EXPANDED SYLLABUS FOR EOSC 550

Overview

1. Geophysical remote sensing
2. Forward problem, model space, data space
3. Linear functionals
4. Inverse Problem: Ill-posed and well-posed problems:
5. Inverse problems with: perfect data, finite number of accurate data, finite number of inaccurate data
6. Global comments on model construction, appraisal and inference.

Review of Essential Mathematics

1. Vector spaces, subspaces, bases and dimensions
2. Norms for infinite and finite dimensional spaces
3. Inner product
4. Hilbert space
5. Decomposition theorem

Minimum norm construction using accurate data

1. Prior information: What do we know about the model?
2. Solution via calculus of variations
3. Solution using the decomposition theorem
4. 2–Data gravity problem:
   a. smallest model
   b. smallest deviatoric model
   c. flattest model with surface constraint
   d. generic objective function
5. Least square minimization problem
6. Matrix norms, conditions numbers,
7. Ill-conditioning of linear inverse problems (effects of noisy data)

Discretizing the Forward Problem and SVD

1. Why discretize?
2. Quadrature and Galerkin Methods
3. Underdetermined, determined, overdetermined systems
4. Singular Value Decomposition (SVD)

Linear Inverse Problems with Inaccurate Data

1. Inaccuracies in geophysical measurements
2. Misfit objective functions
3. Fitting within a desired tolerance
4. Choosing and discretizing the model objective function
5. SVD solution
6. Tikhonov formulation
7. Truncated SVD
8. Filter functions and an introduction to subspace methods
9. Transformation to standard form
10. GIF applet

Choosing a Regularization Parameter

1. Discrepancy principle
2. L-curve
3. GCV

Basics for the Nonlinear Inverse Problems

1. Example problems
2. Gradients, Jacobians, Hessians
3. Newton's method
4. Gauss-Newton method
5. Strategies for solutions:
    a. Cooling approaches
    b. Parker's two-stage approach

Tuning the inversion algorithm: A Practical Example

1. 3D magnetic inversion (Integral equation approach)
    a. depth weighting
    b. positivity
    c. field example

Calculation of Sensitivities and Frechet Derivatives

1. Frechet derivative
2. Sensitivities

Inverse Problems with Partial Differential Equations

1. 1D DC resistivity problem
2. Discretizing with finite volumes
3. Calculating of sensitivities
4. A Gauss-Newton solution
5. Direct and iterative solvers
6. Preconditioning
7. Practical field example: 2D DC resistivity

General Measures of Misfit and Model Norm

1. l_1-norm solutions
2. Linear programming
3. General measures and numerical solutions

Parameter Estimation and Uncertainty

1. Linear Problems
2. Parameter uncertainty estimates
3. Non-linear problems
4. Example: Unexploded Ordnances
5. Direct search methods

Resolution in Linear Inverse Problems

1. Exploring model space
2. Backus-Gilbert Appraisal with accurate data
3. Deltaness criteria
4. Appraisal with inaccurate data
5. Example: linear deconvolution problems
6. Point spread functions
7. Funnel function analysis
8. Model covariance estimates