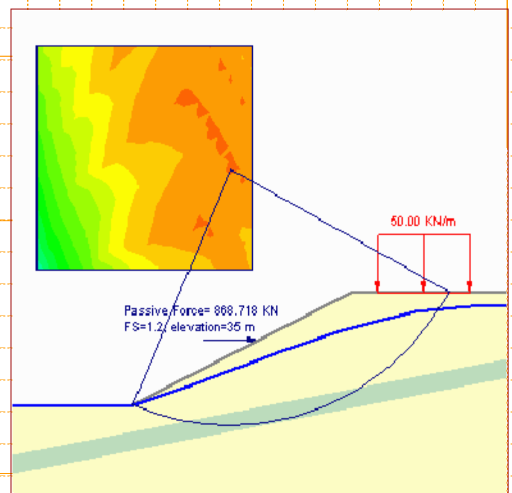
EOSC433/536:

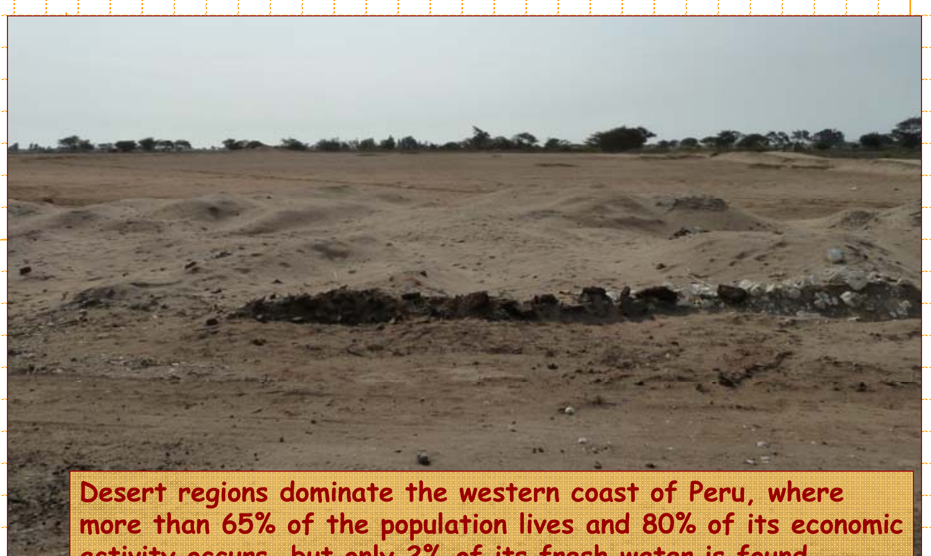
Geological Engineering Practice I – Rock Engineering

# Lecture 6: Limit Equilibrium Analysis






## Sechura Desert, Northern Peru



Desert regions dominate the western coast of Peru, where more than 65% of the population lives and 80% of its economic activity occurs, but only 2% of its fresh water is found.

# Sechura Desert, Northern Peru



# Olmos Water Conveyance and Irrigation Project


First envisioned in the 1920's, pre-feasibility was carried out in the 1960's, and design was completed by soviet engineers in 1982. Construction on the project began in 2006.

## Olmos Water Conveyance and Irrigation Project



## Limon Dam, Peru

The Limón Dam is a 43 m high, concrete-faced rockfill dam. The dam will divert up to 2 million cubic meters of water per year through the 20 km Transandino tunnel to the Olmos River Valley.

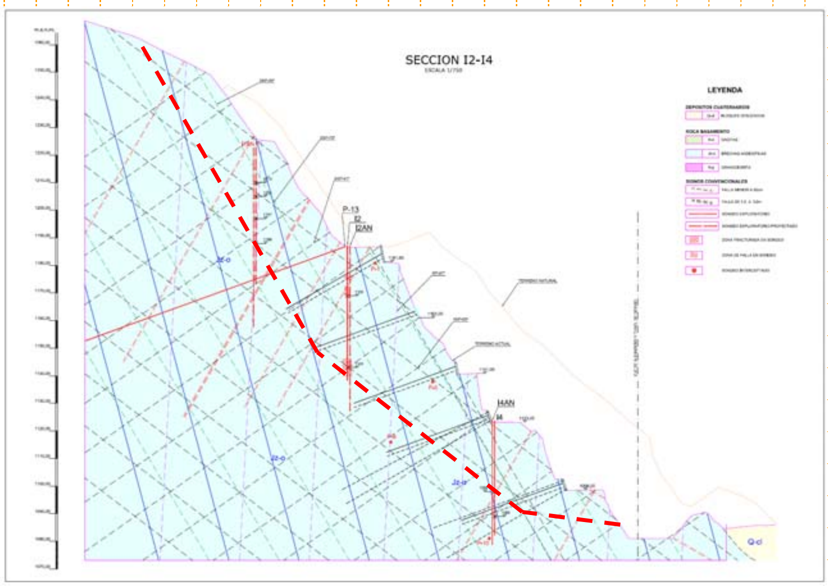
## Limon Dam Abutment



## Limon Dam Abutment

90 m high failure with cutting of slope.

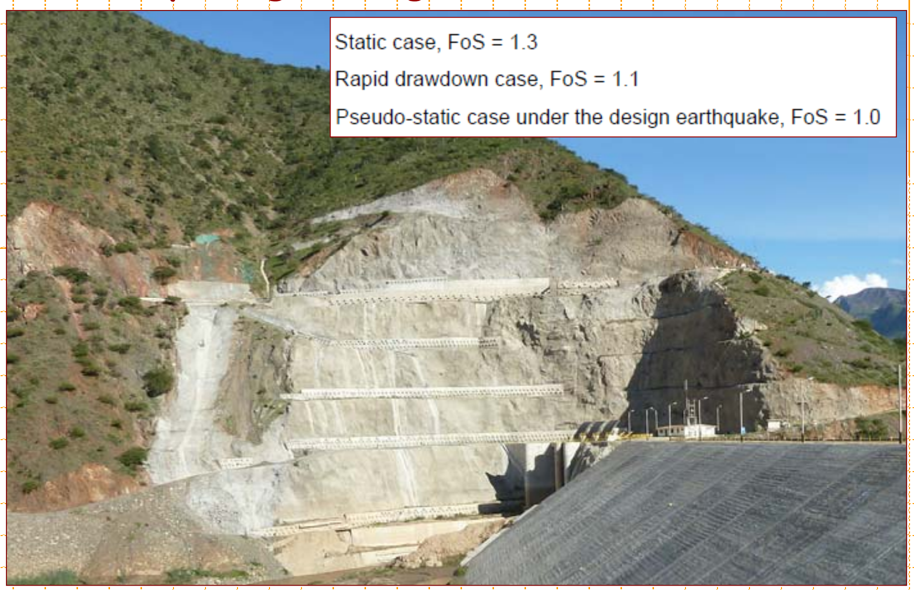
## Rock Slope Engineering – Limon Dam






## Rock Slope Engineering – Limon Dam

# Rock Slope Engineering – Limon Dam



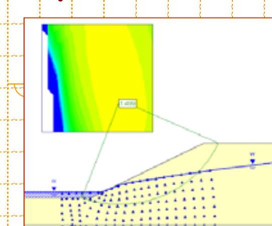
Static case, FoS = 1.3

Rapid drawdown case, FoS = 1.1

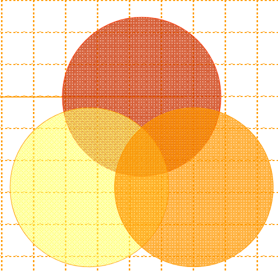
Pseudo-static case under the design earthquake, FoS = 1.0

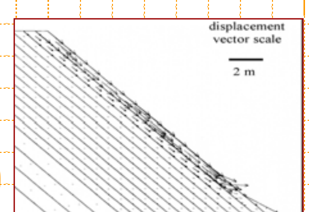


# Analysis in Geotechnical Engineering

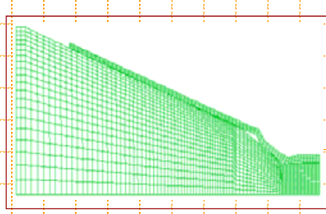


LIMIT EQUILIBRIUM
(infinite slope,
method of slices, etc.)



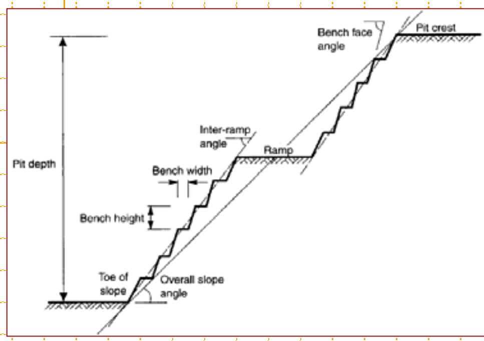



DISCONTINUUM
(distinct element, particle
flow codes, etc.)

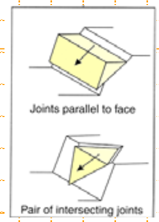


CONTINUUM
(boundary element,
finite element,
finite difference, etc.)

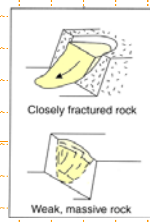
## Rock Slope Engineering – Failure Modes and Structure




Wyllie & Mah (2004)




A function of scale and joint persistence.






## Limit Equilibrium Analysis

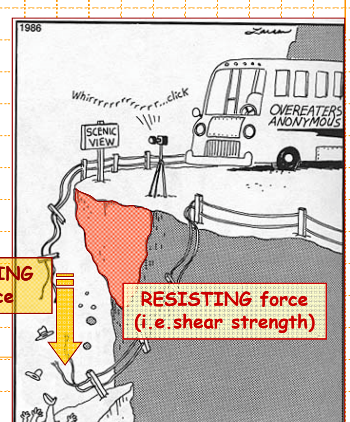
The most widely applied analytical technique used in geotechnical analysis is that of *limit equilibrium*, whereby force or/and moment equilibrium conditions are examined on the basis of statics. These analyses require information about material strength, but not stress-strain behaviour.

The typical output from a limit equilibrium analysis is the "Factor of Safety":

$$FS = \frac{\text{resisting forces}}{\text{driving forces}}$$

$$= \frac{\text{shear strength}}{\text{shear stress}}$$

FS > 1.0 represents a stable situation
FS < 1.0 denotes failure




Group photo disasters

## Limit Equilibrium Analysis

Although limit equilibrium can be applied to many geotechnical problems, it has been most widely used within the context of slope stability analysis. The analysis of slope stability may be implemented at two distinct stages:

*Back analysis – carried out to determine material properties at time of failure; should be responsive to the totality of processes which led to failure.*

*Forward analysis – applied to assess safety in a global sense to ensure that the slope will perform as intended;*

As such, analyses are undertaken to provide either a factor of safety, identify a potential failure surface, or through back-analysis, a range of shear strength parameters at failure.



## Analysis in Geotechnical Design

The fundamental requirement for a meaningful analysis should include the following steps of data collection & evaluation:

- site characterization (geological conditions);
- groundwater conditions (pore pressure distribution);
- geotechnical parameters (strength, deformability, permeability);
- primary stability mechanisms (kinematics, potential failure modes).

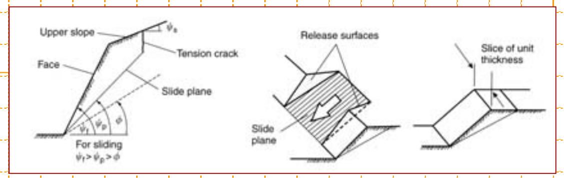
Clayton *et al.* (1995)

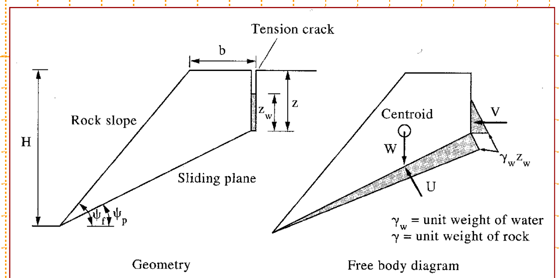
| EVENT | DESCRIPTION |
|---|---|
| 1 | Preliminary desk study or fact-finding survey |
| 2 | Aerial photograph interpretation |
| 3 | Site walkover survey |
| 4 | Preliminary subsurface exploration |
| 5 | Soil classification by description and simple testing |
| 6 | Detailed subsurface exploration and field testing |
| 7 | Physical survey (laboratory testing) |
| 8 | Evaluation of data |
| 9 | Geotechnical design |
| 10 | Field trials |
| 11 | Liaison by geotechnical engineer with site staff during project construction |

Ideal order of events for a site investigation.

# Limit Equilibrium – Translational Sliding (Rock)

The solution for translational sliding requires that the **strikes** of the sliding plane and slope are **parallel** and that **no end restraints** are present. Furthermore, the solution incorporates the assumptions that the rock mass is **impermeable**, the sliding block is **rigid**, the strength of the slide plane is given by the **Mohr-Coulomb shear criterion** and that all forces pass through the **centroid** of the sliding block.

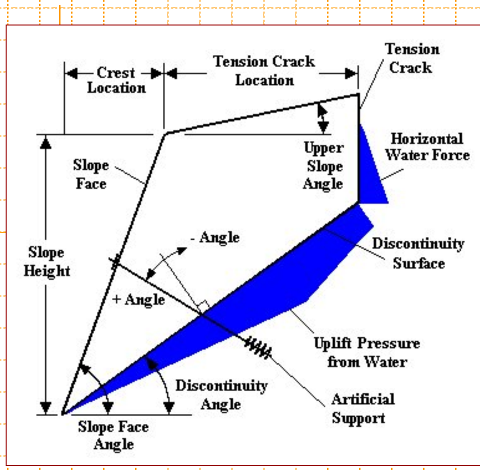



$$F = \frac{c'(H-z)\operatorname{cosec}\psi_p + (W\cos\psi_p - U - V\sin\psi_p)\tan\phi'}{V\cos\psi_p + W\sin\psi_p}.$$



# Limit Equilibrium – Translational Sliding (Rock)




Hoek & Bray (1981)

Factor of safety:

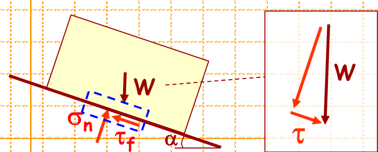
$$FS = \frac{\{cA + [W(\cos\Psi_p - a\sin\Psi_p) - U - V\sin\Psi_p + T\cos\theta]\tan\phi\}}{[W(\sin\Psi_p + a\cos\Psi_p) + V\cos\Psi_p - T\sin\theta]}$$

where

- $H$ = height of slope face;
- $\Psi_f$ = inclination of slope face;
- $\Psi_s$ = inclination of upper slope face;
- $\Psi_p$ = inclination of failure plane;
- $b$ = distance of tension crack from slope crest;
- $a$ = horizontal acceleration, blast or earthquake loading;
- $T$ = tension in bolts or cables;
- $\theta$ = inclination of bolt or cable to normal to failure plane;
- $c$ = cohesive strength of failure surface;
- $\phi$ = friction angle of failure surface;
- $\gamma_r$ = density of rock;
- $\gamma_w$ = density of water;
- $Z_w$ = height of water in tension crack;
- $Z$ = depth of tension crack;
- $U$ = uplift water force;
- $V$ = driving water force;
- $W$ = weight of sliding block; and
- $A$ = area of failure surface.

# Effective Stress

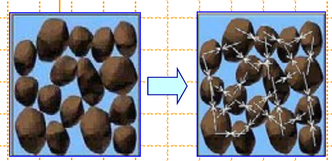
High pore pressures may adversely affect the stability of a slope due to a decrease in effective stresses.



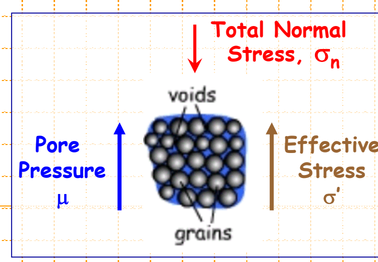

$$FS = \frac{\tau_F}{\tau} = \frac{(\sigma'_n \tan\phi + c)}{\tau}$$

$\tau$ = W sinα

Factor of Safety

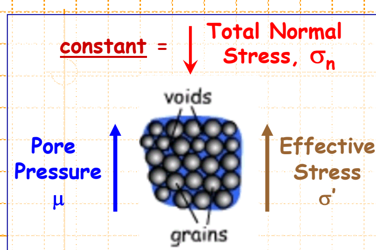


This intergranular stress, or effective stress, may be viewed as the sum of the contact forces divided by the total area.





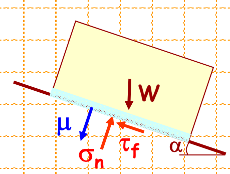

# Effective Stress




The effective stress cannot be measured; it can only be calculated.

However, the total normal stress and pore pressure can be calculated based on the overburden weight and location of the groundwater table.

$$\sigma' = \sigma_n - \mu$$



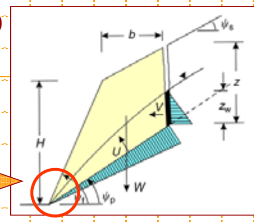

constant

$$FS = \frac{\tau_F}{\tau} = \frac{([\sigma_n - \mu] \tan\phi + c)}{\tau}$$

As precipitation infiltrates the ground, the total normal stress remains relatively unchanged but the pore pressure increases decreasing the effective normal stress acting on the sliding surface (thereby decreasing the frictional strength component).

# Planar Analysis – Water Pressure Scenarios

Wyllie & Mah (2004)




$U = \frac{1}{2}\gamma_w z_w (H + b\tan\psi_s - z)\operatorname{cosec}\psi_p$

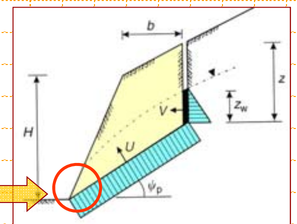
$V = \frac{1}{2}\gamma_w z_w^2$

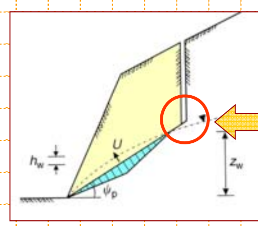
free draining toe

Drainage blocked at toe
(uniform pressure on slide plane)

$U = A\gamma_w z_w$

$V = \frac{1}{2}\gamma_w z_w^2$



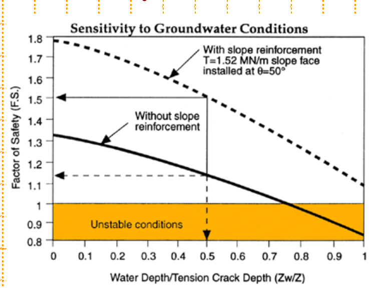



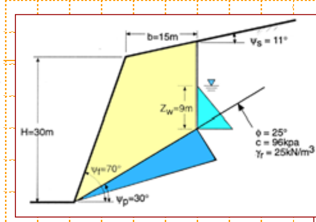

Water table below tension crack
(triangular pressure on slide plane)

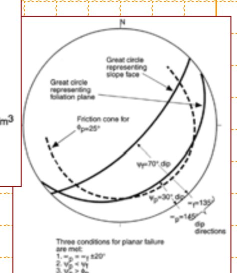
$U = \frac{1}{2}\frac{z_w}{\sin\psi_p}h_w\gamma_w$

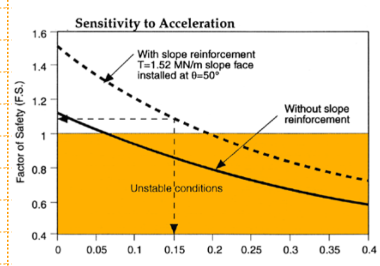


# Limit Equilibrium – Sensitivity Analysis



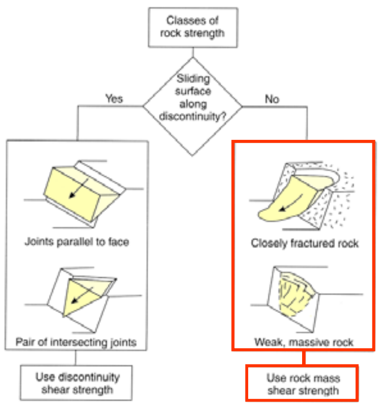




*… calculation of factor of safety vs. different depths of water in the tension crack, and vs. horizontal acceleration.*

# Rotational Slip Surfaces

In weak materials such as highly weathered or closely fractured rock, and rock fills and soils, a strongly defined structural pattern no longer exists, and the shear failure surface develops along the line of least resistance. These slip surfaces generally take a circular shape.






# Limit Equilibrium – Rotational Sliding

The fundamental assumptions of a limit equilibrium analysis as applied to rotational slides include:

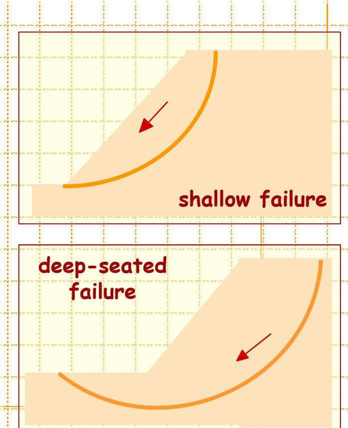
*… slope failure mechanism occurs as a rotational slide (failure mechanism is assumed!);*

*… resisting forces required to equilibrate disturbing forces are found from static solution (summation of forces/momemts);*

*… the shear resistance required for equilibrium is compared to the available shear strength to solve for the Factor of Safety;*

*… the slip surface with the lowest FS is found by iteration;*

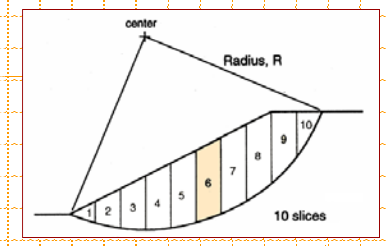
*… the Factor of Safety is assumed to be constant along the entire slip surface*

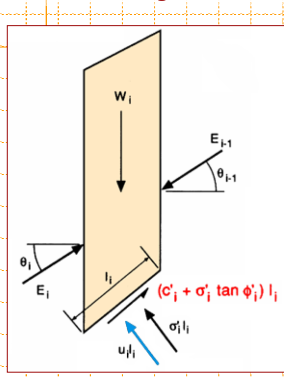



Morgenstern (1995)

# Limit Equilibrium – Method of Slices

The most commonly used solutions divide the mass above an assumed slip surface into vertical slices. This is to accommodate conditions where the soil properties and pore pressures vary with location throughout the slope.




The forces acting on a typical slice, *i*, are:

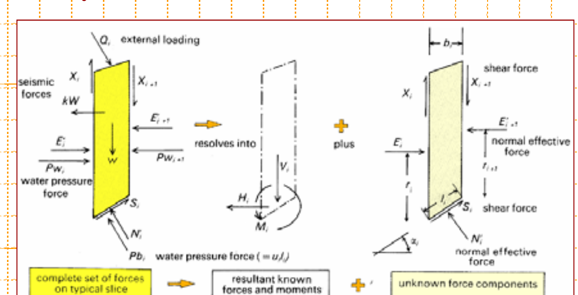
- W = weight of slice
- c,$\phi$ = mobilized shear forces at base of slice
- $\sigma' \cdot l$ = effective normal forces on base
- $u \cdot l$ = water pressure force on base
- E = side forces exerted by neighboring slices.



# Method of Slices – Equations & Unknowns

Analysing the summation of forces and/or moments for these slices (i.e. $\Sigma M=0$, $\Sigma Fx=0$, $\Sigma Fy=0$), it is soon recognized that there are more unknowns than equations.

As such, the forces involved are statically indeterminate. Various methods have therefore been developed to make up the balance between the number of equilibrium equations and the number of unknowns in the problem.




Bromhead (1992)

| Equations | Condition |
|---|---|
| $n$ | Moment equilibrium for each slice |
| $2n$ | Force equilibrium in two directions (for each slice) |
| $n$ | Mohr–Coulomb relationship between shear strength and normal effective stress |
| $4n$ | Total number of equations |

| Unknowns | Variable |
|---|---|
| 1 | FOS |
| $n$ | Normal force at base of each slice, $N'$ |
| $n$ | Location of normal force, $N'$ |
| $n$ | Shear force at base of each slice, $S_m$ |
| $n-1$ | Interslice force, $Z$ |
| $n-1$ | Inclination of interslice force, $\theta$ |
| $n-1$ | Location of interslice force (line of thrust) |
| $6n-2$ | Total number of unknowns |

Duncan (1996)

# Method of Slices - Assumptions

| METHOD | LIMITATIONS, ASSUMPTIONS, AND EQUILIBRIUM CONDITIONS SATISFIED |
|---|---|
| Ordinary method of slices (Fellenius 1927) | Factors of safety low—very inaccurate for flat slopes with high pore pressures; only for circular slip surfaces; assumes that normal force on the base of each slice is $W \cos \alpha$; one equation (moment equilibrium of entire mass), one unknown (factor of safety) |
| Bishop's modified method (Bishop 1955) | Accurate method; only for circular slip surfaces; satisfies vertical equilibrium and overall moment equilibrium; assumes side forces on slices are horizontal; $N+1$ equations and unknowns |
| Janbu's simplified method (Janbu 1968) | Force equilibrium method; applicable to any shape of slip surface; assumes side forces are horizontal (same for all slices); factors of safety are usually considerably lower than calculated using methods that satisfy all conditions of equilibrium; $2N$ equations and unknowns |
| Lowe and Karafiath's method (Lowe and Karafiath 1960) | Generally most accurate of the force equilibrium methods; applicable to any shape of slip surface; assumes side force inclinations are average of slope surface and slip surface (varying from slice to slice); satisfies vertical and horizontal force equilibrium; $2N$ equations and unknowns |
| Janbu's generalized procedure of slices (Janbu 1968) | Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes heights of side forces above base of slice (varying from slice to slice); more frequent numerical convergence problems than some other methods; accurate method; $3N$ equations and unknowns |
| Spencer's method (Spencer 1967) | Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes that inclinations of side forces are the same for every slice; side force inclination is calculated in the process of solution so that all conditions of equilibrium are satisfied; accurate method; $3N$ equations and unknowns |
| Morgenstern and Price's method (Morgenstern and Price 1965) | Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes that inclinations of side forces follow a prescribed pattern, called $f(x)$; side force inclinations can be the same or can vary from slice to slice; side force inclinations are calculated in the process of solution so that all conditions of equilibrium are satisfied; accurate method; $3N$ equations and unknowns |
| Sarma's method (Sarma 1973) | Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes that magnitudes of vertical side forces follow prescribed patterns; calculates horizontal acceleration for barely stable equilibrium; by prefactoring strengths and iterating to find the value of the prefactor that results in zero horizontal acceleration for barely stable equilibrium, the value of the conventional factor of safety can be determined; $3N$ equations, $3N$ unknowns |

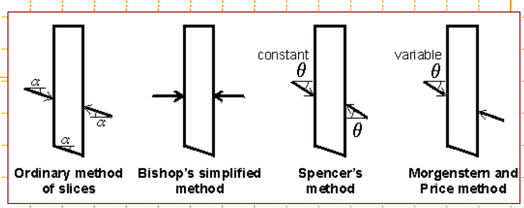
Duncan (1996)

**The various Method of Slices procedures either make assumptions to make the problem determinate (balancing knowns and unknowns), or they do not satisfy all the conditions of equilibrium.**



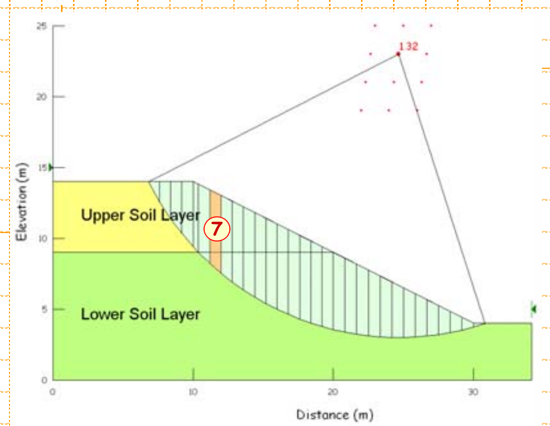
# Method of Slices - Assumptions

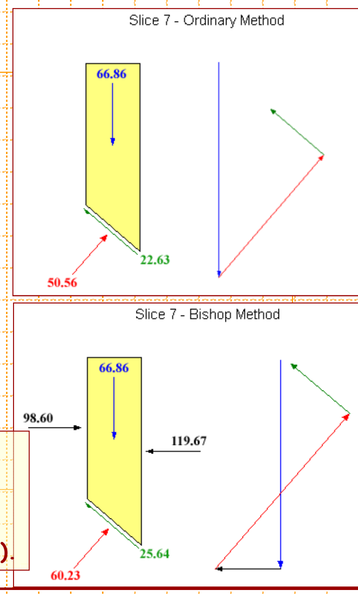
**The treatment of side forces, is one of the key assumptions that differentiate several of the various Method of Slices procedures.**




| Method | Force Equilibrium: 1st Direction (e.g., Vertical) | Force Equilibrium: 2nd Direction (e.g., Horizontal) | Moment Equilibrium |
|---|---|---|---|
| Ordinary or Fellenius | Yes | No | Yes |
| Bishop's Simplified | Yes | No | Yes |
| Janbu's Simplified | Yes | Yes | No |
| Spencer | Yes | Yes | Yes |
| Morgenstern-Price | Yes | Yes | Yes |
| GLE | Yes | Yes | Yes |
| Corps of Engineers | Yes | Yes | No |
| Lowe-Karafiath | Yes | Yes | No |

## Method of Slices - Assumptions

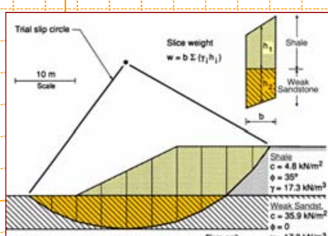



Because different methods use different assumptions to make up the balance between equations and unknowns (to render the problem determinate), some methods do not satisfy all conditions of equilibrium (i.e. force and/or moment).



## Ordinary Method - Computation




| Slice No. | W | l | α | c | φ | u | $N_1$ | $N_2$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 112 | 5.3 | -32.0 | 35.9 | 0.0 | 0 | -60 | 191 |
| 2 | 297 | 4.9 | -22.0 | 35.9 | 0.0 | 0 | -111 | 177 |
| 3 | 499 | 4.7 | -13.0 | 35.9 | 0.0 | 0 | -112 | 169 |
| 4 | 726 | 4.6 | -4.0 | 35.9 | 0.0 | 0 | -51 | 165 |
| 5 | 903 | 4.6 | 4.0 | 35.9 | 0.0 | 0 | 63 | 165 |
| 6 | 1,028 | 4.7 | 13.0 | 35.9 | 0.0 | 0 | 231 | 169 |
| 7 | 1,003 | 4.9 | 22.0 | 35.9 | 0.0 | 0 | 376 | 177 |
| 8 | 818 | 5.3 | 32.0 | 35.9 | 0.0 | 0 | 433 | 191 |
| 9 | 587 | 6.7 | 43.0 | 4.8 | 35.0 | 0 | 400 | 333 |
| 10 | 128 | 5.6 | 55.0 | 4.8 | 35.0 | 0 | 105 | 79 |
| 11 | | | | | | | 0 | 0 |
| 12 | | | | | | | 0 | 0 |
| 13 | | | | | | | 0 | 0 |
| 14 | | | | | | | 0 | 0 |
| 15 | | | | | | | 0 | 0 |
| | | | | | | $\Sigma =$ | 1274 | 1816 |

W = weight of slice - kN/m
c = cohesion intercept - kN/m$^2$
φ = friction angle - degrees
u = pore pressure - kN/m$^2$
α = angle between base of slice and horizontal - degrees
l = length of slip surface segments measured along base of slice - m

$N_1 = W \sin\alpha$
$N_2 = [W \cos\alpha - ul] \tan\phi + cl$
$F = \Sigma (N_2) / \Sigma (N_1) = \underline{1.43}$

The "ordinary method" only resolves the forces acting at the base of the slice. This allows for the side forces to be neglected and for the problem to be easily solved.

# Bishop's Modified Method - Computation

| Slice No. | W | l | α | c | φ | u | $N_1$ | $N_2$ | $N_2$ | $N_2$ | $N_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | $F_a$ = | | | | |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 8 | | | | | | | | | | | |
| 9 | | | | | | | | | | | |
| 10 | | | | | | | | | | | |
| 11 | | | | | | | | | | | |
| 12 | | | | | | | | | | | |
| 13 | | | | | | | | | | | |
| 14 | | | | | | | | | | | |
| 15 | | | | | | | | | | | |
| | | | | | | Σ= | | | | | |
| | | | | | | | $F_C$= | | | | |

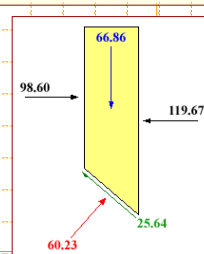
W = weight of slice - kN/m
φ = friction angle - degrees
c = cohesion intercept - kN/m$^2$
u = pore pressure - kN/m$^2$
α = angle between base of slice and horizontal - degrees
l = length of slip surface segments measured along base of slice - m
$F_a$ = assumed F
$F_C$ = calculated F

$$N_1 = W \sin\alpha$$

$$N_2 = \frac{\left\{\left[\frac{W}{\cos\alpha} - ul\right]\tan\phi + cl\right\}}{\left[1 + \frac{\tan\alpha\tan\phi}{F_a}\right]}$$

$$F_C = \Sigma(N_2)/\Sigma(N_1) = \_\_\_$$

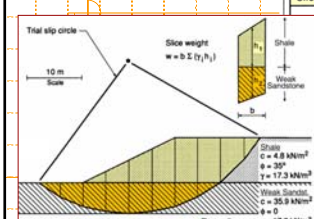
The "Bishop's Modified Method" includes interslice side forces, but requires an iterative procedure to determine the Factor of Safety.




Duncan (1996)




# Bishop's Modified Method - Computation


| Slice No. | W | l | α | c | φ | u | $N_1$ | $N_2$ | $N_2$ | $N_2$ | $N_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | $F_a$ = | 1.43 | 1.51 | 1.52 | |
| 1 | 112 | 5.3 | -32.0 | 35.9 | 0.0 | 0 | -60 | 192 | 192 | 192 | |
| 2 | 297 | 4.9 | -22.0 | 35.9 | 0.0 | 0 | -111 | 177 | 177 | 177 | |
| 3 | 499 | 4.7 | -13.0 | 35.9 | 0.0 | 0 | -112 | 170 | 170 | 170 | |
| 4 | 726 | 4.6 | -4.0 | 35.9 | 0.0 | 0 | -51 | 165 | 165 | 165 | |
| 5 | 903 | 4.6 | 4.0 | 35.9 | 0.0 | 0 | 63 | 165 | 165 | 165 | |
| 6 | 1,028 | 4.7 | 13.0 | 35.9 | 0.0 | 0 | 231 | 170 | 170 | 170 | |
| 7 | 1,003 | 4.9 | 22.0 | 35.9 | 0.0 | 0 | 376 | 177 | 177 | 177 | |
| 8 | 818 | 5.3 | 32.0 | 35.9 | 0.0 | 0 | 433 | 192 | 192 | 192 | |
| 9 | 587 | 6.7 | 43.0 | 4.8 | 35.0 | 0 | 400 | 408 | 415 | 416 | |
| 10 | 128 | 5.6 | 55.0 | 4.8 | 35.0 | 0 | 105 | 108 | 111 | 111 | |
| 11 | | | | | | | | | | | |
| 12 | | | | | | | | | | | |
| 13 | | | | | | | | | | | |
| 14 | | | | | | | | | | | |
| 15 | | | | | | | | | | | |
| | | | | | | Σ= | 1,274 | 1,925 | 1,934 | 1,935 | |
| | | | | | | | $F_C$= | 1.51 | 1.52 | 1.52 | |


W = weight of slice - kN/m
φ = friction angle - degrees
c = cohesion intercept - kN/m$^2$
u = pore pressure - kN/m$^2$
α = angle between base of slice and horizontal - degrees
l = length of slip surface segments measured along base of slice - m
$F_a$ = assumed F
$F_C$ = calculated F


$$N_1 = W \sin\alpha$$

$$N_2 = \frac{\left\{\left[\frac{W}{\cos\alpha} - ul\right]\tan\phi + cl\right\}}{\left[1 + \frac{\tan\alpha\tan\phi}{F_a}\right]}$$

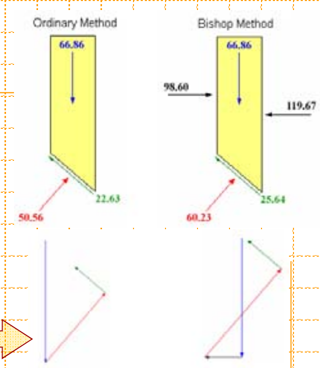
$$F_C = \Sigma(N_2)/\Sigma(N_1) = \_\_\_$$


Duncan (1996)

# General Limit Equilibrium

1. Different methods use different assumptions to make up the balance between equations and unknowns to render the problem determinate; or
2. Some methods, such as the ordinary and Bishop's modified methods, do not satisfy all conditions of equilibrium (i.e. force and/or moment).

The degree to which the force polygon closes indicates whether force equilibrium is achieved.



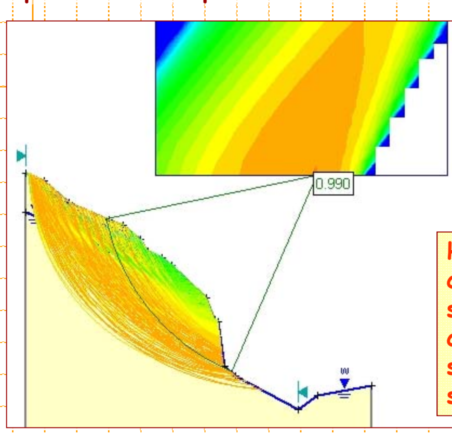

**General Limit Equilibrium (GLE)**: Method that encompasses key elements of several Method of Slice solutions, calculating one Safety Factor based on moment equilibrium and one based on horizontal force equilibrium. The method also allows for a range of interslice shear-normal force conditions, making it the most rigorous of all the methods, satisfying both force and moment equilibrium, for circular and non-circular slip surfaces.

Krahn (2003)



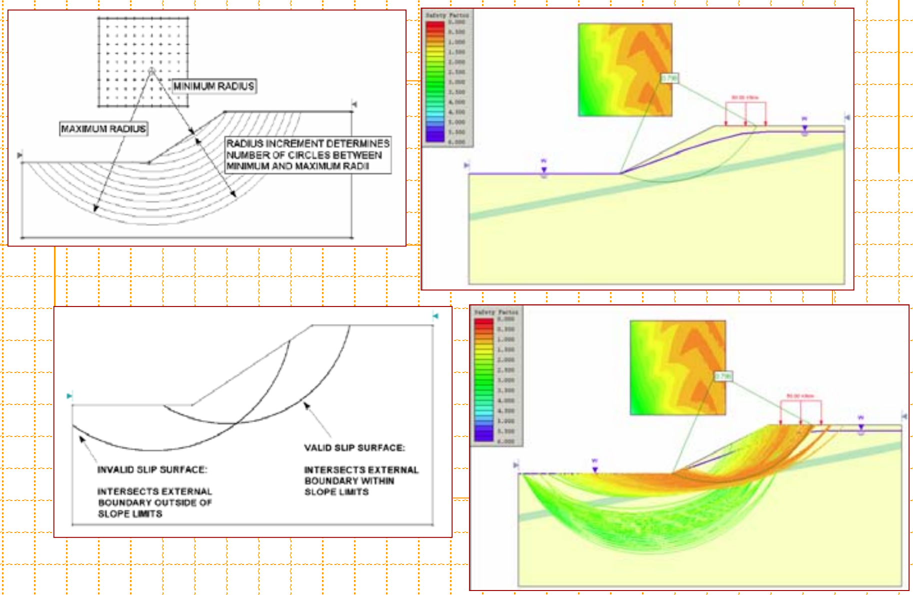
# Computer-Aided Limit Equilibrium Analysis

In cases where the shear failure surface is not known, its anticipated location can be found from analysis of the whole range of possible surfaces, and taking the actual surface to be that which gives the lowest factor of safety. This procedure can be quickly carried out using computer-based slip surface search routines.



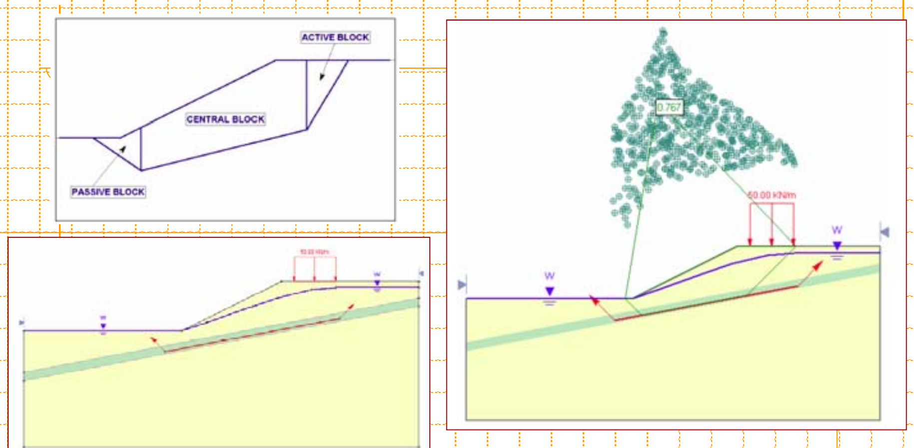

Hand or spreadsheet calculations can take hours to solve for a single slip surface, whereas a computer requires only seconds to solve for hundreds of potential slip surfaces.

# Critical Slip Surface Search





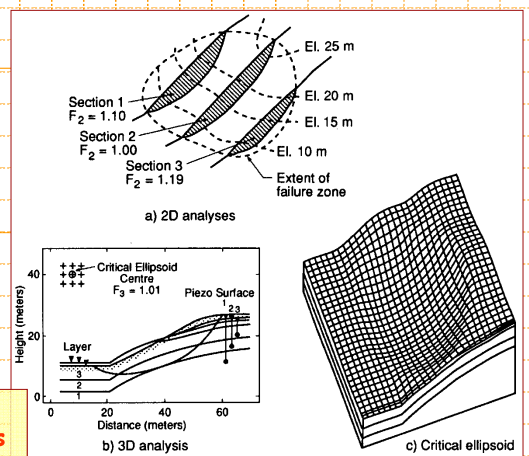

# Analysis of Non-Circular Slip Surface




For a non-circular slip surface, a block search routine is used that analyzes a limited number of slip surfaces relating to the division of the slide mass into an active, central and passive slide block.

## Advanced Limit Equilibrium Analysis - 3D

Most limit equilibrium formulations are two-dimensional even though actual slope failures are three-dimensional. However, there are a few 3-D limit equilibrium programs employing a "method of columns" approach.




Hungr *et al.* (1989)

The 3-D analysis program CLARA divides the sliding mass into columns, rather than slices as used in the 2-D analysis mode.



## Limit Equilibrium Analysis - Limitations

Although limit equilibrium methods are very useful in slope analysis, they do have their limitations and weaknesses:

1. The implicit assumptions of ductile stress-strain behaviour for the material (stress-strain relationships are neglected);
2. Most problems are statically indeterminate;
3. The factor of safety is assumed to be constant along the slip surface (an oversimplification, especially if the failure surface passes through different materials);
4. Computational accuracy may vary;
5. Allow only basic loading conditions (do not incorporate in situ stresses);
6. Provide little insight into slope failure mechanisms (do not consider stress state evolution or progressive failure).

# Uncertainty

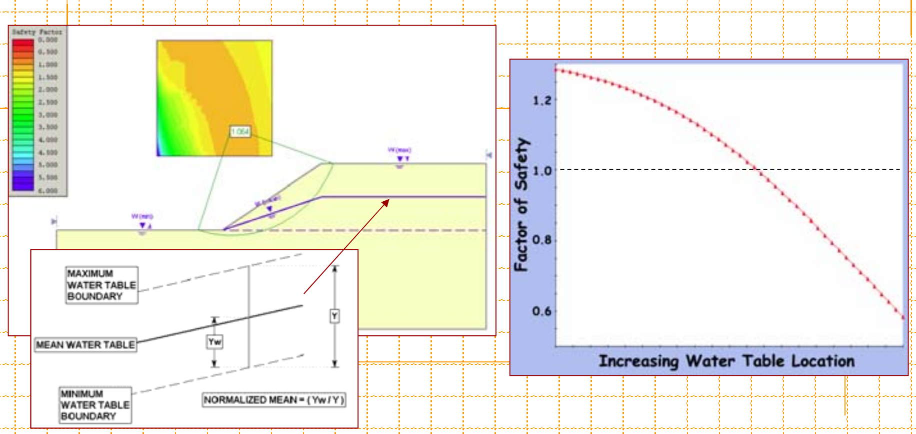
Geotechnical engineers must deal with natural conditions that are largely unknown and must be inferred from limited and costly observations. The principal uncertainties have to do with the accuracy and completeness with which subsurface conditions are known and with the resistances that the materials will be able to mobilize (e.g. strength).

| Uncertainties |
| --- |
| Position of the critical slip surface |
| Modeling of static and cyclic load history |
| Strain-softening |
| Progressive failure |
| Testing procedures in reference tests |
| Scale effect |
| Rate of shear |
| Stress conditions |
| Redistribution of stresses |
| Anisotropy |
| Structure stiffness |
| Model of soil profile |
| Drainage assumptions |
| Plane strain versus 3D analysis |



# Sensitivity Analysis

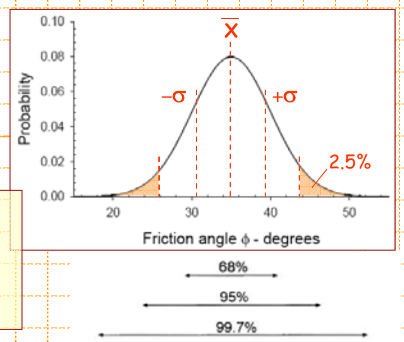
Sensitivity analyses allow for the determination of the "sensitivity" of the safety factor to variation in the input data variables. This is done by varying one variable at a time, while keeping all other variables constant, and plotting a graph of safety factor versus the variable.



# Probability Analysis

**Probabilistic analyses consider the variability of input parameters, and provide the probability of failure based on a given probability distribution function (defined through a known mean and standard deviations).**

**Probability distribution**: A probability density function (PDF) describes the relative likelihood that a random variable will assume a particular value. The area under the PDF is always unity.




The normal distribution is the most common type of PDF. It is used for most probabilistic studies, although for some parameters, a different distribution may be applicable (e.g. joint spacing).

A small standard deviation indicates a tightly clustered data set while a large standard deviation indicates a large scatter about the mean. For a normal distribution, 68% of the test values will fall within an interval defined by the mean ± one standard deviation while 95% will fall within two standard deviations.



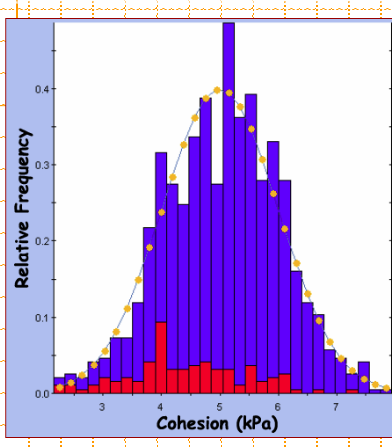
# Probability Distribution Functions

**In addition to the commonly used normal distribution there are a number of alternative distributions which are used in probability analyses. Some of the most useful are:**

- *Beta distributions* (Harr, 1987) are very versatile distributions which can be used to replace almost any of the common distributions and which do not suffer from the extreme value problems discussed above because the domain (range) is bounded by specified values.
- *Exponential distributions* are sometimes used to define events such as the occurrence of earthquakes or rockbursts or quantities such as the length of joints in a rock mass.
- *Lognormal distributions* are useful when considering processes such as the crushing of aggregates in which the final particle size results from a number of collisions of particles of many sizes moving in different directions with different velocities. Such multiplicative mechanisms tend to result in variables which are lognormally distributed as opposed to the normally distributed variables resulting from additive mechanisms.
- *Weibul distributions* are used to represent the lifetime of devices in reliability studies or the outcome of tests such as point load tests on rock core in which a few very high values may occur.

## Probability Analysis – Monte Carlo Simulation

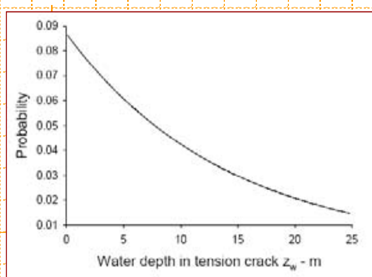
The Monte Carlo method uses random or pseudo-random numbers to sample from the probability distributions and, if sufficiently large numbers of samples are generated and used in a calculation such as that for a factor of safety, a distribution of values for the end product will be generated.

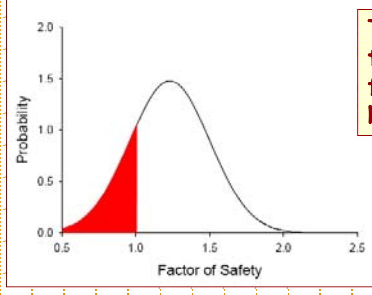



… Monte Carlo sampling (relative frequency) of cohesion taken as a random variable – 1000 samples, with those producing a factor of safety <1 highlighted in red.



## Probability of Failure




Remember that the PF and RI calculated for the Overall Slope, are not associated with a specific slip surface, but include the safety factors of all global minimum slip surfaces from the Probabilistic Analysis.

The Probability of Failure is simply equal to the number of analyses with safety factor less than 1, divided by the total Number of Samples.

The Reliability Index is an indication of the number of standard deviations which separate the Mean Safety Factor from the critical safety factor ( = 1).

## Case History – Usoi Rockslide Dam

In the winter of 1911, a massive 2.2 km$^3$ rockslide in the Pamir Mountains of southeastern Tajikistan was triggered by a magnitude 9.0 earthquake blocking the valley and damming the river running through it.





## Case History – Usoi Rockslide Dam

**Usoi Dam**

volume = 2.2 km$^3$
length = 5 km
average width = 3.2 km
height from the lake bottom = 567 m

**Lake Sarez**

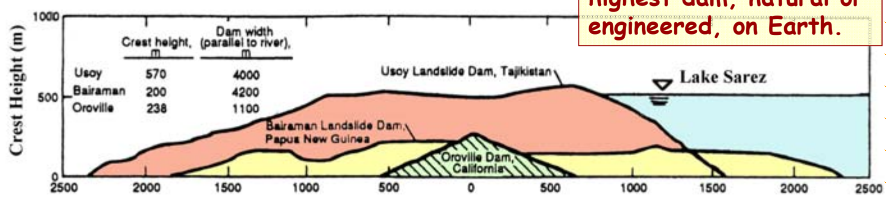
length - 55.8 km
maximum width - 3.3 km
maximum depth - 500 m
maximum water volume - 16,074 km$^3$


## Case History – Usoi Rockslide Dam

Practically immediately after the catastrophe, the question was raised whether Lake Sarez is dangerous or not :

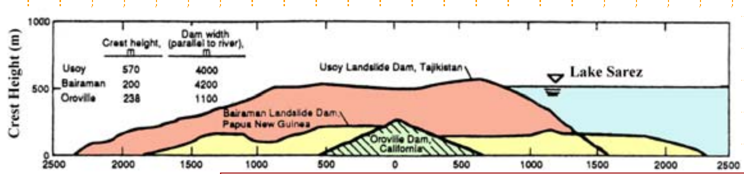
- will the accumulated water break through the dam, causing a catastrophic flood that would sweep 2000 km through the Amu Daryua River basin (inhabited by over 5 million people), demolishing everything on its way; or
- will the lake exist for a long time (several thousand years) in a normal regime of its evolutionary development.

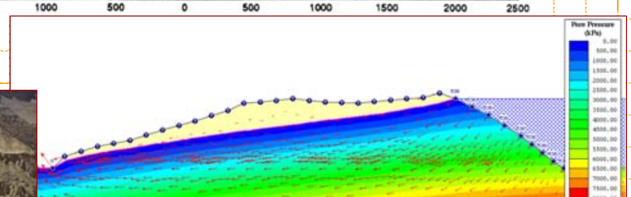
The Usoi Dam is the highest dam, natural or engineered, on Earth.

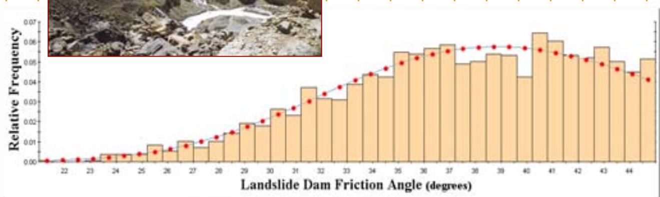





## Case History – Usoi Rockslide Dam



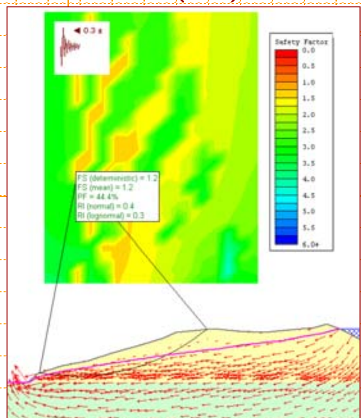


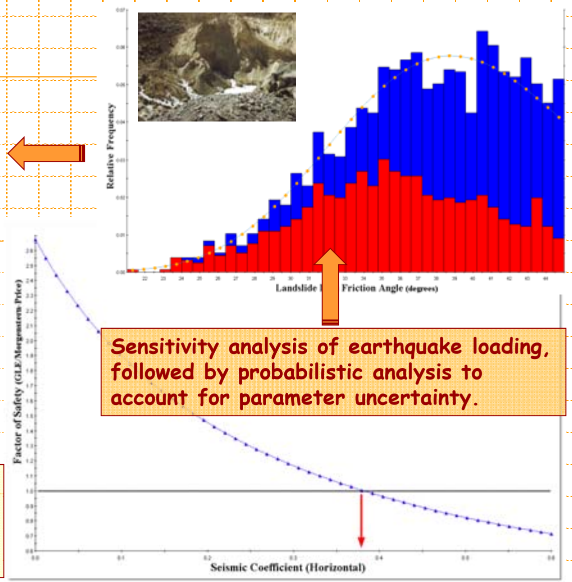

*Probabilistic analysis:*

*'Gamma' distribution skewed towards lower values of $\phi$, with a mean value of 40°.*

# Computer-Aided Probabilistic Analysis

Eberhardt & Stead (2006)




Sensitivity analysis of earthquake loading, followed by probabilistic analysis to account for parameter uncertainty.

FEM groundwater analysis to calculate pore pressures to be used in a stability analysis of a rockslide dam.



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