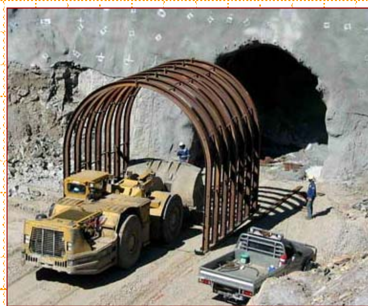
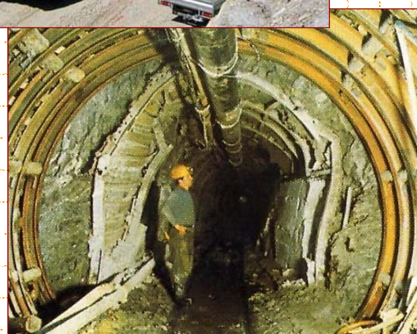




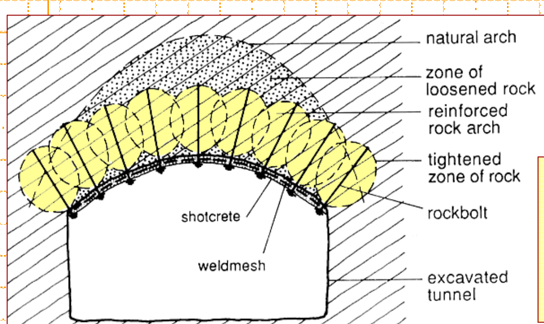
EOSC 547:

Tunnelling & Underground Design

Topic 7:
Ground Characteristic & Support Reaction Curves

Weak Rock - Controlling Ground Deformations

To preserve rock mass strength, by minimizing deformations, it is necessary to apply support early. Support measures may include rock bolts, wire mesh and shotcrete. These first-pass support measures are generally expected to be the major load bearing component, with secondary support being installed as needed.



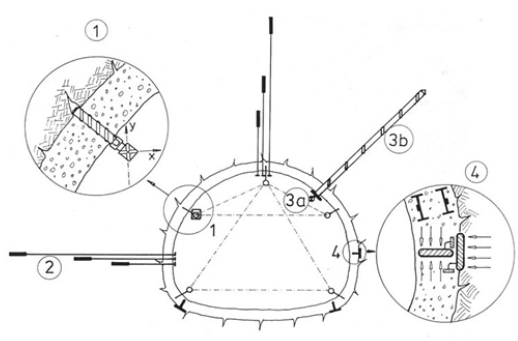
Support is added to create a stable self-supporting arch within the rock mass over the tunnel/drift opening.

New Austrian Tunnelling Method (NATM)

The New Austrian Tunnelling Method (NATM) is an approach integrating the principles of rock mass behaviour and the monitoring of this behaviour during excavation. It involves the monitoring of rock mass deformations and the revision of support to obtain the most stable and economical lining. Thus, the NATM is seen to be advantageous as the amount of support installed is matched to the ground conditions, as opposed to installing support for the expected worst case scenario throughout the entire drift.

Rabcewicz (1964):
"A new tunnelling method – particularly adapted for unstable ground – has been developed which uses surface stabilisation by a thin auxiliary shotcrete lining, suitably reinforced by rockbolting and closed as soon as possible by an invert. Systematic measurement of deformation and stresses enables the required lining thickness to be evaluated and controlled".

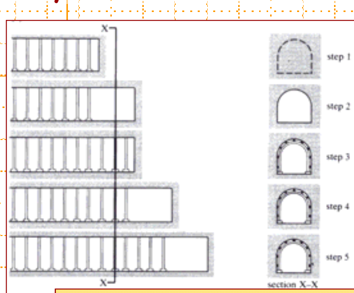
Tunnel Measurement Systems

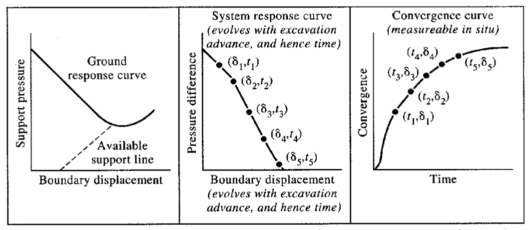


Legend	Measuring objective	Instrument
1	Deformation of the excavated tunnel surface	Convergence tape Surveying marks
2	Deformation of the ground surrounding the tunnel	Extensometer
3	Monitoring of ground support element 'anchor'	Total anchor force
4	Monitoring of ground support element 'shotcrete shell'	Pressure cells Embedments gauge

Ground Reaction - Convergence

In practice, it may not be possible to establish the exact form of the ground response curve, but we can measure the displacement that occurs, usually in the form of convergence across an excavation. The ground response curve and convergence curves are linked because they are different manifestations of a single phenomenon.



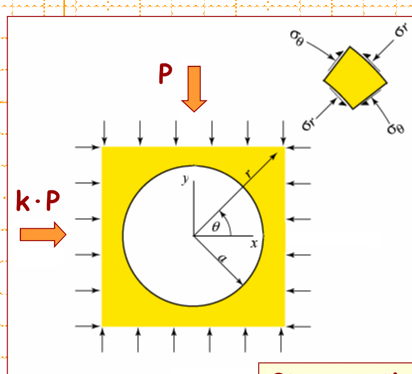


Hudson & Harrison (1997)

Convergence occurs rapidly as excavation proceeds; subsequently the convergence rate decreases as equilibrium is approached.

Stresses & Displacements - Circular Excavations

The Kirsch equations are a set of closed-form solutions, derived from the theory of elasticity, used to calculate the stresses and displacements around a circular excavation.



$$\sigma_{rr} = \frac{p}{2}\left[(1+K)\left(1-\frac{a^2}{r^2}\right) - (1-K)\left(1-4\frac{a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{\theta\theta} = \frac{p}{2}\left[(1+K)\left(1+\frac{a^2}{r^2}\right) + (1-K)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{r\theta} = \frac{p}{2}\left[(1-K)\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta\right]$$

$$u_r = -\frac{pa^2}{4Gr}\left[(1+K) - (1-K)\left\{4(1-\nu) - \frac{a^2}{r^2}\right\}\cos 2\theta\right]$$

$$u_\theta = -\frac{pa^2}{4Gr}\left[(1-K)\left\{2(1-2\nu) + \frac{a^2}{r^2}\right\}\sin 2\theta\right]$$

Brady & Brown (2006)

Stress ratio:
$k = \sigma_h/\sigma_v$

Stresses & Displacements - Circular Excavations

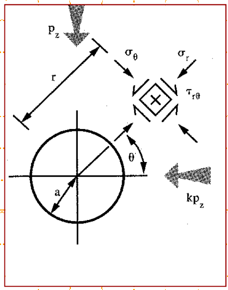
From these equations we can see that the stresses on the boundary (i.e. when r = a) are given by:

$$\sigma_{\theta\theta} = p[(1+k) + 2(1-k)\cos 2\theta]$$

$$\sigma_{rr} = 0$$

$$\tau_{r\theta} = 0$$

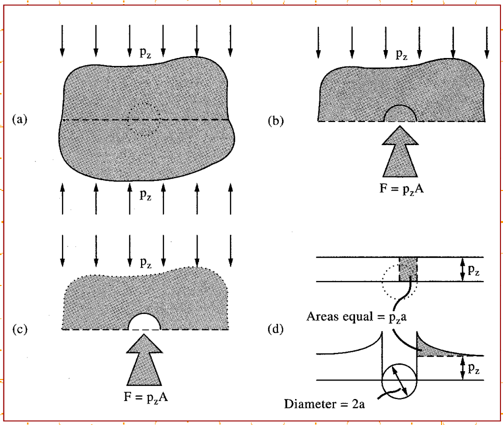
Note that the radial stresses are zero because there is no internal pressure, and the shear stresses must be zero at a traction-free boundary.



Conservation of Load

Another concept that can be elegantly demonstrated from the Kirsch equations is the principle of conservation of load.

... principle of conservation of load before and after excavation. The sketches show how the distribution of vertical stresses across a horizontal plane changes.

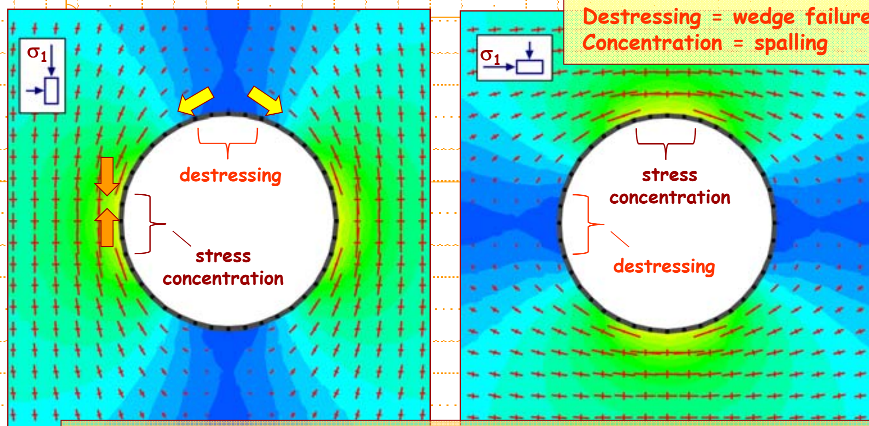


Hudson & Harrison (1997)

Orientation of σ_1 & Induced Stresses

Potential Ground Control Issues:

Destressing = wedge failures
Concentration = spalling



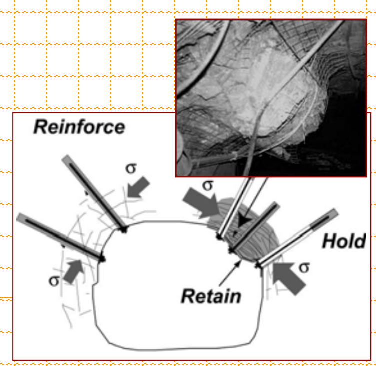
Stresses can be visualized as flowing around the excavation periphery in the direction of the major principle stress (σ_1). Where they diverge, relaxation occurs; where they converge, stress increases occur.

The Stabilization Strategy

The effects of excavation (displacements, stress changes, etc.), and the optimal stabilization strategy to account for them, should not blindly attempt to maintain the original conditions (e.g. by installing massive support or reinforcement and hydraulically sealing the entire excavation). As the displacements occur, engineering judgement may determine that they can be allowed to develop fully, or be controlled later.

Reinforcement: the primary objective is to mobilize and conserve the inherent strength of the rock mass so that it becomes self-supporting.

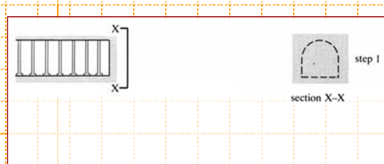
Support: the primary objective is to truly support the rock mass by structural elements which carry, in whole or part, the weights of individual rock blocks isolated by discontinuities or of zones of loosened rock.



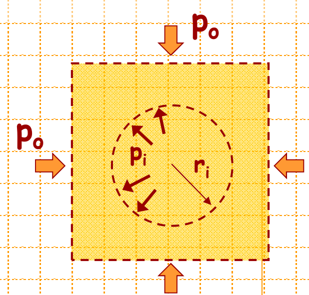
Kaiser *et al.* (2000)

Tunnel Support Principles

Consider a tunnel being advanced by conventional methods, where steel sets are installed after each drill & blast cycle.



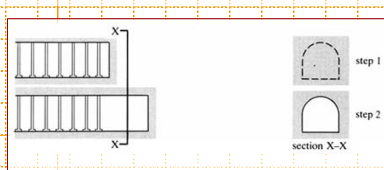
Daemen (1977)



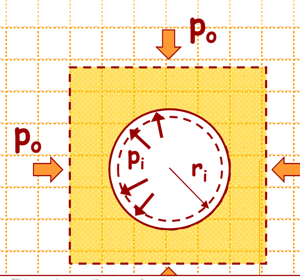
Step 1: The heading has not reached X-X and the rock mass on the periphery of the future tunnel profile is in equilibrium with the internal pressure (p_i) acting equal and opposite to p_o.

Tunnel Support Principles

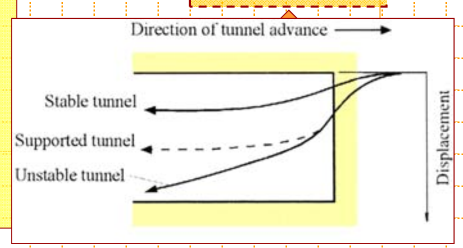
Consider a tunnel being advanced by conventional methods, where steel sets are installed after each drill & blast cycle.



Daemen (1977)



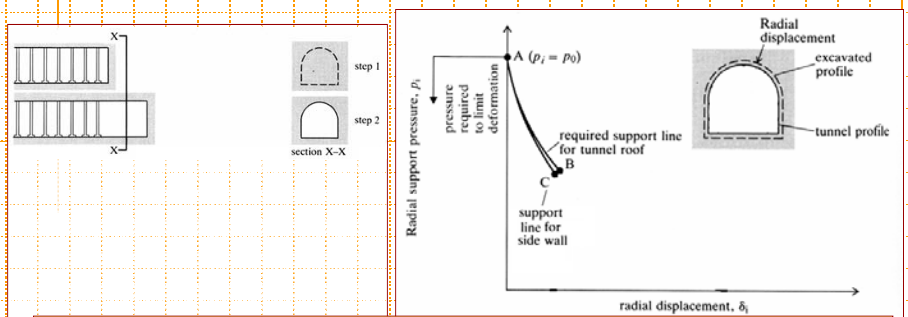
Step 2: The face has advanced beyond X-X and the support pressure (p_i) provided by the rock inside the tunnel has been reduced to zero. Given that the blasted rock must be mucked out before the steel sets can be installed, deformation of the excavation boundaries starts to occur.



Tunnel Support Principles

We can then plot the radial support pressure (p_i) required to limit the boundary displacement (δ_i) to a given value.

Daemen (1977)

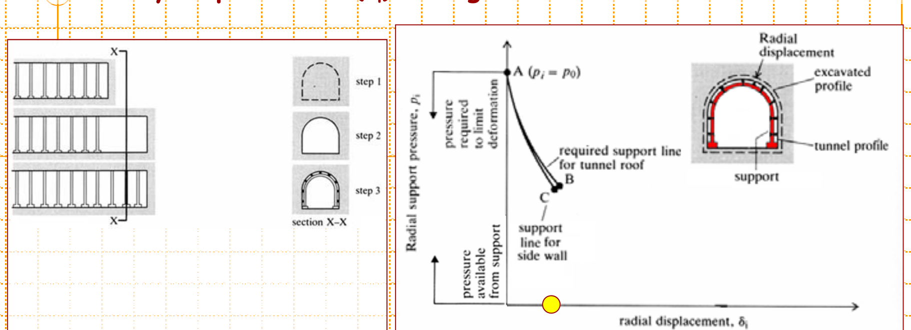


Thus, by advancing the excavation and removing the internal support pressure provided by the face, the tunnel roof will converge and displace along line AB (or AC in the case of the tunnel walls; the roof deformation follows a different path due to the extra load imposed by gravity on the loosened rock in the roof).

Tunnel Support Principles

We can then plot the radial support pressure (p_i) required to limit the boundary displacement (δ_i) to a given value.

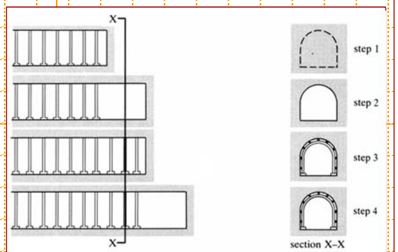
Daemen (1977)

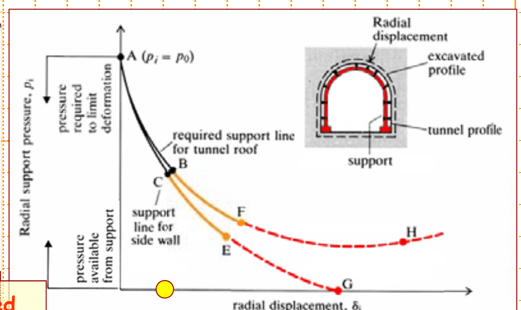


By **Step 3**: the heading has been mucked out and steel sets have been installed close to the face. At this stage the sets carry no load, but from this point on, any deformation of the tunnel roof or walls will result in loading of the steel sets.

Tunnel Support Principles

We can then plot the radial support pressure (p_i) required to limit the boundary displacement (δ_i) to a given value.





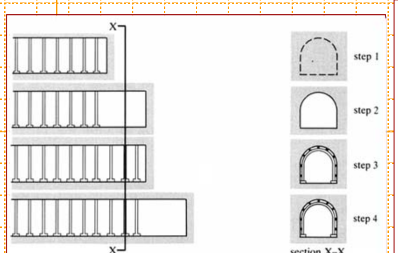
Daemen (1977)

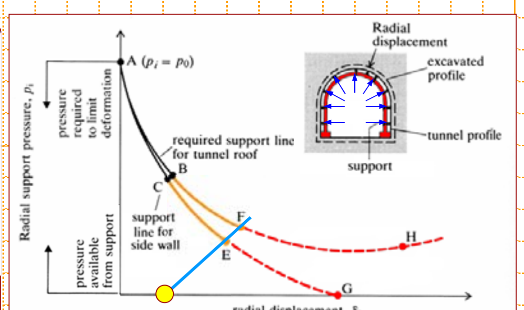
In **Step 4**: the heading is advanced one and a half tunnel diameters beyond X-X by another blast. The restraint offered by the proximity of the face is now negligible, and further convergence of the tunnel boundaries occurs.

If steel sets had not been installed, the radial displacements at X-X would continue increasing along the dashed lines EG and FH. In this case, the side walls would reach equilibrium at point G. However, the roof would continue deforming until it failed.

Tunnel Support Principles

We can then plot the radial support pressure (p_i) required to limit the boundary displacement (δ_i) to a given value.





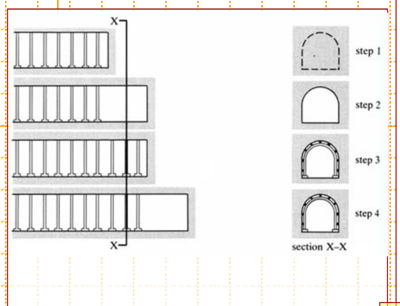
Daemen (1977)

… but with steel sets installed, the tunnel convergence will begin to load the support.

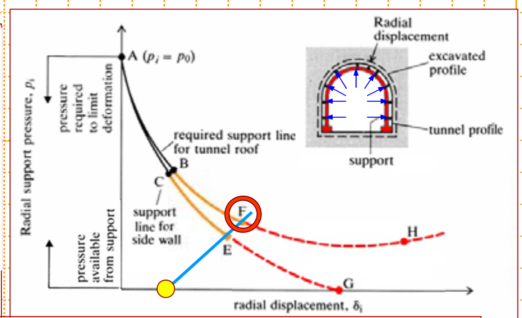
This load path is known as the **support reaction line** (or available support line). The curve representing the behaviour of the rock mass is known as the **ground response curve** (or support required curve).

Tunnel Support Principles

We can then plot the radial support pressure (p_i) required to limit the boundary displacement (δ_i) to a given value.



Daemen (1977)

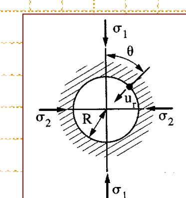


Equilibrium between the rock and steel sets is reached where the lines intersect.

It is important to note that most of the redistributed stress arising from the excavation is carried by the rock and not by the steel sets!!

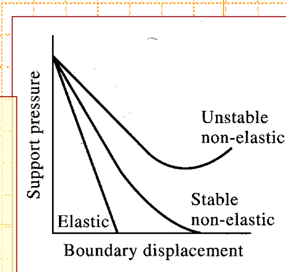
Ground Response Curve

Consider the stresses and displacements induced by excavating in a continuous, homogeneous, isotropic, linear elastic rock mass (CHILE). The radial boundary displacements around a circular tunnel assuming plane strain conditions can be calculated as:



$$u_r = (R/E)[\sigma_1 + \sigma_2 + 2(1 - \nu^2)(\sigma_1 - \sigma_2)\cos 2\theta - \nu\sigma_3]$$

where R is the radius of the opening,
σ_1 and σ_2 are the far-field in-plane principal stresses,
σ_3 is the far-field anti-plane stress,
θ is indicated in the margin sketch, and
E and ν are the elastic constants.

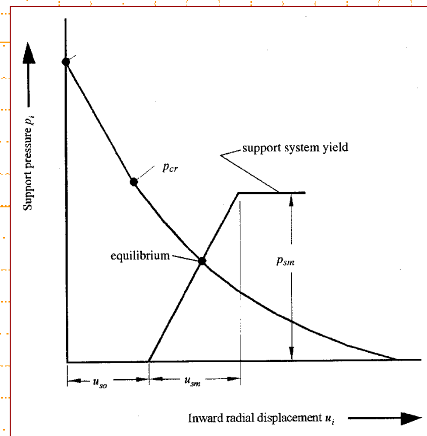


Hudson & Harrison (1997)

Where the ground response curve intersects the boundary displacement axis, the u_r value, represents the total deformation of the boundary of the excavation when support pressure is not provided. Typically only values less than 0.1% of the radius would be acceptable for most rock tunnelling projects.

Support Reaction Curve

If support is required, we can gain an indication of the efficacy of particular support systems by plotting the elastic behaviour of the support, the available support line, on the same axes as the ground response curve. The points of interest are where the available support lines intersect the ground response curves: at these points, equilibrium has been achieved.



Hoek *et al.* (1995)

Worked Example: Rock-Support Interaction

Q. A circular tunnel of radius 1.85 m is excavated in rock subjected to an initial hydrostatic stress field of 20 MPa and provided with a concrete lining of internal radius 1.70 m. Assuming elastic behaviour of the rock/lining, calculate/plot the radial pressure and the radial displacement at the rock lining interface if the lining is installed after a radial displacement of 1 mm has occurred at the tunnel boundary.

A. Given:

$$u_r = -\frac{pa}{2G}$$

$$p_r = k\frac{u_r - u_o}{a}$$

$$k = \frac{E_c}{1+\nu_c}\frac{a^2-(a-t_c)^2}{(1-2\nu_c)\,a^2+(a-t_c)^2}$$

- p = hydrostatic stress
- a = tunnel radius
- G = shear modulus (assume 2 GPa)
- p_r = radial support pressure
- k = lining stiffness
- u_o = rock displacement when support installed
- t_c = concrete lining thickness
- E_c = lining elastic modulus (assume 30 GPa)
- ν_c = lining Poisson ratio (assume 0.25)

Worked Example: Rock-Support Interaction

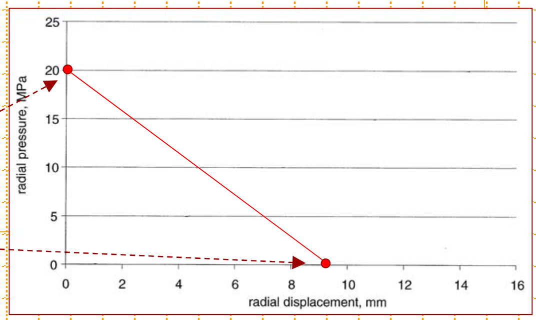
A. To find the ground response curve we need to identify the two end points of the line: one is the *in situ* condition of zero displacement at a radial stress of 20 MPa, the other is the maximum elastic displacement induced when the radial stress is zero.

(1) $u_r = \frac{pa}{2G}$ ⇒ $u_r = \frac{(20e6\,Pa)(1.85m)}{2\cdot(2e9\,Pa)} = 0.00925m$

(2) Plotting our ground response line, we have two known points:

$p_r = 20\ MPa$
$u_r = 0\ mm$

$p_r = 0\ MPa$
$u_r = 9.25\ mm$



Harrison & Hudson (2000)

Worked Example: Rock-Support Interaction

A. To find the support reaction line, we assume the lining behaves as a thick-walled cylinder subject to radial loading. The equation for the lining characteristics in this case is:

$$k = \frac{E_c}{1+\nu_c}\frac{a^2-(a-t_c)^2}{(1-2\nu_c)\,a^2+(a-t_c)^2}$$

(3) Solving for the stiffness of the lining, where t_c = 1.85 – 1.70 = 0.15 m, E_c = 30 GPa and ν_c = 0.25, we get:

$$k = \frac{30\,GPa}{1+0.25}\left[\frac{(1.85m)^2-(1.85m-0.15m)^2}{(1-0.5)(1.85m)^2+(1.85m-0.15m)^2}\right]$$

$$k = \underline{\underline{2.78\,GPa}}$$

Worked Example: Rock-Support Interaction

A. ③ Thus, for a radial pressure of 20 MPa and u_o = 1 mm, the lining will deflect radially by:

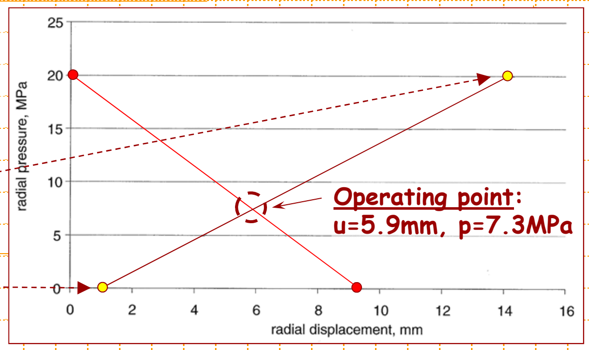
$$p_r = k\frac{u_r - u_o}{a} \Rightarrow u_r = \frac{a}{k}p_r + u_o = \frac{1.85m}{2.78e9\ Pa}20e6\ Pa + 0.001\ m$$

$$u_r = \underline{0.014\ m}$$

④ Plotting our support reaction line, we have two known points:

$p_r = 20\ MPa$
$u_r = 0.014\ mm$

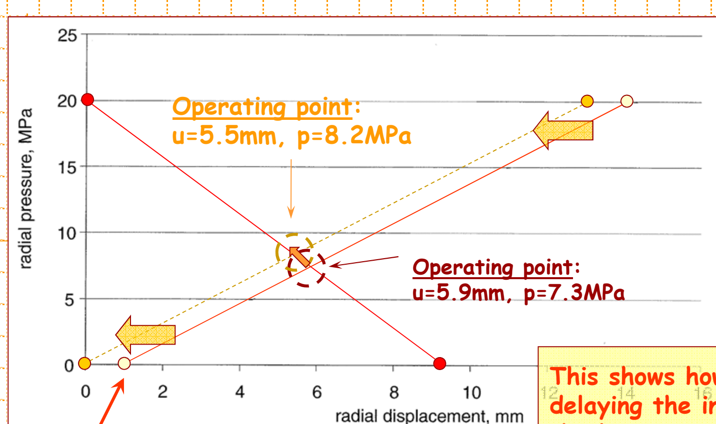
$p_r = 0\ MPa$
$u_r = 1\ mm$



Harrison & Hudson (2000)

Worked Example: Rock-Support Interaction

Harrison & Hudson (2000)



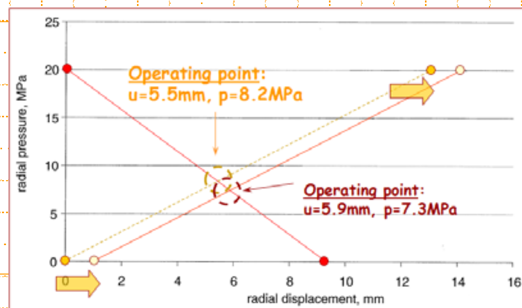
1 mm displacement of tunnel boundary before lining is installed

This shows how, by delaying the installation of the lining, we can reduce the pressure it is required to withstand – but at the expense of increasing the final radial displacement.

Rock Support in Yielding Rock

Thus, it should never be attempted to achieve zero displacement by introducing as stiff a support system as possible – this is never possible, and will also induce unnecessarily high support pressures. The support should be in harmony with the ground conditions, with the result that an optimal equilibrium position is achieved.

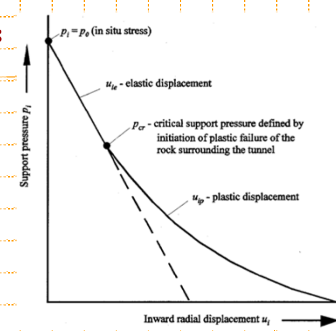
In general, it is better to allow the rock to displace to some extent and then ensure equilibrium is achieved before any deleterious displacement of the rock occurs.



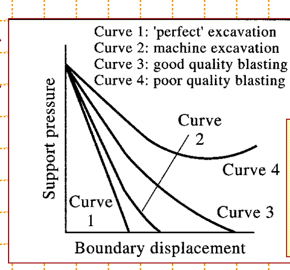
Hudson & Harrison (1997)

Ground Response Curve – Yielding Rock

Note that plastic failure of the rock mass does not necessarily mean collapse of the tunnel. The yielded rock may still have considerable strength and, provided that the plastic zone is small compared with the tunnel radius, the only evidence of failure may be some minor spalling. In contrast, when a large plastic zone forms, large inward displacements may occur which may lead to loosening and collapse of the tunnel.



Hoek et al. (1995)



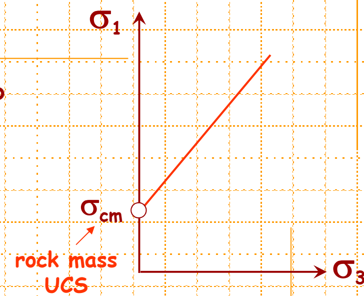
Hudson & Harrison (1997)

Effect of excavation methods on shape of the ground response curve due induced damage and alteration of rock mass properties.

Ground Response Curve – Plastic Deformation

To account for plastic deformations, a yield criterion must be applied. If the onset of plastic failure is defined by the Mohr-Coulomb criterion, then:

$$\sigma_1 = \sigma_{cm} + k\sigma_3$$

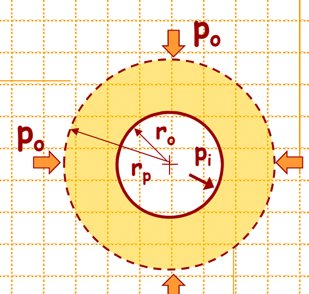


The uniaxial compressive strength of the rock mass (σ_{cm}) and the slope of the failure envelope is in σ_1-σ_3 space is:

$$\sigma_{cm} = \frac{2c \cdot \cos\phi}{(1-\sin\phi)} \qquad k = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Ground Response Curve – Plastic Deformation

Now assuming that a circular tunnel of radius r_o is subjected to hydrostatic stresses (p_o), failure of the rock mass surrounding the tunnel occurs when the internal pressure provided by the tunnel lining is less than the critical support pressure, which is defined by:



$$p_{cr} = \frac{2p_o - \sigma_{cm}}{1+k}$$

If $p_i > p_{cr}$, then the deformation of the rock mass and inward radial displacement is elastic:

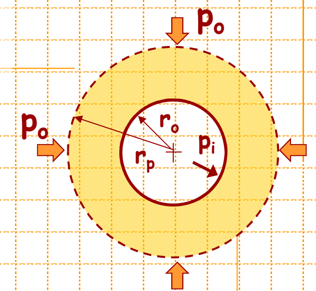
$$u_{ie} = \frac{r_o(1+\nu)}{E}(p_o - p_i)$$

If $p_{cr} > p_i$, then the radius of the plastic zone around the tunnel is given by:

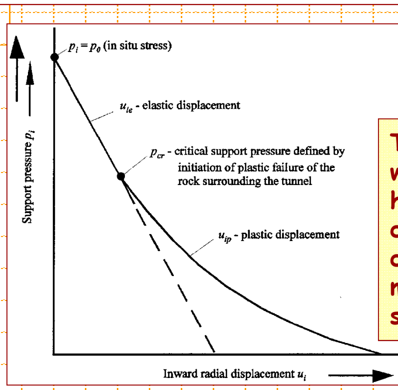
$$r_p = r_o\left[\frac{2(p_o(k-1)+\sigma_{cm})}{(1+k)((k-1)p_i+\sigma_{cm})}\right]^{\frac{1}{(k-1)}}$$

Ground Response Curve – Plastic Deformation

The total inward radial displacement of the tunnel roof and walls is then given by:



$$u_{ip} = \frac{r_o(1+\nu)}{E}\left[2(1-\nu)(p_o - p_{cr})\left(\frac{r_p}{r_o}\right)^2 - (1-2\nu)(p_o - p_i)\right]$$

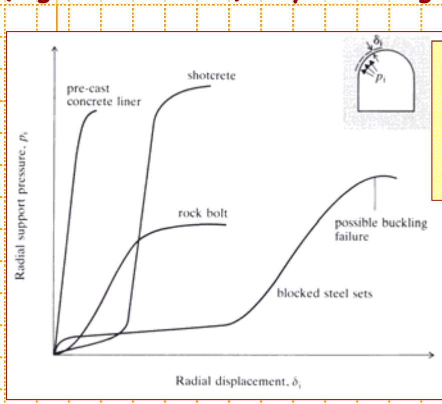


Hoek *et al.* (1995)

This plot shows zero displacement when the support pressure equals the hydrostatic stress (p_i=p_o), elastic displacement for p_o>p_i>p_{cr}, plastic displacement for p_i<p_{cr}, and a maximum displacement when the support pressure equals zero.

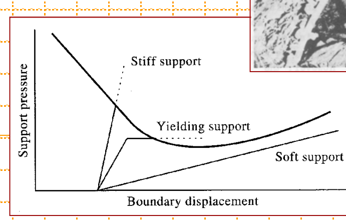
Rock Support in Yielding Rock

Another important conclusion drawn from these curves, for the case of unstable non-elastic conditions, is that stiff support (e.g. pre-cast concrete segments) may be successful, but that soft support (e.g. steel arches) may not bring the system to equilibrium.

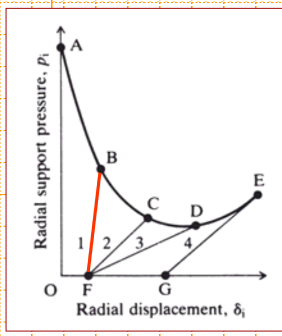


Brady & Brown (2004)

One of the primary functions of the support is to control the inward displacement of the walls to prevent loosening.



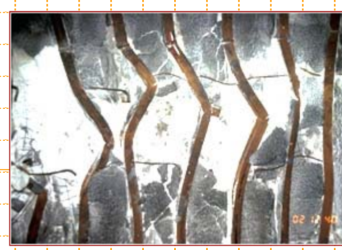
Summary: Rock Support in Yielding Rock



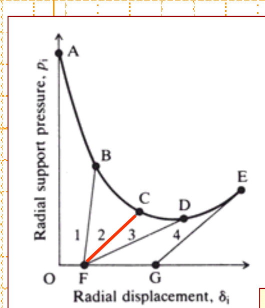
Brady & Brown (2004)

Support 1 is installed at *F* and reaches equilibrium with the rock mass at point *B*:

This support is too stiff for the purpose and attracts an excessive share of the redistributed load. As a consequence, the support elements may fail causing catastrophic failure of the rock surrounding the excavation.



Rock Support in Yielding Rock

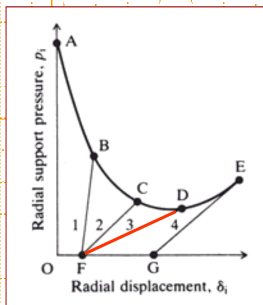


Support 2, having a lower stiffness, is installed at *F* and reaches equilibrium with the rock mass at point *C*:

Provided the corresponding convergence of the excavation is acceptable operationally, this system provides a good solution. The rock mass carries a major portion of the redistributed load, and the support elements are not stressed excessively.

Note that if this support was temporary and was to be removed after equilibrium had been reached, uncontrolled displacement and collapse of the rock mass would almost certainly occur.

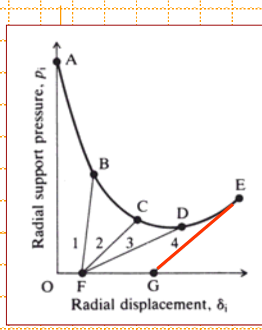
Rock Support in Yielding Rock



Support 3, having a much lower stiffness than support 2, is also installed at *F* but reaches equilibrium with the rock mass at point *D* where the rock mass has started to loosen:

Although this may provide an acceptable temporary solution, the situation is a dangerous one because any extra load imposed, for example by a redistribution of stress associated with the excavation of a nearby opening, will have to be carried by the support elements. In general, support 3 is too compliant for this particular application.

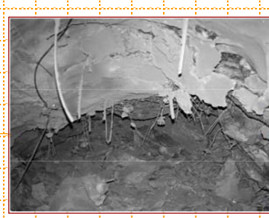
Summary: Rock Support in Yielding Rock



Brady & Brown (2004)

Support 4, of the same stiffness as support 2, is not installed until a radial displacement of the rock mass of *OG* has occurred. :

In this case, the support is installed late, excessive convergence of the excavation will occur, and the support elements will probably become overstressed before equilibrium is reached.



Lecture References

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