 to NE), then followed by i (from 1 to NN). If the surface topography (topo.dat) file is supplied, the bounds for cells above the surface will be ignored. These values should be assigned a negative value (e.g. –1.0) to avoid confusion.

Contents of this page:

3.1 Introduction

In a MS-WindowsXX operating environment codes are best run using the Graphical User Interface (GUI). See the separate GUI instructions.

However, all programs in the package also can be run by typing the program name followed by command line arguments. With such a format, they can be executed directly on the command line or in a shell script. When a program is executed without any arguments, it will print a simple message describing the usage. The command format is described below.

Command format:

```
PROGRAM arg_1 arg_2 arg_3 ...
```

`PROGRAM` = name of the executable program. If the program is not in the current directory, its path must be included also.
`arg_n` = a command line argument which is the name of a file. It is usually one of those described in the preceding section or a control file containing input parameters.

3.2 MAGFOR3D

This program performs forward modelling. Command line usage:

```
magfor3d mesh obs.loc model.sus [topo.dat]
```

Input files :
All files are in ASCII text format - they can be read with any text editor. Input files can have any name the user specifies. Details are in the "Elements" chapter.
`mesh` = the 3D mesh
`obs.loc` = the inducing field parameters, anomaly type and observation locations.
`model.sus` = the susceptibility model.
`topo.dat` = surface topography (optional). If omitted, the surface will be treated as being flat.

Output file:
This file is created by MAGFOR3D - it's name can NOT be specified. Details are in the 'Elements" chapter.
`magfor3d.mag` = the computed magnetic anomaly data. Since the data in this file are accurate, the column of the standard deviations for the error is not included.

3.3 MAGSEN3D

This program performs the sensitivity and depth weighting function calculations. Command line usage:

```
magsen3d magsen3d.inp
```

For a sample input file type: `magsen3d –inp` .

Format of the control file `magsen3d.inp`

```
mesh
obs.mag
topo.dat
iwt
beta   znot
```

```
wvlet
itol   eps
```

Input files and parameters:

`mesh`	3D mesh
`obs.mag`	data file . Contains the inducingfield parameters, anomaly type, observation locations, and the observed magnetic anomalies with estimated standard deviation.
`topo.dat`	surface topography (optional). If null is entered, the surface will be treated as being flat.
`iwt`	an integer identifying the type of generalized depth weighting to use in the inversion. =1 for depth weighting (only for surface data); =2 for distance weighting (surface and/or borehole).
`beta, znot`	parameters defining the depth weighting function. When iwt=1, beta and znot are used as β and z_0 to define the **depth** weighting according to eq.(13) (background). When iwt=2, beta and znot are used as β and z_0 to define the **distanc** e weighting according to eq.(14) (background). If null is entered on this line (line 5), then the program sets beta=3 and calculates the value of znot based upon the mesh and data location. This is true for iwt=1 or 2. For most inversions, however, setting this input line to "null" is recommended . The option for inputing beta and znot is provided for experienced users who would like to investigate the effect of the generalized depth weighting for special purposes. The value of beta should normally be close to 3.0. (Note this is different than the value used by GRAV3D) Smaller values of give rise to weaker weighting.
`wvlet`	a five-character string identifying the type of wavelet used to compress the sensitivity matrix. The types of wavelets available are Daubechies wavelet with 1 to 6 vanishing moments (daub1, daub2, and so on) and Symmlets with 4 to 6 vanishing moments (symm4, symm5, symm6). Note that daub1 is the Haar wavelet and daub2 is the Daubechies-4 wavelet. The Daubechies-4 wavelet should be used for most inversions, while the others are provided for users' experimentation. If null is entered, no compression is performed and the program generates a dense matrix in its original form.
`itol, eps`	an integer and a real number that specify how the wavelet threshold level is to be determined. itol=1: program calculates the relative threshold and eps is the relative reconstruction error of the sensitivity. A reconstruction error of 0.05 is usually adequate. itol=2: the user define the threshold level and eps is the relative threshold to be used. If null is entered on this line, a default relative reconstruction error of 0.05 is used and the relative threshold level is calculated (i.e., itol=1, eps=0.05). The detailed explanation of threshold level and reconstruction error can be found in the Background (section 1.5) of this manual.

Example of magsen3d.inp control file

```
mesh      ! mesh file
obs.nois  ! data file
null      ! topography
2         ! iwt=1 depth, =2 distance
null      ! beta, znot | null
daub2     ! wavelet type
1 0.05    ! itol, eps | null
```

Two output files:

1) `maginv3d.mtx` is the sensitivity matrix file to be used in the inversion. This file contains the sensitivity matrix, generalized depth weighting function, mesh, and discretized surface topography. It is produced by the program and it's name is not adjustable. It is very large and may be deleted once the work is completed.

2) A file `sensitivity.txt` is output after running magsen3d.exe. It contains the average sensitivity for each cell. This file can be used for depth of investigation analysis or for use in designing special model objective function weighting. (Added for mag3d version 4.0)

3.4 MAGINV3D

This program performs 3D magnetic inversion. Command line usage is:

```
maginv3d maginv3d.inp
```

For a sample input file type: `maginv3d –inp`. Format of the control file maginv3d.inp is as follows:

```
irest
mode
par tolc
obs.mag
maginv3d.mtx
ini.sus
ref.sus
bounds.sus
αs αe αn αv
w.dat
idisk
```

Control parameters:

irest	restarting flag: =0: start inversion from scratch. =1: restart inversion after it is interrupted. Restart requires two file written out by MAGINV3D before the interruption: maginv3d.aux and maginv3d.kap (see below).
mode	an integer specifying one of three choices for determining the tradeoff parameter (see Figure 4 of background). • mode=1: the program chooses the tradeoff parameter by carrying out a line search so that the target value of data misfit is achieved. • mode=2: the user inputs the tradeoff parameter. • mode=3: the program calculates the tradeoff parameter by applying the GCV analysis to the inversion without positivity.
par, tolc	two real numbers that are used differently. Their use depends upon the value of mode. • mode=1: the target misfit value is given by the product of par and the number of data N, i.e.,. The second parameter, tolc, is the misfit tolerance. The target misfit is considered to be achieved when the relative difference between the true and target misifts is less than tolc. Normally, the value of par should be 1.0 if the correct standard deviation of error is assigned to each datum. When 0.0 is entered for tolc, the program assumes a default value of tolc=0.02. • mode=2: par is the user-input value of tradeoff parameter. In this case, tolc is not used by the program. • mode=3: none of the two input values are used by the program. However, this line of input still needs to be there. NOTE: When mode=1 both par and tolc are used. When mode=2 only par is used. When mode=3, neither par nor tolc are used. However, the third line should always have two values.
α_s	coefficient for the smallest model component.
α_e	coefficient for the derivative in the easting direction.
α_n	coefficient for the derivative in the northing direction.
α_v	coefficient for the derivative in the vertical direction. If null is entered on the eighth line, then the above four parameters take the following default values: alphas=0.0001, alphae=alphan=alphav=1.0 . None of the alpha's can be negative and they cannot be all equal to 0 at the same time.
idisk	parameter which determines how the sensitivity matrix will be accessed. =0: sensitivity matrix will be stored in memory. If there is not enough memory, idisk will be set to 1 automatically. =1: sensitivity matrix will be accessed from disk when needed. UPDATES FOR Ver4.0 IN THE APPENDIX.

Input files:

Input files can be any file name. If there are spaces in the path or file name, you MUST use quotes ("") around the entire path+filename.

obs.mag	input data file . The file must specify the standard deviations of the error. By definition these are greater than zero.

maginv3d.mtx	|	sensitivity matrix and depth weighting functionfile (calculated by MAGSEN3D).
ini.sus	|	initial model stored in the same way as model.sus. If null is entered, the default value of 0.001 is used. For a constant initial model, enter a value.
ref.sus	|	reference model stored in the same way as model.sus. If null is entered, the default value of 0.0 is used. For a constant reference model, enter a value.
bounds.sus:	|	(Version 4.0 only) Three options are possible here - either a file name, two constants or NULL. - bounds.sus: file name for a two-column file with bounds for each cell. - 2 numbers: two constants representing lower and upper bounds respectively - NULL: default susceptibility bounds of 0.0 and 1.0 will be used.
w.dat	|	weighting function (optional). If null is entered, the program assumes uniform weight of 1.0 .

Output files:

Output files are created by the programs. They have fixed file names.

maginv3d.log	|	The log file containing more detailed information for each iteration and summary of the inversion.
maginv3d.sus	|	Recovered susceptibility model.
maginv3d.pre	|	The predicted data.
maginv3d.aux	|	Auxiliary file listing the data misfit model norm, and Lagrange multiplier at different stages of the inversion. This is used only for the purpose of restarting the inversion.
maginv3d.kap	|	Temporary file containing the susceptibility model produced at different stages of the inversion. This is used only for the purpose of restarting the inversion.
maginv3d_nopos.sus	|	This file is output during the first part of the inversion. It contains the susceptibilities without the bounds constraints imposed. (Added for mag3d version 4.0)
maginv3d_XX.sus	|	These files are output after each beta iteration. (Added for mag3d version 4.0)

Example of maginv3d.inp control file:

The inversion is started from scratch with a zero reference model. The inversion will try to converge to a target misfit equal to the number of data. The sensitivity matrix will be stored in memory.

```
0                 !! irest
1                 !! mode
1.0 0             !! par, tolc
obs.nois          !! obsf
maginv3d.mtx      !! mtx file
0.001             !! initial model
0.0               !! reference model
0.5 0.001         !! Lower and uppper bounds (SI)
null              !! lower, upper bounds
0.0001 1. 1. 1.   !! alphaS, alphaE, alphaN, alphaZ
null              !! 3D weighting
0                 !! idisk
```

Log file explanation

(This explanation comes from the GRAV3D manual - the MAG3D log file is basically identical.) The log file maginv3d.log contains more detailed information about the convergence of the inversion. Depending on how the inversion is set up by the users, the content of the log file is slightly different. In general, there are two stages. In the first stage, the program estimates an approximate regularization parameter. In the second stage, the program performs the inversion with bound constraints using a logarithmic barrier method of minimization, which consists of outer iterations with the barrier parameter and inner iterations of conjugate gradient solution of an linear system within each barrier iteration. The log file information is organized according to these two levels of iterations.

We can refer to the flow chart Figure 3 in the Background chapter to understand the output in the log file. Below we briefly describe the content of the log file according to the parameter mode chosen for the inversion.

Mode 1:

In this mode, a user-defined target misfit is supplied by specifying chifact. The first stage of inversion performs a line search without the bound constraints to estimate an approximate value of the regularization parameter and the slope of the misfit curve. The log file identifies this segment and lists summaries of each linear solution such as the number of conjugate gradient (CG) iterations, data misfit, and model norm. The second stage of inversion carries out a number of logarithmic barrier solutions. Each solution corresponding to a single regularization parameter is obtained by a sequence of barrier iterations, and nested inside each barrier iteration is a set of CG iterations. This portion of the log file begins with the value of regularization parameter, initial values of data misfit and model norm. It is then organized in segments corresponding to barrier iterations and lists the number of CG iterations, data misfit, model norm, barrier parameter, and the value of the barrier function at the conclusion of that iteration. As barrier iterations progress, the data misfit, barrier parameter, and barrier function should decrease monotonically. The model objective function may increase or decrease depending on the nature of the initial model of the logarithmic barrier minimization. However, both data misfit and model objective function should plateau at the end.

One or more solutions may be obtained to complete the line search and to achieve the target misfit. Each solution will have a distinct portion in the log file. The trial values of the regularization are dynamically predicted during the inversion and they are not necessarily in any order. Upon completion of line search and the inversion, the log file will list a summary of the data misfit and model norm corresponding to different values of the regularization parameter sorted in increasing order.

Mode 2:

In this mode, the user specifies a value of regularization parameter and the program performs a single logarithmic barrier minimization to obtained the solution. The log file mainly consists of the information for one logarithmic solution as described in Mode 1.

Mode 3:

In this mode, the program first performs a GCV estimate of data noise to obtain an approximate value of the regularization parameter, and it then carries out a single logarithmic barrier minimization to obtain the inverse solution. During the first stage, a number of trial values of the regularization parameter is tested, and two linear systems are solved for each value. Thus the first part of the log file is organized according to the value of regularization parameter. For
each value, the numbers of CG iterations, data misfit, model norm, and GCV value are listed. At the end of GCV search, the log file lists the data misfit, model norm, GCV value according to the regularization parameter sorted in increasing order. The regularization parameter corresponding to the lowest GCV value is used to obtain the final solution in the second stage. The second part of the log file corresponds to a single logarithmic barrier minimization and it is identical to that when mode 2 is chosen.

3.5 MAGPRE3D

This utility multiplies a model by the sensitivity matrix in maginv3d.mtx to produce the predicted data. This program is included so that users who are not familiar with the wavelet transform and the structure of maginv3d.mtx can utilize the available sensitivity matrix to carry out model studies.

Command line usage:

`magpre3d maginv3d.mtx obs.loc model.sus`

Input files:

`maginv3d.mtx` = the sensitivityfile from MAGSEN3D.

`obs.loc` = the inducing field parameters, anomaly type and observation locations.

`model.sus` = the susceptibility model.

Output file:

`magpre3d.mag` = predicted data.

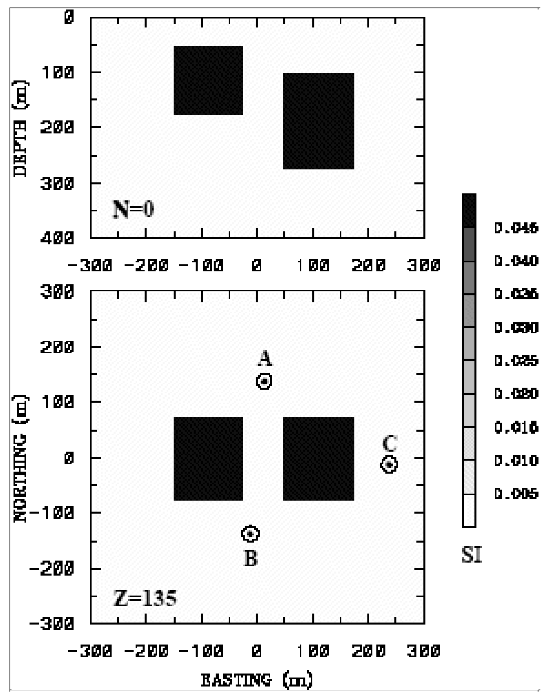
MAG3D manual:
Synthetic examples - Ver. 4.0

Introduction

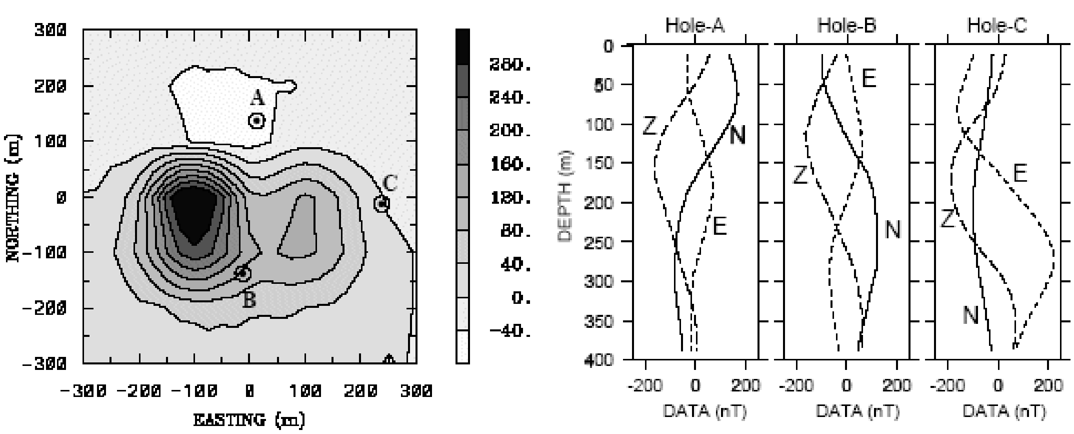
We present two synthetic examples to illustrate Version 3.0 of MAG3D. Important functionalities of MAG3D are the ability to handle multicomponent borehole data and the wavelet compression for large scale problems. The two synthetic examples are constructed to show these features. The first model consists of two vertical prisms, and both surface and borehole data are simulated. We illustrate the inversion of individual data sets and joint inversion of surface and borehole data. The second example is intended to be large and is composed of several prisms buried at different locations and depths beneath a topographic surface. Aeromagnetic data are simulated for this example. The size of this example is too large for the direct approach to handle. We show that the wavelet compression allows the solution of such a large problem with little demand on computing resources.

Example 1

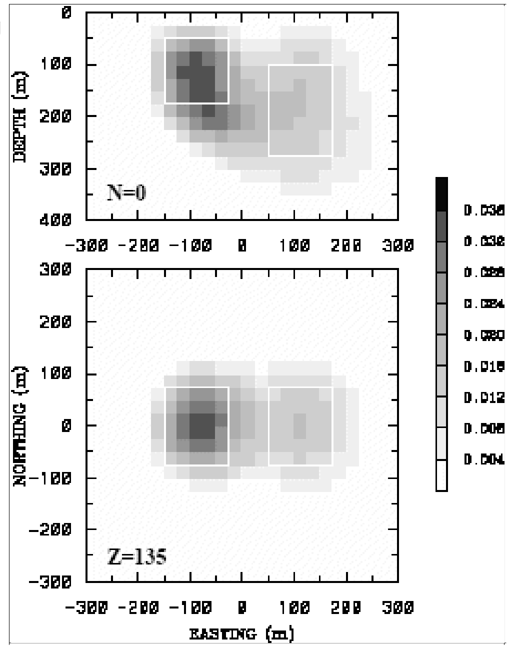
This figure displays the synthetic susceptibility model. It consists of two prisms buried in a non-susceptible background. The collar positions of three vertical boreholes are indicated on the plan-section. An inducing field in the direction I=65 and D=25 is assumed. The total field anomaly was calculated on the surface at an interval of 25 m along east-west lines spaced 100 m apart, resulting in a total of 175 data. Three-component data in easting, northing, and vertical directions are calculated in the boreholes. There are 16 observation locations spaced 25 m vertically in each hole, and the total number of data in three holes is 144.



Gaussian noise having a standard deviation of 2 nT has been added to all data and the resulting simulated observations are shown in the next figure where the surface data are shown as gray-scale contours and the three-component borehole data are plotted as functions of depth. For inversion, we use a m region and divide it into 24x24x16 cubic cells of 25 m on a side. This yields a total of 9216 cells. For all the inversions presented below, the distance weighting is used.



This figure shows the susceptibility model recovered from inverting the surface data alone. The true positions of the prisms are indicated by the white lines. This model clearly shows the presence of the shallow prism at the correct location but it does not give a clear indication of a separate, deeper prism. There is only a broad zone of low susceptibility extending from the high-susceptibility zone. The vertical extent of the anomaly is not well-defined.



Click the button to see the susceptibility model recovered by inverting the three-component borehole data. The model shows two regions of high susceptibility at locations corresponding to the true prisms, and the recovered depths agree well with the true depths. However, the amplitude of the shallow prism is small and there is no clear separation from the deep prism. Despite this, the model provides a good result considering that there are only three widely separated boreholes and that the inversion has no explicit information regarding where to place the magnetic material.

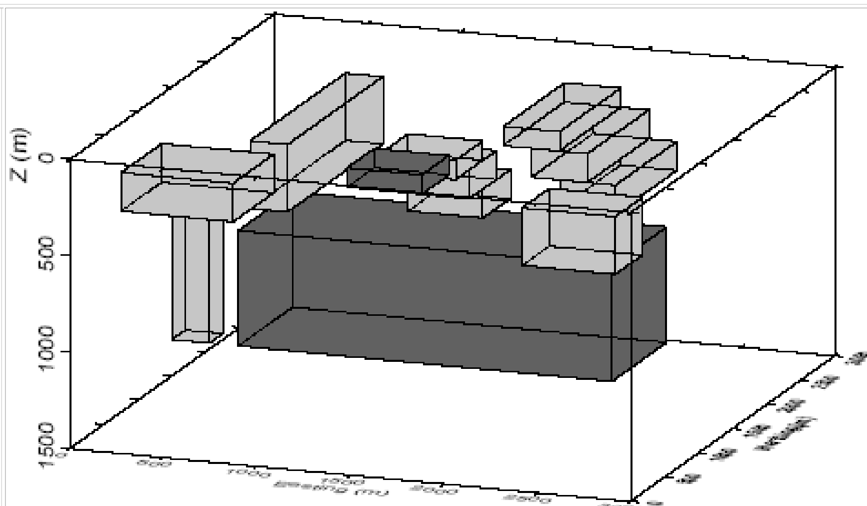
Click to see the susceptibility model obtained from the joint inversion of the surface and three-component borehole data. This model combines the merits of the models from individual inversions. Both target prisms are well define in horizontal and vertical locations and their amplitudes are comparable to those of the true model. This model provides the best representation of the true model.

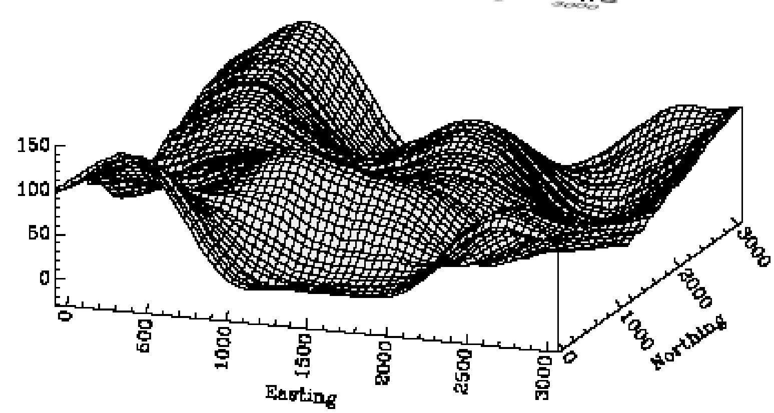
This is the total-field Anomaly Inversion. It shows the model recovered by inverting the surface and borehole total-fieldanomaly. The total field anomaly in the boreholes is first computed from the three-component data shown above. This simulates a more realistic situation since in many practical applications, only the total field anomaly can be extracted accurately from borehole measurements. The recovered model images both prisms. The success of this inversion demonstrates that single component borehole data can provide the information that is complementary to surface data when multicomponent data are not available.

Example 2

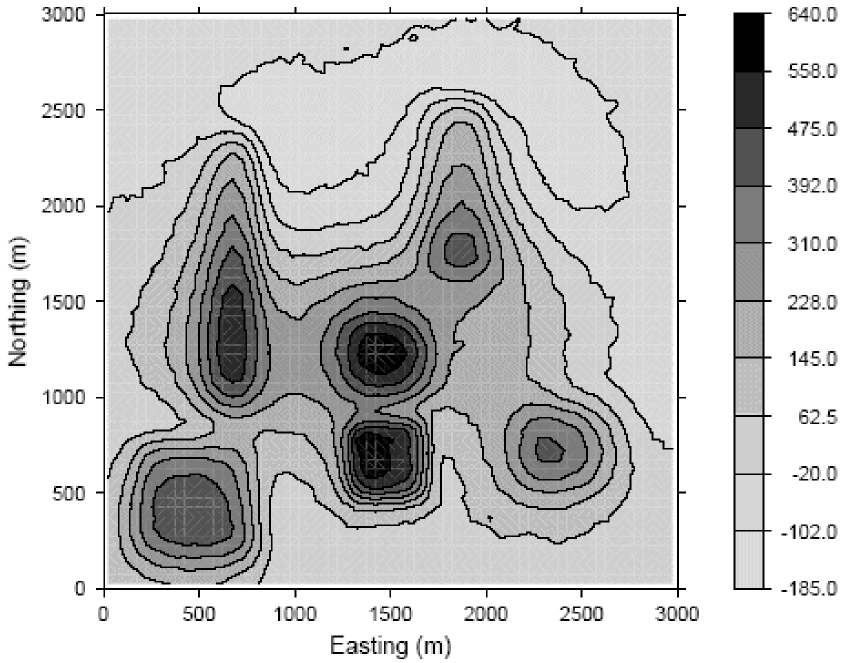
The true model is illustrated to the right. The figure showsthe relative sizes and positions of seven susceptible bodies buried in a non-susceptible background. The six smaller bodies are placed at shallow depths to simulate small-scale anomalies, and the large block is placed at a greater depth to generate a broad anomaly over which the small anomalies are superimposed.

The surface topography above this model is shown in the next figure. The elevation of the surface ranges mostly between 0 and 125 m, with a few points reaching 150 m.



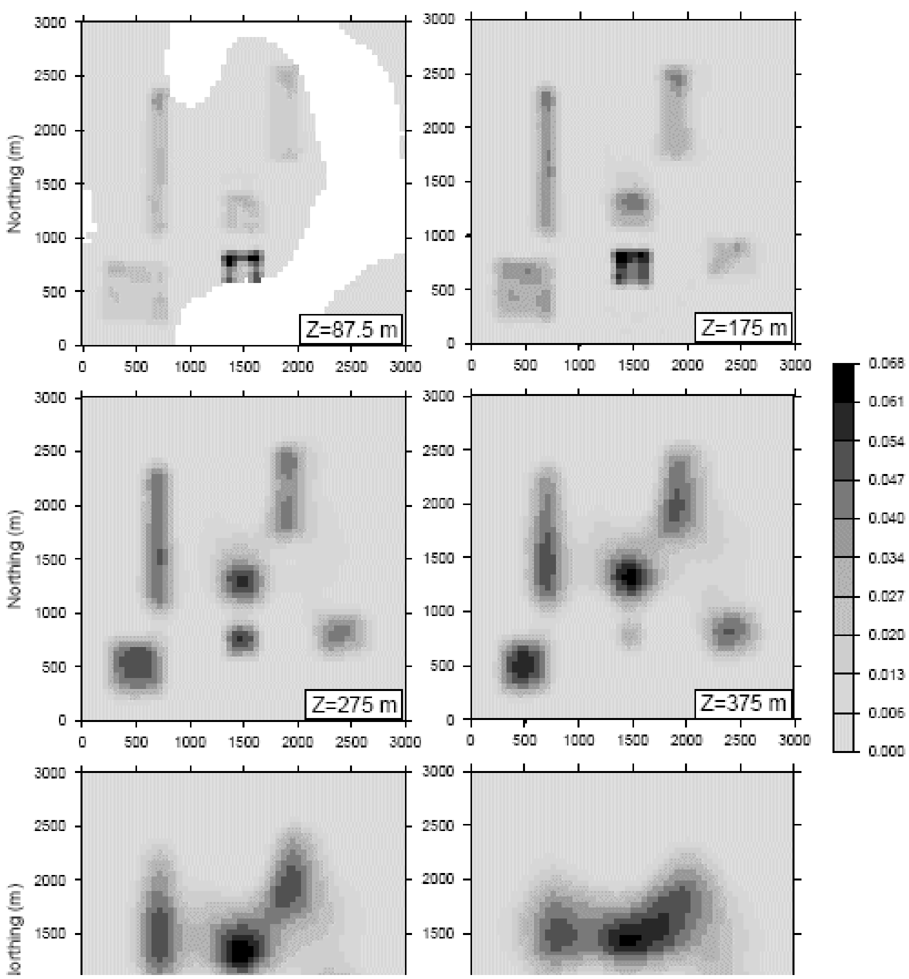


Simulated aeromagnetic data calculated at a constant terrain clearance of 75 m are shown next. The data are located on a grid with a 50–m spacing in both directions. This simulates a normal data set that has been decimated - it is NOT recommended that "gridded" data are used for inversion. The inducing field is assumed to be in the direction $I=65^{o}$ and $D=25^{o}$, and total field anomalies are calculated. The data have been contaminated by independent Gaussian noise having a zero mean and a standard deviation of 5 nT. The data show six anomalies due to the shallow blocks, but they provide no indication about the presence of the deep block.

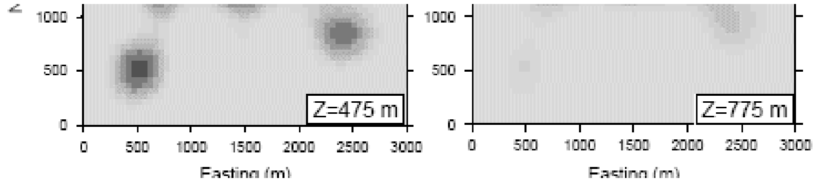


To invert these data, a model region of 3.2 km by 3.2 km by 1.5 km is used. The top of the mesh is placed at the elevation of 125 m. The cell width is 50 m in both horizontal directions and the thickness varies from 25 m near the surface to 100 m at the bottom. After the surface topography is discretized onto the mesh, the resulting model contains a total of 110,000 cells. The corresponding sensitivity matrix requires more than 1.5 Gb to store, and that is beyond the memory limit of many workstations. When compressed using the Daubechies-4 wavelet at a reconstruction accuracy of 5%, a compression ratio of 76 is achieved. The compressed sensitivity matrix requires on 43.5 Mb of storage. The sensitivity calculation takes 245 minutes on a SUN Sparc20 workstation, or 175 minutes on a 233–MHz MMX Pentium PC.

The model objective function is specifie by choosing $\alpha_S=0.0001$, $\alpha_X=\alpha_Y=\alpha_Z=1$, and a zero reference model.
The 3D weighting functions are all set to unity. The inversion is performed by setting the target misfit to the expected value of 3,600 and executing MAG3D with mode=1. The inversion uses 60 Mb of memory and lasts 146 minutes on the SUN Sparc20 workstation, or 110 minutes on a 233-MHz MMX Pentium PC having 64 Mb of memory (1995). Thus the entire procedure from the calculation and compression of the sensitivity to the inversion requires about 392 minutes on the SUN workstation, or 285 minute on the PC.



The last figure displays the susceptibility obtained from the inversion. The model is shown in six plan-sections. The different bodies in the true model are well- imaged. In particular, the large block at depth is cleary visible. The blank area in the section at Z=87.5 indicates the region above the topographic surface. The depth labelled in each section is referenced to the surface elevation of 125 m.

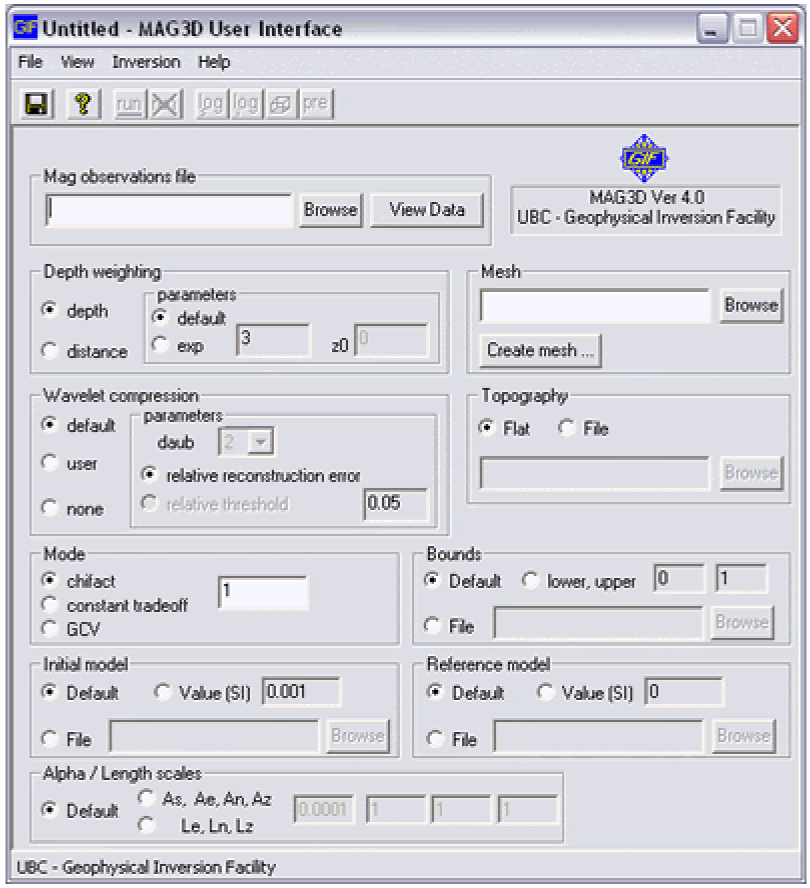


(selected) The large block at depth and the small block near the surface have a susceptibility of 0.08 SI unit, and the other blocks have a susceptibility of 0.05.

Click this button to see the true (synthetic) model.

MAG3D Ver 4.0 Help: Graphical User Interface - One-page version

UBC-GIF



This page contains brief instructions on using the MAG3D graphical user interface (shown to the right). It is provided as a printable version of the HTML "point-and-click" version, available by clicking here.

Menus

The four menus provide essentially the same functions as the tool-bar buttons described below, except for:

- The **File** menu **Open** option permits opening of a parameter file (*.inp) if one already exists.
- The **File** menu **New** option basically clears the interface to make a fresh start.

Save button

The parameter set in the GUI window must be saved before an inversion can be run. The results of inversion will be placed in the directory where this input file is saved.

Important note about directories:
Save each inversion run into a new directory because all output files get saved using default names.

Help button

This basically gives the program name and version number, and some notes about it's development. Please be aware of the copyright notice.

Run and kill buttons

The inversion can be run only after the parameters have been saved using the save button.

Use the "**kill run**" button to stop an inversion before it ends by itself. Do not end an inversion prematurly by closing the DOS window. This button is inactive unless an inversion is underway.

View resulting log, data and model files.

Four buttons here provide access to inversion results, and to log files summarizing the inversion program's performance.

- Clicking **logs** pops up the sensitivity log file into a text editor window. This is only useful for debugging.
- Clicking **logi** pops up the inversion log file into a text editor window. This contains details about each step and iteration of the inversion. It's format depends upon which mode was run (chifact,

constant tradeoff or GCV), and information is more useful to those who understand in detail the theory of the processing algorithms.
- Clicking the **model** button opens the inversion model in a new MeshTools3D window. MeshTools3D has it's own user manual.
- Clicking the **predicted data** button opens a new data viewing window with the observed data at the top, and the predicted data on the bottom. Predicted data were calculated by the inversion program on the final model. You can choose to see errors or misfit instead of predicted data. See the `gm-data-viewer.exe` help notes for more details.

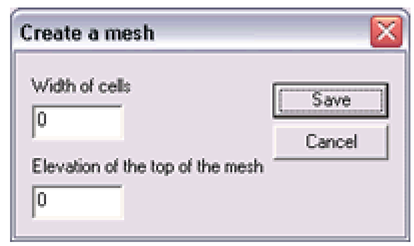
Data file

Specify the input data file here. See the MAG3D user manual for data file format. You can enter the file name (including full path), or drag and drop from Windows Explorer.

The **View Data** button opens the data set in a new window by running the `gm-data-viewer.exe` program.

Mesh

The mesh which defines the model is specified here. See the user manual section 2.2 (FILE: mesh) for details.



If a mesh has not been generated, a default mesh can be created using the **Create mesh ...** button. A data file must already be specified. This dialogue appears:

Specify the size of square cells in the core region (under the data set) by entering their width in units of metres. Cell depths will be half this width. Specify the elevation in metres for the top of the mesh. This must be above the topograpy. There are remarks on topograpy and meshes in several places in the manual, including section 2.2 FILE: mesh, and section 2.2 FILE: topo.dat.

The default mesh created by this dialogue will include 3 padding cells around the core region. These cells increase in size at greater distances from the core.

Wavelet compression parameters.

The compression used to speed up dense matrix multiplication is specified here.

- Use **default** to use parameters that have worked well in nearly all problems. This is the recommended mode.
- Specify **user** to set your own compression parameters. Details are in the user manual section 3.3.
  - The value of **daub** specifies the type of wavelet, as per section 3.3.
  - Selecting **relative reconstruction error** is equivalent to setting itol=1 as per section 3.3
  - Selecting **relative threshold** is equivalent to setting itol=2 as per section 3.3.
  - The **value** is set based upon which of these two choices is made.
- Selecting **none** will result in no compression, and probably rather long computation times.

Topography file

No topography file is necessary if the Earth's surface under the data set is assumed flat. If topography is significant then it should be specified, as per the user manual, section 2.2 FILE: topo.dat.

There are remarks on topograpy and meshes in several places in the manual, including section 2.2 FILE: mesh, and section 2.2 FILE: topo.dat.

Inversion mode

MAG3D Version 3.x can determine the tradeoff parameter in either of three ways.

1. Select **chifact**, and enter a value for *chifact*. The program chooses the tradeoff parameter by carrying out a line search so that a target value, ϕ_d^*, of data misfit is achieved. This target value is ϕ_d^* = *chifact* x *N*, where *N* is the number of data points. This is a common option if errors on data are assumed to be Gaussian and un-correlated.
2. Select **user** and enter a fixed value for the tradeoff parameter. This option is rarely used in practice.
3. Select **GCV** to ask the program to calculate a tradeoff parameter by applying the GCV analysis to the inversion without positivity. This option is computing intensive.

See the user manual section 1.6 for details regarding these three methods of choosing regularization.

Setting bounding constraints

This section allows for specification of upper and/or lower bounding values for susceptibilities of each cell. There are three ways of specifying bounds:

1. **Default:** If the user chooses to **not** define upper and lower bounds, the program employs default bounds for susceptibilities in every cell of 1.0 and 0.0 respectively (S.I. units). While it is true that some rocks have susceptibility greater than 1.0, MAG3D Version 4.0 still assumes small susceptibilities, as this code has done since the original version. When there are very high susceptibilities, the relation between incident and induced magnetization is no longer linear, and the problem becomes more complicated. Inverting data in the presence of very high susceptibilities is still a topic of research, and MAG3D Version 4.0 does not allow for high susceptibilities in the solution.
2. **Upper and lower values:** Use this option if you want to set a single upper bound value and a single lower bound for all cells.
3. **Using a file:** Each cells can have individual upper and lower bounds if a bounds.sus file is specified. See the MAG3D manual Chapter 2.

Initial model

This is the first susceptibility model the program works with. The outcome of the inversion should not depend much on this model, but if the initial model is close to the final one then convergence can be expected much more quickly.

- Selecting **default** causes the program to use a default initial model of zero susceptibility.
- Select **Value** and enter a susceptibility if the host rocks are not non-magnetic.
- Select **File**, and specify the file name, if a susceptibility model (defined on the mesh in use for inversion) is available.

Reference model

This is the reference susceptibility model that is part of the model objective function. The inversion will try to minimize the difference between the final model and this reference.

- Selecting **default** causes the program to use a default reference model of zero susceptibility.

- Select **Value** and enter a susceptibility if you know the background susceptibility of host rocks.
- Select **File**, and specify the file name, if a susceptibility model (defined on the mesh in use for inversion) is available for use in the model objective function as a reference.

Model objective function coefficients.

These are the parameters that control how much emphasis is placed on each part of the model objective function.

- Select default to specify default alpha parameters of *alphas=0.0001, alphax = alphay = alphaz = 1.0*
- Alternatively specify your own values, using either alpha parameters or length scales, and following these guidelines:

Influence of Alphas on results

Consider the ratios $\frac{\alpha_x}{\alpha_s}$, $\frac{\alpha_y}{\alpha_s}$ and $\frac{\alpha_z}{\alpha_s}$. Larger ratios result in smoother models. As a rule of thumb, keep this table in mind:

$\frac{\alpha_x}{\alpha_s}, \frac{\alpha_y}{\alpha_s}, \frac{\alpha_z}{\alpha_s} >> 1$	Structure is penalized. Constant reference models has little effect.
$\frac{\alpha_x}{\alpha_s}, \frac{\alpha_y}{\alpha_s}, \frac{\alpha_z}{\alpha_s} \Rightarrow 0$	Smallest term dominates, so models are rough.

To estimate values for the α's for a specific case, start by defining two length scale terms as follows:

$$L_x = \sqrt{\frac{\alpha_x}{\alpha_s}},\ L_y = \sqrt{\frac{\alpha_y}{\alpha_s}} \text{ and } L_z = \sqrt{\frac{\alpha_z}{\alpha_s}}$$

Then consider these rules of thumb to help choose the α's: In general, keep $L_{x \text{ and } y}$ approximately the same as the shortest array separation, and keep $L_{x \text{ and } y}$ approximately equal to L_Z. Also, the following relations are useful: '

$L_{X \text{ or } Y}$ > mesh cell width
L_Z > mesh cell thickness
$L_{X \text{ or } Y}$ < total width of the mesh
L_Z < total depth of the mesh

Top of MAG3D GUI help.
The user manual for MAG3D (PDF format) has complete details on every parameter and all file structures.