Numerical modelling for underground excavation design

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Synopsis

Provided that they are used correctly, numerical models can be of considerable assistance to mining engineers in designing underground excavations. Guidance is given to the engineer or geologist who is not a specialist in numerical analysis but who is interested in using numerical modelling tools or in employing someone to analyse the stresses, displacements and failure in the rock surrounding underground excavations. A description of the theoretical background to a number of these numerical models and discussion of the advantages and disadvantages of each are followed by guidelines on the selection of appropriate models for each stage in the development of a mine, from exploration to the completion of mining.

The creation of underground excavations results in significant changes in the stresses in the surrounding rock and an understanding of the behaviour of such excavations requires that the stresses and displacements in this rock mass be analysed. Such analysis has posed a long-standing challenge to rock mechanics engineers, and the historical development of rock mechanics has been closely associated with the development of methods for the analysis of stresses and displacements in rock structures.

Some of the earliest techniques of stress analysis utilized closed-form mathematical solutions or photoelastic models. However, even quite simple problems required mathematically skilled analysts and a significant amount of time for their solution and, as a result, analyses remained largely academic exercises.

The advent of computers in the 1960s made feasible the development of programs that were based on fairly simple concepts of elasticity but which required considerable computational effort. The results of these early attempts, combined with increasing computational power and efficiency, encouraged the development of programs that were capable of modelling increasingly sophisticated problems. Many such numerical techniques have now been developed, and the computational power that is available at modest cost makes possible the practical application of the majority of them.

The discussion that follows is intended to help the engineer or geologist who is interested in the analysis of stresses, displacements and failure in the rock surrounding underground excavations to become familiar with the types of modelling tools available and the strengths and limitations of each. The discussion is divided into two parts. In the first a framework is developed for understanding the theoretical basis of the most common methods and the practical limitations of various implementations of the methods. In the second part guidelines are provided for assessing where various models are best applied.

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Theoretical background

Fundamentals

The response of a rock mass to the creation of an excavation is a complex process: intact pieces of rock may deform elastically or fracture; joint surfaces may open, close or slip; and individual blocks may undergo rigid body displacements. As it is not practical to model each of these distinct processes in detail, the objective of the analyst is to determine which processes need to be considered explicitly and which can be represented in an averaged way.

The deformation of a body due to imposed loads can be regarded as being either continuous or discontinuous. The deformation illustrated in Fig. 1(*a*) is continuous, whereas that shown in Fig. 1(*b*) involves discontinuous behaviour. When investigating a particular problem the analyst must decide which regions of the rock mass may be treated as continuous and which surfaces should be explicitly represented as discontinuities. This decision is usually based on the relative amounts of sliding that may take place on various planes and the spacing between these planes relative to the dimensions of the excavation.

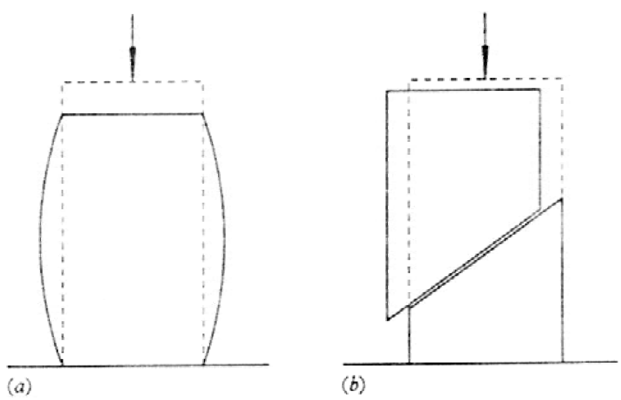


Fig. 1 (*a*) Continuous and (*b*) discontinuous behaviour of uniaxially loaded specimen

Once a representation of discontinuous and continuous behaviour has been established expressions are required that relate stress and strain. Such expressions are called 'constitutive relations'. A region composed of a single material is referred to as 'homogeneous', whereas one that consists of several materials is called 'heterogeneous'. Each homogeneous portion of a region requires a constitutive relation. Most available constitutive relations assume that the material is 'isotropic' (i.e. that material properties are invariant with respect to direction) or, less commonly, 'orthotropic' (i.e. that material properties are different in mutually perpendicular directions). It is also possible that material properties could vary along some non-orthogonal set of axes, but constitutive models capable of representing such behaviour are not widely available.

For the purposes of the present discussion constitutive relations will be broadly classed as elastic and elasto-plastic.

In elastic models an expression is developed that relates stress and strain uniquely. Linear elasticity is commonly used, but non-linear elastic models also exist. The term 'plasticity' is generally used to describe some residual or irrecoverable deformation that occurs as a result of loading. Elasto-plastic models use the concept of a 'yield function': the response of a material is elastic when the state of stress is within the bounds defined by the yield function, whereas plastic (and, possibly, some additional elastic) deformation occurs when the state of stress lies on the bound defined by the yield function. A variety of 'post-yield' behaviours may be postulated, such as strength increasing, remaining unchanged or decreasing as plastic deformation continues.

The first concern of the user is to determine how the particular problem at hand is best represented as a combination of explicit joint surfaces and regions of continua, each with appropriate constitutive definitions. Therefore, the focus here will be on reviewing the ways in which various methods of analysis treat these issues; detailed consideration of the formulations of various elements, the development of constitutive relations, and so on, is beyond the scope of the present discussion, although references will be made to more detailed works where appropriate.

Although all of the analytical methods that will be reviewed have been used to investigate certain problems with varying degrees of success, no single method has proved appropriate for all types of modelling. Thus, the reader should view the following discussion as though surveying the contents of a tool-box, each tool being best suited for particular aspects of the job at hand. Finally, the practical application of any computer-based method is greatly enhanced by the availability of efficient pre- and post-processors (the interfaces that the user works with to enter modelling data and retrieve solution results). As the pre- and post-processors can influence modelling efficiency significantly, a commercial package's implementation of these features should be scrutinized as carefully as its computational capabilities when the overall performance of the package is being considered.

Closed-form solutions

For simple materials and simple excavation geometries it is sometimes possible to develop closed-form solutions, which are mathematical relations between stresses and displacements for every point in the surrounding material. In 1898 Kirsch published a solution for the stresses and displacements around a circular hole in a biaxially loaded elastic plate, and this formed the basis for many early studies of rock behaviour around tunnels and shafts.[1] This solution is still used as a check for numerical solutions.

Following along the path pioneered by Kirsch, researchers such as Love, Muskhelishvili and Savin[2,3,4] published solutions for excavations of various shapes in elastic plates. A useful summary of these solutions and their application in rock mechanics was published by Brown in an introduction to a publication on analytical and computational methods in engineering rock mechanics.[5]

As rock mechanics developed into a discipline in its own right closed-form solutions that recognized the heterogeneous nature of rock and which explored some of the consequences of failure began to emerge. The progressive development of plastic failure around a circular tunnel in a hydrostatic stress field has been considered by a number of authors and forms a basis for the understanding of the interaction between a failing rock mass and the support installed in the tunnel. The work in this field has been summarized by Brown *et al.*[6]

Closed-form solutions still possess great value for a conceptual understanding of behaviour and for testing numerical models. For design purposes, however, these models are limited to very simple geometries and constitutive relations.

Limit equilibrium solutions

An important type of analysis, particularly for near-surface excavations in hard rock that contains well-developed joints and bedding planes, is the limit equilibrium technique. In this technique the gravitational stresses acting on a rigid wedge or block, separated from the surrounding rock mass by intersecting discontinuities, are calculated and are checked against the shear resistance offered by the contact surfaces to determine whether the block can fall or slide. Since confining stresses are difficult to incorporate into a limit equilibrium model, this technique is limited to analyses in which surrounding stress fields can be ignored. Work on the application of limit equilibrium techniques to the analysis of the stability of structurally defined wedges in dam foundations by Londe[7] laid much of the groundwork on which subsequent rock mechanics research-workers have built. In particular, the work of Warburton[8,9] Goodman and Shi[10] and Lin and Fairhurst[11] has resulted in sophisticated three-dimensional models that can be used in a wide variety of circumstances encountered in underground excavations. These solutions are difficult to follow and to implement because of the inherently complex three-dimensional geometry of actual rock masses. It is expected that these models will be most useful for research on general trends of rock mass behaviour and that simpler and more user-friendly models will eventually be developed for use in the design of typical underground excavations.

Numerical methods

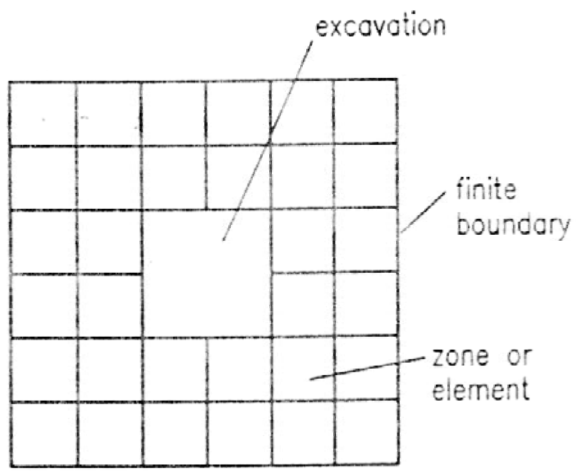
Most underground mining excavations are irregular in shape and are frequently adjacent to other excavations. These groups of excavations—which may be stopes or the various service openings associated with a ramp or shaft system—form a set of complex three-dimensional shapes. In addition, since orebodies are frequently associated with such geological features as faults and intrusions, the rock properties are seldom uniform within the rock volume of interest. Consequently, the closed-form solutions described earlier are of limited value in calculating the stresses, displacements and failure of the rock mass surrounding such mining excavations. Fortunately, a number of computer-based numerical methods have been developed over the past few decades and these provide the means for obtaining approximate solutions to such problems.

The approach adopted in all of the numerical methods is to divide the problem into smaller physical and mathematical components and then sum the influence of the components to approximate the behaviour of the whole system. The series of equations formed in this process require some method of solution; the most common is to formulate the problem as a set of simultaneous equations that are assembled as matrices and vectors and solved for accordingly with the use of a variety of techniques for solving matrices. This—the matrix or implicit solution technique—is most efficient for solving problems with comparatively simple constitutive laws. Where behaviour is more complex a solution requires multiple steps and matrix reformulations, a procedure that lowers the efficiency of solution. An alternative technique for solution is based on the assumption that a disturbance at a point in space is initially felt only by points in the immediate vicinity. With 'time' (i.e. over succeeding computational steps) the disturbance spreads from point to point throughout the region until equilibrium is established. Even for simple problems, this requires a long computational process and care has to be taken in applying damping to control numerical oscillations. It does not, however, require the formulation or solution of a matrix. This explicit or dynamic relaxation technique has practical implications in terms of improved efficiency, numerical stability and accuracy when the problem includes complex constitutive relations and severe gradients in the quantity of interest over the model region.

Either solution technique can be used for any of the following methods although, for most individual modelling packages, convention has led to the exclusive use of one scheme or the other.

The numerical methods used in rock mechanics can be divided into two classes—boundary and domain methods. In the first only the boundary of the excavation is divided into elements and the interior of the rock mass is represented mathematically as an infinite continuum. Domain methods divide the interior of the rock mass into geometrically simple zones, each with assumed properties. The collective behaviour and interaction of these simplified zones model the more complex and otherwise unpredictable overall behaviour of the rock mass. Finite-element and finite-difference techniques are domain methods that treat the rock mass as a continuum, and the distinct-element method is a domain method that models each individual block of rock as a unique element.

These two distinct classes of analysis can be combined in the form of hybrid models to maximize the advantages and minimize the disadvantages of each method.



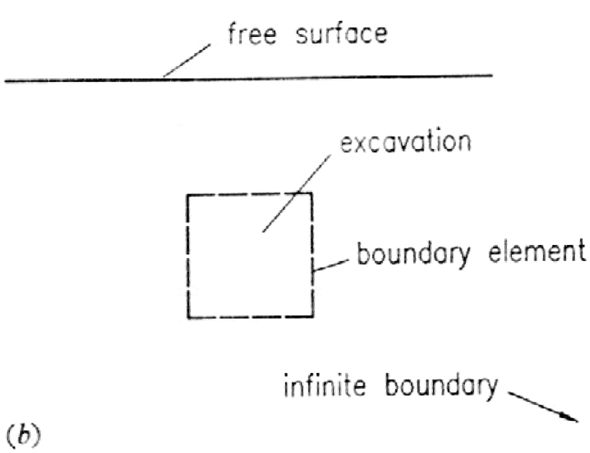


Fig. 2 Domain and boundary methods of modelling rock surrounding underground excavation: (*a*) division of model into elements in domain methods; (*b*) definition of excavation boundary in boundary methods

Some general observations can be made on the basis of the two types of simplified models illustrated in Fig. 2. In domain methods a significant amount of effort is required to discretize the model (divide it into zones). With complex models, such as those containing multiple openings, discretization can become extremely difficult. In contrast, boundary methods require that only the excavation boundary be discretized and the surrounding rock mass is treated as an infinite continuum. Since fewer elements are required in the boundary method, the demand on computer memory and on the skill and experience of the user is reduced.

In domain methods the outer boundaries of the discretized model must be placed sufficiently far away from the excavations that errors arising from the interaction between these outer boundaries and the excavations are reduced to an acceptable minimum. On the other hand, since boundary methods treat the rock mass as an infinite continuum, the far-field conditions need only be specified as stresses that act on the entire rock mass and no outer boundaries are required. The main strength of boundary methods lies in the simplicity that is achieved by representing the rock mass as a continuum of infinite extent. It is this representation, however, that generally precludes the incorporation of variable material properties and the modelling of interactions between rock and support. Although techniques have been developed to allow some representation of variable rock properties in boundary-element modelling, these types of problems are more conveniently modelled by domain methods.

Before selecting the appropriate modelling technique for particular types of problems it is necessary to understand the basic components of each technique.

Boundary-element method

The boundary-element method derives its name from the fact that the user 'discretizes', or divides into elements, only the boundaries of the problem geometry (i.e. excavation surfaces, the free surface for shallow problems, joint surfaces where joints are explicitly considered and material interfaces for multi-material problems). In fact, several types of boundary-element methods are collectively referred to as 'the boundary-element method' since they all possess the feature just described.

These methods may be divided into three groups: first, the indirect (fictitious stress) method, so named because the stress conditions on the boundary are found first and separate relations are subsequently applied to find boundary displacements; second, the direct method, so named because unknown stresses and/or displacements are solved for directly from the specified boundary conditions; and, third, the displacement discontinuity method, so named because it represents the result of pulling apart a 'slit' in an elastic continuum.

The differences between the first two methods are, typically, transparent to the user. The direct method has certain advantages in terms of program development, as will be discussed later in the section on 'hybrid approaches'.

Central to the development of the boundary-element method is the concept of a 'fundamental solution'—for example, the solution for the stress and displacement fields in an infinite medium due to the application of a load at a point. This solution can be integrated over lines or areas to develop two-dimensional and three-dimensional elements, respectively. Thus, a relationship is established between the conditions on the surface of the element and the conditions at all points within the remaining medium. This implies that each element can have an effect on every other element.

A system of linear equations is assembled, the members of the coefficient matrix representing the influence of one element on another. Because each element can influence every other element, the coefficient matrix is generally fully populated. In contrast, the matrices typically formed by a finite-element package are banded. For a given number of equations fully populated matrices take longer to solve than do banded matrices. However, for a particular problem the boundary-element method typically requires fewer elements since only the boundary is discretized. In practice, for elastic analyses the time required to arrive at a solution by either method is negligible compared with that required to construct the model and interpret its results.

Because of the nature of the fundamental solution, the state at a point in the medium can be determined solely from the conditions on the discretized boundaries. This is used to advantage in modelling underground excavations by devising the problem so that the solution reflects the fact that far-field stresses are not influenced by the excavation's creation. In contrast, in the other numerical techniques the far-field conditions must be approximated in some way—for example, by discretizing beyond the 'zone of influence' of the excavation and fixing the outer boundary or applying to it the complement of the *in-situ* stresses.

The fact that a boundary element represents an influence 'to infinity' can also be a disadvantage. For example, a heterogeneous rock mass consists of regions of finite, not infinite, extent. Special techniques must be derived to handle these situations. Although such techniques may work well for simple problems, it is usually desirable to use one of the other numerical methods for increasingly sophisticated models.

Joints are modelled explicitly in the boundary-element method by means of the displacement discontinuity approach. Reasonably general constitutive modelling of joint surfaces is possible, though at increased computational expense. Also, numerical convergence is often found to be a problem for models that incorporate many joints. For these reasons problems that require the explicit consideration of several joints and/or sophisticated modelling of the constitutive behaviour of joints are often better handled by one of the remaining numerical methods.

A widely used application of displacement discontinuity boundary elements is in the modelling of tabular orebodies. Here the entire ore seam is represented as a 'joint', initially filled with ore that may have a different stiffness from that of the host rock. Mining is simulated by reducing the stiffness of the ore to zero in those areas where mining has occurred, and the resulting stress redistribution to the surrounding pillars may be examined.[12,13]

Further details of boundary-element methods can be found in the work of Crouch and Starfield.[14]

Finite-element and finite-difference methods

In practice the finite-element method is usually indistinguishable from the finite-difference method; thus, they will be treated here as one and the same. For the boundary-element method it was seen that conditions on a surface could be related to the state at all points throughout the remaining medium, even to infinity. In comparison, the finite-element method relates the conditions at a few points within the medium (nodal points) to the state within a finite closed region formed by these points (the element). The physical problem is modelled numerically by discretizing (i.e. dividing into zones or elements) the problem region (Fig. 3).

The finite-element method is well suited to solving problems that involve heterogeneous or non-linear material properties since each element explicitly models the response of its contained material. However, finite elements are not well suited to modelling 'infinite' boundaries, such as occur in problems relating to underground excavation. As

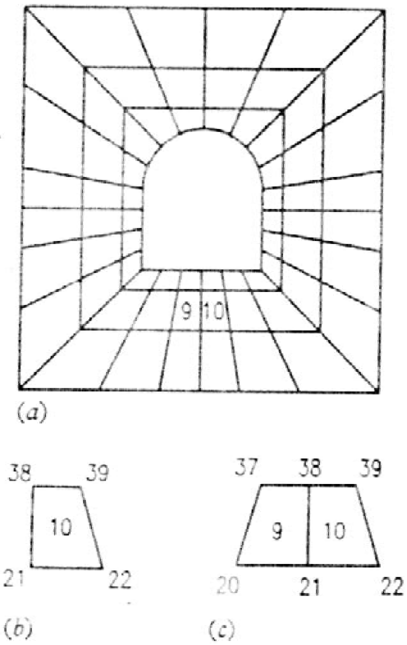


Fig. 3 Development of finite-element model of continuum problem: (*a*) finite-element mesh; (*b*) single element; (*c*) shared nodes

previously noted, one technique for handling infinite boundaries is to discretize beyond the 'zone of influence' of the excavation and to apply appropriate boundary conditions to the outer edges. Another approach has been to develop elements one of whose edges extends to infinity (i.e. the so-called 'infinity' finite elements). In practice, efficient pre- and post-processors allow the user to perform parametric analyses quickly and to assess the influence of approximated far-field boundary conditions, the time required for this process being negligible compared with the total time of analysis.

Joints can be represented explicitly by means of specific 'joint elements'. Different techniques have been proposed for handling such elements, but no single technique has found universal favour. Joint interfaces may be modelled with the use of quite general constitutive relations—though possibly at increased computational expense, depending on the technique employed for solution.

Once the model has been discretized, material properties have been assigned and loads have been prescribed some technique must be used to redistribute any unbalanced loads and thus determine the solution to the new equilibrium state. The techniques available for solution can broadly be divided into two classes—implicit and explicit—with respect to time. Implicit techniques assemble systems of linear equations, which are then solved by standard techniques of matrix reduction. Any material non-linearity is accounted for by modifying stiffness coefficients (secant approach) and/or by adjusting prescribed variables (initial stress or initial strain approach). These changes are made in an iterative manner such that all constitutive and equilibrium equations are satisfied for the given load state. The response of a non-linear system generally depends on the sequence of loading, and thus it is necessary that the load path modelled be representative of the actual load path experienced by the body. This is achieved by breaking the total applied load into increments, each increment being sufficiently small to ensure solution convergence for the increment after only a few iterations. However, as the modelled system becomes increasingly non-linear and individual load increments represent ever smaller portions of the total load the incremental solution technique increasingly resembles modelling the quasi-dynamic behaviour of the body as it responds to gradual application of the total load. In recognition of this a 'dynamic relaxation' solution technique was proposed by Otter and co-workers[15] and first applied to geomechanics modelling by Cundall.[16] In this technique no matrices are formed. Rather, solution proceeds explicitly in the time domain—unbalanced forces acting at a material integration point result in acceleration of the mass that is associated with the point; the application of Newton's law of motion expressed as a difference equation yields incremental displacements; applying the appropriate constitutive relation produces the new set of forces, and so on marching in time, for each material integration point in the model. (Some intermediate steps have been omitted for brevity; see Cundall[17] for a detailed discussion.) This solution technique has the advantage that both geometric and material non-linearities are accommodated with little additional computational effort relative to a corresponding linear analysis, and computational expense increases only linearly with the number of elements used. A further practical advantage lies in the fact that numerical divergence is usually manifest in the form of obviously anomalous physical behaviour. Thus, even quite inexperienced users may recognize numerical divergence.

Most commercially available finite-element packages use implicit (i.e. matrix) solution techniques. For linear problems and problems of 'moderate non-linearity' implicit techniques tend to perform faster than explicit solution techniques. However, as the 'degree of non-linearity' of the system increases imposed loads must be applied in smaller increments, which implies a greater number of matrix reformulations and reductions and, therefore, increased computational expense. Hence, highly non-linear problems are best handled by packages that employ an explicit solution technique.

Distinct-element method

In ground conditions that are conventionally described as 'blocky' (i.e. where the spacing of the joints is of the same order of magnitude as the dimensions of the excavation), intersecting joints form blocks and wedges of rock that may be regarded as rigid bodies. That is, individual blocks may be considered rigid (i.e. not deformable) because individual bodies of rock may be free to rotate and translate and the deformation that takes place at the contacts between blocks may be significantly greater than the deformation of the intact rock. For such ground conditions it is usually necessary to model many joints explicitly. However, the behaviour of such systems is so highly non-linear that even a jointed finite-element code employing an explicit solution technique may perform quite inefficiently.

An alternative modelling approach is to develop data structures that represent the blocky nature of the system being analysed. Each block is considered to be a unique free body that may interact at its contacts with surrounding blocks. Contacts may be represented by the overlaps of adjacent blocks, thereby avoiding the necessity of unique 'joint elements'. This has the added advantage that arbitrarily large relative displacements may occur at the contact, a situation that is not generally tractable in finite-element programs.

Because of the high degree of non-linearity of the systems being modelled explicit solution techniques are favoured for distinct-element codes. As for finite-element codes employing explicit solution techniques, this permits a very general constitutive modelling of joint behaviour with little increase in computational effort, results in computation time being only linearly dependent on the number of elements used and reduces the demands on the skills and experience of the user by comparison with codes that employ implicit solution techniques.

An example of the use of the distinct-element method by Vogele and co-workers[18] to model the behaviour of a jointed rock mass surrounding an underground opening is illustrated in Fig. 4.

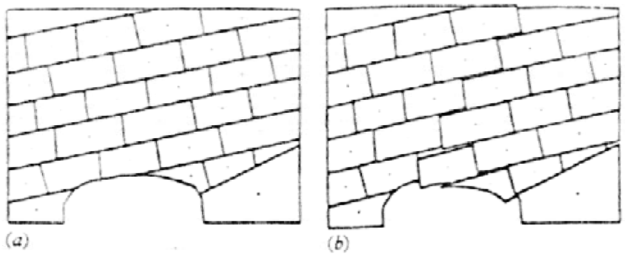


Fig. 4 Distinct-element modelling of jointed rock mass: model of rock mass surrounding excavation (*a*) before failure and (*b*) after failure. (After Vogele and co-workers[18])

Although the distinct-element method has until now been used most extensively in academic environments, it is quickly finding its way into the offices of consultants and mine planners and designers as well. Further experience in the application of this powerful modelling tool to practical design situations (and the subsequent documentation of such application) is required so that an understanding may be developed of where, when and how the distinct-element method is best applied.

Hybrid approaches

The relative strengths and weaknesses of each of the preceding numerical methods can be summarized as in Table 1.

Again, it is emphasized that suitable pre- and post-processors enhance productivity considerably. For example, the time required to mesh surfaces (boundary-element models) is comparable to the time required to mesh volumes (finite-element models) when efficient mesh generators are available. Therefore, the ease of meshing has not been considered as an inherent advantage/disadvantage of a particular numerical method.

The objective of a hybrid method is to combine the above methods so as to eliminate as many of the undesirable characteristics as possible while retaining as many of their advantages as possible. For example, in modelling an underground excavation most non-linearity will occur close to the excavation boundary, whereas the rock mass at some distance will behave in an elastic fashion. Thus, the near-field rock mass might be modelled by means of a distinct-element or finite-element method, which is then linked at its outer limits to a boundary-element method so that the far-field boundary conditions are modelled 'exactly'. In such an approach the direct boundary-element technique is favoured as it gives increased programming and solution efficiency. As another example, a highly fractured rock mass may contain more widely spaced, preferentially weak joint sets. The weak joint sets delineate blocks and wedges—as described for the distinct-element method—but now the individual blocks are fractured and cannot be considered rigid. A suitable approach in such situations may be to discretize each distinct-element block internally into finite elements. Thus, the response of the preferentially weak joints is modelled explicitly, whereas the response of the more competent joint sets is modelled implicitly as continuum behaviour. Fig. 5 illustrates an example in which Lorig and Brady[19] used a hybrid model consisting of a discrete-element model for the

Table 1 Relative strengths and weaknesses of numerical methods

	Advantages	Disadvantages
Boundary-element method	Far-field conditions inherently represented	Coefficient matrix fully populated
	Only boundaries require discretization, resulting in smaller number of solution variables than for finite-element methods	Solution time increases exponentially with number of elements used
		Limited potential for handling heterogeneous and non-linear materials
Finite-element and finite-difference methods	Material heterogeneity easily handled	Entire volume must be discretized, resulting in larger number of solution variables than for boundary-element methods
	Material and geometric non-linearity handled efficiently, especially when explicit solution technique is used	Far-field boundary conditions must be approximated
	When implicit solution techniques are used matrices are banded	For linear problems explicit solution techniques are relatively slow
	When explicit solution techniques are used less skill is required of user in assessing numerical convergence	Solution time increases exponentially with number of elements used for implicit solution techniques
Distinct-element method	Data structures well suited to modelling systems with high degree of non-linearity resulting from multiple intersecting joints	Solution times seem much slower than for linear problems
	Very general constitutive relations may be used with little penalty in terms of computational expense	Results can be sensitive to assumed values of modelling parameters (these 'disadvantages' are natural consequence of nature of system being modelled; as there is currently no modelling alternative for such problems, the term 'disadvantage' must be interpreted accordingly)
	Solution times increase only linearly with number of elements used	

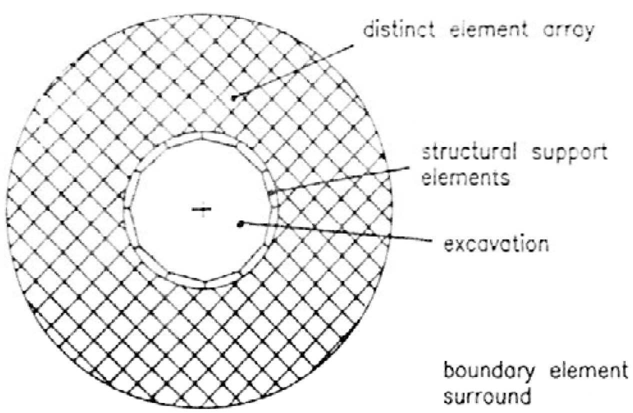


Fig. 5 Hybrid model composed of distinct elements, surrounding excavation, coupled to boundary-element model. (After Long and Brady[19])

near-field conditions and a boundary-element model for the far-field conditions in a rock mass around a circular tunnel. Other examples have been discussed by Brady.[20]

Guidelines for selection of appropriate models

Some of the discussion that follows is based on an excellent review by Starfield and Cundall[21] in which the historical and future perspectives of rock mechanics modelling are critically examined. Anyone who is seriously considering investing in or embarking on a rock mechanics modelling project is urged to read this in its entirety. In the context of the present discussion the emphasis will be placed on the selection of appropriate rock mechanics models.

Fig. 6 shows a simple classification of modelling problems from Starfield and Cundall[21] after Holling.[22] The diagram divides modelling problems into four regions.

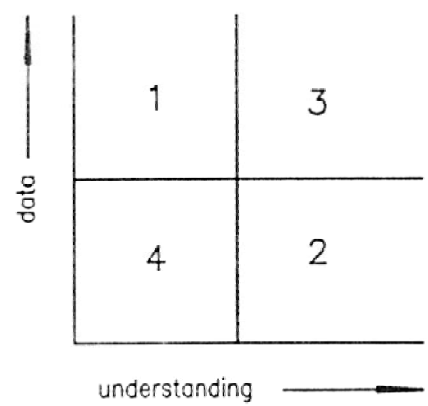


Fig. 6 Classification of modelling problems. (After Holling[22])

Region 1 is characterized by an abundance of data but a lack of understanding of the problem. This type of problem is very rare in rock mechanics, the only example that comes to mind being the abundance of microseismic data that are collected in mines prone to rockburst and the limited progress that has been made in their interpretation.

In region 3 both data and understanding are available and models can be developed, validated and used with confidence. Unfortunately, rather few problems in rock mechanics fall into this category, which is more appropriate to such fields as structural and mechanical engineering, where designed components of manufactured materials are analysed.

Region 2, where the database is poor but the level of understanding is high, may be applicable to a few situations in rock mechanics. Unfortunately, much of the understanding is in the form of practical experience and is difficult to quantify for incorporation into models. A typical example may be in blasting where successful techniques have been developed on the basis of trial and error and where the measurement of all of the parameters required to develop adequate numerical models is very difficult.

Region 4, which is characterized by low levels of both data and understanding, is the region into which most rock mechanics problems fall. Problems in this region are often ill posed, leading to difficulties in interpreting the results of modelling and to questions as to whether the correct problem has been modelled. Adequate validation of these models may be impossible and they should never be used routinely as, for example, models used by structural engineers are. Rock mechanics models should be used cautiously and thoughtfully at all times and they should never be run once only; it is in the sensitivity of the results to changes in parameters and assumptions that the model is most informative.

On the basis of their review of the current state of rock mechanics modelling and their own extensive experience as modellers Starfield and Cundall offered the following comments on modelling in rock mechanics.

(1) A model is a simplification, rather than an imitation, of reality. It is an intellectual tool that has to be designed or chosen for a specific task.

(2) The design of the model should be driven by the questions that the model is supposed to answer rather than by the details of the system that is being modelled. This helps to simplify and control the model.

(3) It might even be appropriate to build a few simple models rather than one complex model; the simple models would either relate to different aspects of the problem or address the same questions from different perspectives.

(4) The aim should not be to attempt to validate a model but to gain confidence in it and modify it with use. One's approach to the model should be like that of a detective rather than a mathematician.

Progress is slow and painful, from region 4 of Holling's diagram towards region 3, and is accomplished by a kind of 'bootstrap' operation. First one builds a simple model and exercises it in a conjectural way. The results almost always suggest new ways of obtaining data or new ways of interpreting available data. New data, in turn, suggest improvements to the model or ideas for new models. Implementing those improvements leads to requirements for new data or to insights, and so on. The whole process may be termed 'adaptative modelling'.

On the assumption that the reader is a user rather than a modeller, the discussion that follows is provided in an attempt to translate Starfield and Cundall's comments[21] into practical guidelines for mining engineers. The problem that is considered encompasses the complete process of feasibility study, detailed design and continuing analysis during the mining of an underground base-metal deposit in a hard rock environment.

Guide to application of rock mechanics modelling over life of underground mine

Exploration

During the very early stages of the feasibility study the amount of information available is minimal. The orebody has probably been explored by diamond drilling and its possible extent roughly defined. The cores have given a good indication of the types of rock that are likely to be encountered in the hanging-wall, orebody and footwall, but, because of the nature of exploration drilling, probably do not provide very much quantitative information on the spacing of discontinuities, the degree of alteration and other factors that can have a significant influence on the characteristics of the rock mass.

Rock mechanics modelling has very little to offer at this stage in helping the owner to decide whether mining is feasible and what mining method should be used. These decisions will probably have to be based on an assessment of the type of orebody, its depth and three-dimensional shape, the types of rock surrounding the orebody and the presence of known faults and sources of subsurface water. Comparison of these factors with similar deposits that are already being mined will suggest the range of mining methods that should be considered and whether mining of the deposit is likely to be economically viable.

Preliminary feasibility studies

Once these basic decisions have been made and it has been decided to proceed with the feasibility study a number of possible underground mining schemes will be evaluated and some detailed questions will be addressed to the rock mechanics engineers in the company or to the consultants brought in to assist with the feasibility study. Typical questions concern the maximum probable stope spans, the optimum location of shafts and service excavations, pillar dimensions and the possibility of pillar recovery, the potential for rockbursts and the possibility of using backfill as a support medium. It is unlikely that sufficient data are available at this stage to permit the use of detailed rock mechanics modelling tools and only the simplest of conceptual models should be considered.

On the basis of the types of rocks encountered in the exploration drilling a first estimate of the possible range of rock mass characteristics will have to be made. Hoek and Brown[23,24] have given some very approximate guidelines for estimating rock mass strengths and deformation moduli, and these guidelines, together with any other available information, will have to suffice for such estimates. The *in-situ* stresses will almost certainly be unknown at this stage, and the trends published by Brown and Hoek,[25] along with any information gathered from adjacent mines, will probably form the basis for estimates of the probable range of *in-situ* stresses. Because of the wide range of values for rock mass properties and *in-situ* stresses it is essential that a large number of options should be investigated in an attempt to find bounds for the problem. This dictates that the model should be comparatively simple, very user-friendly, fast to operate and that it should provide some form of graphic output that is easy to evaluate and can be used as a communication tool in discussions with mine planners and management.

Of the currently available rock mechanics models, the group that is best suited to the task outlined above is that comprising the two-dimensional boundary-element models. Many of these will run efficiently on microcomputers and satisfy all of the other requirements in terms of graphics input and output and simplicity of operation. In spite of the limitations imposed by the facts that these models are two-dimensional and that they can only be used to model elastic rock masses, they can still be used to give a first approximation of zones of possible overstress around the proposed mine openings and of the possible influence of variations in rock mass properties and *in-situ* stresses. Details of support, whether in the form of rockbolts and cables or backfill, cannot be adequately modelled by two-dimensional boundary-element models and such studies will have to be left to a later stage of the evaluation.

Detailed feasibility studies

At some stage in the feasibility study more geological information will become available. This may be in the form of core recovered from a drilling programme that has been specifically designed to include geotechnical information or from an exploration shaft or adit that has been excavated to explore the orebody in greater detail. In addition to establishing the types of rock that are present, this geological information will give an indication of the nature, spacing and orientation of discontinuities, such as bedding planes, joints or faults, in the rock mass. Additional information on subsurface groundwater will be available and, possibly, some measurements of *in-situ* stresses will have been obtained by means of hydraulic fracturing or overcoring techniques. All this information will provide a basis for a critical re-evaluation of the results obtained from the earlier two-dimensional elastic analyses.

If the orebody is located at a significant depth below surface (say, more than 1000 m) in hard, massive rock, it is probable that most of the problems that will be encountered in mining the deposit will be associated with stress-induced failure. Rockbursts will have to be accepted as a potential threat and consideration will have to be given to the layout and sequencing of the mining operation in such a way that zones of high stress are minimized. Except in the vicinity of major geological features (such as faults), it is unlikely that local support will play a significant role in the stability of the mine. On the other hand, backfilling could have a major influence on the stability of individual or groups of stopes and on the potential for rockbursts in the surrounding rock. In these circumstances two-dimensional elastic modelling, such as that provided by boundary-element models, will probably be of limited value, although the use of these models in support of more sophisticated models should certainly be continued.

Three-dimensional boundary-element or finite-element models may have to be used if complex three-dimensional mine layouts are required;[26,27] otherwise, where the shape of the orebody is less complicated, displacement discontinuity models may provide a better option.[13] It is probable that, in selecting these models, the advice of a specialist consultant should be sought since a great deal of time and money can be lost if an inappropriate model is selected. Care should be taken that the modelling programme is not too ambitious since the project is still at the feasibility stage and the available data do not justify very detailed analysis.

If the orebody is located at shallow depth and if the geological features in the rock mass are well defined, the stability of the underground openings will probably be controlled by such geological features rather than by the stress-induced failures described earlier. Under these circumstances emphasis should be placed on modelling the potential for failure of wedges or blocks that can fall or slide under gravitational loading. Simple manual stereographic analyses or the use of closed-form analytical solutions, such as those published by Hoek and Brown,[23] are probably adequate for such analyses.

Detailed design

Once the decision has been made to proceed with mining the study moves from the feasibility to the detailed design stage. The process of detailed design should continue throughout the life of the mine and should be responsive to specific problems that are encountered as stopes are developed and pillars recovered. Some of the studies undertaken for this stage of development will relate to 'permanent' openings, such as shaft stations, underground crusher chambers or main access openings, such as shafts, ramps and haulages. These studies are similar to many of those carried out for civil engineering purposes and generally take the form of two-dimensional elastic analyses by means of boundary-element techniques or, where rock failure occurs, finite-difference or finite-element studies.

Early years of mining

Unless the characteristics of the rock mass have been seriously misjudged during the feasibility stage or the wrong mining method has been chosen, the first few years of mining are generally relatively trouble-free. Extraction ratios are low, mining depths are generally moderate and the owners can enjoy a reasonable return on their investment without having to worry about underground stability problems, groundwater control and rockbursts. Unfortunately, it is during these trouble-free years that much of the expertise accumulated during the feasibility stage is lost and much of the knowledge that was generated is filed away and forgotten. In terms of economic return on investment, it may not be worthwhile for an owner to attempt to retain this expertise if ten or more years of trouble-free mining can be anticipated.

Problems in mature mines

The next stage in the process occurs when the mine has reached maturity and when extraction ratios are high, pillars are being recovered and the occurrence of rockbursts is quite frequent. It will be clear to even the most optimistic mine owner that all of these problems are going to get worse as mining progresses and the mine nears the end of its life. Rock mechanics studies at this stage of the mine operation will probably be aimed at two distinct levels. The first of these is modelling the overall layout of the mine in which the sequencing of the mine openings has to be considered in an effort to optimize the layout so as to minimize the problems that have been experienced. The second level is details of support on the scale of individual stopes—mainly from the point of view of minimizing dilution and maintaining the safety of men and equipment in that location.

At this stage of the mine's life a great deal of information will be available on the characteristics of the rock mass and on *in-situ* stresses, groundwater flow patterns, backfill behaviour and the performance of support. Some of this information may not have been systematically collected, particularly if the mine has enjoyed a long period of trouble-free mining, but it will be available to anyone who is sufficiently determined to find it. In some cases quantitative information will be available from the monitoring of rock mass movements, changes in *in-situ* stress and microseismic events or from systematic visual observation. If such information is not available, it is generally advisable to initiate some systematic monitoring programmes at this time.

Depending on the nature of those problems which have been encountered, specialists in rock mechanics can now call on their most sophisticated models. The economic justification for using these models generally lies in the value of the ore that is tied up in pillars or that is inaccessible unless safe and efficient mining techniques are devised. At this stage many of the experience-based, empirical rules that were so helpful during the feasibility and design stages are no longer of much value. Each mine has its own set of unique problems and the mining methods have to be tailored to overcome these. If rockbursts are the major problem, rock mechanics models that permit the evaluation of alternative three-dimensional layouts or the use of destressing excavations must be considered. These models may involve the use of three-dimensional boundary- or finite-element techniques. If pillar failures are the major concern and techniques for their recovery have to be devised, displacement discontinuity models may be the most appropriate. Where fault-generated rockbursts or failures are a major problem, three-dimensional distinct-element modelling may be required to build up an understanding of the overall behaviour. Two-dimensional finite-difference analysis may be the most appropriate technique to use on a local scale when failure around individual excavations has been controlled by the installation of support in the form of grouted cables or backfill.

In using these models all the precautions outlined by Starfield and Cundall[21] should be kept in mind and it should be remembered that a numerical model can never imitate reality. In spite of their limitations, these models can provide real insights into the behaviour of complex rock masses and they can be used to assist mining engineers in making some of the difficult design decisions that they face in a mature mine.

Conclusions

Numerical modelling has played and will continue to play an important role in the design of underground mine openings. In their simplest form these models can be used to explore whether or not stress or structurally controlled instabilities are likely to develop in the rock mass surrounding the openings. In their more advanced forms they can be used to optimize excavation sequences and support strategies.

The basic principles of numerical modelling, the advantages and disadvantages of different types of models and the selection of appropriate models for the analysis of the problems that arise at different stages in the life of a mine have been discussed. In all cases numerical models should be regarded as tools to be used in conjunction with empirical mine design rules and engineering judgement to assist the mining engineer in the optimization of underground opening design.

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