When considering different failure mechanisms underground, we generally distinguish between those that are primarily structurally-controlled and those that are stress-controlled. Of course some failure modes are composites of these two conditions, and others may involve the effect of time and weathering on excavation stability.

### Underground Instability Mechanisms

<table>
<thead>
<tr>
<th>Stress Conditions</th>
<th>Massively Fractured</th>
<th>Moderately Fractured</th>
<th>Highly Fractured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low in-situ Stress</td>
<td>Linear elastic response</td>
<td>Failure caused by blocks</td>
<td>Sliding failure of the rock mass</td>
</tr>
<tr>
<td>Moderate in-situ Stress</td>
<td>Brittle failure adjacent to excavation boundary</td>
<td>Localized brittle failure of rock mass</td>
<td>Localized sliding failure of rock mass and surrounding soil structures</td>
</tr>
<tr>
<td>High in-situ Stress</td>
<td>Brittle failure around the excavation</td>
<td>Brittle failure around the excavation and movement of blocks</td>
<td>Brittle failure around the excavation and movement of blocks</td>
</tr>
</tbody>
</table>

Martin et al. (1999)
... another way of viewing the problem
(Continuum -vs- Discontinuum)

As a continuum, the failure path passes through the rock mass. As a discontinuum, the failure surface is dictated more directly by the presence of specific pre-existing discontinuities. It is also possible to have intermediate cases where the failure occurs partly along discontinuities and partly through bridges of intact rock.

Although most soil instabilities are of the continuous nature, the majority of rock mass instabilities are caused by individual discontinuities. This is because the strength of the intact rock can be high, with the result that the pre-existing discontinuities are the weakest link.

Underground Instability Mechanisms

- Relaxation-driven failure when $\sigma_{\text{min}}$ is low

Kaiser et al. (2000)
Discontinuity Persistence

Persistence refers to the areal extent or size of a discontinuity plane within a plane. Clearly, the persistence will have a major influence on the shear strength developed in the plane of the discontinuity, where the intact rock segments are referred to as 'rock bridges'.

<table>
<thead>
<tr>
<th>Description</th>
<th>Modal trace length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>very low persistence</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>low persistence</td>
<td>1–3</td>
</tr>
<tr>
<td>medium persistence</td>
<td>3–10</td>
</tr>
<tr>
<td>high persistence</td>
<td>10–20</td>
</tr>
<tr>
<td>very high persistence</td>
<td>20</td>
</tr>
</tbody>
</table>

Discontinuity Spacing

Spacing is the perpendicular distance between adjacent discontinuities; and is usually expressed as the mean spacing of a particular set of joints.
Discontinuity Roughness

From the practical point of view of quantifying joint roughness, only one technique has received some degree of universality - the Joint Roughness Coefficient (JRC). This method involves comparing discontinuity surface profiles to standard roughness curves assigned numerical values.

Discontinuity Roughness - Subjectivity

Beer et al. (2002)
Stereonets - Pole Plots

Plotting dip and dip direction, pole plots provide an immediate visual depiction of pole concentrations. All natural discontinuities have a certain variability in their orientation that results in scatter of the pole plots. However, by contouring the pole plot, the most highly concentrated areas of poles, representing the dominant discontinuity sets, can be identified.

The equatorial projection “equal area” stereonet is the most favoured for plotting/analyzing discontinuity data. It is ideal for contouring the concentration of discontinuity poles.

Discontinuity Data - Probability Distributions

Discontinuity properties can vary over a wide range, even for those belonging to the same set. The distribution of a property can be described by means of probability distributions.

A normal distribution is applicable where a particular property's mean value is the most commonly occurring. This is usually the case for dip and dip direction.

A negative exponential distribution is applicable for properties of discontinuities, such as spacing and persistence, which are randomly distributed.

Negative exponential function: 
\[ f(x) = \frac{1}{X} e^{-x/X} \]

Wyllie & Mah (2004)
A negative exponential distribution is applicable for properties of discontinuities, such as spacing and persistence, which are randomly distributed. 

\[
f(x) = \frac{1}{x} \cdot e^{-x/\lambda}
\]

For example, for a discontinuity set with a mean spacing of 2 m, the probabilities that the spacing will be less than:

- 1 m: \( F(x) = (1 - e^{-1/2}) = 40\% \)
- 5 m: \( F(x) = (1 - e^{-5/2}) = 92\% \)

Structurally-controlled instability means that blocks formed by discontinuities either fall or slide from the excavation periphery as a result of the body forces (usually gravity) enabled by the process of excavation. To assess the likelihood of such failures, an analysis of the kinematic admissibility of potential wedges or planes that intersect the excavation face(s) can be performed.
To consider the kinematic admissibility of plane instability, five necessary but simple geometrical criteria must be met:

(i) The plane on which sliding occurs must strike near parallel to the slope face (within approx. \( \pm 20^\circ \)).

(ii) Release surfaces (that provide negligible resistance to sliding) must be present to define the lateral slide boundaries.

(iii) The sliding plane must "daylight" in the slope face.

(iv) The dip of the sliding plane must be greater than the angle of friction.

(v) The upper end of the sliding surface either intersects the upper slope, or terminates in a tension crack.

---

Similar to planar failures, several conditions relating to the line of intersection must be met for wedge failure to be kinematically admissible:

(i) The dip of the slope must exceed the dip of the line of intersection of the two wedge forming discontinuity planes.

(ii) The line of intersection must "daylight" on the slope face.

(iii) The dip of the line of intersection must be such that the strength of the two planes are reached.

(iv) The upper end of the line of intersection either intersects the upper slope, or terminates in a tension crack.
**Kinematic Analysis - Daylight Envelopes**

**Daylight Envelope:** Zone within which all poles belong to planes that daylight, and are therefore potentially unstable.

---

**Friction Cone:** Zone within which all poles belong to planes that dip at angles less than the friction angle, and are therefore stable.

---
Pole Plots - Kinematic Admissibility

Having determined from the daylight envelope whether block failure is kinematically permissible, a check is then made to see if the dip angle of the failure surface (or line of intersection) is steeper than the with the friction angle.

Thus, for poles that plot inside the daylight envelope, but outside the friction circle, translational sliding is possible.

Wyllie & Mah (2004)
Wedge Failure - Direction of Sliding

Scenario #1: If the dip directions of the two planes lie outside the included angle between $\alpha_i$ (trend of the line of intersection) and $\alpha_f$ (dip direction of face), the wedge will slide on both planes.

Example scenario #2: If the dip directions of one plane (e.g. Plane A) lies within the included angle between $\alpha_i$ (trend of the line of intersection) and $\alpha_f$ (dip direction of face), the wedge will slide on only that plane.

Kinematic Analysis - Underground Wedges

The minimum requirement to define a discrete block is four non-parallel planes, which give rise to a tetrahedral block. In terms of the instability analysis, such a block can be formed by three discontinuity planes and one plane representing the excavation periphery. On a hemispherical projection, these blocks may be identified as spherical triangles where the plane of projection represents the excavation surface.

Given that a tetrahedral block exists, there are three kinematic possibilities to be examined: the block falls from the roof; the block slides (either along the line of maximum dip of a discontinuity, or along the line of intersection of two discontinuities); or the block is stable.
Analysis of Kinematic Admissibility - Falling

Falling occurs when a block detaches from the roof of an excavation without sliding on any of the bounding discontinuity planes. In the case of gravitational loading, the direction of movement is vertically downwards.

This is represented on the projection as a line with a dip of 90°, i.e. the centre of the projection. Thus, if this point falls within the spherical triangle formed by the bounding discontinuities, falling is kinematically admissible.

Analysis of Kinematic Admissibility - Sliding

Kinematic methods used to analyze blocks sliding from the roof, either on one discontinuity plane (planar failure) or on a line of intersection (wedge failure), generally consider the spherical triangle and whether any part of it has a dip greater than the angle of friction.

Assuming that each discontinuity plane has the same friction angle, the sliding direction will occur along a line of maximum dip (either that of a plane or a line of intersection of two planes). No other part of the spherical triangle represents a line of steeper dip than these candidates.
Analysis of Kinematic Admissibility - Sliding

However, not all lines of maximum dip on a stereonet projection will be candidates for the sliding direction. Although some planes/lines of intersection may be dipping at angles greater than the friction angle, sliding is not kinematically admissible if the line of maximum dip is outside the spherical triangle formed by the intersecting planes (i.e. the wedge).

The spherical triangle, therefore, represents the region of kinematically admissible directions of movement and any other direction represents directions directed into the rock surrounding the block.

Hudson & Harrison (1997)

Analysis of Kinematic Admissibility - Sliding

... hence, the shaded blocks above represent (a) planar sliding along $\beta_2$; and (b) wedge sliding along $\beta_3$.

... of course, if the spherical triangles fall completely outside the friction circle, then the blocks are identified as being stable.
Discontinuity Shear Strength

Strength along a discontinuity surface is mostly provided by asperities. For shear failure to occur, the discontinuity surfaces must either dilate, allowing asperities to override one another, or shear through the asperities.

As normal stresses increase, dilatancy is gradually reduced as a greater proportion of the asperities are damaged during shearing. Here, the friction angle progressively diminishes to a minimum value (residual friction).

A rough surface that is initially undisturbed and interlocked will have a peak friction angle of \((\phi + \iota)\), where \(\iota\) is the roughness angle.

Discontinuity Shear Strength - Example

The following tests were obtained in a series of direct shear tests carried out on 100 mm square specimens of granite containing clean, rough, dry joints.

<table>
<thead>
<tr>
<th>Normal stress (MPa)</th>
<th>Peak shear strength (\tau_{p}) (MPa)</th>
<th>Residual shear strength (\tau_{r}) (MPa)</th>
<th>Displacement at peak shear strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{n}) (MPa)</td>
<td>(\tau_{p}) (MPa)</td>
<td>(\tau_{r}) (MPa)</td>
<td>Normal displacement (\delta(n))</td>
</tr>
<tr>
<td>0.25</td>
<td>0.25</td>
<td>0.15</td>
<td>0.54</td>
</tr>
<tr>
<td>0.50</td>
<td>0.50</td>
<td>0.30</td>
<td>0.67</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>0.60</td>
<td>0.65</td>
</tr>
<tr>
<td>2.00</td>
<td>1.55</td>
<td>1.15</td>
<td>0.45</td>
</tr>
<tr>
<td>3.00</td>
<td>2.15</td>
<td>1.70</td>
<td>0.39</td>
</tr>
<tr>
<td>4.00</td>
<td>2.60</td>
<td>–</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Direct shear tests give normal and shear values which may be plotted directly.
Plotting the peak strength data we can see that it takes the form of a bilinear strength envelope.

At higher normal stresses, however, these asperities are sheared.

The initial slope of this envelope has an apparent friction angle of (φ+i).
φ + i = 45°

Thus… roughness angle

i = 45° - 30° = 15°

If we were to repeat this for the residual strength values…

<table>
<thead>
<tr>
<th>Normal stress (MPa)</th>
<th>Peak shear strength (MPa)</th>
<th>Residual shear strength (MPa)</th>
<th>Displacement at peak shear strength (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.25</td>
<td>0.15</td>
<td>0.24</td>
</tr>
<tr>
<td>0.50</td>
<td>0.50</td>
<td>0.30</td>
<td>0.82</td>
</tr>
<tr>
<td>1.00</td>
<td>0.90</td>
<td>1.15</td>
<td>1.65</td>
</tr>
<tr>
<td>2.00</td>
<td>1.55</td>
<td>1.75</td>
<td>3.30</td>
</tr>
<tr>
<td>4.00</td>
<td>2.90</td>
<td>3.15</td>
<td>4.00</td>
</tr>
</tbody>
</table>
Dilatancy and Shear Strength

In the case of sliding of an unconstrained block of rock from a slope, dilatancy will accompany shearing of all but the smoothest discontinuity surfaces. If a rock block is free to dilate, then the second-order asperities will have a diminished effect on shear strength.

Thus, by increasing the normal force across a shear surface by adding tensioned rock bolts, dilation can be limited and interlocking along the sliding surface maintained, allowing the second-order asperities to contribute to the shear strength.

Residual Strength

For the residual strength condition, any cohesion is lost once displacement has broken the cementing action. Also, the residual friction angle is less than the peak friction angle because the shear displacement grinds the minor irregularities on the rock surface and produces a smoother, lower friction surface.
Once a series of joint sets have been identified as potentially forming tetrahedral wedges, several questions may arise as to whether they will be problematic or not:

- In the case of a falling wedge, how much support will be required to hold it in place (what kind of loads on the added support can be expected, how dense will the bolting pattern have to be, etc.);

- In the case of a sliding wedge, do the shear stresses arising due to gravitational forces exceed the shear strength along the sliding surface, i.e. provided by friction and sometimes cohesion (in the form of intact rock bridges or mineralized infilling), and if so, how much support will be required to stabilize the block, how dense will the bolting pattern have to be, etc..

In both cases, the volume/weight of the maximum wedge that may form is required. This can be determined through further geometrical constructions.

To calculate the maximum wedge volume:

1) Identify the joint planes/great circles on the stereonet plot that form the wedge. In this example, the three persistent, planar discontinuity sets have dip directions/dips of: (1) 138/51, (2) 355/40, (3) 219/67.

Together, these joints are known to form wedges within the horizontal, planar roof of an excavation in sedimentary rock.

The stereonet construction is finished by drawing lines passing through the corners of the spherical triangle and centre of the stereonet.
2) On a separate sheet of paper, construct a scaled plan view, where the width of the window represents the width of the excavation. As such, the analysis will consider the largest block that could be released from the excavation roof.

In this particular example, the roof is rectangular in shape, is 6 m wide, and has its long axis orientated at an azimuth of 025°.

Given that the great circle representing the horizontal plane through the tunnel coincides with that of the stereonet projection, it is convenient to construct the window aligned parallel to the tunnel axis.

3) On the scaled window, mark an arbitrary horizontal reference line and starting point. For example, about halfway along the western margin of the roof.

Inspection of the spherical triangle in the stereonet plot suggests that the corner of the face triangle formed by planes 2 and 3 will touch the western margin of the roof, and the corner formed by planes 1 and 2 will touch the eastern margin when the largest possible tetrahedral block is considered.

As such, the arbitrary reference point can represent the corner of the face triangle formed by planes 2 and 3.
4) The lines associated with planes 2 and 3 can now be added to the window construction by counting off the angles between the horizontal reference line on the stereonet plot (at 025°) and the diametral lines for planes 2 and 3 (striking at 085° and 129°, respectively).

These angles can then be transferred to the window construction and measured off relative to the starting point and reference line along the western margin of the roof.
5) The point where the line for plane 2 intersects the eastern margin of the roof in the window construction represents the corner of the face triangle formed by planes 1 and 2. Thus, the line for plane 1 can be added by measuring the angle between the two planes on the stereonet and transferring it to the window construction. The outline/trace of the wedge on the tunnel roof is now complete.

6) The next step is to add the corner edges of the wedge to complete the 3-D trace of the tetrahedron in the window construction box. This can be done following a similar procedure by transferring the lines of intersection between the planes (i.e. I_{12}, I_{23}, I_{13}) and their measured angles from the stereonet to the window construction.
7) Since this construction can be completed graphically by overlaying the stereonet with the window construction, or geometrically by measuring the angles off the stereonet and transferring them onto the window construction, several checks can be made to find any errors that may have arisen.

The final step involving the finding of the location of the wedge's apex also gives a valuable check since the area of the triangle of error formed by these converging lines is a measure of any imprecision in the construction.

8) The dimensions of the face triangle appearing on the excavation surface can now be scaled off directly from the construction. It's area, $A_f$, can be found by taking any pair of adjacent sides and their included angles:

$$A_f = \frac{1}{2}l_1l_2 \sin \theta_{12} = \frac{1}{2}l_2l_3 \sin \theta_{23} = \frac{1}{2}l_3l_1 \sin \theta_{31}$$

This gives a face area of 10.1 m$^2$. 

---

Maximum Wedge Volume

Priest (1985)
The areas of the three internal block surfaces can be found in a similar way from the edge lengths and appropriate internal angles:

\[ A_1 = \frac{1}{2} \left( \frac{l_{12}}{\cos \beta_{12}} - \frac{l_{13}}{\cos \beta_{13}} \right) \sin \theta_1 \]

\[ A_2 = \frac{1}{2} \left( \frac{l_{21}}{\cos \beta_{21}} - \frac{l_{23}}{\cos \beta_{23}} \right) \sin \theta_2 \]

\[ A_3 = \frac{1}{2} \left( \frac{l_{31}}{\cos \beta_{31}} - \frac{l_{32}}{\cos \beta_{32}} \right) \sin \theta_3 \]

To find the volume of the wedge, the wedge height and the face area are required. The face area, \( A_f \), has already been found. The wedge height, \( h \), is given by:

\[ h = l_{12} \tan \beta_{12} = l_{23} \tan \beta_{23} = l_{31} \tan \beta_{31} \]

which for this example problem comes to 1.47 m.

The volume, \( V \), of the tetrahedral block is then given as:

\[ V = A_f h / 3 \]

resulting in a block volume of approximately 5 m³.
11) Now assuming a unit weight of 25 kN/m³ for sedimentary rock, the block would have a weight of approximately 124 kN.

By dividing this value through by the face area, it can be seen that a support pressure of only 12.3 kN/m², distributed over the face triangle, would be required to keep it in place.

This support pressure could, for example, be provided by rock bolts anchored beyond the block at a distance of 2 to 3 m above the excavation roof.

Wedge Analysis – Computer-Aided

The three-dimensional nature of wedge stability problems (i.e. size and shape of potential wedges in the rock mass surrounding an opening) necessitates a set of relatively tedious calculations. While these can be performed by hand, it is far more efficient to utilise computer-based techniques.
Computer-Aided Wedge Analysis in Design

The speed of computer-aided wedge analyses allow them to be employed within the design methodology as a tool directed towards "filter analysis". This is the type of analysis which is carried out during the preliminary design stage of a project to determine whether or not there are stability issues for a number of different problem configurations (e.g. a curving tunnel, or the different slopes of an open pit mine or along a pipeline, etc.).

Evert Hoek notes that this type of analysis is extremely important in defining slopes in which there are no problems so that the limited geotechnical resources available on such projects are not wasted on meaningless data collection and analysis. Those slopes that are identified as having potential structural stability problems can then be categorized into "deal with now" or "study later" groups, depending on the severity of the problem and the consequences of failure. Severe problems will then require more detailed analyses.

Key Block Analysis

The underlying axiom of block theory is that the failure of an excavation begins at the boundary with the movement of a block into the excavated space. The loss of the first block augments the space, possibly creating an opportunity for the failure of additional blocks, with continuing degradation possibly leading to massive failure.

As such, the term key-block identifies any block that would become unstable when intersected by an excavation. The loss of a key-block does not necessarily assure subsequent block failures, but the prevention of its loss does assure stability.

Key-block theory therefore sets out to establish procedures for describing and locating key blocks and for establishing their support requirements.
Distinct-Element Methods - a type of domain method adapted for jointed rock masses, distinct-element methods model individual blocks of rock as deformable but also allow the blocks to rotate and slide relative to one another. Numerically, each block is considered a unique free body that may interact at contact locations with surrounding blocks.