Ground Reaction - Convergence

A key principle in underground construction involving weak rock is the recognition that the main component of tunnel support is the strength of the rock mass and that it can be mobilized by minimizing deformations and preventing rock mass "loosening".

Whittaker & Frith (1990)

During construction of a tunnel, some relaxation of the rock mass will occur above and along the sides of the tunnel.
Controlling Ground Deformations

In order to preserve the rock mass strength, by minimizing rock mass deformations, it is necessary to apply temporary support early. Temporary support measures may include steel sets, rock bolts, wire mesh and shotcrete. These temporary support measures are generally seen as the major load bearing component, with the primary concrete lining being erected after the tunnel has become stable. The primary role of this lining is to seal the tunnel and to provide a partial load bearing component.

Support is added to create a stable self-supporting arch within the rock mass over the tunnel opening.

Forepoling is used to provide an arching effect in the 3rd dimension to control ground deformations ahead of the tunnel face.

Early Tunnel Experiences in Weak Rock

Austrian method

Breaking out of the tunnel to full width then began at the shoulders, working down.

Once the excavation was fully opened, the masonry lining was built up from the foundations to the crown of the arch in consecutive 5 m long sections.
The New Austrian Tunnelling Method (NATM) is an approach or philosophy integrating the principles of rock mass behaviour and the monitoring of this behaviour during tunnel excavation. The word ‘method’ is a poor choice of word usage, as the NATM is not a set of specific excavation and support techniques. Instead, the NATM involves a combination of many established ways of excavation and tunnelling, but the difference is the continual monitoring of the rock movement and the revision of support to obtain the most stable and economical lining.

What the NATM is not:
- A method (i.e., a set of specific excavation and support guidelines).
- Simply the employment of shotcrete as support.

Rabcewicz (1964):
“A new tunnelling method – particularly adapted for unstable ground – has been developed which uses surface stabilisation by a thin shotcrete lining, suitably reinforced by rockbolting and closed as soon as possible by an invert. Systematic measurement of deformation and stresses enables the required lining thickness to be evaluated and controlled.”
New Austrian Tunnelling Method (NATM)

Key Elements of the NATM Philosophy

1) Mobilization of Strength: The inherent strength of the rock surrounding the tunnel should be conserved and mobilised to the maximum extent possible (i.e., controlled deformation of the ground is required to develop its full strength). Primary support is directed to enable the rock to support itself. It follows that the support must have suitable load-deformation characteristics and be placed at the correct time.
Key Elements of the NATM Philosophy

2) Primary Support:
   Minimization of ground loosening and excessive deformations may be achieved in various ways, but generally a primary support system consisting of systematic rock bolting and a thin semi-flexible shotcrete lining is used. Whatever support is used, it is essential that it is placed and remains in physical contact with the ground and deforms with it.

   While the NATM generally includes shotcrete, it does not mean that the use of shotcrete constitutes the NATM.

3) Flexible Support: The NATM is characterized by versatility/adaptability leading to flexible rather than rigid tunnel support. Thus strengthening is not by a thicker concrete lining but a flexible combination of rockbolts, wire mesh and steel ribs. The primary support will partly or fully represent the total support required and the dimensioning of the secondary support will depend on measurement results.

4) Measurements: The NATM requires the installation of instrumentation at the time the initial support is installed to monitor deformations and support loads. This provides information on tunnel stability and enables optimization of the load bearing rock mass ring.
Key Elements of the NATM Philosophy

5) Closing of Invert: Closing of the invert to form a load-bearing ring of the rock mass is essential. In soft ground tunnelling, the invert must be closed quickly and no section of the excavated surface should be left unsupported even temporarily. For rock tunnels, the rock mass must be permitted to deform sufficiently before the support takes full effect.

A review of NATM failures found that in most cases, failure was a result of collapse at the face where the lining is still weak and cantilevered.

The 1994 Heathrow tunnel collapse.

A review of NATM failures found that in most cases, failure was a result of collapse at the face where the lining is still weak and cantilevered.

The builder and an Austrian engineering firm was fined a record £1.7m for the collapse, which put lives at risk and caused the cancellation of hundreds of flights.
Key Elements of the NATM Philosophy

6) Excavation Sequencing: The length of the tunnel left unsupported at any time during construction should be as short as possible. Where possible, the tunnel should be driven full face in minimum time with minimum disturbance of the ground by blasting.

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Key Elements of the NATM Philosophy

7) Contractual Arrangements: Since the NATM is based on monitoring (i.e., observational approach), changes in support and construction methods should be possible and worked into the contractual system. All parties involved in the design and execution of the project – design and supervisory engineers and the contractor’s engineers and foremen – must understand and accept the NATM approach and adopt a cooperative attitude to decision making and the resolution of problems.

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<table>
<thead>
<tr>
<th>Class</th>
<th>Approx. G range</th>
<th>Approx. SDR range</th>
<th>Tunnel Section Diameter</th>
<th>Rock Mass</th>
<th>Support Measures</th>
</tr>
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<tbody>
<tr>
<td>F1</td>
<td>10-100</td>
<td>15-60</td>
<td>Long term stability</td>
<td>Low support, unexcavated, controlled excavation</td>
<td>Up to 5.0</td>
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<tr>
<td></td>
<td>F2</td>
<td>4-10</td>
<td>Local mobilization</td>
<td>Low support, controlled excavation</td>
<td>Up to 5.0</td>
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<tr>
<td></td>
<td>F3</td>
<td>1-4</td>
<td>Frequent mobilization</td>
<td>System support, ventilation, pumping</td>
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<tr>
<td></td>
<td>F4</td>
<td>0.1-1</td>
<td>Frequent mobilization</td>
<td>Bored tunnels, excavated face</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>F5</td>
<td>0-0.1</td>
<td>Frequent mobilization</td>
<td>Conventional excavation</td>
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<tr>
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<td>F6</td>
<td>0-0.01</td>
<td>Long term stability</td>
<td>In-situ support, tunnelling, etc.</td>
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<tr>
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<td>F7</td>
<td>0-0.003</td>
<td>No support</td>
<td>In-situ support, tunnelling, etc.</td>
<td>None</td>
</tr>
</tbody>
</table>

Payment for support is often based on a rock mass classification completed after each drill and blast round.
**NATM: Advantages/Limitations**

**Advantages:** The primary advantage of NATM is the economy resulting from matching the amount of support installed to the ground conditions, as opposed to installing support for the expected worst case scenario throughout the entire tunnel. The safety of the work is more easily assured because the sizes and configurations of the headings making up the total tunnel cross section can be adapted to the degree of instability of the working face.

**Disadvantages:** One of the chief problems is the need for cooperation between the Owner’s and Contractor’s engineers in deciding the amount of support to be installed from day to day. It is not easy to achieve this in the adversarial conditions often encountered. Also, the ‘one man, one job’ philosophy of union contracting tends to spoil the economic advantages since most of the tasks are necessarily performed sequentially, some of them by other trades. Daily production rates are often lower, and in soft ground, more support is generally required to support the working face, than with shield driven tunnels.

McCusker (1991)

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**Weak Rock - Controlling Ground Deformations**

To preserve rock mass strength, by minimizing deformations, it is necessary to apply support early. Support measures may include rock bolts, wire mesh and shotcrete. These first-pass support measures are generally expected to be the major load bearing component, with secondary support being installed as needed.
New Austrian Tunnelling Method (NATM)

The New Austrian Tunnelling Method (NATM) is an approach integrating the principles of rock mass behaviour and the monitoring of this behaviour during excavation. It involves the monitoring of rock mass deformations and the revision of support to obtain the most stable and economical lining. Thus, the NATM is seen to be advantageous as the amount of support installed is matched to the ground conditions, as opposed to installing support for the expected worst case scenario throughout the entire drift.

Rabcewicz (1964):
"A new tunnelling method – particularly adapted for unstable ground – has been developed which uses surface stabilisation by a thin auxiliary shotcrete lining, suitably reinforced by rockbolting and closed as soon as possible by an invert. Systematic measurement of deformation and stresses enables the required lining thickness to be evaluated and controlled".

Ground Reaction - Convergence

In practice, it may not be possible to establish the exact form of the ground response curve, but we can measure the displacement that occurs, usually in the form of convergence across an excavation. The ground response curve and convergence curves are linked because they are different manifestations of a single phenomenon.

Convergence occurs rapidly as excavation proceeds; subsequently the convergence rate decreases as equilibrium is approached.

Hudson & Harrison (1997)
Stresses & Displacements - Circular Excavations

The Kirsch equations are a set of closed-form solutions, derived from the theory of elasticity, used to calculate the stresses and displacements around a circular excavation.

\[ k = \frac{\sigma_h}{\sigma_v} \]

From these equations we can see that the stresses on the boundary (i.e. when \( r = a \)) are given by:

\[ \sigma_{\theta\theta} = p \left[ (1+k) + 2(1-k) \cos 2\theta \right] \]
\[ \sigma_{rr} = 0 \]
\[ \tau_{r\theta} = 0 \]

Note that the radial stresses are zero because there is no internal pressure, and the shear stresses must be zero at a traction-free boundary.
**Conservation of Load**

Another concept that can be elegantly demonstrated from the Kirsch equations is the principle of conservation of load. The sketches show how the distribution of vertical stresses across a horizontal plane changes.

-Hudson & Harrison (1997)-

**Orientation of \( \sigma_1 \) & Induced Stresses**

Potential Ground Control Issues: Destressing = wedge failures, Concentration = spalling.

Stresses can be visualized as flowing around the excavation periphery in the direction of the major principle stress \( (\sigma_1) \). Where they diverge, relaxation occurs; where they converge, stress increases occur.
The Stabilization Strategy

The effects of excavation (displacements, stress changes, etc.), and the optimal stabilization strategy to account for them, should not blindly attempt to maintain the original conditions (e.g., by installing massive support or reinforcement and hydraulically sealing the entire excavation). As the displacements occur, engineering judgement may determine that they can be allowed to develop fully, or be controlled later.

**Reinforcement:** The primary objective is to mobilize and conserve the inherent strength of the rock mass so that it becomes self-supporting.

**Support:** The primary objective is to truly support the rock mass by structural elements which carry, in whole or part, the weights of individual rock blocks isolated by discontinuities or of zones of loosened rock.

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Tunnel Support Principles

Consider a tunnel being advanced by conventional methods, where steel sets are installed after each drill & blast cycle.

**Step 1:** The heading has not reached X-X and the rock mass on the periphery of the future tunnel profile is in equilibrium with the internal pressure \( p_i \) acting equal and opposite to \( p_o \).
Tunnel Support Principles

Consider a tunnel being advanced by conventional methods, where steel sets are installed after each drill & blast cycle.

Step 2: The face has advanced beyond X-X and the support pressure \( (p_i) \) provided by the rock inside the tunnel has been reduced to zero. Given that the blasted rock must be mucked out before the steel sets can be installed, deformation of the excavation boundaries starts to occur.

Daemen (1977)

Tunnel Support Principles

We can then plot the radial support pressure \( (p_i) \) required to limit the boundary displacement \( (\delta_i) \) to a given value.

Daemen (1977)

Thus, by advancing the excavation and removing the internal support pressure provided by the face, the tunnel roof will converge and displace along line AB (or AC in the case of the tunnel walls; the roof deformation follows a different path due to the extra load imposed by gravity on the loosened rock in the roof).
We can then plot the radial support pressure ($p_r$) required to limit the boundary displacement ($\delta_i$) to a given value.

**Tunnel Support Principles**

By Step 3: the heading has been mucked out and steel sets have been installed close to the face. At this stage the sets carry no load, but from this point on, any deformation of the tunnel roof or walls will result in loading of the steel sets.

**Tunnel Support Principles**

In Step 4: the heading is advanced one and a half tunnel diameters beyond X-X by another blast. The restraint offered by the proximity of the face is now negligible, and further convergence of the tunnel boundaries occurs.

If steel sets had not been installed, the radial displacements at X-X would continue increasing along the dashed lines EG and FH. In this case, the side walls would reach equilibrium at point G. However, the roof would continue deforming until it failed.
Tunnel Support Principles

We can then plot the radial support pressure ($p_r$) required to limit the boundary displacement ($\delta_i$) to a given value.

This load path is known as the support reaction line (or available support line). The curve representing the behaviour of the rock mass is known as the ground response curve (or support required curve).

Equilibrium between the rock and steel sets is reached where the lines intersect.

It is important to note that most of the redistributed stress arising from the excavation is carried by the rock and not by the steel sets!
**Ground Response Curve**

Consider the stresses and displacements induced by excavating in a continuous, homogeneous, isotropic, linear elastic rock mass (CHILE). The radial boundary displacements around a circular tunnel assuming plane strain conditions can be calculated as:

\[ u_r = \frac{R}{E} [\sigma_1 + \sigma_2 + 2(1 - \nu) (\sigma_1 - \sigma_2) \cos 2\theta - \nu \sigma_3] \]

where \( R \) is the radius of the opening,
\( \sigma_1 \) and \( \sigma_2 \) are the far-field in-plane principal stresses,
\( \sigma_3 \) is the far-field anti-plane stress,
\( \theta \) is indicated in the margin sketch, and
\( E \) and \( \nu \) are the elastic constants.

Where the ground response curve intersects the boundary displacement axis, the \( u_r \) value, represents the total deformation of the boundary of the excavation when support pressure is not provided. Typically only values less than 0.1% of the radius would be acceptable for most rock tunnelling projects.

**Support Reaction Curve**

If support is required, we can gain an indication of the efficacy of particular support systems by plotting the elastic behaviour of the support, the available support line, on the same axes as the ground response curve. The points of interest are where the available support lines intersect the ground response curves: at these points, equilibrium has been achieved.
A worked example: Rock-support interaction

Q. A circular tunnel of radius 1.85 m is excavated in rock subjected to an initial hydrostatic stress field of 20 MPa and provided with a concrete lining of internal radius 1.70 m. Assuming elastic behaviour of the rock/lining, calculate/plot the radial pressure and the radial displacement at the rock lining interface if the lining is installed after a radial displacement of 1 mm has occurred at the tunnel boundary.

A. Given:

\[ p = \text{hydrostatic stress} \]
\[ a = \text{tunnel radius} \]
\[ G = \text{shear modulus (assume 2 GPa)} \]
\[ p_r = \text{radial support pressure} \]
\[ k = \text{lining stiffness} \]
\[ u_o = \text{rock displacement when support installed} \]
\[ t_c = \text{concrete lining thickness} \]
\[ E_c = \text{lining elastic modulus (assume 30 GPa)} \]
\[ \nu_c = \text{lining Poisson ratio (assume 0.25)} \]

Harrison & Hudson (2000) plot the ground response line, we have two known points:

1. \[ u_r = \frac{pa}{2G} \]
\[ u_r = \frac{(20 \text{MPa})(1.85 \text{m})}{2 \cdot (2 \times 10^9 \text{Pa})} = 0.00925 \text{m} \]

2. Plotting our ground response line, we have two known points:

\[ p_r = 20 \text{ MPa} \]
\[ u_r = 0 \text{ mm} \]
\[ p_r = 0 \text{ MPa} \]
\[ u_r = 9.25 \text{ mm} \]
Worked Example: Rock-Support Interaction

A. To find the support reaction line, we assume the lining behaves as a thick-walled cylinder subject to radial loading. The equation for the lining characteristics in this case is:

\[ k = \frac{E_c}{1 + \nu_c} \frac{a^2 - (a - t_c)^2}{a^2 + (a - t_c)^2} \]

3. Solving for the stiffness of the lining, where \( t_c = 1.85 - 1.70 = 0.15 \) m, \( E_c = 30 \) GPa and \( \nu_c = 0.25 \), we get:

\[ k = \frac{30 \text{ GPa}}{1 + 0.25} \left[ \frac{(1.85m)^2 - (1.85m - 0.15m)^2}{(1 - 0.5)(1.85m)^2 + (1.85m - 0.15m)^2} \right] \]

\[ k = 2.78 \text{ GPa} \]

B. Thus, for a radial pressure of 20 MPa and \( u_o = 1 \) mm, the lining will deflect radially by:

\[ p_r = k \frac{u_c - u_o}{a} \]

\[ u_r = \frac{a}{k} p_r + u_o = \frac{1.85m}{2.78 \times 10^6 \text{ Pa}} \times 20 \times 10^6 \text{ Pa} + 0.001 \text{ m} \]

\[ u_r = 0.014 \text{ m} \]

4. Plotting our support reaction line, we have two known points:

\[ p_r = 20 \text{ MPa} \]
\[ u_r = 0.014 \text{ mm} \]

\[ p_r = 0 \text{ MPa} \]
\[ u_r = 1 \text{ mm} \]
Worked Example: Rock-Support Interaction

Operating point: 
\[ u=5.9\text{mm}, \ p=7.3\text{MPa} \]

1 mm displacement of tunnel boundary before lining is installed. This shows how, by delaying the installation of the lining, we can reduce the pressure it is required to withstand – but at the expense of increasing the final radial displacement.

Operating point: 
\[ u=5.5\text{mm}, \ p=8.2\text{MPa} \]

Rock Support in Yielding Rock

Thus, it should never be attempted to achieve zero displacement by introducing as stiff a support system as possible – this is never possible, and will also induce unnecessarily high support pressures. The support should be in harmony with the ground conditions, with the result that an optimal equilibrium position is achieved.

In general, it is better to allow the rock to displace to some extent and then ensure equilibrium is achieved before any deleterious displacement of the rock occurs.
Ground Response Curve – Yielding Rock

Note that plastic failure of the rock mass does not necessarily mean collapse of the tunnel. The yielded rock may still have considerable strength and, provided that the plastic zone is small compared with the tunnel radius, the only evidence of failure may be some minor spalling. In contrast, when a large plastic zone forms, large inward displacements may occur which may lead to loosening and collapse of the tunnel.

Hudson & Harrison (1997)

Effect of excavation methods on shape of the ground response curve due induced damage and alteration of rock mass properties.

Ground Response Curve – Plastic Deformation

To account for plastic deformations, a yield criterion must be applied. If the onset of plastic failure is defined by the Mohr-Coulomb criterion, then:

\[ \sigma_1 = \sigma_{cm} + k\sigma_3 \]

The uniaxial compressive strength of the rock mass (\(\sigma_{cm}\)) and the slope of the failure envelope is in \(\sigma_1-\sigma_3\) space is:

\[ \sigma_{cm} = \frac{2c \cdot \cos\phi}{(1-sin\phi)} \]

\[ k = \frac{(1+sin\phi)}{(1-sin\phi)} \]
Now assuming that a circular tunnel of radius $r_o$ is subjected to hydrostatic stresses ($p_o$), failure of the rock mass surrounding the tunnel occurs when the internal pressure provided by the tunnel lining is less than the critical support pressure, which is defined by:

$$p_{cr} = \frac{2p_o - \sigma_{sat}}{1+k}$$

If $p_i > p_{cr}$, then the deformation of the rock mass and inward radial displacement is elastic:

$$u_{ie} = \frac{r_o(1+v)}{E}(p_o - p_i)$$

If $p_{cr} > p_i$, then the radius of the plastic zone around the tunnel is given by:

$$r_p = r_o \left[ \frac{2(p_i(k-1) + \sigma_{sat})}{(1+k)(k-1)p_i + \sigma_{sat}} \right]^{(k-1)}$$

The total inward radial displacement of the tunnel roof and walls is then given by:

$$u_{io} = \frac{r_o(1+v)}{E} \left[ 2(1+v)(p_o - p_i) \left( \frac{r_o}{r_p} \right)^2 - (1-2v)(p_o - p_i) \right]$$

This plot shows zero displacement when the support pressure equals the hydrostatic stress ($p_i = p_o$), elastic displacement for $p_i > p_{cr}$, plastic displacement for $p_i < p_{cr}$, and a maximum displacement when the support pressure equals zero.
Rock Support in Yielding Rock

Another important conclusion drawn from these curves, for the case of unstable non-elastic conditions, is that stiff support (e.g. pre-cast concrete segments) may be successful, but that soft support (e.g. steel arches) may not bring the system to equilibrium.

One of the primary functions of the support is to control the inward displacement of the walls to prevent loosening.

Summary: Rock Support in Yielding Rock

Support 1 is installed at F and reaches equilibrium with the rock mass at point B:

This support is too stiff for the purpose and attracts an excessive share of the redistributed load. As a consequence, the support elements may fail causing catastrophic failure of the rock surrounding the excavation.
Rock Support in Yielding Rock

Support 2, having a lower stiffness, is installed at F and reaches equilibrium with the rock mass at point C:

Provided the corresponding convergence of the excavation is acceptable operationally, this system provides a good solution. The rock mass carries a major portion of the redistributed load, and the support elements are not stressed excessively.

Note that if this support was temporary and was to be removed after equilibrium had been reached, uncontrolled displacement and collapse of the rock mass would almost certainly occur.

Rock Support in Yielding Rock

Support 3, having a much lower stiffness than support 2, is also installed at F but reaches equilibrium with the rock mass at point D where the rock mass has started to loosen:

Although this may provide an acceptable temporary solution, the situation is a dangerous one because any extra load imposed, for example by a redistribution of stress associated with the excavation of a nearby opening, will have to be carried by the support elements. In general, support 3 is too compliant for this particular application.
Support 4, of the same stiffness as support 2, is not installed until a radial displacement of the rock mass of \(OG\) has occurred.

In this case, the support is installed late, excessive convergence of the excavation will occur, and the support elements will probably become overstressed before equilibrium is reached.


Squeezing Ground Behaviour
Squeezing Ground Behaviour

Squeezing ground refers to weak rock under high stresses, which causes the rock mass to undergo large deformations. This squeezing action may result in damage or failure of the ground support system, or require the costly re-excavation of the tunnel section.

Field observations from several tunnels in Taiwan.

Assuming no support

\[ C_{q ef f}/C_p = \text{rock mass strength / in situ stress} \]
Extreme squeezing requires the use of yielding support in order to accommodate these large deformations.
Lecture References


