Brittle Fracture & Stress-Controlled Failure

Brittle -vs- Plastic Failure Mechanisms

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Spalling and Rockburst in Tunnelling

Kaiser et al. (2000)

Orientation of $\sigma_1$ & Induced Stresses

Potential Ground Control Issues:
- Destressing = wedge failures
- Concentration = spalling

Stresses can be visualized as flowing around the excavation periphery in the direction of the major principle stress ($\sigma_1$). Where they diverge, relaxation occurs; where they converge, stress increases occur.
Stress Driven Spalling

Falling slabs of rock a hazard to workers.

Problem for TBM as gripper pads cannot be seated on the side wall.
Stress Driven Spalling and Popping

Rockbursting

JAN, 25 1990
Rockbursting - Strainbursts

Rockburst: A sudden and violent failure of rock where rock fragments are ejected into the excavation. Energy is released as seismic energy radiated in the form of strain waves.

- Usually occurs after blasting, as face is unable to adjust to the immediately stress increase
- Immediate unloading of confinement from a triaxial to uniaxial stress condition, stored energy released as seismic energy
- Commonly occurs when drifting through contact between a brittle and relative soft rock (i.e. highly dependent on local mine rock stiffness)

Strainburst: A self-initiated rockburst that develops due to a disequilibrium between high stresses and rock strength (i.e. dynamic unstable fracturing).

Rockbursting - Slip Bursts

Slip burst: Slip bursts are characterized as a stick-slip shear movement on a discontinuity. These bursts are less likely to be triggered by a particular blast, and more likely to occur afterwards. Slip occurs when the ratio of shear to normal (effective) stress along the fault plane reaches a critical value (its shear strength).

- Similar to mechanics of an earthquake
- Fault slip typically intersects the mine openings
- In most cases, mining activity causes slip by removing normal stress, although some local intensification of shear stress may also occur
- Changes in stress along a fault are often linked to mine activities by time-dependent deformation processes. These time-dependent processes can act over long periods of time, regardless of continued mining
AECL’s Underground Research Laboratory

240 m Level

σ_3

420 m Level

AECL’s URL - Brittle Failure

1.2m diameter

300mm diameter

Martin (1997)
**Failure Criterion in Solid Mechanics**

To understand the mechanisms that contribute to stress-induced brittle failure (spalling, bursting, etc.), we need to understand the basic principles of rock strength and the initiation and propagation of brittle fractures.

Traditionally, there have been two approaches to analyzing rock strength:

- **Experimental approach** (i.e. phenomenological)
- **Stress-based**
- **Strain-based**
- **Energy-based**

**Mechanistic approach**

**Analysis of Rock Strength**

**Phenomenological Approach**

Relies on generalization of large scale observations.

Theories include:
- Maximum Stress theory
- Tresca theory
- Coulomb theory
- Mohr-Coulomb failure criterion
- Hoek-Brown failure criterion

**Mechanistic Approach**

Derives its theories from elements of fracture at the microscopic scale.

Theories include:
- Griffith Crack theory
- Linear Elastic Fracture Mechanics (LEFM)
Deviatoric Stress and Shear Failure

Lab testing and field observations suggest that a shear failure criterion may be more applicable than a maximum stress criterion. In 2-D, the maximum shear stress is related to the difference in the major and minor principal stresses (i.e. deviatoric stress).

Mohr-Coulomb Failure Criterion

Failure occurs if:
\[ \tau_{\text{max}} \geq c + \sigma \tan \phi \]

\( \tau_{\text{max}} \) is the maximum shear stress, \( c \) is the cohesion, and \( \phi \) is the friction angle.
Mohr-Coulomb: Mechanistic Perspective

The Mohr-Coulomb criterion is widely applied for describing shear failure of rock. However, mechanistically speaking:

- Friction develops only on differential movement. Such movement can take place freely in a cohesionless material, but hardly in a cohesive one like rock prior to the development of a failure plane. In other words, mobilization of friction only becomes a factor once a failure plane is in the latter stages of development;

- Many brittle failures observed in the lab and underground appear to be largely controlled by the development of microfractures. Since these fractures initiate on a microscopic scale at stresses below the peak strength, the dismissal of all processes undetectable to the naked eye and prior to peak strength leaves the phenomenological approach lacking.

This is not to say that phenomenological approaches like Mohr-Coulomb are not useful. Remember: Mohr-Coulomb is the most widely used failure criterion, but its limitations need to be recognized.

Analysis of Brittle Rock Strength

Phenomenological Approach

- Relies on generalization of large scale observations.

  Theories include:

  - Maximum Stress theory
  - Tresca theory
  - Coulomb theory
  - Mohr-Coulomb failure criterion
  - Hoek-Brown failure criterion

Mechanistic Approach

- Derives its theories from elements of fracture at the microscopic scale.

  Theories include:

  - Griffith Crack theory
  - Linear Elastic Fracture Mechanics (LEFM)
At the atomic level, the development of interatomic forces is controlled by the atomic spacing which can be altered by means of external loading.

On extension, the structure fractures where the interatomic force is exhausted (i.e. the theoretical tensile strength).

In compression...

... displacement is countered by an inexhaustible repulsive force.

Thus, interatomic bonds will only break when pulled apart (i.e. in tension).
Theoretical Strength

Strength is therefore a function of the cohesive forces between atoms, where if $F > F_{\text{max}}$, then the interatomic bonds will break. As such, we can derive the following:

$$F_{\text{max}} = \frac{1}{2} \left[ C \left( r - r_0 \right) + \frac{1}{3} \right] \sigma^2$$

$$N = a^2 \quad \text{number of atoms per unit area}$$

$$\sigma_{\text{max}} = \frac{F_{\text{max}}}{A}$$

Now for most rocks, the Young’s modulus, $E$, is of the order $10^{-10}$ GPa. If so, then the theoretical tensile strength of these rocks should be $1-10$ GPa.

However, this is at least $1000$ times greater than the true tensile strength of rock!!!

Griffith Theory

To explain this discrepancy, Griffith (1920) postulated that in the case of a linear elastic material, brittle fracture is initiated through tensile stress concentrations at the tips of small, thin cracks randomly distributed within an otherwise isotropic material.
Crack Propagation in Compression

Under uniaxial compressive loading conditions, the highest tangential stress concentration on an elliptical crack boundary was inclined 30° to the major principal stress. As these cracks develop, they will rotate to align themselves with the major principal stress, $\sigma_1$.

Experimentally, it has been shown that brittle fractures propagate in the direction of $\sigma_1$. Cracks develop in this way to allow the newly forming crack faces to open/dilate in the direction of least resistance (i.e. normal to $\sigma_1$ in the direction of $\sigma_3$).

This is most easily accommodated in uniaxial compression since $\sigma_3 = 0$. For example, along a free surface!!
Linear Elastic Fracture Mechanics

Griffith's energy instability concept forms the basis for the study of fracture mechanics, in which the loading applied to a crack tip is analyzed to determine whether or not the crack will propagate.

1) Associated with a crack tip in a loaded material is a stress intensity factor, $K_I$, corresponding to the induced stress state surrounding the crack (and likewise $K_{II}$ and $K_{III}$ depending on the crack displacement mode).

2) For a given crack, the boundary material will have a critical stress intensity factor, $K_{IC}$, corresponding to the material strength at the crack tip.

3) The criterion for crack propagation can then be written as $K_I \geq K_{IC}$. Laboratory testing for the $K_{IC}$ parameter is referred to as fracture toughness testing.

4) The crack will continue to propagate as long as the above expression is met, and won’t stop until: $K_I < K_{IC}$. 

Ingraffea (1987)

Crack Interaction & Coalescence

crack interaction

crack tip stresses increase

cracks propagate & interact

yielding of bridging material

cracks coalesce: energy released

localization & development of rupture surface

Eberhardt et al. (1998a)
**Damage Around an Underground Excavation**

- $\sigma_1 = 14 \text{ MPa}$
- $\sigma_3 = 14 \text{ MPa}$
- Microseismic events (Martin, 1997)
- 420 m Level

**Failure Around Underground Excavations**

Observations from underground mining in massive brittle rocks suggest that failure initiates when the maximum tangential boundary stress reaches approximately 40% of the unconfined compressive strength.

- $\sigma_{ci} = 0.4 \sigma_{UCS}$
- $\sigma_{\max} = 0.4 \sigma_{UCS}$

This correlates with experimental studies of brittle rock failure that show that stress-induced damage initiates at approximately 40%.
Damage Around an Underground Excavation

In other words, stress-induced failure process begins at stress levels well below the rock’s unconfined compressive strength.

Example: Tunnel Spalling & Depth of Failure

Problem: A 14-m diameter, 100-m deep tunnel is to be excavated in a weak but massive sedimentary rock unit with an average compressive strength of 25 MPa. The tunnel will be excavated by a tunnel boring machine. In-situ stress tests revealed that the major principal stress is horizontal and three times higher than the vertical stress. This has raised concerns of potential ground control problems related to stress-induced fracturing and slabbing of the rock.

As such, the designers need to estimate the potential depth of stress-induced slabbing in order to select the proper rock support measures.
Example: Tunnel Spalling & Depth of Failure

Assuming a vertical stress of 2.5 MPa (calculated from the overburden), and adopting a horizontal to vertical stress ratio of 3, a maximum tangential stress of 20 MPa in the tunnel roof is calculated.

\[ \sigma_{\text{max}} = \frac{20}{25} \text{ MPa} = 0.8 \]

Using Martin et al. (1999)'s empirical relationship

\[ D_f = a = 4 \text{ m} \]

This means that, potentially, the slabbing may extend 4 m into the roof.

Tunnel Spalling & Depth of Failure

Using Martin et al. (1999)'s empirical relationship
Unstable Crack Propagation

As crack-induced damage accumulates, the stress level associated with crack initiation remains essentially unchanged; however, the stress level required for rupture reduces dramatically.

Stable propagation: controlling the applied load can stop crack growth.

Unstable propagation: relationship between the applied stress and the crack length ceases to exist and other parameters, such as the crack growth velocity, take control of the propagation process.

Under such conditions, crack propagation would be expected to continue even if loading was stopped and held constant.

Bieniawski (1967) correlated the threshold for unstable crack growth, also referred to as the point of critical energy release and the crack damage threshold, with the point of reversal in the volumetric stress-strain curve.

Stiffness, Energy & Failure

The violence and completeness of failure once unstable crack propagation is reached will depend on the relationship between the stiffness of the loaded component and that of the surrounding rock.

Capacity of the pillar to sustain load = P_{\text{max}}

However, the post peak behaviour is also influenced by the surrounding rock stiffness (through which the pillar is being loaded).
Unstable Crack Propagation - Pillar Loading

Thus, if we have an increase of convergence $\Delta s$ beyond $P_{\text{max}}$, to accommodate this displacement, the load on the pillar must reduce from $P_A$ to $P_B$.

1. The amount of energy, $\Delta W_{\text{pillar}}$, absorbed in the process is given by the area $ABED$.

2. However, in displacing by $\Delta s$ from point $A$, the mine rock only unloads to $F$ and releases stored strain energy, $\Delta W_{\text{mine}}$, given by the area $AFED$.

3. In this case, $\Delta W_{\text{mine}} > \Delta W_{\text{pillar}}$ and catastrophic failure occurs at, or shortly after, peak strength because the energy released by the mine rock during unloading is greater than that which can be absorbed by the pillar.

Brittle Failure in Tunnelling

Diederichs (2007)

Cai et al. (2004)
Grain Size - Spalling or Rockburst

Tonalite (intrusive)  Hornfels (fine-grain metamorphic)

SPALLING

Grain Size - Spalling or Rockburst

Tonalite (intrusive)  Hornfels (fine-grain metamorphic)

BURST
**Influence of Confining Stress**

Under low confinement, cracks can more easily open (in the $\sigma_3$ direction) and therefore propagate, leading to increased crack interactions and an acceleration of brittle failure. In contrast, the addition of confining stress works to make crack opening more difficult and therefore suppresses crack propagation. Confining stress therefore plays an important role in mitigating brittle failure.

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**Ground Control through Confinement**

![Image of ground control through confinement]

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Lecture References


